CONTROL OF PIEZOELECTRIC ACTUATORS USING ANALOG SLIDING MODE CONTROLLER

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ABSTRACT
This work aims the precise control of the piezoelectric actuators that can theoretically provide unlimited motion resolution. For this purpose Sliding Mode Controller (SMC) is used. The proposed algorithm is known to have order of sampling time square and therefore faster application than DSP was required. The application of the algorithm via analog electronics is suggested. Preliminary work, simulation and experimental results are presented.

KEY WORDS
Sliding Mode Control, Analog Control, Piezoelectric actuator, Hysteresis, Nonlinear Control

1. Introduction

Piezoelectric actuators (PEA), based on crystalline effects do not suffer from "stick slip" effect and theoretically provide unlimited resolutions. Therefore, they are already widely used in many applications, some requiring sub-micrometer resolution, such as in ultrasonic motors, sports materials like skis and bikes [1], in aerospace [2], in hard disk drives [3] etc… Another main application of these ceramics is the scanning tunneling microscope (STM) and atomic force microscope (AFM) [4].
Precision control of piezoelectric actuators is hardly acquired due to highly nonlinear input/output behavior dominated by hysteresis behavior between electrical voltage and strain [5,[6,[7]. Hysteresis yields a rate-independent lag and residual displacement near zero input, significantly reducing the precision of the actuators [8]. Another undesired characteristic of piezoelectric actuators is the "creep effect" [7]. Both hysteresis and creep effects are shown in Figure 1.
Most of the piezoelectric actuator applications require high precision motion control where closed loop control is the only answer. Despite that fact, many attempts to drive the piezoelectric actuator as open loop system with compensation of the nonlinearities are investigated [5,[6,[7,[8]. However, the position tracking of those systems hardly satisfied the requirements.

In order to design a control scheme with successful tracking performance without precise dynamic modeling some fuzzy logic and neural network solutions are presented in the literature. However, due to the limited performance, this research area did not find much popularity [9].
On the other hand, sliding-mode control is one of the effective nonlinear robust control approaches. One of the most important aspect of SMC is the discontinuous nature of the control action which switches between two values to move the system motion on so-called "sliding mode" that exist in a manifold. SMC provides system invariance to uncertainties once the system is in the sliding mode [10,[11]. Beside switching control action, continuous control is also possible with appropriate formulation.

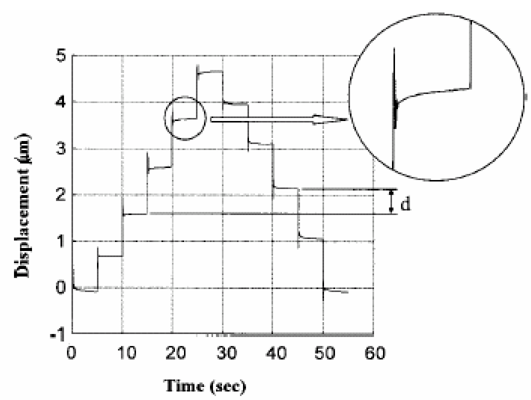

Figure 1: Open loop step response of the piezoelectric actuator: "d" is the displacement difference due to hysteresis. Creep effect is magnified in the circular view [7].

Abidi et al. used SMC with continuous control action in conjunction with the disturbance observer for both position and force tracking in piezoelectric actuators [11]. In their work, the discrete time formulation and application of the SMC is shown and applied for PEA control. Abidi et al. showed that discrete time SMC can track nanometer size references with strain gage feedback in the presence of a disturbance observer. Proven that the controller has error on the order of the square of the sampling time, improvements on the speed of the

controller are required. Although DSP based controllers are easier to build and run, their speed is limited. Analog circuits based on large bandwidth op-amp circuits on the other hand, promise much more speed.
The aim of this work is to apply the discrete time SMC algorithm on analog control circuit. By the way speed limitation introduced by the DSP will be eliminated and improvement of the tracking performance is expected.

2. Discrete Time Sliding Mode Control

With the assumption that the PEA can be modeled as a linear lumped parameters *T*, *m*, *b*, *k* second order electromechanical system with voltage as the input *u(t)*, position *x* as the output and hysteresis nonlinearity *h(x,u)* being the major disturbance, the model can be written as [5,[6,[12],

$$m \cdot \ddot{x} + c \cdot \dot{x} + k \cdot x = T\big(u(t) - h(x,u)\big) - F_{ext} \quad (1)$$

For such a system, we can design a SMC both on discrete and continuous time domains [11]. To start it is possible to write (1) in a more general form;

$$\dot{x} = f\left(x, h(x,u), F_{ext}\right) + B \cdot u \quad (2)$$

The aim is to derive the states of the system into the set

$$S = \left\{x : G \cdot \left(x^{ref} - x\right) = \sigma\left(x^{ref}, x\right) = 0\right\} \quad (3)$$

Where $x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$ is the state vector, $x^{ref} = \begin{bmatrix} x_1^{ref} & x_2^{ref} \end{bmatrix}^T$ is the reference vector, $\sigma\left(x^{ref}, x\right)$ is the function defining the sliding manifold and $G = \begin{bmatrix} C & 1 \end{bmatrix}$ with C being a positive constant. The derivation starts with the selection of a positive definite Lyapunov function $V(\sigma)$ with negative derivative

$$V(\sigma) = \sigma \cdot \sigma^T / 2 \geq 0 \quad (4)$$

$$\Rightarrow \dot{V}(\sigma) = \sigma \cdot \dot{\sigma} \quad (5)$$

In order to guarantee the asymptotic stability, $\dot{V}(\sigma)$ may be selected as

$$\dot{V}(\sigma) = -D \cdot \sigma \cdot \sigma^T \leq 0\,,\ D \in \Re^+ \quad (6)$$

If the control can be determined from (5) and (6), the asymptotic stability of solution (3) can be obtained. Combining those two equations

$$\sigma \cdot (\dot{\sigma} + D \cdot \sigma) = 0 \quad (7)$$

A solution for this last equation is

$$\dot{\sigma} + D \cdot \sigma = 0 \quad (8)$$

The derivative of the sliding function is as follows

$$\dot{\sigma} = G \cdot \left(\dot{x}^{ref} - \dot{x}\right) = G \cdot \dot{x}^{ref} - G \cdot \dot{x} \quad (9)$$

From this last equation and using (2) we obtain

$$\dot{\sigma} = G\dot{x}^{ref} - Gf - GBu(t) = GB\big(u_{eq} - u(t)\big) \quad (10)$$

where u_{eq} is defined as the control on the sliding surface (when $\dot{\sigma} = 0$). If this last equation is inserted in (8) and the result is solved for the control

$$u(t) = u_{eq} + (GB)^{-1} D\sigma \quad (11)$$

is obtained. It can be seen from (11) that u_{eq} is difficult to calculate. Using the fact that u_{eq} is a continuous function, (10) can be written in discrete-time form after applying Euler's approximation,

$$\dot{\sigma} = \frac{\sigma_{k+1} - \sigma_k}{T_s} = GB\left(u_{eq,k} - u_k\right) \quad (12)$$

where T_s is the sampling time and $k \in Z^+$. It is also possible to write (11) in discrete-time form just as it was done before

$$u_{eq,k} = u_k + (GB)^{-1} \cdot D \cdot \sigma_k \quad (13)$$

If (12) is solved for u_{eq}, the following is obtained

$$u_{eq,k} = u_k + (GB)^{-1} \cdot \left(\frac{\sigma_{k+1} - \sigma_k}{T_s}\right) \quad (14)$$

Since the system is causal and it is required to avoid calculation of the predicted value for σ, control cannot be dependent on a future value of σ. Having equivalent control as a continuous function, the current value of the equivalent control will be approximated by a single time-step backward value,

$$\hat{u}_{eq,k} \cong \hat{u}_{eq,k-1} = u_{k-1} + (GB)^{-1} \cdot \left(\frac{\sigma_k - \sigma_{k-1}}{T_s}\right) \quad (15)$$

Here $\hat{u}_{eq,k}$ is the estimate of the current value of the equivalent control. If (15) is inserted in (13);

$$u_k = u_{k-1} + K\left(D \cdot \sigma_k + \dot{\sigma}_k\right),\ K = (GB)^{-1} \quad (16)$$

It easily seen that the above control law is derived from discrete-time approximations based on the continuous-time equations. Hence, these approximations will introduce errors in the control that must be analyzed carefully. Closed loop behaviour of this control is investigated by Abidi et al. and proven that the maximum deviation of the system (1), from manifold (3) under the control (16), both at each sampling time and at the intersampling time, is on the order of the square of the sampling time: $O\left(T_s^2\right)$ [11]. Moreover the same work proves the Lyapunov stability of the closed loop system.

3. Analog Solution Design and Simulations

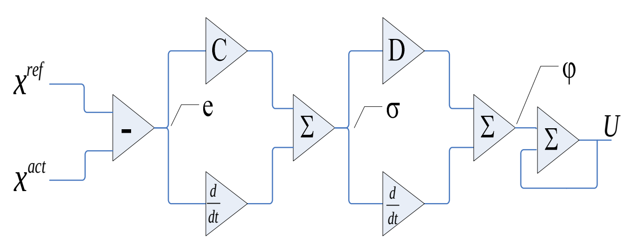

Figure 2: Draft analog circuit scheme to calculate the control.

A. Analog Solution Design

Control presented in previous section can algorithmically be summarized as follows;

- Calculate the error: $e = x^{ref} - x$
- Calculate sliding manifold function: $\sigma = C \cdot e + \dot{e}$
- Calculate: $\varphi = \left(D \cdot \sigma + \dot{\sigma}\right)$
- Finally calculate the control $u_k = u_{k-1} + K \cdot \varphi$

A draft circuit for the algorithm can be drawn as shown in Figure 2. From that draft it is then easy to generate analog circuit scheme for the SMC using differentiation and summing operational amplifier blocks.

Table 1: Assumed maximum values for signals and parameters.

Parameter	Determined Value	Signal	Assumed Max Value
C	400	e	0.05V
D	120	$\dot{e}$	10V
K	3.10^{-3}	σ	25V
		$\dot{\sigma}$	5000V
		φ	5000V
		U	1V

B. Analog Scaling

Once the scheme is present and C, D and K constants are roughly known, designing the circuit is straight forward. However there is one key issue about the signal magnitudes: the amplifier outputs (or signal strengths) cannot be numerically equal to the problem variables they represent, except in very special cases, since the outputs of the amplifiers are limited to the supply voltage which is generally smaller than the problem variables. Therefore, represented signals may reach higher values than the supply voltage and saturate the outputs. Moreover, some gains used in the algorithm, namely the problem variables, may be too high or too small for a practical realization. Therefore, signal strengths cannot be numerically equal to the program variables, but merely proportional to them. They must be multiplied by appropriate coefficients, called "scale factors" to assure that the amplifier output or magnitude of the parameters is realistic.

To determine the scaling factors, so called "analog scaling" technique is used as it was once used in analog computers. In this technique each signal is normalized according to its estimated maximum value. Then signal equations are rewritten to find actual op-amp gains. As an example we can study the following equation $\sigma = C \cdot e + \dot{e}$ with assumed maximum values for signals and parameters that are presented on the Table 1.

$$25 \cdot \left[\frac{\sigma}{25}\right] = 400 \cdot 0.05 \cdot \left[\frac{e}{0.05}\right] + 10 \cdot \left[\frac{\dot{e}}{10}\right] \tag{17}$$

$$\Rightarrow \left[\frac{\sigma}{25}\right] = 0.8 \cdot \left[\frac{e}{0.05}\right] + 0.4 \cdot \left[\frac{\dot{e}}{10}\right] \tag{18}$$

Equation (18) is equivalent to $\sigma = C \cdot e + \dot{e}$ since we have substituted equivalent expressions. But it guarantees that the signals will be in the range of the op-amps unless the expected maximum values are exceeded. To obtain problem variables, one must rescale those values according to the obtained equations.

C. Circuit Design

All 4 control equations together with the two derivative equations, one for $\dot{e}$ and the other for $\dot{\sigma}$, must be scaled (equations 18-24).

$$\left[\frac{e}{0.05}\right] = 20.0 \cdot \left(\left[\frac{x^{ref}}{1.0}\right] - \left[\frac{x^{act}}{1.0}\right]\right) \tag{19}$$

$$\left[\frac{\dot{e}}{10}\right] = 5.10^{-3} \frac{d}{dt} + \left[\frac{e}{0.05}\right] \tag{20}$$

$$\left[\frac{\sigma}{25}\right] = 0.8 \cdot \left[\frac{e}{0.05}\right] + 0.4 \cdot \left[\frac{\dot{e}}{10}\right] \tag{21}$$

$$\left[\frac{\dot{\sigma}}{5000}\right] = 5.10^{-3} \frac{d}{dt} + \left[\frac{e}{25}\right] \tag{22}$$

$$\left[\frac{\varphi}{5000}\right] = 0.6 \cdot \left[\frac{\sigma}{25}\right] + 1.0 \cdot \left[\frac{\dot{\sigma}}{5000}\right] \tag{23}$$

$$\left[\frac{u_k}{1}\right] = \left[\frac{u_{k-1}}{1}\right] + 15 \cdot \left[\frac{\varphi}{5000}\right] \tag{24}$$

The analog application of those equations in analog electronics is straight forward. The circuit is composed of summing and difference amplifiers, differentiators and a low pass filter.

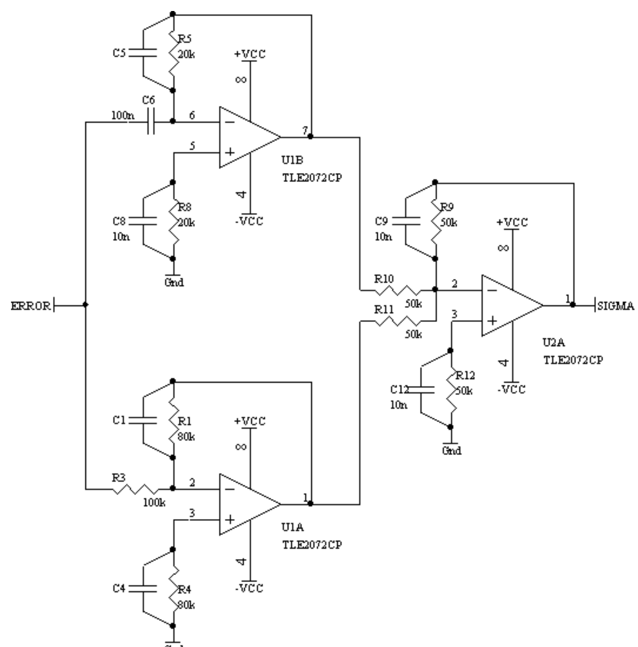

Figure 3: Analog circuit scheme for the calculation of σ..

The part calculating $\sigma = C \cdot e + \dot{e}$ can be seen in Figure 3 above. $\varphi = D \cdot \sigma + \dot{\sigma}$ calculation is the repetition of the same block with different gains, therefore it is not given as separate schematic.

The realization of the last part is shown in Figure 4. In this part, an inverting summing amplifier, *amplifier B*, is adding the scaled φ value by *amplifier A* to the previous value of the output which is the output signal retarded by a low pass filter (*amplifier C*).

Amplifier C, is a low pass first order filter with corner frequency, $f_{corner} \cong 318Hz$, high enough not to interfere with the motion. As a transfer function this circuit is equivalent to;

$$F(s) = \frac{1}{\tau s + 1}, \ \tau = 5 \cdot 10^{-4} \tag{25}$$

This filter creates a phase delay in the signal. Due to this delay, the output signal can be fed back to the amplifier B as the delayed or “previous value” of the control U .

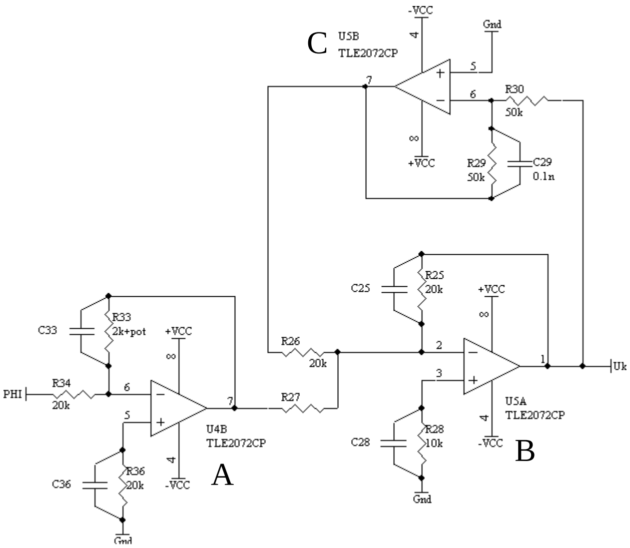

Figure 4: Realization of the integration part.

D. Circuit Simulations

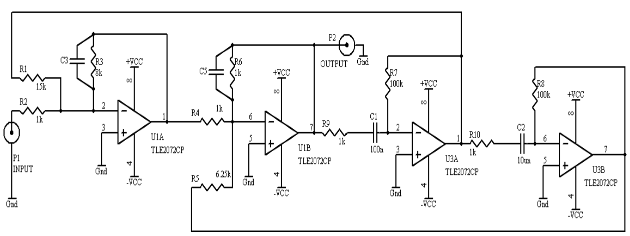

Figure 5: 2nd order plant used for simulations.

Designed circuit is first simulated using PSpice v9.2 by Cadence Design Systems. To check the functionality a sample plant is required. Therefore a 2nd order system transfer function is imitated with a plant circuit. The plant transfer function given in (26) is equivalent to the mass-spring-damper system with mass m, damping b, spring constant k and input gain τ. This plant is close to the lumped parameter PEA model except the hysteresis that is missing (see Figure 5 for the circuit realization).

$$H(s) = \frac{9000}{s^2 + 480s + 900} = \frac{\tau}{ms^2 + bs + k} \qquad (26)$$

Once the circuit design is complete simulations for sinusoidal reference inputs are studied. The position tracking, control voltage and related error for the tracking of a 0.2V sinusoidal input reference. The tracking error is on the order of 0.17%.

Another simulation is run for exponentially increasing and decreasing pulse input of 0.8V maximum amplitude. Related figures are below.

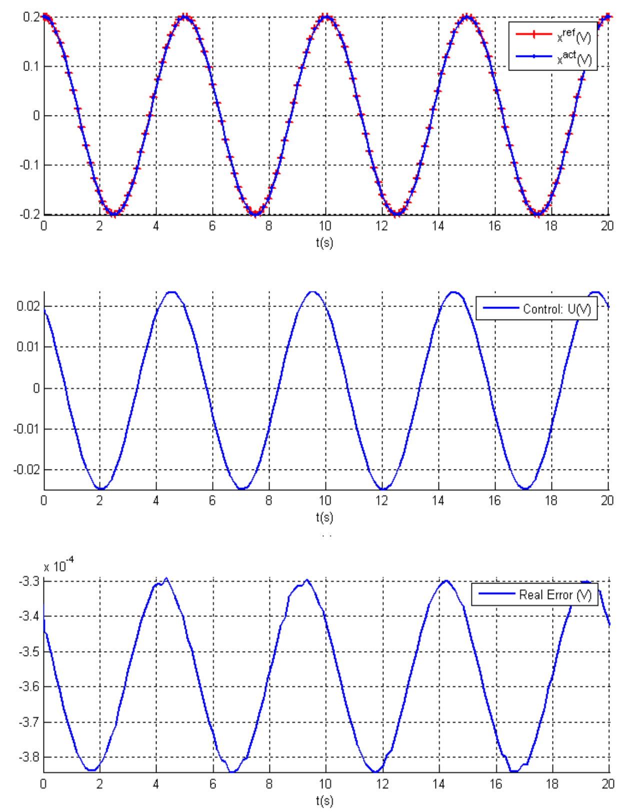

Figure 6: Real and actual position signals (top), control input (middle) and tracking error (bottom) for a 200mV sinusoidal input.

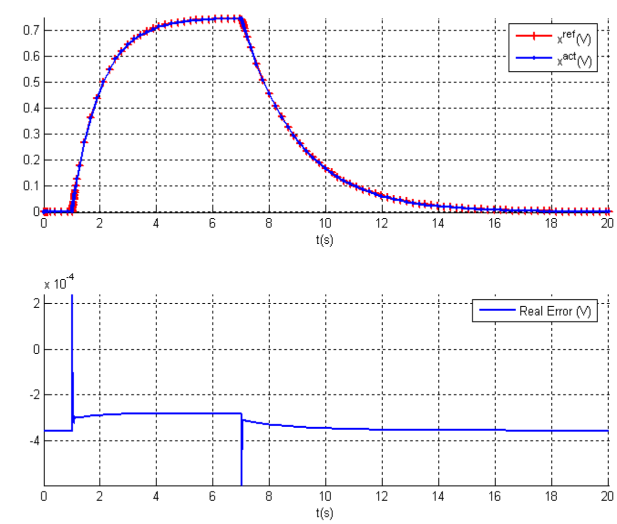

Figure 7: real and actual position signals (top) and tracking error (bottom) for a 0.8V exponential pulse.

4. FPAA Application & Experiments

Once the design is completed and simulations verified the circuit design, real experiments are aimed. For this purpose a print circuit board (PCB) that assembles designed circuits is designed and produced. This board contains 4 integrated circuits with 2 operational amplifiers in each. The size of the board is 20x14cm. Position tracking experiments for the piezoelectric actuator with DSP and produced analog circuit are realized separately for comparison purposes.

A. Experimental Setup

For experimental purposes, the setup shown in Figure 8 below is constructed; voltage amplifier is the Piezomechanik SVR 150-3 voltage amplifier, PEA is the piezoelectric actuator with embedded strain gage for position measurement and the strain gage amplifier is the BA501 strain gage amplifier from Vishay's Measurement Group. Here SMC is the designed sliding mode control algorithm implemented in DSP (for DSP experiments) or is the analog circuit (for analog controller experiments).

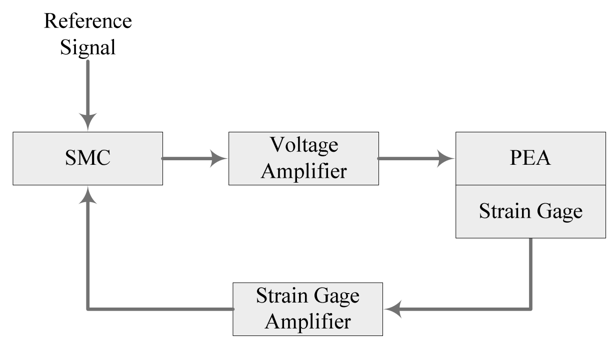

Figure 8: Piezoelectric actuator control setup.

In those experiments, both DSP and analog control, we should remind that the effect of the voltage amplifier, designed and produced by Piezomechanik GMBH, is unknown. We assume that it is limited with a low pass filter.

B. DSP Position Tracking Experiments

For comparison of the results DSP application of the algorithm is realized on dSpace DS1102 platform which possesses TMS320C31 DSP chip running at 40 MHz with 50ns cycle time. The platform does have 2 16-bit ADC (Input) ±10V and 4 12-bit DAC (Outputs) ±10V. Results for the position tracking of the 100*nm* peak to peak, 2.25Hz sinusoidal reference is presented. From the experiment we can see that tracking of this reference is realized with maximum error less than 4*nm* or 7.11*nm* peak to peak. The error is 7.11% for the algorithm that runs at 8kHz as maximum speed.

C. Analog Circuit, Position Tracking Experiments

In analog circuit experiments, the data is captured by Agilent Technologies 54622D digital oscilloscope. The reference and actual signals are given without offset to better have feeling on the tracking error. The third channel shown in figures is the difference between the signals, the error, as calculated by the oscilloscope.

According to our experiments, the tracking of a 600mVpp (Volt peak to peak) sinusoidal voltage reference is successfully tracked with 18mVpp tracking error corresponding to 3.00%. Naturally those voltage values are the readings of the strain gage amplifier. The performance can be better understood when those values are converted to metric correspondents. 17.96um (micrometers) corresponds to 1V of the strain gage amplifier reading, and 1um position deflection results 55.68mV. According to those conversion values the given reference is 10.776um and the tracking error is 323nm. Very similar results are obtained with a triangular wave reference; the error is 58.6mVpp (1.053um) for the reference of 2.016Vpp (36.21um); 2.91%.

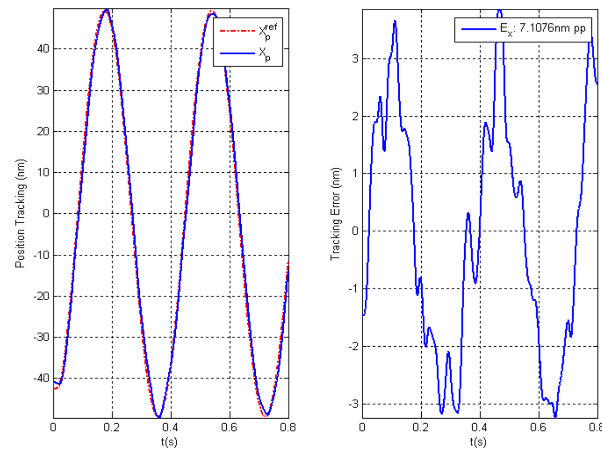

Figure 9: Tracking of 100nm pp sinus reference; tracking on the left and error on the right.

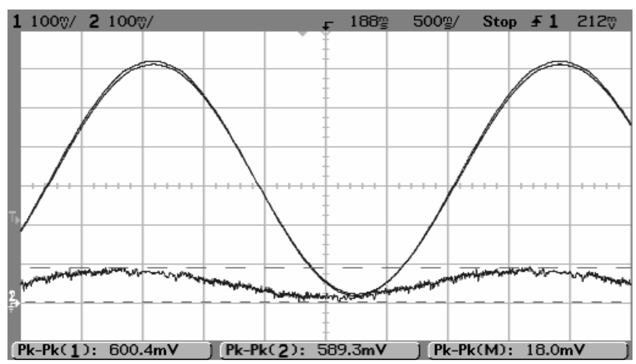

Figure 10: Position tracking of the piezoelectric actuator for 300mHz, 600mVpp sinusoidal reference.

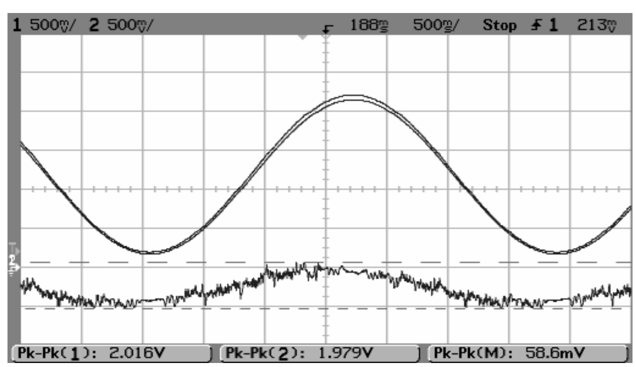

Figure 11: Position tracking of the piezoelectric actuator for 300mHz, 2Vpp sinusoidal reference.

5. Conclusion

In this work we have presented the design of a SMC for PEAs. The control is known to have order of sampling time square error. DSP application of the control could run at speeds up to 8kHz. For the improvement of the performance analog application of the control is proposed. Circuit simulations showed good performance. For the experimental works PCB design and prototyping is realized.

Experimental results proved that the analog production of the proposed SMC is possible and its performance is acceptable. Although the error in DSP application is around 7%, analog application resulted with around 3% error.

The main problem in the system is the noise due to the interaction with the environment and the twice derivation that amplifies the noise.

With the proposed analog circuit good tracking performance is obtained. Moreover the cost for such a controller is less than 25$. Compare to 5000$ DSP solution, this analog controller deserves further improvements.

Finally, the use of such a controller in systems controlled by digital controllers including DSP, PC, microchip, FPGA etc… will help users to save from heavy computational load.

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