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Spin bath decoherence of quantum entanglement

Zafer Gedik *

Faculty of Engineering and Natural Sciences, Sabanci University, Tuzla, Istanbul 34956, Turkey

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Abstract

We study an analytically solvable model for decoherence of a two spin system embedded in a large spin environment. As a measure of entanglement, we evaluate the concurrence for the Bell states (Einstein–Podolsky–Rosen pairs). We find that while for two separate spin baths all four Bell states lose their coherence with the same time dependence, for a common spin bath, two of the states decay faster than the others. We explain this difference by the relative orientation of the individual spins in the pair. We also examine how the Bell inequality is violated in the coherent regime. Both for one bath and two bath cases, we find that while two of the Bell states always obey the inequality, the other two violate the inequality at early times.

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Entanglement, nonlocal quantum correlations between subsystems, is not only one of the basic concepts in quantum mechanics [1] but also central to quantum computation and quantum information [2]. Decoherence, loss of phase relations between the states, is essential in understanding how a quantum system becomes effectively classical [3]. Therefore, how an entangled system undergoes decoherence or how the entanglement changes as a result of interaction with the environment is an important issue and for two entangled spins subject to quantum noise created by a bosonic bath the problem has already been studied [4,5].

In this work, we concentrate on decoherence of two spins as a result of an interaction with a spin bath. This problem is closely related to electron spin dynamics, due to hyperfine interaction with surrounding nuclear spins, in quantum dots [6]. Decoherence of various systems, including superconducting quantum interference devices (SQUIDs) coupled to nuclear and paramagnetic spins, can be described by similar models [7]. Many spin systems can exhibit interesting behaviors including parity dependent decoherence where some non-diagonal elements of the density matrix survive the initial decay of other entries due to environmental spins [8,9]. For the central spin model, which describes a localized spin coupled to a spin bath, the quasiclassical equations of motion are integrable [10].

Quantification of entanglement is a major challenge in quantum information theory. A well known measure for a pure state of a pair of quantum systems is the von Neumann entropy or equivalently the Shannon entropy of the squares of the Schmidt coefficients [11]. The entropy of the partial density matrix, which is obtained by tracing out one of the members from the total density matrix, can be used to parametrize the entanglement. For a pair of binary quantum objects (qubits) an alternative parameter is the concurrence which is related to the von Neumann entropy bijectively [12]. To quantify the entanglement between the two spins, we are going to use the concurrence because of its mathematical simplicity. Our main results related to entanglement will turn out to be independent of the choice of the measure.

Decoherence of the two spins can be viewed as a generation of entanglement between the pair and the spin bath (or baths) and hence any measure of the entanglement can also be used to parametrize the decoherence. As the members of the pair lose their entanglement with each other, they start to entangle with the bath spins. What we are going to evaluate is the concurrence corresponding to the entanglement between the two partners.

Our aim is to understand how two entangled spins lose their correlation due to other spins interacting with them. For this purpose, we start with a very simple model where we can

* Tel.: +90 2164839610; fax: +90 2164839550.
E-mail address: gedik@sabanciuniv.edu

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observe decoherence effects. The model Hamiltonian

$$H = c_{1z}\sum_{k=1}^{N_1}\hbar\omega_{1k}\sigma_{1kz} + c_{2z}\sum_{k=1}^{N_2}\hbar\omega_{2k}\sigma_{2kz} \tag{1}$$

describes two central spins, with z-component operators c_{1z} and c_{2z}, coupled to bath spins represented by σ_{nkz}, where $n=1,2$ labels the baths and $k=1,2,3,\ldots,N_n$ labels the individual spins. All spins are assumed to be 1/2 and c_{1z}, c_{2z}, and σ_{nkz} denote the corresponding Pauli matrices. Hamiltonian (1) is a simple two spin generalization of the model proposed by Zurek to study decoherence in spin systems [13]. First, we are going to consider two different spin baths where each spin couples only one of them. Later, we are going to examine how our results change when the pair interacts with a single bath. Since, the Hamiltonian (1) involves only the z-components of the spins, it can also be used to study decoherence of other two-state systems.

We are going to assume that at $t=0$, the central spins are not entangled to the spin baths so that the state is in the product form $|\Psi(0)\rangle = |\Psi_c(0)\rangle|\Psi_{\sigma 1}(0)\rangle|\Psi_{\sigma 2}(0)\rangle$ where

$$|\Psi_c(0)\rangle = \left(a_{\uparrow\uparrow}|\uparrow\uparrow\rangle + a_{\uparrow\downarrow}|\uparrow\downarrow\rangle + a_{\downarrow\uparrow}|\downarrow\uparrow\rangle + a_{\downarrow\downarrow}|\downarrow\downarrow\rangle\right) \tag{2}$$

with obvious notation for the two spins and

$$|\Psi_{\sigma n}(0)\rangle = \bigotimes_{k=1}^{N_n}\left(\alpha_{nk}|\uparrow_{nk}\rangle + \beta_{nk}|\downarrow_{nk}\rangle\right) \tag{3}$$

where $|\uparrow_{nk}\rangle$ and $|\downarrow_{nk}\rangle$ are eigenstates of σ_{nkz} with eigenvalues $+1$ and -1, respectively, and $|\alpha_{nk}|^2+|\beta_{nk}|^2=1$. At later times, the state is no more in the product form due to entanglement of the pair with environmental spins but instead it is given by

$$\begin{aligned}|\Psi(t)\rangle = &(a_{\uparrow\uparrow}|\uparrow\uparrow\rangle|\Psi_{\sigma 1}(+t)\rangle|\Psi_{\sigma 2}(+t)\rangle \\ &+ a_{\uparrow\downarrow}|\uparrow\downarrow\rangle|\Psi_{\sigma 1}(+t)\rangle|\Psi_{\sigma 2}(-t)\rangle \\ &+ a_{\downarrow\uparrow}|\downarrow\uparrow\rangle|\Psi_{\sigma 1}(-t)\rangle|\Psi_{\sigma 2}(+t)\rangle \\ &+ a_{\downarrow\downarrow}|\downarrow\downarrow\rangle|\Psi_{\sigma 1}(-t)\rangle|\Psi_{\sigma 2}(-t)\rangle)\end{aligned} \tag{4}$$

where

$$|\Psi_{\sigma n}(t)\rangle = \bigotimes_{k=1}^{N_n}(\alpha_{nk}\mathrm{e}^{-i\omega_{nk}t}|\uparrow_{nk}\rangle + \beta_{nk}\mathrm{e}^{i\omega_{nk}t}|\downarrow_{nk}\rangle). \tag{5}$$

We are going to see that it is the randomness of the interaction strengths and the expansion coefficients that will lead to decoherence of the pair.

The total, central spin and the baths, density matrix is given by $\rho(t)=|\Psi(t)\rangle\langle\Psi(t)|$ but what we are interested in is the reduced density matrix which is obtained from the former by tracing out the bath degrees of freedom as $\rho_c(t)=Tr_\sigma\rho(t)$ where subscript σ means that trace is evaluated by summing over all possible nk states. We can write the resulting density matrix in the product basis $\{|\uparrow\uparrow\rangle,|\uparrow\downarrow\rangle,|\downarrow\uparrow\rangle,|\downarrow\downarrow\rangle\}$ as

$$\rho_c = \begin{pmatrix} |a_{\uparrow\uparrow}|^2 & a_{\uparrow\uparrow}a^*_{\uparrow\downarrow}r_2 & a_{\uparrow\uparrow}a^*_{\downarrow\uparrow}r_1 & a_{\uparrow\uparrow}a^*_{\downarrow\downarrow}r_1r_2 \\ a^*_{\uparrow\uparrow}a_{\uparrow\downarrow}r^*_2 & |a_{\uparrow\downarrow}|^2 & a_{\uparrow\downarrow}a^*_{\downarrow\uparrow}r_1r^*_2 & a_{\uparrow\downarrow}a^*_{\downarrow\downarrow}r_1 \\ a^*_{\uparrow\uparrow}a_{\downarrow\uparrow}r^*_1 & a^*_{\uparrow\downarrow}a_{\downarrow\uparrow}r^*_1r_2 & |a_{\downarrow\uparrow}|^2 & a_{\downarrow\uparrow}a^*_{\downarrow\downarrow}r_2 \\ a^*_{\uparrow\uparrow}a_{\downarrow\downarrow}r^*_1r^*_2 & a^*_{\uparrow\downarrow}a_{\downarrow\downarrow}r^*_1 & a^*_{\downarrow\uparrow}a_{\downarrow\downarrow}r^*_2 & |a_{\downarrow\downarrow}|^2 \end{pmatrix} \tag{6}$$

where * means complex conjugation and decoherence factors $r_1(t)$ and $r_2(t)$ are given by

$$r_n(t) = \prod_{k=1}^{N_n}\left(|\alpha_{nk}|^2\mathrm{e}^{-i2\omega_{nk}t} + |\beta_{nk}|^2\mathrm{e}^{i2\omega_{nk}t}\right). \tag{7}$$

In general both expansion coefficients α_{nk}, β_{nk} and interaction strengths ω_{nk} are random. If the bath spins point randomly at $t=0$ we can write the expansion coefficients as $\alpha_{nk}=\cos(\theta_{nk}/2)\mathrm{e}^{-i\phi_{nk}/2}$ and $\beta_{nk}=\sin(\theta_{nk}/2)\mathrm{e}^{i\phi_{nk}/2}$, where θ_{nk} and ϕ_{nk} are spherical polar coordinates determining the direction of the spins, and we assume that the angles θ_{nk} and ϕ_{nk} have uniform distributions in the intervals $[0,\pi]$ and $[0,2\pi]$, respectively. It is possible to show that for sufficiently short times $|r_n(t)|$ exhibits a Gaussian time dependence $\mathrm{e}^{-a_nt^2}$ rather than exponential [14]. In our case

$$a_n = 8\sum_k|\alpha_{nk}|^2|\beta_{nk}|^2|\omega_{nk}|^2. \tag{8}$$

We are going to obtain several coherence factors given by expressions similar to Eq. (7). We first note that the larger the interaction strengths $|\omega_{nk}|$, the faster the decay. Secondly, for a given set of $\{\omega_{nk}\}$, the fastest decoherence is attained when $|\alpha_{nk}|$ and $|\beta_{nk}|$ become equal.

To evaluate the concurrence [12], we need to find the time-reversed or spin-flipped density matrix $\tilde{\rho}_c$ which is given by

$$\tilde{\rho}_c = (\sigma_y\otimes\sigma_y)\rho^*_c(\sigma_y\otimes\sigma_y). \tag{9}$$

Here σ_y is the Pauli spin matrix and $\otimes$ stands for the Kronecker product, and ρ^*_c is obtained from $\tilde{\rho}_c$ via complex conjugation. We can write the spin-flipped density matrix as

$$\tilde{\rho}_c = \begin{pmatrix} |a_{\downarrow\downarrow}|^2 & -a_{\downarrow\uparrow}a^*_{\downarrow\downarrow}r_2 & -a_{\uparrow\downarrow}a^*_{\downarrow\downarrow}r_1 & a_{\uparrow\uparrow}a^*_{\downarrow\downarrow}r_1r_2 \\ -a^*_{\downarrow\uparrow}a_{\downarrow\downarrow}r^*_2 & |a_{\downarrow\uparrow}|^2 & a_{\uparrow\downarrow}a^*_{\downarrow\uparrow}r_1r^*_2 & -a_{\uparrow\uparrow}a^*_{\downarrow\uparrow}r_1 \\ -a^*_{\uparrow\downarrow}a_{\downarrow\downarrow}r^*_1 & a^*_{\uparrow\downarrow}a_{\downarrow\uparrow}r^*_1r_2 & |a_{\uparrow\downarrow}|^2 & -a_{\uparrow\uparrow}a^*_{\uparrow\downarrow}r_2 \\ a^*_{\uparrow\uparrow}a_{\downarrow\downarrow}r^*_1r^*_2 & -a^*_{\uparrow\uparrow}a_{\downarrow\uparrow}r^*_1 & -a^*_{\uparrow\uparrow}a_{\uparrow\downarrow}r^*_2 & |a_{\uparrow\uparrow}|^2 \end{pmatrix} \tag{10}$$

The final step in evaluation of the concurrence C is to find the four eigenvalues $\{\lambda_i\}$ of the product matrix $\rho_c\tilde{\rho}_c$ in the decreasing order so that

$$C = \max\left\{0, \sqrt{\lambda_1}-\sqrt{\lambda_2}-\sqrt{\lambda_3}-\sqrt{\lambda_4}\right\}. \tag{11}$$

We are going to evaluate the above expression for the Bell states (Einstein–Rosen–Podolsky pairs)

$$|e_1\rangle = \frac{|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle}{\sqrt{2}} \tag{12}$$

$$|e_2\rangle = \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}$$

$$|e_3\rangle = \frac{|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle}{\sqrt{2}}$$

$$|e_4\rangle = \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}.$$

As we are going to see, the Bell states have the property that the concurrence is the same for all of them. In fact, any other basis obtained from the Bell states by replacing the coefficients $\pm 1/\sqrt{2}$ with $e^{i\theta}/\sqrt{2}$ (θ being a real number) has the same property.

For two baths, the concurrence, which is the same for all of the Bell states, turns out to be

$$C = |r_1||r_2|. \tag{13}$$

Since, $r_1(0) = r_2(0) = 1$, the concurrence is also unity at $t = 0$. On the other hand, both r_1 and r_2, and hence the concurrence, decay with time and vanish. For the special case where only one of the spins, say the first one, interacts with a spin bath so that $r_2(t) = r_2(0) = 1$, we still observe a decay in the concurrence. This is an expected result because the entangled pair must be treated as a single system rather than individual spins.

We next consider the case where both spins undergo decoherence due to interaction with the same spin bath so that the Hamiltonian becomes

$$H = \hbar \sum_{k=1}^{N} (\omega_{1k} c_{1z} + \omega_{2k} c_{2z}) \sigma_{kz} \tag{14}$$

This time the state at $t = 0$ is given by $|\Psi(0)\rangle = |\Psi_c(0)\rangle |\Psi_\sigma(0)\rangle$ where

$$|\Psi_\sigma(0)\rangle = \bigotimes_{k=1}^{N} (\alpha_k |\uparrow_k\rangle + \beta_k |\downarrow_k\rangle). \tag{15}$$

Similar to the two bath case, we can write the density matrix as

$$\rho_c = \begin{pmatrix} |a_{\uparrow\uparrow}|^2 & a_{\uparrow\uparrow} a^*_{\uparrow\downarrow} r_2 & a_{\uparrow\uparrow} a^*_{\downarrow\uparrow} r_1 & a_{\uparrow\uparrow} a^*_{\downarrow\downarrow} r^+_{12} \\ a^*_{\uparrow\uparrow} a_{\uparrow\downarrow} r^*_2 & |a_{\uparrow\downarrow}|^2 & a_{\uparrow\downarrow} a^*_{\downarrow\uparrow} r^-_{12} & a_{\uparrow\downarrow} a^*_{\downarrow\downarrow} r_1 \\ a^*_{\uparrow\uparrow} a_{\downarrow\uparrow} r^*_1 & a^*_{\uparrow\downarrow} a_{\downarrow\uparrow} r^{-*}_{12} & |a_{\downarrow\uparrow}|^2 & a_{\downarrow\uparrow} a^*_{\downarrow\downarrow} r_2 \\ a^*_{\uparrow\uparrow} a_{\downarrow\downarrow} r^{+*}_{12} & a^*_{\uparrow\downarrow} a_{\downarrow\downarrow} r^*_1 & a^*_{\downarrow\uparrow} a_{\downarrow\downarrow} r^*_2 & |a_{\downarrow\downarrow}|^2 \end{pmatrix} \tag{16}$$

where decoherence factors are given by

$$r_n(t) = \prod_{k=1}^{N} \left(|\alpha_k|^2 e^{-i2\omega_{nk} t} + |\beta_k|^2 e^{i2\omega_{nk} t} \right) \tag{17}$$

and

$$r^{\pm}_{12}(t) = \prod_{k=1}^{N} \left(|\alpha_k|^2 e^{-i2(\omega_{1k} \pm \omega_{2k}) t} + |\beta_k|^2 e^{i2(\omega_{1k} \pm \omega_{2k}) t} \right). \tag{18}$$

As we have discussed in the paragraph after Eq. (8), the larger the interaction strengths, the faster the decay. Therefore, we should compare $\omega_{1k} + \omega_{2k}$ with $\omega_{1k} - \omega_{2k}$. If all of the interaction constants ω_{nk} have the same sign, $r^+_{12}(t)$ goes to zero faster than $r^-_{12}(t)$. For the special case, $\omega_{1k} = \omega_{2k}$ for all k, $r^-_{12}(t)$ does not decay at all but remains constant. This is a trivial manifestation of decoherence free subspace [15–17]. When $\omega_{1k} = \omega_{2k}$, $|e_2\rangle$ and $|e_4\rangle$ become immune to the environment as we will see in Eq. (19).

After finding the spin-flipped density matrix $\tilde{\rho}_c$ and eigenvalues of the product $\rho\tilde{\rho}_c$, we can evaluate the concurrence for each of the Bell states. In single bath case the Bell states exhibit different decay rates with the concurrence expressions

$$C_1 = C_3 = |r^+_{12}| \tag{19}$$

$$C_2 = C_4 = |r^-_{12}|.$$

Although we have obtained this two by two grouping of the Bell states in terms of the concurrence, any other measure, like the von Neumann entropy, which depends upon the eigenvalues of the density matrices will yield the same result. Eqs. (13) and (19) show that the concurrence, which is a measure of entanglement is given by nothing but the coherence factor.

We can explain the different results for one bath and two baths decoherence processes in terms of different characters of the Bell states. When the spins interact with separate baths, relative orientation of spins is not important because the only difference between the up and down configurations is complex conjugation of the coherence factor and it is the modulus of the coherence factor which enters the concurrence expression. On the other hand, in the single bath case there is no simple relation between the opposite spin terms. In $|e_1\rangle$ and $|e_3\rangle$ states, spins are always parallel while in $|e_2\rangle$ and $|e_4\rangle$ states, they are always antiparallel. That is why two groups have different decoherence behaviors. It also possible to interpret the difference from correlation point of view. In our calculations we assume that not only the baths but also the spins in same bath are not initially entangled. In one bath case averaging over bath spins is performed independently for individual spins. However, for single bath this is not the case. In fact, single bath can be thought as two separate baths interacting with each member of the central pair where the baths have identical initial spin configuration. In a sense, the baths are correlated in contrast to the two bath case.

Finally, we are going to examine how the Bell inequality is violated in the quantum regime and how it is satisfied in the classical domain [18]. The Bell inequality, or in fact inequalities are satisfied if there exists a local realistic theory [19]. There are a large number of Bell inequalities, all resulting from local realistic assumptions, but following Ref. [20] we will focus our attention on the quantity

$$S = E(\theta_1, \theta_2) - E(\theta_1, \theta'_2) + E(\theta'_1, \theta'_2) + E(\theta'_1, \theta_2), \tag{20}$$

where the correlation function $E(\theta_1, \theta_2)$ is given by,

$$E(\theta_1, \theta_2) = Tr\{\hat{c}_1(\theta_1) \otimes \hat{c}_2(\theta_2) \rho_c\}, \tag{21}$$

with

$$\hat{c}_i(\theta_i) = c_{iz}\cos\theta_i + c_{ix}\sin\theta_i. \tag{22}$$

The Bell inequality is violated if $|S| > 2$. In calculating whether the inequality is violated, the choice of the angles θ_1 and θ_2 is crucial. It is known that not all entangled states violate a Bell inequality [21,22]. That is why θ_is must be chosen carefully. In our case, for the Bell states $\{|e_i\rangle\}$, we can find the corresponding $\{S_i\}$ expressions. To simplify the equations, for a given set of angles θ_1, θ_2, θ_1', and θ_2' we will introduce the notation

$$A = (\cos\theta_1\cos\theta_2 - \cos\theta_1\cos\theta_2' + \cos\theta_1'\cos\theta_2' + \cos\theta_1'\cos\theta_2) \tag{23}$$

$$B = (\sin\theta_1\sin\theta_2 - \sin\theta_1\sin\theta_2' + \sin\theta_1'\sin\theta_2' + \sin\theta_1'\sin\theta_2)$$

so that, for two separate baths,

$$S_1 = A + B\Re\{r_1 r_2\} \tag{24}$$

$$S_2 = -A + B\Re\{r_1 r_2^*\}$$

$$S_3 = A - B\Re\{r_1 r_2\}$$

$$S_4 = -A - B\Re\{r_1 r_2^*\}$$

where r_1 and r_2 are again given by Eq. (7), and $\Re\{z\}$ denotes the real part of the complex number z. For a single bath very similar expressions hold. In this case, $\Re\{r_1 r_2\}$ and $\Re\{r_1 r_2^*\}$ are replaced by $\Re\{r_{12}^+\}$ and $\Re\{r_{12}^-\}$, respectively.

The angles θ_1, θ_2, θ_1', and θ_2' can take arbitrary values. We are going to pick up a particular set for which $\{S_i\}$ are easy to calculate. We will assume that $\theta_1=0$, $\theta_2=\pi/4$, $\theta_1'=\pi/2$, and $\theta_2'=3\pi/4$. For this choice of angles, $A=B=\sqrt{2}$. At $t=0$ where all decoherence factors are unity, for both two bath and single bath cases, S_2 and S_3 vanish, and therefore, they satisfy the Bell inequality $|S|\leq 2$. When the system becomes completely incoherent so that all coherence factors vanish, again for both two bath and single bath cases, we obtain $|S_2|=|S_3|=\sqrt{2}$. Although there is an increase in $|S|$ values, the inequality is still satisfied. In $|e_1\rangle$ and $|e_4\rangle$ states, however, the Bell inequality is violated at $t=0$, since $|S_1|=|S_4|=2\sqrt{2}$. As the decoherence factors vanish, they both decay to $\sqrt{2}$. For two baths, $|S_1|$ and $|S_4|$ exhibit different decay rates. In single bath case, the corresponding factors coming from decoherence for $|e_1\rangle$ and $|e_4\rangle$ states are given by $\Re\{r_{12}^+\}$ and $\Re\{r_{12}^-\}$, respectively. As we have discussed above, the two factors decay at different rates.

In conclusion, using the concurrence and the Bell inequality, we demonstrated that a pair of entangled spins show different decoherence behaviors when the spins interact with a common spin bath or separate baths. Some entangled states can be more vulnerable than others. For example, two entangled electrons in the same quantum dot will have a different coherence characteristics than two in separate dots. Recent proposal by Beenakker et al. for the creation of entangled electron-hole pairs might be an interesting system to look for such decoherence effects [23].

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