Bayesian Trained Rational Functions for Electromagnetic Design Optimization

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Introduction

Design Optimization has been a difficult, demanding but necessary task for the development of novel commercial and wireless applications. Among others, particular applications include miniaturized antennas without sacrifice in their bandwidth and radiation efficiency. The need for design, preferably design optimization is pertinent to the competing physics of these metrics, which has been the focus of researchers for the past two decades. It is reasonable to expect that designs resulting from global design optimization studies that allow for full design space exploration including antenna shape, size, feed location and material will lead to novel configurations with enhanced performance. However, global synthesis via heuristic/search techniques is a challenging task due to the bottlenecks of fast and accurate reanalysis and proper definition of the optimization model. Therefore, unless design studies are limited to only a few number of design variables, heuristic/evolutionary search studies can become impractical. To address this issue, in this paper an approximation scheme suitable for frequency sampled FEM response of electromagnetic systems such as multi-resonant antennas is investigated. The goal is to develop an efficient and reliable scheme that will allow for fast and numerous reanalysis in large scale global design optimization studies. Polynomials, multiquadrics [1], kriging [2] and artificial neural networks (ANNs) [3] are some examples of the interpolating techniques/surrogate models used so far, and the 'virtual' objective function they provide, can be called by the optimization algorithm. In the literature, many Response Surface Methods [4], [5], space mapping techniques [6] or combinations thereof [7] are documented recently coupling especially the aforementioned approximation techniques with stochastic algorithms. The scheme in this paper is based on the Bayesian trained quadratic rational functions. Unlike other naïve techniques such as linear approximation that fails to predict resonance behavior and is highly dependent on sample data, the proposed quadratic rational function effectively emulates resonance behavior with an overall better accuracy than even sampled data with twice as many frequency points. When compared with other known data training approaches, the Bayesian trained rational function proves to have a powerful yet simple approximation capability based on statistics and just a single controlling parameter, *coef*. Naturally, enhancements are expected with the incorporation of effective gradient estimator via the adjoint variable method. An in-depth analysis of the proposed interpolation's efficiency and reliability will follow its use in designing concurrent dielectric and conductor topologies for miniaturized novel antennas.

Interpolation Method: Bayesian Trained Quadratic Rational Functions

The trained data in this paper relates to the response of an antenna model that comprises a geometry discretized with 400 volumetric and 400 surface finite elements. Properties such as the permittivity, permeability and conductivity of each cell could be assigned as design variables of a topology optimization problem. Conductors can attain discrete values of 0 or 1 representing the on/off feature of a conductor. The use of popular heuristic/global based techniques to optimize the device and locate the global optimum will call for multiple reanalysis of the full-wave bandwidth response of a binary encoded large scale design problem. Hence, to predict the return loss response accurately, a frequency sampling of 10 MHz is needed. The on/off nature of conductors is observed to result in high oscillations/multiple resonances within the frequency range of interest (1-2GHz) for various topologies. This makes the reliable interpolation of the response an extremely challenging task. The optimization model with 101 frequency points and N individuals/function of a micro-GA would call the FEA model 101N times for each generation until it reaches convergence. Carefully reselecting design variables, speeding-up the simulation tool and the optimization model are among remedies to reduce computation time. In this paper we focus on sampling with lower number of frequency points (11 vs. 101) and investigate the possibility of predicting the cost function reliably by interpolation using rational quadratic functions. The numerator and denominator of chosen rational function are of second order, and are therefore termed as 'quad-quad'. The order is chosen to closely follow the behavior of the return loss curve. To ensure continuity of successive intervals for highly oscillatory response, we imposed equality of function its first derivative values at interval end points. Coefficients in the quad-quad in Eq. 1 satisfying these conditions are solved analytically.

$$y = \frac{\beta_1 + \beta_2 x + \beta_3 x^2}{1 + \beta_4 x + \beta_5 x^2} \tag{1}$$

In addition to 5 unknown coefficients and 4 boundary conditions (BC), a constraint that restricts the denominator of having a root inside the interval of interest is imposed to emulate nulls of the return loss response. This is achieved via a tuning coefficient, *coef that* relates β_4 to β_5. No root case is ensured with *coef* > 1 constraint, which in turn excludes singularities other than, nulls. The BCs in a non-dimensionalized frequency interval $x \in [0\ 1]$ are given by $y(0)=y_0$, $y'(0)=y'_0$, $y(1)=y_1$, $y'(1)=y'_1$. Solving for β's:

$$\begin{aligned}
&\beta_1 = y_0 \\
&\beta_4 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \begin{cases} a = -(coef/4)\,y'_1 \\ b = y_1 - y_0 - y'_1 \\ c = 2(y_1 - y_0) - (y'_1 + y'_0) \end{cases} \\
&\beta_2 = y'_0 + y_0 \beta_4 \\
&\beta_3 = y_1\left(1 + \beta_4 + (coef/4)\beta_4^2\right) - \beta_1 - \beta_2
\end{aligned} \tag{2}$$

There is no single *coef* that suits all intervals and it can be shown that *coef* depends greatly on the BC's. Therefore, to predict the optimum *coef* in the interval, knowing optimum *coef*s of the training set, the use of Bayesian classifier [8] seems to be appropriate since it recognizes the probabilistic nature of the training set and assigns values and classes to the test sets accordingly. The goal is to classify the intervals' BCs so as to minimize the probability of *coef* misclassification. In *d*-dimensions the general multivariate normal probability density can be written as:

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{d/2}|\boldsymbol{\Sigma}|^{1/2}} \exp\left\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right\} \tag{3}$$

where $\mathbf{x}$ is the variable vector (BC), $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are, the *mean* vector and *covariance* matrix of the training set and d refers to the dimension of the problem (number of BC). Interval parameter *coefs* are assigned to the class with maximum probability.

Interpolation Results

The quad-quad response relies on first derivatives at sampled data points. These are computed numerically at 'close (~0.1%)' points via finite difference. Alternatively, they can be computed analytically via the adjoint method to reduce the computation time by half. The efficiency and reliability of the quad-quad is assessed by comparing it with 2 fitting schemes: 1) Linear interpolation with exact same sampled frequency and 2) linear interpolation with the same number of data points. The latter requires twice as many number of function calls, equivalent to the quad-quad interpolation time due to additional calls for each point's finite difference calculation. All three interpolations are compared in Fig.1 for a selected group of 11 different designs. The error measure corresponds to the square sum of error difference between predicted and original return loss curves. 550 intervals obtained for all designs were used to train the Bayesian classifier and predict the optimum *coef* of the fitted curves. The results indicate an overall error increase of the quad-quad vs. the double sampled linear fitting by 32.8% and a decrease by 22.5% vs. single sampled one. However, better matched nulls with quad-quad prompt for reinvestigation of the error measure. As a result, a more appropriate error measure to predict nulls and their bandwidth would be the bandwidth itself. Here, bandwidth is defined based on matched nulls in the original and fitted curves. The difference between these two values is summed up over the entire frequency range to calculate the overall error (see Fig.2). Results show that quad-quad predicts an error decrease by 27.6% and 55.9% with respect to double and single sampled linear fitting, respectively -a significant improvement over the previous case. The design cases with larger error are re-examined and displayed for a specific interval in Fig.3. Although the center plot is in favor of the double sampled linear interpolation, perturbing sampling points to the left or to the right (left and right plots, respectively) shows that double sampled linear approach is more viable to data change whereas the quad-quad consistently predicts the null and approximates bandwidth. This qualitatively shows that the quad-quad is more robust with respect to sample data. More detailed studies are needed and will be presented at the conference. To evaluate the Bayesian's training capability for extreme designs, it is applied to measured data of a different antenna. The BW prediction is superior for this case by 32.36% and 62.16%, respectively (see Fig.4), motivating further work and use of quad-quad based curve fitting in large scale heuristic based optimization studies.

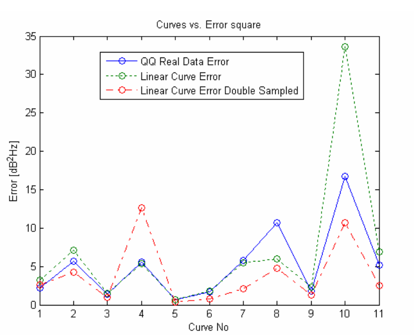

Fig. 1: Square of error vs. design number

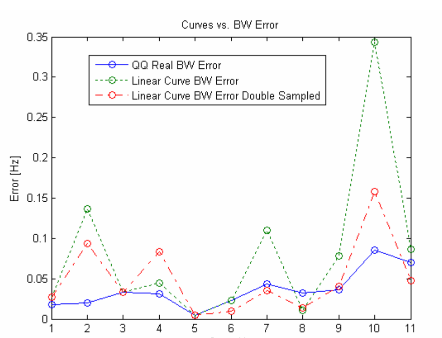

Fig. 2: Bandwidth error vs. design number

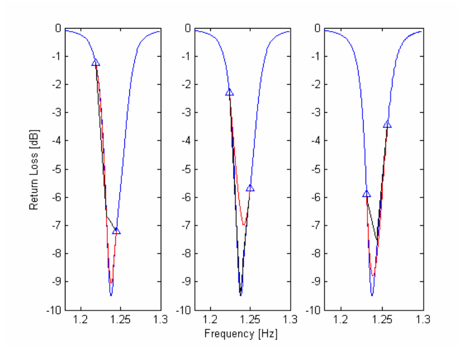

Fig. 3: Effect of data point perturbation on quad-quad (red) and linear double sampled (black) interpolation

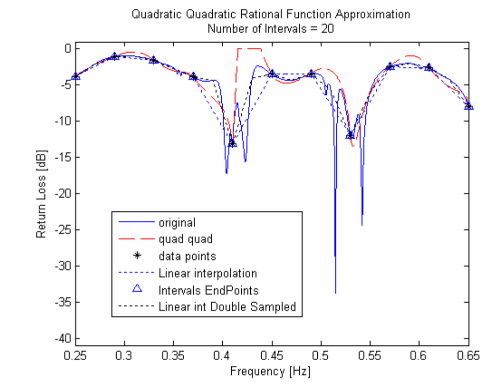

Fig. 4: Measured data set fitted via quad-quad interpolation

Conclusions

This paper presented an interpolation scheme based on Bayesian trained quadratic rational functions for approximating frequency based electromagnetic return loss responses. Initial results indicate that this scheme is an efficient tool in catching nulls and characterizing resonance behavior. With the implementation of the adjoint variable method for effective gradient evaluations, this may be an alternative tool to predict the nulls and corresponding BW values for practical heuristic design optimization studies. Future work includes elaborating on *coef* and adaptive selection of sample points and finally applying it to a global design optimization example.

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