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# SMC Framework in Motion Control Systems

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SUMMARY

Design of a motion control system should take into account both the unconstrained motion performed without interaction with environment or other system, and the constrained motion where system is in contact with environment or has certain functional interaction with another system. In this paper control systems design approach, based on siding mode methods, that allows selection of control for generic tasks as trajectory and/or force tracking as well as for systems that require maintain some functional relation – like bilateral or multilateral systems, establisment of virtual relation among mobile robots or control of haptic systems is presented. It is shown that all basic motion control problems - trajectory tracking, force control, hybrid position/force control scheme and the impedance control - can be treated in the same way while avoiding the structural change of the controller and guarantying stable behavior of the system In order to show applicability of the proposed techniques simulation and experimental results for high precision systems in microsystems assembly tasks and bilateral control systems are presented. 

KEY WORDS: *Motion Control, Sliding Mode Control, Bilateral Control, Interconnected Systems.*

## 1. INTRODUCTION

Modern motion control systems are more and more acting as "agents" between skilled human operator and environment (surgery, microparts handling, teleoperation, etc.), thus design of control should encompass wide range of very demanding tasks. At the lower level one should consider tasks of controlling individual systems - like single DOF systems, motor control, robotic manipulator or mobile robot. On the system level control of bilateral or multilateral interaction between systems of the same or different nature, the remote control in master-slave systems, haptics etc. should be considered. Motion control systems such as robots, vehicles and so on are expected to be applied in environment where presence of humans is natural. Such a complexity of motion control system functions poses a challenge for control systems designers due to the diversity of the tasks and changing structure of the system and the interaction with environment. In general design of motion control system should take into account (i) unconstrained motion - performed without interaction with environment or other system - like trajectory tracking, (ii) motion in which system should maintain its trajectory despite of the interaction with other systems - disturbance rejection tasks, (iii) constrained motion



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where system should modify its behavior due to interaction with environment or another system and should maintain specified interconnection - virtual or real - with other system and (iv) in remote operation control system should be able to reflect the sensation of unknown environment to the human operator.

There are many applications of decentralized control to motion systems, with concepts such as subsumption architecture [1], multi-agent system [2, 3], cell structure [4], and fault tolerant systems [5]. Although design methods for decentralized control systems are interesting as concepts a simple framework in view of controller design is desired to cope with complexity of systems in interaction. Decomposition block control [6] simplifies the design problem. Arimoto and Nguyen [7] showed that under certain conditions overall control input can be designed by linear superposition, Lee and Li proposed a decoupled design method that makes a bilateral control system behave as a common passive rigid mechanical tool [8]. Tsuji et al. proposed a framework of controller design based on functionality [9], Onal et al. implemented a bilateral control using sliding mode control applying functionality [10]. Basic approach in control of bilateral system widely used in literature [11,12,13,14] is based on the design of the controllers for the master and the slave side separately and than adding interacting terms in order to reach the transparency requirements.

The most salient feature of the SMC is a possibility to constrain system motion on the selected manifold in the state space [15]. In discrete-time this control that enforces sliding mode is continuous in a sense of the discrete-time systems and the resulting inter-sampling motion for systems with smooth disturbances is constrained to the $o(T^2)$ vicinity of the sliding manifold [16,17,18,19,20]. The application of SMC in motion control systems [21,22] range from control of power converters, electrical machines, robotic manipulators, mobile robots, PZT based actuators etc In this paper a framework for sliding mode application in motion control systems with or without contact with environment is presented. The possibility to enforce certain functional relations between coordinates of one or more motion systems represents a basis of the proposed algorithm. It will be shown that all basic motion control problems can be treated in the same way while avoiding the structural change of the controller and guarantying stable behavior of the system. This framework can be naturally extended to the control of mechanical systems in interaction, like bilateral or multilateral control.

The organization of the paper is as follows. In Section 2 application of SMC methods to motion control systems are discussed for $n$-degrees of freedom (DOF) fully actuated mechanical system with and/or without motion modification due to interaction with environment. In Section 3 the problems related to the modification of the systems motion due to the contact with environment are discussed and a possible solution in the framework of SMC control is proposed. In Section 4 an extension of the solution presented in Section 3 is applied to motion control systems in interaction and as one example, the application to bilateral control is shown.

## 2. SLIDING MODES IN MOTION CONTROL SYSTEMS

### *2.1. Control Problem Formulation*

For fully actuated mechanical system $S$ being in interaction with environment or another mechanical system mathematical model may be found in the following form

$$S: \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{N}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{F} - \mathbf{F}_{ext}(\mathbf{q},\mathbf{q}_e);$$
$$\mathbf{F}_{ext}(\mathbf{q},\mathbf{q}_e) = \begin{cases} \mathbf{g}_{ie}(\mathbf{q},\mathbf{q}_e) & \text{when in contact with environment} \\ 0 & \text{without contact with environment} \end{cases} \tag{1}$$

where $\mathbf{q} \in \Re^n$ stands for vector of generalized positions, $\dot{\mathbf{q}} \in \Re^n$stands for vector of generalized velocities, $\mathbf{M}(\mathbf{q}) \in \Re^{nxn}$ is the generalized positive definite inertia matrix with bounded parameters hence $M^- \leq \|\mathbf{M}(\mathbf{q})\| \leq M^+$, $\mathbf{N}(\mathbf{q},\dot{\mathbf{q}}) \in \Re^{nx1}$ represents vector of coupling forces including gravity and friction and is bounded by $\|\mathbf{N}(\mathbf{q},\dot{\mathbf{q}})\| \leq N^+$, $\mathbf{F} \in \Re^{nx1}$ with $\|\mathbf{F}\| \leq F_0$ is vector of generalized input forces, $\mathbf{F}_{ext} \in \Re^{nx1}$ with $\|\mathbf{F}_{ext}\| \leq F_{ext}$ is vector of interaction forces being zero when system $S$ is not interacting with environment and $\mathbf{q}_e \in \Re^l$ stands for the vector of generalized positions of environment. $\mathbf{F}_{ext} \in \Re^{nx1} M^-, M^+$, $N^+$, $F_0$ and $F_{ext}$ are known scalars. In system (1) vectors $\mathbf{F}_{ext}$ and $\mathbf{N}(\mathbf{q},\dot{\mathbf{q}})$ satisfy matching conditions [23]. External force can be treated as an additional input to the system (1) able to modify the system configuration in the same way as the control input does.

Vector of generalized positions and generalized velocities defines configuration $\xi(\mathbf{q},\dot{\mathbf{q}})$ of a mechanical system. The control tasks for the system (1) are usually formulated as selection of the generalized input such that: (i) system executes desired motion specified as position tracking, (ii) system exerts a defined force while in the contact with environment and (iii) system reacts as a desired impedance on the external force input or in contact with environment. In literature these problem are generally treated separately [24, 25] and motion that requires transition from one to another task is treated in the framework of hybrid control [26]. The SMC framework can be applied if one defines a sliding mode manifold $\sigma = \mathbf{0}_{nx1}$ in terms of the difference between the desired system configuration (reference) and the actual system configuration.

Without loss of generality, in this paper, it will be assumed that system configuration can be expressed as a linear combination of generalized positions and velocities $\xi(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{C}\mathbf{q} + \mathbf{Q}\dot{\mathbf{q}}$ and the sliding mode manifold is described by :

$$\sigma\left(\xi(\mathbf{q},\dot{\mathbf{q}}), \xi^{ref}(t)\right) = \xi(\mathbf{q},\dot{\mathbf{q}}) - \xi^{ref}(t) = \mathbf{C}\mathbf{q} + \mathbf{Q}\dot{\mathbf{q}} - \xi^{ref}(t) = \mathbf{0},$$
$$\sigma, \xi, \xi^{ref} \in \Re^{nx1}; \mathbf{C}, \mathbf{Q} \in \Re^{n\times n}; \mathbf{C}, \mathbf{Q} > 0, \sigma = [\sigma_1, \sigma_2, ...., \sigma_n]^T \tag{2}$$

where $\xi^{ref}(t) \in \Re^{nx1}$ stands for the reference configuration of the system and is assumed to be smooth bounded function with a continuous first order time derivative, matrices $\mathbf{C}, \mathbf{Q} \in \Re^{n\times n}$ have full rank $rank(\mathbf{C}) = rank(\mathbf{Q}) = n$. In the sliding mode framework requirement (2) is equivalent to the enforcing sliding mode on the manifold $S_q$ defined by

$$S_q = \left\{(\mathbf{q},\dot{\mathbf{q}}): \ \mathbf{C}\mathbf{q} + \mathbf{Q}\dot{\mathbf{q}} - \xi^{ref}(t) = \sigma\left(\mathbf{q},\dot{\mathbf{q}},\xi^{ref}\right) = 0\right\}, \tag{3}$$

Assume that matrices $\mathbf{C}, \mathbf{Q} \in \Re^{nxn}$ are constant and that inverse $\left(\mathbf{Q}\mathbf{M}^{-1}\right)^{-1}$ exists and can be expressed as $\left(\mathbf{Q}\mathbf{M}^{-1}\right)^{-1} = \mathbf{M}\mathbf{Q}^{-1}$. The application of the equivalent control method for system (1) with the sliding mode enforced on the manifold (3) leads to the equations of motion

$$\mathbf{M}\ddot{\mathbf{q}} = \left(\mathbf{Q}\mathbf{M}^{-1}\right)^{-1}\left(\dot{\xi}^{ref}(t) - \mathbf{C}\dot{\mathbf{q}}\right) = \mathbf{M}\ddot{\mathbf{q}}^{des} \Rightarrow \ddot{\mathbf{q}} = \ddot{\mathbf{q}}^{des}$$
$$\ddot{\mathbf{q}}^{des} = \mathbf{Q}^{-1}\left(\dot{\xi}^{ref}(t) - \mathbf{C}\dot{\mathbf{q}}\right) \tag{4}$$

 

The sliding mode motion (4) is equivalent to the acceleration control [25] with desired acceleration $\ddot{\mathbf{q}}^{des} = \mathbf{Q}^{-1}\left(\dot{\xi}^{ref}(t) - \mathbf{C}\dot{\mathbf{q}}\right)$ and the closed loop system behaves as a "nominal plant" defined by design parameters $\mathbf{C}$ and $\mathbf{Q}$. Equations (4) show that in the ideal case motion of the system will not be modified when it comes in contact with environment. If closed loop motion (4) should be modified due to the contact with environment than the reference configuration must depend on the interaction force The structure of the sliding mode control system is depicted in Fig. 1.

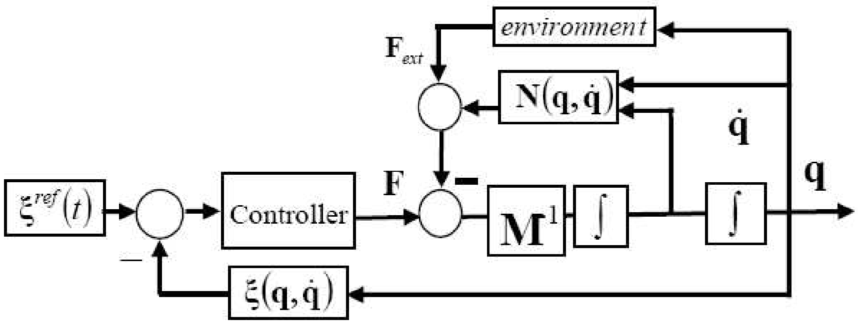


Figure 1. Structure of the SMC motion control system

*2.2. Selection of the control input*

The simplest and the most direct method is to derive a control, which enforces Lyapunov stability conditions for solution $\sigma\left(\xi, \xi^{ref}\right) = \mathbf{0}_{nx1}$ on the trajectories of system (1). A Lyapunov function candidate may be selected as $v = \frac{1}{2}\sigma^T\sigma > 0, v(0) = 0$ and one has to design control that enforces the following structure $\dot{v} = \sigma^T\dot{\sigma} = -\sigma^T\mathbf{\Psi}(\sigma) < 0$ of the Lyapunov functions derivative. For example if . $-\sigma^T\mathbf{\Psi}(\sigma) = -\rho v^{\delta} < 0$ with $\rho > 0$ and $\frac{1}{2} \leq \delta < 1$ stability conditions are satisfied and finite time convergence to the sliding mode manifold is obtained [27]. (For $\delta = 1$ we can say only that the convergence is not slower than exponential.) From $\dot{v} = \sigma^T\dot{\sigma} = -\sigma^T\mathbf{\Psi}(\sigma)$ one can derive $\sigma^T(\dot{\sigma} + \mathbf{\Psi}(\sigma)) = 0$. From $\dot{\sigma} + \mathbf{\Psi}(\sigma)|_{\sigma\neq 0} = 0$ one can find

$$\begin{aligned}\mathbf{F} &= \mathbf{F}_{eq} - \left(\mathbf{Q}\mathbf{M}^{-1}\right)^{-1}\mathbf{\Psi}(\sigma) = \mathbf{F}_{eq} - \mathbf{M}\mathbf{Q}^{-1}\mathbf{\Psi}(\sigma) \\ \mathbf{F}_{eq} &= (\mathbf{F}_{ext} + \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}},)) - \left(\mathbf{Q}\mathbf{M}^{-1}\right)^{-1}\left(\mathbf{C}\dot{\mathbf{q}} - \dot{\xi}^{ref}(t)\right)\end{aligned} \tag{5}$$

For continuous-time systems function $\mathbf{\Psi}(\sigma)$is most often selected to satisfy $-\sigma^T\mathbf{\Psi}(\sigma) = -\rho v^{1/2}$. The resulting control is discontinuous and in mechanical systems may cause chattering. There are many possibilities to deal with chattering problem in mechanical systems [28].

Suppose that control can take values within $\|\mathbf{F}\| \leq F_0$ and available control resources are such that $\|\mathbf{F}_{eq}\| \leq F_0$ thus the control should be $\mathbf{F} = sat\left(\mathbf{F}_{eq} - \mathbf{M}\mathbf{Q}^{-1}\mathbf{\Psi}(\sigma)\right)$ where $sat(\bullet)$ stands for saturation function with bounds $\mathbf{F}_b = F_0\frac{\mathbf{F}}{\|\mathbf{F}\|}$. As shown above for $\|\mathbf{F}\| < F_0$ the sliding mode conditions on manifold (3) are enforced. For $\|\mathbf{F}\| \geq F_0$ one can find the derivative of the sliding mode function as $\dot{\sigma} = \left(\mathbf{QM}^{-1}\right)(\mathbf{F}_b - \mathbf{F}_{eq}) = \left(\mathbf{QM}^{-1}\right)\left(F_0 \frac{\mathbf{F}}{\|\mathbf{F}\|} - \mathbf{F}_{eq}\right)$. By inserting (5) and taking into account that in this region $\|\mathbf{F}\| = F_0$ one can find. $\dot{\sigma} = -\left(1 - \frac{F_0}{\|\mathbf{F}\|}\right) F_0 - \frac{F_0}{\|\mathbf{F}\|}\mathbf{\Psi}(\sigma)$ hence $\sigma$ decreases and after finite time region $\|\mathbf{F}\| < F_0$ is reached.

### *2.3. Discrete-time implementation of control*

The discrete-time implementation of control (5) requires evaluation of the equivalent control at the end of every sampling interval. The equivalent control is smooth and one can resort of using its value in $t = (k-1)T$ instead of the exact value at $t = kT$. By evaluating $\dot{\sigma} = \mathbf{QM}^{-1}(\mathbf{F} - \mathbf{F}_{eq})$ at $t = (k-1)T$ it is easy to derive $\mathbf{F}_{eq}(k-1) = \left(\mathbf{F}(k-1) - \mathbf{MQ}^{-1}\dot{\sigma}(k-1)\right)$. Approximation $\dot{\sigma}(k-1) = (\sigma(k) - \sigma(k-1))/T$ leads to $\mathbf{F}_{eq}(k-1) = \left(\mathbf{F}(k-1) - \mathbf{MQ}^{-1}T^{-1}(\sigma(k) - \sigma(k-1))\right)$ with an approximation error of $o(T^2)$ order. The approximated control input can be expressed as

$$\begin{aligned} \mathbf{F}(k) &\cong \mathbf{F}_{eq}(k-1) - \mathbf{MQ}^{-1}\mathbf{\Psi}(\sigma_k) \\ \mathbf{F}(k) &= \underbrace{\left(\mathbf{F}(k-1) - \mathbf{MQ}^{-1}T^{-1}(\sigma(k) - \sigma(k-1))\right)}_{\mathbf{F}_{eq}(k-1)} - \mathbf{MQ}^{-1}\mathbf{\Psi}(\sigma(k)) \end{aligned} \tag{6}$$

By inserting (6) into (1) one can evaluate system dynamics at $t = kT$ as

$$\mathbf{M}\ddot{\mathbf{q}}(k) = \mathbf{F}_{eq}(k-1) - \mathbf{MQ}^{-1}\mathbf{\Psi}(\sigma(k)) - (\mathbf{F}_{ext}(k) + \mathbf{N}(\mathbf{q}(k), \dot{\mathbf{q}}(k))) \tag{7}$$

The error introduced by this approximation of control can be estimated from the following relation

$$\dot{\sigma}(k) + \mathbf{\Psi}(\sigma(k)) = -\mathbf{QM}^{-1}(\mathbf{F}_{eq}(k) - \mathbf{F}_{eq}(k-1)) \tag{8}$$

The thickness of the boundary layer of the sliding mode manifold can be determined by evaluating $\sigma(kT+\tau) - \sigma(kT) = -\int_{kT}^{kT+\tau}\mathbf{\Psi}(\sigma(t))\,dt + o\left(T^2\right)$. From here one can see why the relay control in discrete-time implementation will result in motion with chattering within a boundary layer having thickness of the $o(T)$ order.

### *2.4. Trajectory tracking and force control*

In literature behavior of a motion control system is mostly analyzed in three separated frameworks: (i) the trajectory tracking, (ii) the force control and (iii) impedance control. Due to the fact that in fully actuated systems interaction forces and system configuration cannot be set independently hybrid schemes had been developed to cope with position-force control tasks and the transitions from one to another [26].

In SMC framework the reference configuration for the trajectory tracking should be selected as $\xi_q^{ref}(t) = -\left(\mathbf{Q}\dot{\mathbf{q}}^{ref} + \mathbf{C}\mathbf{q}^{ref}\right)$ and consequently the sliding mode manifold becomes $S_q = \left\{\mathbf{q}, \dot{\mathbf{q}} : \mathbf{Q}\dot{\mathbf{q}} + \mathbf{C}\mathbf{q} - \left(\mathbf{Q}\dot{\mathbf{q}}^{ref} + \mathbf{C}\mathbf{q}^{ref}\right) = \sigma_q\left(\xi, \xi^{ref}\right) = \mathbf{0}\right\}$ thus the control (5) can be directly applied to obtain $\mathbf{F} = \mathbf{F}_{eq} - \mathbf{MQ}^{-1}\mathbf{\Psi}(\sigma_q)$. For example, if one selects $\mathbf{\Psi}(\sigma_q) = -\mathbf{D}\sigma_q; \mathbf{D} > 0$ the equations of motion can be determined in the following form

$\mathbf{Q}\left(\ddot{\mathbf{q}}-\ddot{\mathbf{q}}^{ref}\right)+(\mathbf{C}+\mathbf{DQ})\left(\dot{\mathbf{q}}-\dot{\mathbf{q}}^{ref}\right)+\mathbf{CD}\left(\mathbf{q}-\mathbf{q}^{ref}\right)=\mathbf{0}$. This can be interpreted as a system with the mechanical impedance having mass $\mathbf{Q}$, damping $(\mathbf{C}+\mathbf{DQ})$ and spring coefficient $\mathbf{CD}$. If the matrix $\mathbf{D}$ is selected diagonal and its elements such that the transient $\dot{\sigma}_q+\mathbf{D}\sigma_q=\mathbf{0}$ is fast as compared to the sliding mode dynamics (defined by matrices $\mathbf{Q}$ and $\mathbf{C}$), after initial transient the resulting motion remains in the $\varepsilon$- thick boundary layer of $\sigma_q=\mathbf{0}$. This result is the same as the one obtained by application of the disturbance observer and PD controller as discussed in [25].

In the force control with the reference $\mathbf{F}^{ref}(t)$ the sliding mode manifold can be defined as $S_F=\left\{(\mathbf{q},\dot{\mathbf{q}}):\ \mathbf{F}(\mathbf{q},\dot{\mathbf{q}})-\mathbf{F}^{ref}(t)=\sigma_F=\mathbf{0}\right\}$. If measured force is modeled as $\mathbf{F}(\mathbf{q},\dot{\mathbf{q}})=\mathbf{K}_P\Delta\mathbf{q}+\mathbf{K}_D\Delta\dot{\mathbf{q}}$, $\Delta\mathbf{q}=\mathbf{q}-\mathbf{q}_e$, $\mathbf{K}_P,\mathbf{K}_D$ are diagonal matrices of appropriate dimensions, then by defining the reference configuration as $\xi_F^{ref}(t)=\left(\mathbf{F}^{ref}(t)+\mathbf{K}_P\mathbf{q}_e+\mathbf{K}_D\dot{\mathbf{q}}_e\right)$ the sliding mode manifold becomes $S_F=\left\{(\mathbf{q},\dot{\mathbf{q}}):\ \mathbf{K}_P\mathbf{q}+\mathbf{K}_D\dot{\mathbf{q}}-\xi_F^{ref}(t)=\sigma_F=\mathbf{0}\right\}$. This manifold has the same form as the one derived for the trajectory tracking, thus the structure of the control input should have the form $\mathbf{F}=\mathbf{F}_{eq}-\mathbf{M}\mathbf{K}_D^{-1}\mathbf{\Psi}(\sigma_F)$, which is the same as for trajectory tracking with appropriate changes of the variables.

## 3. MODIFICATION OF SYSTEM CONFIGURATION IN SMC

### 3.1. Motion modification due to the interaction force

Assume two mechanical systems $S_i:\ \mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i+\mathbf{N}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)=\mathbf{F}_i-\mathbf{g}_{ij}(\mathbf{q}_i,\mathbf{q}_j)$ and $S_j:\ \mathbf{M}_j(\mathbf{q}_j)\ddot{\mathbf{q}}_j+\mathbf{N}_j\left(\mathbf{q}_j,\dot{\mathbf{q}}_j\right)=\mathbf{F}_j+\mathbf{g}_{ij}(\mathbf{q}_i,\mathbf{q}_j)$ with reference configurations $\xi_{iq}^{ref}(t)$ and $\xi_{jq}^{ref}(t)$ respectively. These interconnected systems may be perceived as mobile robots or robotic arms in cooperative works. Let the interaction forces between systems $S_i$ and $S_j$ be denoted as $\mathbf{g}_{ij}(\mathbf{q}_i,\mathbf{q}_j)\in\Re^{n\times1}$, which becomes $\mathbf{g}_{ij}(\mathbf{q}_i,\mathbf{q}_j)=\mathbf{0}$ if systems are not in interaction. This force can be modeled as $\mathbf{g}_{ij}\left(\mathbf{q}_i,\mathbf{q}_j\right)=\mathbf{K}_{Pi}\Delta\mathbf{q}+\mathbf{K}_{Di}\Delta\dot{\mathbf{q}}$, $\Delta\mathbf{q}=\mathbf{q}_i-\mathbf{q}_j$ and should be maintained at the desired value $\mathbf{g}_{ij}^{ref}(t)$ while the systems are in interaction. Assume that only system $S_i$ should change its configuration as a result of the interaction. In this arrangement the system $S_j$ is assumed to be controlled in the trajectory tracking mode and the system $S_i$ should modify its configuration in order to maintain the desired profile of the interaction force. By making reference configuration of system $S_i$ dependent of the desired trajectory $\xi_{iq}^{ref}(t)$ and of the interaction force $\mathbf{g}_{ij}\left(\mathbf{q}_i,\mathbf{q}_j\right)$ one can make motion of the system reacting on both of them. The modification of the trajectory of the system $S_i$ could be selected (i) to be proportional with the interaction force (so-called compliant motion), (ii) to ensure that the interaction force tracks its reference (force tracking), and (iii) the combination of the cases (i) and (ii). For all three cases the sliding mode manifold has the following form

$$S_{iq}=\left\{\mathbf{q}_i,\dot{\mathbf{q}}_i:\mathbf{C}_i\mathbf{q}_i+\mathbf{Q}_i\dot{\mathbf{q}}_i-\xi_{iqF}^{ref}(t)=\sigma_{iqF}=\mathbf{0}\right\} \tag{9}$$

where the configuration of the system $S_i$ is selected as $\xi_{iq}(\mathbf{q}_i,\dot{\mathbf{q}}_i)=\mathbf{C}_i\mathbf{q}_i+\mathbf{Q}_i\dot{\mathbf{q}}_i$. Depending on the specific task, the reference configuration $\xi_{qF}^{ref}(t)$ can take one of the following forms

(i) $\xi_{iqF}^{ref}(t)=\xi_{iq}^{ref}(t)-\mathbf{\Gamma}\mathbf{g}_{ij}(\mathbf{q}_i,\mathbf{q}_j)$ or

(ii) $\xi_{iqF}^{ref}(t)=\xi_{iq}^{ref}(t)-\vartheta\left(\mathbf{g}_{ij}^{ref}(t),\mathbf{g}_{ij}(\mathbf{q}_i,\mathbf{q}_j)\right)$ or

(iii) $\xi_{iqF}^{ref}(t)=\xi_{iq}^{ref}(t)-\left(\vartheta\left(\mathbf{g}_{ij}^{ref}(t),\mathbf{g}_{ij}(\mathbf{q}_i,\mathbf{q}_j)\right)+\mathbf{\Gamma}\mathbf{g}_{ij}(\mathbf{q}_i,\mathbf{q}_j)\right)$

where the reference configuration for position tracking task is selected as $\xi_{iq}^{ref} = \mathbf{C}_i \mathbf{q}_i^{ref} + \mathbf{Q}_i \dot{\mathbf{q}}_i^{ref}$, matrix $\mathbf{\Gamma}$ is the diagonal compliance matrix with elements different from zero in the directions in which compliance is to be maintained, and zero in the directions in which either contact force or trajectory tracking should be maintained. The output of the force tracking controller $\vartheta\left(\mathbf{g}_{ij}^{ref}, \mathbf{g}_{ij}\right)$ enforces sliding mode on the manifold $S_{ijF} = \left\{(\mathbf{q}_i, \dot{\mathbf{q}}_i):\ \mathbf{K}_{Pi}\mathbf{q}_i + \mathbf{K}_{Di}\dot{\mathbf{q}}_i - \xi_F^{ref}(t) = \sigma_{Fij} = \mathbf{0}\right\}$ with the reference configuration defied by $\xi_F^{ref}(t) = \left(\mathbf{g}_{ij}^{ref}(t) + \mathbf{K}_{Pi}\mathbf{q}_j + \mathbf{K}_{Di}\dot{\mathbf{q}}_j\right)$ and is determined as $\vartheta = \vartheta_{eq} - \mathbf{M}_i \mathbf{K}_{Di}^{-1} \mathbf{\Psi}(\sigma_{Fij})$ when systems are in interaction and $\vartheta\left(\mathbf{g}_{ij}^{ref}, \mathbf{g}_{ij}\right) = 0$ when systems are not in interaction.

The control input that enforces sliding mode on the manifold (9) has structure as defined in (5) with appropriate changes of variables and can be written as $\mathbf{F}_i = \mathbf{F}_{eqi} - \mathbf{M}_i \mathbf{Q}_i^{-1} \mathbf{\Psi}(\sigma_{iqF})$. The equivalent control can be expressed in the following form $\mathbf{F}_{eqi} = (\mathbf{F}_{exti} + \mathbf{N}_i) - \left(\mathbf{M}_i \mathbf{Q}_i^{-1}\right)\left(\mathbf{C}_i \dot{\mathbf{q}}_i - \left(\xi_{iq}^{ref} + \dot{\vartheta} - \mathbf{\Gamma}\mathbf{g}_{ij}\right)\right)$. The structure of the control system is depicted in Fig. 2

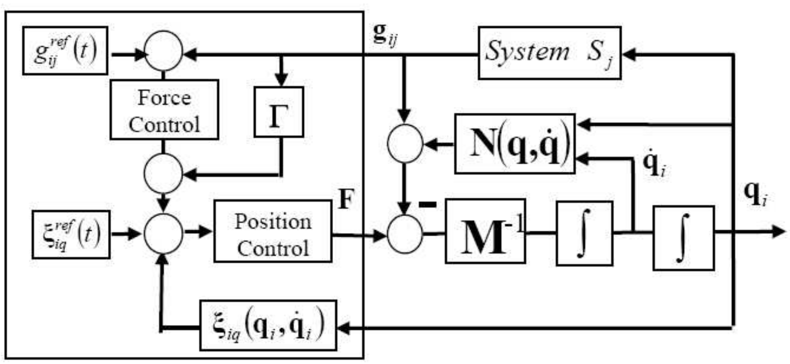


Figure 2. Structure of the control system with modification of the configuration due to the interaction with another system

### *3.2. Examples*

In order to illustrate above results, motion of single DOF system in contact with moving obstacle is simulated

$$\begin{aligned} \dot{x} &= v \\ m\dot{v} &= K_t u - F_{dis} \end{aligned} \tag{10}$$

The parameters of the system (10) were selected as follows: the mass $m = 0.1\,(1 + 0.25\cos(64.28t))$, the gain $K_t = 0.64\,(1 + 0.2\cos(128.28t))$ and the nonlinear disturbance $F_{dis} = 15\,(1 + \cos(4\pi t) + \sin(12\pi t))$ with the stiction force modeled as $\{F_r(t) = K_t u - F_{dis} - 0.5v$ if $\left\{|v| < 1e^{-10} \& |K_t u - F_{dis} - 0.5v| < 15\right\}$ and $F_r(t) = \left(11.25 + 3.75e^{-5|v|} F_0\ \right) sign(v)$ otherwise.$\}$

Controller was selected as $u(k) = sat(u(k-1) + \eta((1-DT)\sigma(k) - \sigma(k-1)))$, with $-100 < u(k) < 100$. Parameters $D = 250$ and $\eta = 120$ and sampling interval $T = 0,0001\,[s]$ are kept constant for all experiments. Sliding mode manifold is selected as $\sigma = C\Delta x + \Delta v - \vartheta_{\Delta F} - \alpha F_e$ with reference trajectory $x^{ref}(t) = 0.35\sin(2\pi t)$ and $C = 100$. The experiments are simulated with $\alpha = 0.25$ and no force limit, and with the reaction force limited by $F_e^{ref}(t) = F_0\ (1 + 0.25\sin(10\pi t))$ with $F_0 = 40$ , $F_0 = 12$ respectively. A moving obstacle with position set as $x_e(t) = 0.15(1 + 0.3\sin(8\pi t))$ is modeled. The interaction force is modeled as $F_e(x) = 1000(x - x_e) + 5(\dot{x} - \dot{x}_e)$ and the interaction force controller is selected as $\vartheta_{\Delta F}(k) = sat(\vartheta_{\Delta F}(k-1) + \eta_F((1-D_F T)\sigma_F(k) - \sigma_F(k-1)))$, with $\sigma_F = F_e - F_e^{ref}$, $D_F = 100$, $\eta_F = 0.1$. Results are depicted in Fig. 3. The modification of the system behavior due to the interaction with environment shows capability of the proposed control structure to keep stability of the system under different control tasks and smooth transition between them.

Despite large changes of the system parameters and disturbance, the motion of the system is tracking the reference and modulation of the system motion is fully confirmed. In contact with environment the controlled system reacts as a virtual impedance creating the force $F_e$ due to the contact.

Experimental verification of the above results is performed on the nonlinear system which consists of the Piezomechanik's PSt150/5/60 stack actuator ($x_{\max} = 60\mu$m, $F_{\max} = 800$N, $v_{\max} = 150$Volt) connected to SVR150/3 low-voltage, low-power amplifier. Force measurement is accomplished by a load cell placed against the actuator. The entire setup is connected to dSPACE® DS1103module hosted in a PC. In experiments the parameters of the sliding mode manifold is $\sigma = C\Delta x + \Delta\dot{x} - \vartheta_{\Delta F}$ and controller parameters $D = 2500$ ,$C = 800$. and the interaction force controller parameters are selected as $D_F = 1900$, $\eta_F = 0.25$ and $\sigma_F = F_e - F_r$. The peculiarity of the PZT actuator is related to the hysteresis characteristics of the system gain [22, 29] what normally requires a nonlinear compensation. Experiments depict the trajectory tracking and force control while in contact with environment. Structure of the experimental setup is depicted in Fig. 4. and experimental results in Fig. 5. The position reference is $x_r = 20 + \cos(0.5t)$ micrometers and the force reference is either 11,5 or 21,5 N. The transitions from position tracking to force tracking and vice versa are clearly shown in figures. In all situations systems behaves as predicted.

### *3.3. Extension to the General Systems in Interactions*

For motion control systems of particular interest is to maintain desired functional relation between subsystems (for example bilateral control or cooperating robots etc.) by acting on all of the subsystems. Assume a set of $n$ single DOF motion systems each described by $S_i : m_i(q_i)\ddot{q}_i + n_i(q_i, \dot{q}_i) = f_i - g_{iext} \quad i = 1, 2, ..., n$ interconnected in such a way so the motion of the overall system can be described by the following model

$$S : \mathbf{M(q)\ddot{q}} + \mathbf{N}(\mathbf{q}, \mathbf{\dot{q}}) = \mathbf{BF} - \mathbf{Bg}_{ij} \tag{11}$$

where $\mathbf{q} \in \Re^n, rank\mathbf{B} = rank\mathbf{M} = n$, vectors $\mathbf{N}$ and $\mathbf{Bg}_{ij}$ satisfy matching conditions. Assume also that the required role $\mathbf{\Phi} \in \Re^m$ may be represented as a set of smooth linearly independent functions $\zeta_1(\mathbf{q}), \zeta_2(\mathbf{q}), ..., \zeta_n(\mathbf{q})$. . Consider a problem of designing control for system $S$ such that the role vector $\mathbf{\Phi}^T = \left[\begin{array}{ccc} \zeta_1(\mathbf{q}) & ... & \zeta_n(\mathbf{q}) \end{array}\right]$ tracks its smooth reference $\mathbf{\Phi}^{ref}$. By differentiating the first time derivative of the role vector $\mathbf{\dot{\Phi}} = \left[\frac{\partial \Phi}{\partial \mathbf{q}}\right]\mathbf{\dot{q}} = \mathbf{J}_\Phi \mathbf{\dot{q}}$ one can

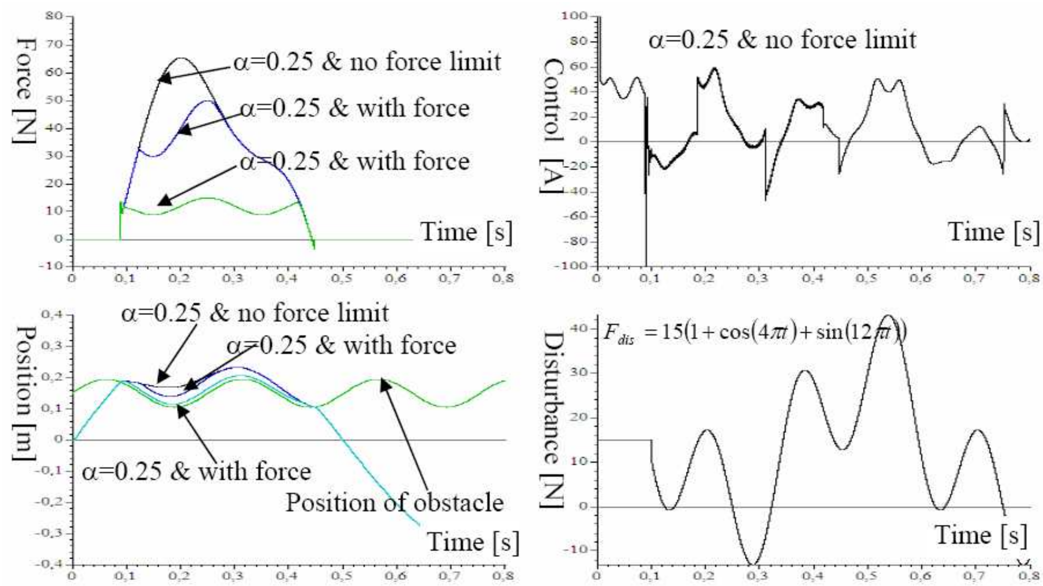


Figure 3. The trajectory tracking and force control in contact with unknown obstacle

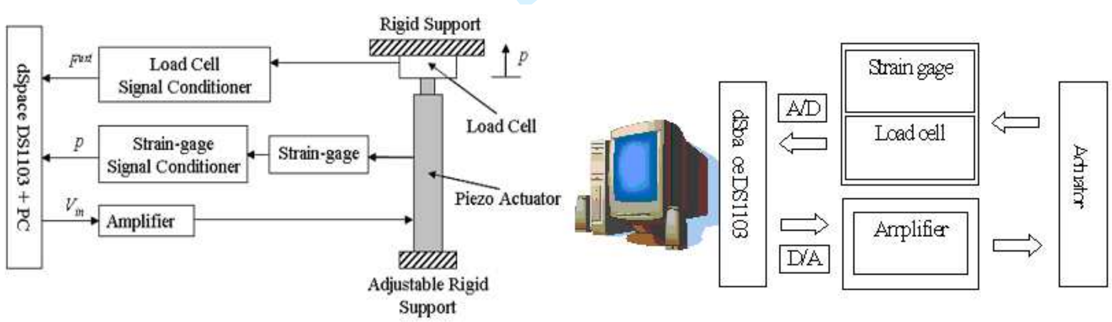


Figure 4. The structure of the experimental setup for PZT actuator control

determine the change of the role vector as

$$\ddot{\mathbf{\Phi}} = \mathbf{J}_\Phi \mathbf{M}^{-1}\mathbf{B}\mathbf{F} - \mathbf{J}_\Phi \mathbf{M}^{-1}\left(\mathbf{B}\mathbf{g}_{ij} + \mathbf{N}\right) + \dot{\mathbf{J}}_\Phi \dot{\mathbf{q}} \qquad (12)$$

A more compact form can be derived by denoting the control vector as $\mathbf{F}_\Phi = \mathbf{B}_\Phi \mathbf{F} = \mathbf{J}_\Phi \mathbf{M}^{-1}\mathbf{B}$ and the disturbance vector as $\mathbf{d}_\Phi = \mathbf{J}_\Phi \mathbf{M}^{-1}\left(\mathbf{B}\mathbf{g}_{ij} + \mathbf{N}\right) + \dot{\mathbf{J}}_\Phi \dot{\mathbf{q}}$.

$$\ddot{\mathbf{\Phi}} = \mathbf{F}_\Phi - \mathbf{d}_\Phi \qquad (13)$$

From (13) one can select control $\mathbf{F}_\Phi = \mathbf{F}^*_\Phi$ sach that the role vector tracks its reference and than,

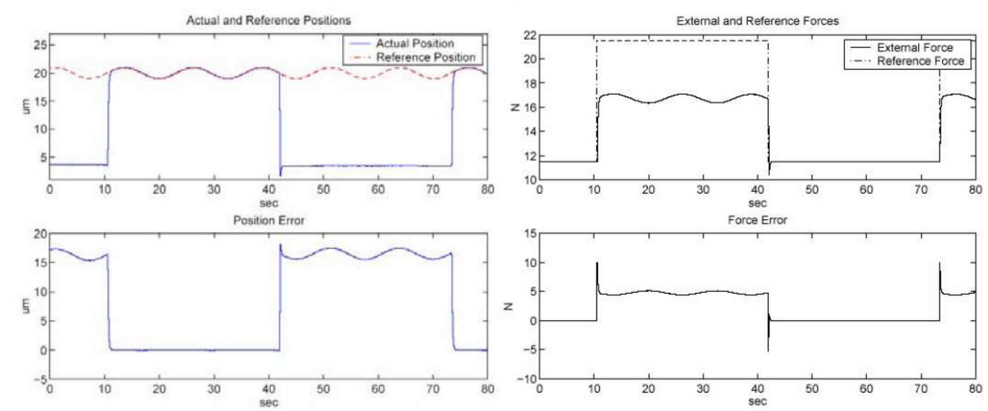


Figure 5. The experimental behavior of the PZT actuator with SMC control



if $(\mathbf{B}_\Phi)^{-1}$ exists determine the original control input as $\mathbf{F} = (\mathbf{B}_\Phi)^{-1}\mathbf{F}^*_\Phi = \left(\mathbf{J}_\Phi\mathbf{M}^{-1}\mathbf{B}\right)^{-1}\mathbf{F}^*_\Phi$. In the general case one has to ensure the existence of the inverse for matrix $\mathbf{B}_\Phi$ by properly selecting matrix $\mathbf{J}_\Phi$. Similar situation had been examined in so-called “function control” framework [30,31,32], There it was assumed that $\mathbf{B} = \mathbf{I}$ and $\mathbf{J}_\Phi$ was selected as a Hadamard matrix along with the compensation of the disturbances $\mathbf{g}_{ij}$ and $\mathbf{N}$ on the plant level. This way control design is greatly simplified and one deals with simple double integrator plants without disturbance.

The same problem can be treated in the SMC framework by selecting the sliding mode manifold $\sigma_\Phi \in \Re^n$ with the system configuration $\xi_\Phi\left(\mathbf{\Phi}, \dot{\mathbf{\Phi}}\right)$ and the reference configuration $\xi^{ref}_\Phi(t)$. Assume that the configuration is expressed as a linear combination of the function vector and its derivative $\xi\left(\mathbf{\Phi}, \dot{\mathbf{\Phi}}\right) = \mathbf{C}_\Phi\mathbf{\Phi} + \mathbf{Q}_\Phi\dot{\mathbf{\Phi}}$. Then the sliding mode manifold becomes

$$S_\Phi = \left\{(\mathbf{q}, \dot{\mathbf{q}}) :\ \mathbf{C}_\Phi\mathbf{\Phi} + \mathbf{Q}_\Phi\dot{\mathbf{\Phi}} - \xi^{ref}_\Phi(t) = \sigma_\Phi = 0\right\} : \tag{14}$$

By applying procedure discussed in Section 2.2, and assuming that $\mathbf{Q}^{-1}_\Phi$ exists, the control input enforcing sliding mode on the manifold (14) can be determined as

$$\begin{array}{l} \mathbf{F}^*_\Phi = \mathbf{F}_{\Phi eq} - \mathbf{Q}^{-1}_\Phi\mathbf{\Psi}(\sigma_\Phi) \\ \mathbf{F}_{\Phi eq} = \mathbf{d}_\Phi - \mathbf{Q}^{-1}_\Phi\left(\mathbf{C}_\Phi\dot{\mathbf{\Phi}} - \dot{\xi}^{ref}_\Phi(t)\right) \end{array} \tag{15}$$

Inverse transformation $\mathbf{F} = \left(\mathbf{J}_\Phi\mathbf{M}^{-1}\mathbf{B}\right)^{-1}\mathbf{F}^*_\Phi$ gives control in the original state space.

### *3.4. Bilateral control*

Behavior of an ideal bilateral system requires the tracking of the master position by the slave and the forces on master and slave side to be equal but with opposite signs [30, 33] (so-called transparency requirements). Below the application of the method discussed in Section 3.3 on bilateral system will be shown. Assume single DOF mechanical systems $S_i : m_i\ddot{x}_i + n_i(x_i, \dot{x}_i) = f_i - g_{iext} \quad i = m, s$ playing role of the master system (index $m$) and the slave system (index $s$). The interaction force on slave side is $g_{iext}$. Let $\alpha$ be the position scaling coefficient and $\beta$ the force scaling coefficient. The force sensed by a human operator is $F_m = Z_m x_m = C_{\mathrm{m}} x_m + Q_m \dot{x}_m$ where $Z_m$ stands for the human operator impedance. The coefficients $Q_m$ and $C_m$ can be selected so that impedance perceived by the human operator is *shaped* in order to give a feeling of a virtual tool in the operator's hand. Let $\mathbf{\Phi}_B^T = \begin{bmatrix} \zeta_x & \zeta_F \end{bmatrix}$ be the function vector with $\zeta_x$ standing for position tracking error and $\zeta_F$ standing for force tracking error. The bilateral system operational conditions can be met if the sliding mode motion is enforced on manifold $S_B = \left\{(x_m,x_s) : \ \xi_B(\mathbf{\Phi}_B) - \xi_B^{ref}(t) = \sigma_B = 0\right\}$. The components of the function vector can be defined as $\zeta_x = x_m - \alpha x_s$ and $\zeta_F = Z_m x_m + \beta g_{sext}$ and the corresponding sliding mode manifolds can be defined in the following way

$$\begin{array}{l} S_x = \left\{(x_m, x_s) : Q_x\dot{\xi}_x + C_x\xi_x = \sigma_x = 0\right\} \\ S_F = \{(x_m, x_s) : Z_m x_m + \beta g_{sext} = C_{\mathrm{m}}\varepsilon_F + Q_m\dot{\varepsilon}_F - (C_{\mathrm{m}} x_s + Q_m \dot{x}_s - \beta g_{sext}) = \sigma_F = 0\} \end{array} \tag{16}$$

where $\varepsilon_F = x_m + x_s$ stands for the control error. Taking the controls $f_x = Q_x(f_m/m_m - \alpha f_s/m_s)_s$, $f_F = Q_m(f_m/m_m + \beta f_s/m_s)$ and the disturbances $d_x = (n_m/m_m - \alpha(n_s + g_{sext})/m_s) + C_x\dot{\varepsilon}_x$, $d_F = (n_m/m_m + (n_s + g_{sext})/m_s) + ((Q_m\ddot{x}_s + C_m\dot{x}_s) - \dot{g}_{sext})$ the projection of the master and the slave systems on the sliding mode manifolds $S_x$ and $S_F$ can be described by

$$\begin{array}{l} \dot{\sigma}_x = f_x - d_x \\ \dot{\sigma}_F = f_F - d_F \end{array} \tag{17}$$

Equations (17) describe two simple first order systems. The selection of controls $f_x = f_x^*$ and $f_F = f_F^*$ that enforce the sliding mode on each of the sliding mode manifolds $S_x$ and $S_F$ and thus on the intersection $S_B = S_x \cap S_F$ can follow the same steps as discussed in Section 2.4 The controller described by (5) is a suitable solution. Stability of solution $\sigma_x = 0$ and $\sigma_F = 0$ will guaranty the fulfillment of the requirements for bilateral system. The control inputs for master and slave systems can then be determined from

$$\begin{array}{l} f_m = \frac{m_m}{\alpha+\beta}\left(\frac{\beta}{Q_x} f_x^* + \frac{\alpha}{Q_m} f_F^*\right) \\ f_s = \frac{m_s}{\alpha+\beta}\left(\frac{1}{Q_m} f_F^* - \frac{1}{Q_x} f_x^*\right) \end{array} \tag{18}$$

For verification of the proposed approach the experimental system consisting of: two 400 W 3-phase Maxon brushless motors ($J$=831 g/cm$^2$,$K_T$= 85 mNm/A, Maxon 4-Q-EC servo amplifier DES 70/10) in current regulation mode; 10.000 ppr encoders; a dSPACE® 1103 real-time controller with 100 $\mu$sec measurement sampling rate and a 1 msec control output sampling rate is used. Structure of the overall system is depicted in Fig. 6. One of the motors is used by the human operator – master system, and the other one is used as a slave system. In order to make contact with different environment the obstacles are put on the right side - hard – steal rode, and on the left side – a sponge. This way experiments related to contact with very different environment are available.

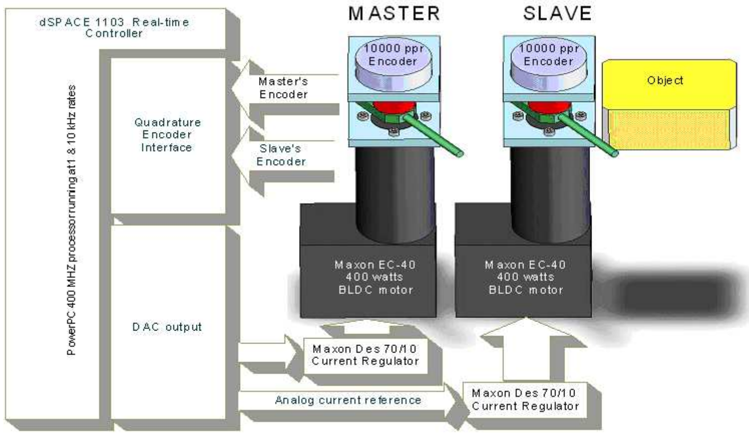


Figure 6. Experimental system for bilateral operation

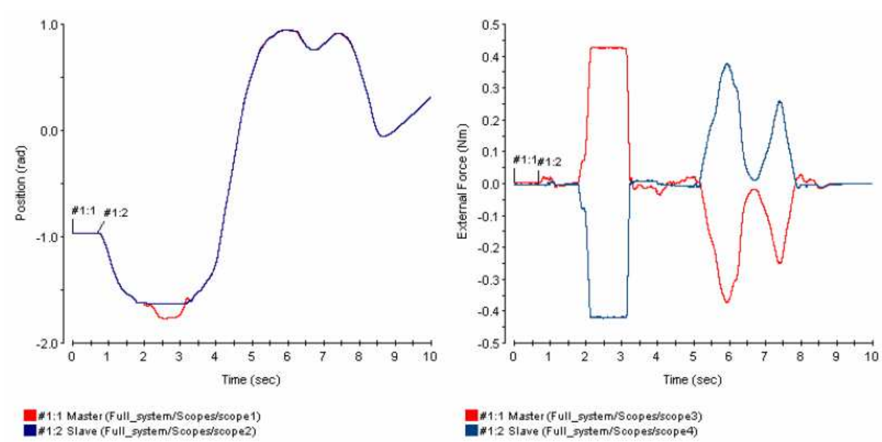


Figure 7. Transients in bilateral control system (a) position response of master and slave sides, (b) forces of master and slave side

Results depicted in Fig. 7., show that the system controller is capable of handling both, contact with soft and contact with hard environments on the slave side. The proposed structure guaranties the functional relation between master and slave system (equality of positions and forces) but it does not influence amplitude for any of these variables. The additional loops (possibly in SMC framework as discussed in this section) are needed for the slave side force limit as discussed in [22].

## 4. CONCLUSIONS

In this paper application of the sliding mode control framework in motion control systems is discussed. The approach is applicable for systems with and without contact with environment that leads to unified formulation of the control tasks. It has been shown that the same approach can be used in controlling systems in interaction and establishing desired functional relation between systems and allowing application of the same framework to bilateral and "function control" systems. All tasks are basically realized by modulating the reference system configuration in such a way that interaction (virtual or real) is maintained. This allows using the same structure of the controller for all tasks. The structure of the controller is selected to fulfill Lyapunov stability criteria and enforce the sliding mode motion on the sliding mode manifold. The realization of the sliding mode control in the discrete-time framework is discussed and it is shown that under proposed control the motion of the system remains within boundary layer of the $o(T^2)$ order. Experiments on high precision PZT based system with a nonlinear gain and on the bilateral control system confirm all theoretical predictions.



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