Reliable Multi-hop Routing with Cooperative Transmissions in Energy-Constrained Networks

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Abstract—We present a novel approach in characterizing the optimal reliable multi-hop virtual multiple-input single-output (vMISO) routing in ad hoc networks. Under a high node density regime, we determine the optimal cardinality of the cooperation sets at each hop on a path minimizing the total energy cost per transmitted bit. Optimal cooperating set cardinality curves are derived, and they can be used to determine the optimal routing strategy based on the required reliability, transmission power, and path loss coefficient. We design a new greedy geographical routing algorithm suitable for vMISO transmissions, and demonstrate the applicability of our results for more general networks.

Index Terms—virtual MIMO, energy-efficiency, space-time block codes

I. INTRODUCTION

In wireless networks, energy efficiency is a dominating design criterion. It is well-known that for the same through-put requirement multi input multi output (MIMO) systems require less transmission energy than single input single output (SISO) systems in the presence of fading [1]. However, it is usually infeasible to mount multiple antennas on small wireless devices due to the required minimum separation of these antennas. To achieve MIMO gains in wireless networks, cooperative (virtual) MIMO techniques have been proposed [2]. There is an increasing interest in translating the advantages of using virtual MIMO at the physical layer into higher layer performance benefits to maximize network throughput, or minimize total energy consumption and end-to-end delay [3] - [8]. In previous works, energy efficiency of cooperative transmissions over a single hop was investigated and compared to the traditional SISO transmissions [4], [5]. The capacity of a large gaussian relay network, where a source cooperates with relay nodes to transmit to a sink node is investigated in [8]. In our work, we investigate energy efficient routing in multi-hop wireless networks with cooperative transmissions when the channel is slowly-varying. Unlike [8], transmissions are required to satisfy an outage probability requirement, which is a suitable metric for this channel model. Also, [8] includes unreliable transmissions between source and relay nodes, which is omitted in the present work.

The key advantage provided by the cooperative transmissions considered in this work is the increase in the transmission range due to diversity gain when all radios transmit at the same fixed power level as in traditional SISO systems. Our objective is to determine in a multi-hop network, the optimal number of cooperating nodes per hop to minimize the end-to-end total energy consumption while satisfying an outage probability requirement at each hop. In order to identify the effect of the number of cooperating nodes on energy consumption, all other parameters, i.e., transmission power, rate and reliability are kept constant. The theoretical analysis of this problem is performed for networks with unlimited node density. Our results indicate that cooperative transmission is especially useful in multi-hop networks with low propagation loss coefficient, stricter outage probability requirement, and lower transmission power level. A new greedy geographical routing algorithm suitable for vMISO transmissions is designed to demonstrate the applicability of our results for more general networks.

The letter is organized as follows: In Section II, we discuss the system model and give the necessary background on vMISO systems. In Section III, we calculate and compare the energy consumption of vMISO and SISO systems under high node density assumption. In Section IV, we develop and analyze a greedy vMISO geographical routing algorithm. Finally, we conclude in Section V.

II. PRELIMINARIES

The channel is modeled as a Rayleigh flat-fading channel, where each node transmits with a fixed power level, \( P_0 \). The receiver has the full channel state information (CSI), but the transmitters do not estimate the channel. Let \( N_0 \) be the one-sided noise spectral density and \( \alpha_0 \) be the complex Gaussian distributed random variable, \( N_\alpha(0, 1) \), characterizing the Rayleigh flat fading channel. The instantaneous signal-to-noise-ratio (SNR) at a SISO receiver is, \( SNR_{SISO} = \frac{P_0}{N_0 |\alpha_0|^2 d_0^{-\beta}} \), where \( \beta \) is the path loss coefficient and \( d_0 \) is the transmission range.

In vMISO systems, a set of cooperating nodes emulate the antenna array of real MISO systems. vMISO systems can provide diversity gain over SISO systems due to the transmission of data over multiple independent channels. We consider decode and forward cooperation scheme, where initially, the head node transmits the original data to the relay nodes [2]. The cooperative transmission begins once all relay nodes receive, and correctly decode the original data. In order to leverage the benefits of space-diversity, data is encoded by a space time block code (STBC) with code rate \( r_n = \frac{k}{k_n} \leq 1 \). The head node and \( n - 1 \) relay nodes simultaneously transmit over \( k_n \) time slots in order to transfer \( k \) information bits. Let \( \alpha_i \) and \( d_i \) be the fading coefficient of the channel, and the distance
between the \(i\)th cooperating node and the destination. The received SNR in vMISO systems with \(n\) cooperating nodes is \([1]\),

\[
SNR^{vMISO} = \frac{P_0}{N_0} \sum_{i=0}^{n-1} |\alpha_i|^2 d_i^{-\beta}.
\]

Depending on the relative locations of the relay nodes and the head node, the symbols may arrive at the receiver with different received powers and relative delays. A numerical analysis on whether this difference can cause significant performance degradation was provided in \([6]\). In \([6]\), the cumulative distribution function for relative delay and power difference is numerically determined for line-of-sight propagation model and assuming that the relay nodes are uniformly distributed within the SISO range (taken as 250m). The results indicate that when the next hop node is further away than the SISO range, then the delay difference between the signals from any two relay nodes is at most 0.6\(\mu\)s for more than 80% of the time, and the power difference is less than 5dB for more than 85% of the time. The asynchronous reception at the receiver can cause inter-symbol interference (ISI), but this can be overcome with such methods as time-reverse space-time codes \([9]\), or space-time OFDM \([10]\). Therefore, in almost all cases wherein cooperative transmissions are used, the diversity gain is only dependent on the number of cooperating transmitters, and not on the physical location of these transmitters.

Spatial diversity can help transmit to larger distances, while satisfying the same bit error rate (BER) requirement with the same transmit power. However, the analysis involving BER involves complicated mathematical functions. A more general and tractable way to capture the link quality is by outage probability, \(d_c\). By a simple change of variables in (1),

\[
Pr \left( \sum_{i=0}^{n-1} |\alpha_i|^2 \leq \gamma_v(n,p) \right) = Pr \left( |\alpha_0|^2 \leq \gamma_0(p) \right) = p.
\]

Proof: Under high node density regime, one can find relay nodes at arbitrary locations with respect to the source node, and thus, effectively emulating a physical antenna array. In this setting, the distance between cooperating nodes and the destination node can be assumed to be approximately the same, i.e., \(d_c\). A similar change of variables in (1),

\[
Pr \left( \sum_{i=0}^{n-1} |\alpha_i|^2 \leq \gamma_v(n,p) \right) = Pr \left( |\alpha_0|^2 \leq \gamma_0(p) \right) = p,
\]

where \(\gamma_v(n,p) = SNR_{th} \frac{K^2}{\beta} d_v^\beta\) and \(\gamma_0(p) = SNR_{th} \frac{K}{\beta} d_0^\beta\). Note that \(\sum_{i=0}^{n-1} |\alpha_i|^2\) is a chi-square random variable with \(2n\) degrees of freedom, and \(\gamma_0\) and \(\gamma_v(n,p)\) can be calculated numerically. Hence, vMISO distance extension factor is calculated from \(I_v(n,p) = \frac{d_v(n,p)}{d_0(p)} = \left( \frac{\gamma_v(n,p)}{\gamma_0(p)} \right)^{1/\beta}\).

In order to facilitate vMISO transmissions in wireless networks a new MAC protocol should also be developed. One such protocol was discussed in detail in \([6]\] which is an evolved version of the four-way RTS/CTS/DATA/ACK handshake. This protocol has a more complex procedure than the one employed in SISO systems due to the necessary coordination between the cooperating nodes. In this work, we do not further elaborate on the MAC layer issues. We assume that the transmissions of multiple source-destination pairs are scheduled such that there is no interference between them. Thus, we focus on the energy-efficiency of cooperative multi-hop transmissions between a source and destination node pair.

### III. Energy Efficiency of vMISO Routing

We assume that all nodes transmit with a fixed transmission power level, \(P_0\), which is the required power to transmit to a distance of \(d_{nom}\) meters by a SISO system when the required outage probability is \(p_{nom}\). Fixing \(d_{nom}\) and \(p_{nom}\) fixes \(P_0\) which in turn with the outage probability \(p\), determines the range \(d_0\) for SISO and the range \(d_v\) for vMISO. We adopt the energy model in \([14]\), where the energy consumed by the

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1 Although some recent works have argued that in real deployments of wireless LANs, the locations of access points exhibit power-law behavior, e.g., \([13]\), we believe that high node density assumption may serve as a good direct approximation of dense sensor networks, where it provides a continuous surface of transport nodes.
amplifier is given as $100p J/\text{bit/m}^2$. Therefore, the energy consumed to transmit a bit to a distance of $d_{\text{nom}}$ meters at outage probability $p_{\text{nom}}$ is $E_a = 100d_{\text{nom}}^3 p J/\text{bit}$. The energy consumed by transmitter and receiver antenna circuitries are given respectively as $E_t = E_r = 50n J/\text{bit}$ in [14]. We neglect the processing energy cost at the transmitter and receiver.

A. Single-Hop vMISO vs. Multi-Hop SISO

We first compare the energy efficiency of direct vMISO transmission and multi-hop SISO transmission. Let $E_v\text{-MISO}(k, d_v, n)$ be the total energy cost of transmitting $k$ bits of information to a distance of $d_v$ in a single vMISO transmission with $n$ cooperating nodes. Initially, the head node broadcasts $k$ bits of original data to its relay nodes consuming $k(E_t + E_a)$ units of energy in the process. Also, $n-1$ relay nodes consume $k(n-1)E_r$ units of energy during reception. We assume that all nodes are pre-loaded with a table of space-time block code matrices, such as those given in [15] for different levels of transmit diversity. Each column of the matrix corresponds to a block duration (time), whereas, each row holds the symbols to be transmitted by each relay node. Next, each cooperating node transmits blocks of in total of $k_n$ bits when code rate is $r_n = k/k_n$, collectively consuming $nk_n(E_t + E_a)$ units of total energy. The destination combines received bits linearly, and consumes approximately $k_nE_r$ units of energy. Therefore, when $E_v = E_r = E_t$, $E_v\text{-MISO}(k, d_v, n) = kE_v \left[ n + \frac{E_a}{E_t} + \frac{1}{r_n} \left( n + 1 + \frac{E_a}{E_r} \right) \right]$, for $n \geq 2$.

Similarly, a SISO transmission consumes $E_v\text{SISO}(k, d_v) = kE_v + kE_a + kE_r$ units of energy while transmitting $k$ bits to a distance $d_v(p)$, where $d_v(p) = d_{\text{nom}} \left( \frac{\gamma_n(p)}{\gamma_n(p_{\text{nom}})} \right)^{1/\beta}$.

**Theorem 1 (Efficiency of direct vMISO transmission):**
When there are $n$ cooperating nodes, a direct vMISO transmission consumes less energy than the multi-hop SISO system, if

$$[I_v(n,p)] > \frac{n + \frac{E_a}{E_t} + \frac{1}{r_n} \left( n + 1 + \frac{E_a}{E_r} \right)}{2 + \frac{E_a}{E_t}}. \quad (2)$$

**Proof:** When SISO transmissions are used to reach the same distance as a single hop vMISO transmission, $d_v(n, p) = I_v(n, p) d_0(p)$, at least $[I_v(n, p)]$ hops are needed. Thus, the total energy consumed with multi-hop SISO is $E_v\text{SISO}(k, d_v) = [I_v(n, p)] (2kE_v + kE_a)$. Comparing $E_v\text{-MISO}(k, d_v, n)$ and $E_v\text{SISO}(k, d_v)$, we observe that $n + E_a/E_r + \frac{1}{r_n} \left( n + 1 + nE_a/E_r \right) < [I_v(n, p)] (2 + E_a/E_r)$, if vMISO is more energy efficient.

B. Multi-Hop vMISO vs. Multi-Hop SISO

Now, our objective is to find the optimal transmission strategy at each hop of a multi-hop vMISO system that minimizes the total energy consumption. We first determine the optimal number of cooperating nodes, $n_{\text{opt}}$, at each hop for a given outage probability requirement, $p$. Note that under a high node density regime, $n_{\text{opt}}$ is the same at each hop by symmetry.

1) Optimal transmission strategy given the required link outage probability: Let $E_{\text{mhop}}(k, D, n)$ be the total energy consumption of transmitting $k$ bits to a distance of $D$ meters with a multi-hop SISO or a multi-hop vMISO system with $n$ cooperating nodes, and $M = \left[ \frac{D}{d_0(p)} \right]$ and $K = \left[ \frac{D}{d_v(n, p)} \right]$ be the number of hops needed to transmit a symbol to a distance of $D$ with multi-hop SISO and vMISO systems, respectively. Then, the optimal number of cooperating nodes is determined by solving the following optimization problem:

$$\min_{n \in Z^+ : n \geq 2} \left\{ M E_{\text{SISO}}(k, d_v(n, p), n) \right\}. \quad (3)$$

Note that the solution of (3) may give SISO as the optimal solution. (3) is a nonlinear integer program, since $n$, $K$, and $M$ are nonnegative integers. The effective solution space of (3) is a narrow domain in integer space, since STBC is not efficient for large $n$. Therefore, we determine $n_{\text{opt}}$ by enumeration. In Figure 2, we calculated $n_{\text{opt}}$ numerically for varying $p$, where we used $r_n$ values given in [15]. It is observed that cooperation is especially preferred when $p$ is low, since the number of hops taken by vMISO is much lower than SISO. This reduction in number of hops compensates the higher per hop energy consumption of vMISO. For high outage probability, $n_{\text{opt}}$ decreases, and converges to $n = 1$, when $p \approx 0.06$, because for $p > 0.05$, $I_v(n, p)$ remains approximately constant. Thus, for $p > 0.05$, and $\beta = 2$, SISO is preferred. Also, $n_{\text{opt}}$ is lower for higher $\beta$, since the transmission energy increases with $\beta$.

2) Optimal transmission strategy and end-to-end reliability: Now, we also consider the end-to-end reliability of the transmissions. If a transmission fails on a link, it is retransmitted. Link failure is presumed to be independent and unpredictable, so our objective is to minimize the total average energy cost by determining $n_{\text{opt}}$ at every hop. Assuming that the channel is slowly varying, a vMISO transmission fails with outage probability, $p$. The number of transmissions until the first success is a geometric random variable, and the
expected number of cooperative transmissions is calculated from \(1/(1 - p)\). The cooperative transmission only begins if all of the relay nodes correctly decode the original symbol. The probability that none of the relay nodes can decode the symbol correctly is \(1 - (1 - p)^{n_a}\). Thus, the average number of broadcasts at the first phase of vMISO transmissions is calculated as \(1/(1 - p)^{n_a}\). Then, the total average energy cost of multi-hop vMISO is

\[
E \left[ E_{\text{vMISO}}(k, D, n, p) \right] = K E_e k \left[ \frac{n + \frac{E_e}{1 - p}}{(1 - p)^{n_a - 1}} + \frac{n(1 + \frac{E_e}{1 - p}) + 1}{(1 - p)r_n} \right].
\]

Similarly, the total average energy cost of multi-hop SISO is

\[
E \left[ E_{\text{SISO}}(k, D, p) \right] = M E_e k \left( \frac{\frac{E_e}{1 - p} + 1}{1 - p} \right).
\]

In Figures 3 and 4, we give the optimal cardinality of cooperation sets \(n_{opt}\) and the corresponding energy costs for varying nominal transmission ranges \(d_{nom}\), respectively. It is observed that there is an optimal transmission power level corresponding to the \(d_{nom}\) value for which the energy consumption is the minimum. This is reasonable because there is a trade-off between the number of hops on the path and the energy consumed at each hop. It is also observed that the optimal \(d_{nom}\) decreases as \(\beta\) increases, since per hop energy cost increases exponentially with respect to \(\beta\). On a similar note, we see that for higher values of \(d_{nom}\) SISO is preferable to vMISO unless \(\beta = 2\).

IV. GREEDY COOPERATIVE GEOGRAPHICAL ROUTING

In this section, we investigate whether our results determined under high node density regime are applicable to more general networks. For this purpose, we modify the well-known greedy geographical routing (GR) algorithm to incorporate vMISO transmissions. We run extensive simulation studies and compare the energy consumption for varying number of cooperating nodes. Recall that the diversity gain does not change significantly according to the physical locations of the nodes [6], and thus, we can use the result of Lemma 1 for networks with lower node densities.

In greedy GR, packets are stamped with the positions of their destinations; all nodes know their own positions; and a node forwards a packet to its neighbor that is geographically closest to the destination, so long as that neighbor is closer to the destination [16]. In order to forward data from source, \(s\), to destination, \(t\), the positions of the neighbor nodes are gathered via periodic “HELLO” message broadcasts. In a network employing vMISO links, all nodes within the vMISO transmission range of a node, i.e., \(d_v(n, p) = I_v(n, p) \beta d(p)\), are considered as the vMISO neighbors of that node. Thus, each node collects its \(I\)-hop neighbor information, where \(I = I_v(n, p)\). This can be realized by setting time-to-live (TTL) value of “HELLO” messages to \(I\), and having HELLO messages re-broadcast by each receiving node until TTL is decremented to 1.

In order to approach the destination as much as possible at each hop, the greedy forwarding strategy aims to select a next-hop node that is \(d_v(n, p)\) meters closer to \(t\) than the forwarding node itself, and also has at least \(n - 1\) SISO neighbors. If there is no such node among neighbors, the algorithm chooses the node that has at least \(n - 1\) SISO neighbors, and makes the most progress towards \(t\). However, as shown by Theorem 1, vMISO transmissions are not always energy efficient. Therefore, a vMISO link between two arbitrary nodes \(a\) and \(b\) is established only if the energy cost of vMISO link is lower than that of a multi-hop SISO path from \(a\) to \(b\). If vMISO is not energy efficient, then the node forwards its data to one of its SISO neighbors with degree \(n - 1\) that makes the most progress towards \(t\).

We perform simulation studies with the proposed vMISO GR algorithm. We consider a 30m × 30m square area, where the nodes are randomly distributed, and \(s\) and \(t\) lie at the opposite corners of this area. We perform the simulations for varying node densities, and our results represent the average of the measurements over 20 random topologies for each node density. The transmissions are attenuated by a random
Rayleigh distributed amount, and if a transmission is unsuccessful, i.e., received SNR is less than $SNR_{th}$, it is repeated. We used the values $SNR_{th} = 14$dB, $N_0 = -40$dBm, $d_{nom} = 10$m, $p_{nom} = 0.1$, $p = 5 \times 10^{-3}$, and $\beta = 3$ in the simulations.

In Figure 5, the average energy consumed per bit routed from $s$ to $t$ with vMISO and SISO are shown for varying node densities. From our results given in the previous section, we expect that the energy cost would be minimum if there are two cooperating nodes per hop. Indeed, the simulation results verify this expectation, and we also see that vMISO energy consumption decreases as the node density gets higher.

In Figure 6, the number of hops taken by vMISO, $K$, and SISO systems, $M$, is presented. The number of hops of a path is an important measure, since the paths with lower number of hops can provide higher end-to-end throughputs. It is observed that the number of hops decreases for increasing node densities, since at each hop near-maximum progress can be made due to the availability of nodes at ideal locations between $s$ and $t$. When $n = 2$, we observe that $K$ is less than half of $M$ due to longer vMISO transmission range. However, when $n = 4$, the number of hops is not significantly lower than the number of hops with $n = 2$, which explains why the network is much more energy efficient when $n = 2$.

V. CONCLUSIONS

In this letter, we studied the energy-efficiency of a general multi-hop vMISO system. Under high node density regime, we determined the optimal cardinality of cooperation sets according to the required outage probability, transmission power and path loss coefficient. We demonstrated that our results can be used to design practical vMISO routing algorithms suitable for more general networks. As a future work, we will investigate the trade-off between the maximum number of non-interfering simultaneous vMISO transmissions possible and total energy consumed in the network to understand the bits-per-joule capacity of such networks.

REFERENCES


