Continuous Time Controller Based on SMC and Disturbance Observer for Piezoelectric Actuators

S. Yannier¹, A. Sabanovic²

Abstract – In this work, analog application for the Sliding Mode Control (SMC) to piezoelectric actuators (PEA) is presented. DSP application of the algorithm suffers from ADC and DAC conversions and mainly faces limitations in sampling time interval. Moreover piezoelectric actuators are known to have very large bandwidth close to the DSP operation frequency. Therefore, with the direct analog application, improvement of the performance and high frequency operation are expected. Design of an appropriate SMC together with a disturbance observer is suggested to have continuous control output and related experimental results for position tracking are presented with comparison of DSP and analog control application.

Keywords: Sliding Mode Control, Disturbance Observer, Piezoelectric Actuator, Analog Electronics

I. Introduction

Piezoelectric effect [1] is a crystalline effect and therefore piezoelectric actuators do not suffer from “stick slip” effect mainly caused by the friction between elements of a mechanical system. This property theoretically offers an unlimited resolution and therefore the use in many applications to provide sub-micrometer resolution; ultrasonic motors, sports materials like skis and bikes [2], aerospace [3], hard disk drives [4] etc… Other main application of these ceramics is the scanning tunneling microscope (STM) and atomic force microscope (AFM) [5]. However, the achievable resolution in practice can be limited by a number of other factors such as the piezo control amplifier (electronic noise), sensor (resolution, noise and mounting precision) and control electronics (noise and sensitivity to EMI).

In naturally occurring piezoelectric materials, such as quartz, piezoelectric effect is too small to be of practical use. Modern man-made piezoelectric polycrystalline ceramics, such as lead zirconate titanate (PZT), lanthanum modified lead zirconate titanate (PLZT) and polyvinylidene fluoride (PVDF), are much more suitable for actuator purposes [1, 6].

Beside their high speed, high bandwidth, high stiffness, high electrical-mechanical transformation efficiency and little heat generation properties making piezoelectric materials agreeable actuators, all piezoelectric materials are ferroelectric and as all ferroelectric materials they exhibit fundamental hysteresis phenomena in the polarization versus the applied electric field, as well as in all the material properties coupled to the polarization [4-12].

Hysteresis yields a rate-independent lag and residual displacement near zero input [13]. Hysteresis is nondifferential, multivalued, and is usually unknown. The existence of hysteresis often limits the performance of the piezoelectric actuator, leads to the severe inaccuracies (up to 10%-15% of the traveling path) and causes undesirable oscillation or even instabilities when the piezo actuator is operated in an open loop manner [14] Achieving high speed, large-range precision positioning of piezo actuators is therefore challenging [15], hence how to design an effective controller for dealing with the hysteretic feature becomes a very important topic.

Another undesired characteristic of piezoelectric actuators is the “creep effect”, that can be observed when a step input voltage is applied to the input of the actuator [16, 17]. Figure 1 shows a step response of a general stack-type piezoelectric actuator. It has been known that the creep response has a logarithmic shape over time that can be represented by the following equation [16]:

\[ \text{Displacement} = \frac{d}{4.5 \times \text{Time}} \]

Fig. 1: Open loop step response of the piezoelectric actuator. “d” is the displacement difference due to hysteresis. Creep effect is magnified in the circular view [16].

Piezoelectric actuators shows dominant capacitive behavior and therefore can be controlled with either
Some charge control methods are investigated by researchers. In [18], a current source is directly used for charge steering, while in [19] a simple configuration is proposed consisting of a voltage source, an operational amplifier, an external capacitance, and a high voltage amplifier. It is known that this simple configuration has two apparent drawbacks: both sides of the piezoelectric actuator are floating with respect to ground and the configuration is very sensitive to op-amp bias current [10].

In voltage steering on the other hand, much effort is done to model the hysteresis with invertible functions that will be used for compensation purposes [5, 10, 12, 13, 20, 21]. The choice of open loop control is generally shown as a “must” since the position change measurement in the orders of micrometers requires expensive devices. However, open-loop techniques have been successful in providing results up to 2% error which is acceptable compare to the 15-20% hysteresis error in the nature of the piezoelectric actuator. Still due to the difficulties involved in modeling the actuator precisely, those techniques seem to approach the boundaries.

In [4], disturbance compensation based on a hysteresis model is used. However, unmodeled disturbances required the addition of a robust $H_{\infty}$ controller. A similar work is done by Tamer and Dahleh [22]. They tried both the $H_{\infty}$ method with estimated velocity. Experimental results show that although the error boundaries decreased from 8% to 6%, hysteresis is still a problem even at low frequencies. $H_{\infty}$ control on the other hand gives better tracking and eliminates the high frequency oscillations for the cost of rounding the corners of the triangular waves and noticeable delay increasing with frequency.

In order to design a control scheme that will achieve successful tracking performance without precise dynamic modeling, some fuzzy logic and neural network solutions are presented in the literature. However, due to the limited performance, this research area did not find much popularity [23].

Originally designed as system motion for dynamic systems whose essential open-loop behavior can be sufficiently modeled with ordinary differential equations, Sliding Mode Control (SMC) is one of the effective nonlinear robust control approaches that provides system invariance to uncertainties once the system is in the sliding mode [24, 25].

Bonnail et al. applied SMC on piezoelectric actuated scanning tunneling microscope to precisely follow the sample surface with the feedback of the tunneling current [26]. Compare to the commercial PI controlled motion, their solution shows less oscillating tunneling current due to the better electric performance of the surface.

Woronko et al. used SMC to control additional degree of freedom added to tool position on conventional CNC turning centers to improve machining precisions [27]. Similarly Chiang et al. worked on the control of a piezoelectric actuator mounted on the tip of a pneumatic cylinder. That is how they obtained a large stroke yet precise linear actuation system using adaptive discrete variable structure control.

Abidi et al. used SMC in conjunction with the disturbance observer for both position and force tracking in piezoelectric actuators [28, 29]. In their work, they used the lumped parameter model of the piezoelectric actuator to estimate the external disturbances via disturbance observer. Feedforward application of the estimated disturbance improved the results.

The aim of this work is to design appropriate control to drive piezoelectric actuators on “analog control circuit” to improve the tracking performance of piezoelectric actuators.

The rest of the paper is organized as follows. Section II describes the controller design with disturbance observer and plat model. The analog controller electronics that is designed is presented in Section III. Section IV presents the experimental results while conclusions and areas for future research are presented in Section V.

II. Sliding Mode Control

In this paper we will consider dynamical systems that can be represented as a class of nonlinear systems linear with respect of control as described by the following equation

$$\dot{x} = f(x) + B(x)u + d$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control vector, $f(x) \in \mathbb{R}^n$ is an unknown, continuous and bounded nonlinear function, $B(x) \in \mathbb{R}^{m \times n}$ is a known input matrix whose elements are continuous and bounded and $\text{rank}(B(x))_{0x} = m$, with $d \in \mathbb{R}^n$ being an unknown, bounded external disturbance. Both $f(x) \in \mathbb{R}^n$ and $d \in \mathbb{R}^n$ satisfy the matching conditions and all their components are bounded $\|f_i(x)\|_{0x} \leq M$ and $\|d_i(t)\|_{0t} \leq N$. Fully actuated mechanical systems belong to the class of systems described by (1). Such systems can be interpreted as $m$ interconnected subsystems $\dot{x}_i = h_i(x_i, x_j) + h_i(x_i, t) \cdot u_i + g_i(x_i, x_j)$, $h_i(x_i, x_j)$ in general represents Coulomb friction term, $g_i(x_i, x_j)$ represents the interaction term and is regarded as a disturbance.

The aim is to determine the control input $u = [u_1, ..., u_m]^T$ such that the system states
x₁(t),...,xₙ(t) track the desired trajectories x₁d(t),...,xₙd(t) while control error satisfies selected dynamical constraints.

II.1. Controller Design

The controller will be designed in the SMC framework by firstly selecting a suitable sliding manifold that will ensure desired systems dynamics and then selecting control such that the Lyapunov stability conditions are satisfied. Selecting the Lyapunov function candidate in terms of the sliding function is a natural way of guaranteeing the sliding mode existence on the selected manifold and thus having desired closed loop dynamics. Finally, the necessary control input should be selected that will fulfill the requirements of the Lyapunov stability criteria.

Sliding Manifold

For system (1) the natural selection of the sliding manifold is in the following form

\[ \sigma = G e_i = 0, \quad (2) \]

where tracking error vector is defined as \( e_i = [e_1, \ldots, e_n]^T \in \mathbb{R}^n \), \( e_i = x_i - x_i \), and the sliding surface satisfies \( \sigma = [\sigma_1, \ldots, \sigma_m]^T \in \mathbb{R}^m \), \( G \in \mathbb{R}^{m \times n} \).

Computing the Necessary Control Input

A Lyapunov Function candidate can be selected as

\[ V = \frac{1}{2} \sigma^T \sigma \quad (3) \]

where, \( V \in \mathbb{R} \). This function can also be stated as \( V = (1/2) \| \sigma \|_2^2 \), where \( \| \cdot \|_2 \) indicates Euclidian norm with \( V(0) = 0 \). The time derivative of the candidate Lyapunov function \( \dot{V} \) should be negative definite. In order to use this condition in selection of the control, we may require that the \( \dot{V} \) satisfies some preselected form. Equating the time derivative of this function to a negative definite function like in (4),

\[ \dot{V} = -\sigma^T D \sigma - \mu \frac{\sigma}{\sigma^T \sigma} \quad (4) \]

where \( D \) is a positive definite symmetric matrix, and \( \mu > 0 \) thus Lyapunov conditions are satisfied. By substituting (3) into (4), the following requirement is found.

\[ \sigma^T (\sigma + D \sigma + \mu \frac{\sigma}{\sigma^T \sigma}) = 0 \quad (5) \]

Therefore, for \( \sigma \neq 0 \), the control law can be calculated by satisfying the following equation.

\[ \left( \sigma + D \sigma + \mu \frac{\sigma}{\sigma^T \sigma} \right) = 0 \quad (6) \]

and the sliding mode conditions are satisfied. The discontinuous term can be selected as small in order to avoid chattering. It had been proven [30, 31] that in the discrete time implementation the sliding mode is guaranteed with continuous control action. We are targeting analog application for which controller will be implemented in continuous time domain so in our application the discontinuous term \( \mu \frac{\sigma}{\sigma^T \sigma} \) will be omitted and we will be determining the control action that satisfies conditions \((\sigma + D \sigma) = 0\) but all further analysis can be easily adopted for application of expression (6) if the term \( D \sigma \) is replaced with \( [D \sigma + \mu \sigma / \sigma^T \sigma] \).

For system (1) with sliding mode manifold (2) the control that satisfies \((\sigma + D \sigma) = 0\) can be determined as

\[ u = -(GB)^{-1} \left( G \left( f + d \right) - \dot{x}_d \right) \]

\[ = u_{eq} + (GB)^{-1} D \sigma \quad (7) \]

where \( x_d = [x_1, \ldots, x_d] \) and \( u_{eq} \) is so-called equivalent control obtained as a solution of the equation \( \sigma = GB(u_{eq} - u) = 0 \). By substituting (7) into (1) the equations of motion of system (1) in manifold (2) are obtained as \( \sigma = G e_i = 0 \) and the approach to this solution is governed by equation (6). This is a result of the specific structure of the plant (1) in which states is selected as the derivatives of the measurable outputs and each sub-block is represented in the canonical form.

To implement this control input, information about the plant dynamics and external disturbances are needed, which is hard to achieve. Hence, this solution needs the information on the equivalent control thus may be applied for the plants when \( u_{eq} \) is known or can be estimated with sufficient accuracy. In this paper we will be using a fact proven in [32, 33] that the solution of the differential equation

\[ \tau \dot{z} + z = u - (GB)^{-1} \cdot \sigma \quad (8) \]

with small enough filtering time constant \( \tau \) is close to the equivalent control. In this paper we will be using this result in order to avoid direct calculation of the equivalent control from \( u_{eq} = -(GB)^{-1} G \left( f + d \right) \dot{x}_d \) but instead to use approximated result \( u_{eq} \).

Equation (8) can be used together with the control presented in equation (7) to obtain the simplified controller equation;

\[ u = \frac{u_{eq}}{\tau + s + 1} + K \cdot \left( \frac{D \sigma + s \sigma}{\tau + s + 1} \right) \quad (9) \]

Do not use Laplace “s” and derivative “d/dt0dot” in the same equation

where \( K = (GB)^{-1} \).
II.2. Piezoelectric Actuator Model

In this work a piezoelectric actuator (PEA) that consists of a piezo-drive integrated with a sophisticated flexure structure for motion amplification is used. The flexure structure is wire-EDM-cut and is designed to have zero stiction and friction.

In addition to the absence of internal friction, flexure stages exhibit high stiffness and high load capacity. Flexure stages are also insensitive to shock and vibration. However, since the piezo-drive exhibits non-linear hysteresis behavior, the overall system will also exhibit the same behavior.

The dynamics of the piezo-stage can be represented by the following second-order differential equation coupled with hysteresis in the presence of external forces

$$m_{eff}\ddot{x} + c_{eff}\dot{x} + k_{eff}x = T\cdot[u(t) - h(x,u)] - F_{ext} \tag{10}$$

where $x$ denotes the displacement of the stage, $m_{eff}$, $c_{eff}$ and $k_{eff}$ denotes the effective mass, effective damping and effective stiffness of the stage respectively, $T$ denotes the electromechanical transformation ratio, $u$ denotes the input voltage and $h(x,u)$ denotes the unknown, bounded, non-linear hysteresis that has been found to be a function of $x$ and $u$, and $F_{ext}$ is the external force acting on the stage [12, 34].

The model represented by (10) shows that the hysteresis may be perceived as a disturbance force that satisfies matching conditions. This means that the sliding mode based control should be able to reject the influence of the hysteresis nonlinearity on the mechanical motion. At the same time it is obvious that the lumped disturbance consisting of the external force acting on the system and the hysteresis can be estimated, thus allowing the application of the disturbance rejection method in the overall system design.

II.3. Disturbance Observer

The structure of the observer is based on (10) under the assumption that all the plant parameter uncertainties, nonlinearities and external disturbances can be represented as a lumped disturbance. To show that assume all plan parameters have nominal values denoted with subscript $N$ and uncertainties shown with $\Delta$

$$m_{eff} = m_N + \Delta m \quad c_{eff} = c_N + \Delta c \quad k_{eff} = k_N + \Delta k \quad T = T_N + \Delta T \tag{11}$$

As obvious, $x$ the displacement of the plant and $u$ the input to the plant are measurable. Hence, the nominal structure of the plant is found as follows

$$m_N\ddot{x} = T_N\cdot u - F_d \tag{12}$$

where $F_d = T_N \cdot h(\Delta T(h + u)) + \Delta m \cdot \ddot{x} + \Delta c \cdot \dot{x} + \Delta k \cdot x + c_N\dot{x} + k_N x$

is the disturbance on the system.

For simplicity the disturbance observer proposed by Ohnishi et al. can be used [35]. The derivation of the observer is as follows. From equation (b) the disturbance for is

$$F_d = T_N \cdot u - m \cdot \ddot{x} \tag{14}$$

Ohnishi et al. proposes that;

$$\hat{F}_d = \frac{g}{s + g} \left( T_N \cdot u - m \cdot s^2 \ddot{x} \right) \tag{15}$$

where $\hat{F}_d$ is the observed disturbance force, $g$ is the constant determining the corner frequency of the first order filter. Since $x$ and $u$ are measurable, $\hat{u}$, the correction to the control output, can then be calculated as

$$\hat{u} = \frac{\hat{F}_d}{T_N} = \frac{g}{s + g} \left( u - \frac{m}{T_N} \cdot s^2 \ddot{x} \right) \tag{16}$$

The same remark as earlier

III. Analog Circuit Design

III.1. Sliding Mode Controller

Following the derivation in section II.1 above, the analog controller should control the input $u$ from the two inputs; desired trajectory $x_d$ and the actual trajectory $x$. Selecting the sliding manifold as

$$\sigma = G \cdot e_t = G \cdot \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \tag{17}$$

where tracking error vector is defined as $e_t = [e_1 \ e_2]^T \in \mathbb{R}^2$ with $e_1 = e = x_d - x$ and $e_2 = \dot{e} = \dot{x}_d - \dot{x}$. Accordingly the control $u$ is calculated as below

$$u = \hat{u} + K \cdot \left( D\sigma + \frac{s \sigma}{s + 1} \right) \tag{18}$$

where $\hat{u} = \frac{u}{T \cdot s + 1}$. Equation (2) can further be integrated to this last equation for simplification.

$$u = \hat{u} + K \cdot \left( D \cdot (C \cdot e + \dot{e}) + \frac{(C \cdot e + \dot{e})}{s + 1} \right) \tag{19}$$

The same remark as earlier

Only for very small time constants $\tau$, this last equation can further be simplified to obtain

$$u = \hat{u} + K \cdot (D \cdot C \cdot e + (D + C) \cdot \dot{e}) \tag{20}$$

where $\phi = D \cdot C \cdot e + (D + C) \cdot \dot{e}$. From this last equation the algorithm can be deduced immediately. Fig. 2 depicts this algorithm as functional blocks of the
The circuit is based on mathematical operations like summation, differentiation and will be realized with op-amp circuits. Therefore the algorithm also includes the inverting behavior of the op-amps.

Accordingly the calculation starts with the error calculation of the error $e$ and then two successive differentiation blocks to calculate $\dot{e}$ and $\ddot{e}$. Then the calculation of the intermediate variable $\varphi$ is just the summation of those variables with appropriate coefficients.

Finally the control is summed up with the filtered control $\hat{u}$ and disturbance observers output $\tilde{u}$ before fed to the high voltage amplifier (H.V.).

### III.2. Disturbance Observer

The designed disturbance observer can simply be integrated to the system to use the actual position $x$ and control $u$ as inputs and calculate the correction term to the control output; $\tilde{u}$. The interconnection is depicted in Fig. 3.

### IV. Experimental Results

#### IV.1. Experimental Setup

For experimental purposes, the setup shown in Fig. 5 is constructed; high voltage amplifier is built using MP108 power operational amplifier from Apex Microtechnology, PEA is the piezoelectric actuator with embedded strain gage for position measurement and the strain gage amplifier is the SCM5B38-03 wide band strain gage amplifier from Dataforth Corporation. Here SMC is the designed sliding mode control algorithm implemented in DSP (for DSP experiments) or is the analog circuit (for analog controller experiments).

The algorithm of the observer is also straightforward as shown in Fig. 4. The observer’s output is added to the last summation block before high voltage amplifier shown in Fig. 2.
54622D digital oscilloscope. The reference and actual signals are given without offset to have better feeling on the tracking error. The third channel shown in some figures is the error signal. Peak to peak values are given at the image captions in metric correspondents: 17.96um (micrometers) corresponds to 1V of the strain gage amplifier reading, or in other words 1um position deflection results 55.68mV.

IV.2 Position Tracking Experiments Using DSP

For comparison of the results DSP application of the control is realized on dSpace DS1102 platform which possesses TMS320C31 DSP chip running at 40 MHz with 50ns cycle time. The platform does have two 16-bit ADC (Input) ±10V and four 12-bit DAC (Outputs) ±10V. The algorithm runs at 10kHz. discretization is made based on Euler’s method.

![Fig. 6: DSP tracking experiment for 4.5um pp 1Hz sinusoidal reference.](image)

Position tracking of 1Hz sinusoidal inputs is studied. First 4.5um peak to peak and then 10.8um peak to peak inputs are tested. Results are shown on Fig. 6 and on Fig. 7 respectively. The errors for comparison are 110nm and 200nm, corresponding to 2.5% and 1.9% respectively.

![Fig. 7: DSP tracking experiment for 10.8um pp 1Hz sinusoidal reference.](image)

IV.3 Analog Circuit, Position Tracking Experiments

Similar experiments are conducted for analog circuit realization of the SMC with disturbance observer. The tracking of a 35.60um peak to peak 1Hz sinusoidal reference is resulted with 50nm peak to peak tracking error corresponding to only 0.14%. Compare to 2.5% tracking error of the DSP implementation, this result is almost 18 times better.

![Fig. 8: Tracking of 35.69um pp 1Hz sinusoidal reference. The error is 50nm pp (0.14%).](image)

Tracking of a single sinusoidal pulse of period 1 second and peak to peak amplitude 35.60um is shown on Fig. 9. According to the experiment the peak tp peak error value is 54nm corresponding to 0.15% tracking error.

![Fig. 9: Tracking of 35.60um pp 1Hz sinusoidal reference. The error is 54nm pp (0.15%).](image)

Triangular wave shapes are also studied. As an example tracking of 35.60um pp 1Hz triangular reference is presented on Fig. 10. The resulting peak to peak error is 54nm pp corresponding to 0.15%. Triangular wave shape are in particular importance since they constitutes non-continuous references.

![Fig. 10: Tracking of 35.60um pp 1Hz triangular reference. The error is 54nm pp (0.15%).](image)

To present the tracking of an abstract but continuous wave form the tracking of human beat signal is presented on Fig. 11. For this waveform with peak to peak amplitude of 21.70um the tracking resulted with
controller can track a reference signal with a good performance. The proposed SMC is possible and that the disturbance observer as introduced in [35].

The use of such a controller in systems controlled by digital controllers, including DSP, PC, microchip and FPGA, will help users to save from heavy computational load. Moreover, due to the operation frequency limitation in digital systems mainly due to the analog to digital conversion, the controller is expected to have better performance than the DSP application. Experimental results proved that the analog production of the proposed SMC is possible and that the controller can track a reference signal with a good degree of accuracy.

References


**Authors’ information**

1Sabanci University, Faculty of Engineering and Natural Sciences, Mechatronics Program.
2 Sabanci University, Faculty of Engineering and Natural Sciences, Mechatronics Program.

Selim Yannier was born in Istanbul, Türkiye on 1978. He received his B.S. degree in Physics with honors from Orta Doğu Teknik Üniversitesi (Middle East Technical University), Ankara, Türkiye in Fall 2001. He then began M.S. studies at Mechatronics Engineering Program, Sabanci University, Istanbul, Türkiye in Spring 2001 and completed at Spring 2002. He is currently Ph.D. student at the same institution. Since January 2001 he is working as a Research and Teaching Assistant in Sabanci University, Istanbul, Türkiye. His research interests are motion control, mobile robotics and control for nanotechnologies.

Asif Sabanovic is a full professor of mechatronics at Sabanci University, Istanbul. He received B.S. degree in electrical engineering (in 1970) , M.S. degree and Dr. Eng. degree from University of Sarajevo. His fields of interest include nonlinear control, robust control, robotics, electric drives, power electronics and mechatronics.

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The second paragraph uses the pronoun of the person (he or she) and not the author’s last name. Information concerning previous publications may be included. Current and previous research interests ends the paragraph.

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