Effect of Thin-Walled Workpiece Dynamics on Milling Stability: A 3D Spectral Element Method-Based Study

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Abstract

Chatter stability is a major challenge in milling thin-walled components due to their evolving dynamics during machining. In this study, we predicted the workpiece dynamics along the tool path using a three-dimensional spectral element method (3D-SEM) based on a spectral-Chebyshev formulation. The workpiece was discretized into coarse brick elements, the elemental mass and stiffness matrices were computed spectrally, and at each material-removal step an eigenvalue problem was solved to obtain the instantaneous natural frequencies and mode shapes. The corresponding frequency-response functions (FRFs) at the tool–workpiece interface were determined, capturing the dynamic changes due to tool position and progressive material removal. Using these FRFs, we constructed 3D stability-lobe diagrams for each milling pass and identified chatter-free cutting parameters, which were validated by time-domain simulations and showed excellent agreement. The results demonstrate that the 3D-SEM accurately and efficiently captures the variations in workpiece dynamics and provides a robust tool for reliable chatter-stability prediction in the milling of thin-walled structures.

Keywords: Thin-walled components, Workpiece dynamics, Spectral element method, Chatter stability

1. Introduction

Thin-walled structures are key components in many industries, including aerospace, aviation, energy, and others. Due to their high flexibility, they are prone to chatter vibrations during milling operations. These vibrations are strongly influenced by the dynamic behavior of the thin-walled workpiece along the tool path. As extensively established in previous studies, chatter in milling can be mitigated by employing numerical process models and stability lobe diagrams to identify stable cutting conditions. However, the main difficulty in utilizing those models or diagrams is the varying thin-walled workpiece dynamics along the tool path. As material is removed during the milling process, the dynamics, specifically the frequency response functions (FRFs), of the thin-walled workpiece continuously change, leading to corresponding variations in the stable cutting parameters. Therefore, capturing and accounting for the evolving workpiece dynamics is crucial to achieving both geometric accuracy and high productivity.

A number of approaches have been proposed to address this challenge. For instance, Dang et al. [1], proposed an in-process workpiece dynamics prediction approach based on reduction and structural modification methods. They generated a finite-element (FE) model of the workpiece once and then updated it continuously along the tool path, verifying their approach through impact and chatter tests. Similarly, Hamann and Eberhard [2] implemented a parametric model order reduction technique to the FE model to predict varying workpiece dynamics depending on tool position, and later incorporated these data in stability analysis of thin-walled milling. On the other hand, Li et al. [3] introduced a systematic updating methodology combining FE model reduction, free-interface, and structural-modification techniques to accelerate the computations, and validated it through experimental chatter tests. Tuysuz and Altintas [4] proposed a reduced order substructuring method for thin-walled-workpiece frequency response functions (FRFs) during milling operations, and later presented a time-domain dynamics update model for the same purpose [5]. On the other hand, the thin-walled dynamics along the tool path were analytically updated using structural modification technique in the study completed by Alan et al. [6]. Similarly, Budak et al. [7] employed this method for the thin-walled blade geometry, and then they conducted chatter stability analysis using predicted frequency response functions of this geometry.

As highlighted in the literature, varying thin-walled workpiece dynamics poses significant challenges in milling operations. As an alternative to the methods implemented in the literature, this study uses three-dimensional (3D) spectral element method (SEM) to predict the thin-walled workpiece dynamics considering both material removal and tool position along the tool path. Based on these predictions, 3D stability lobe diagrams, accommodating evolving dynamics of thin-walled workpieces along the tool path, are generated for chatter stability predictions. For verification of chatter stability predictions, time domain simulations were performed, and stable cutting parameters were identified for each material removal pass.

2. Modeling of Thin-Walled Workpiece Dynamics

To achieve an accurate and computationally efficient evaluation of the varying dynamics of thin-walled workpieces, the 3D SEM, which is developed by the authors in [8], is utilized. In this method, first the thin-walled workpiece is discretized into coarse brick elements using a coarse meshing algorithm, as shown in Fig. 1.

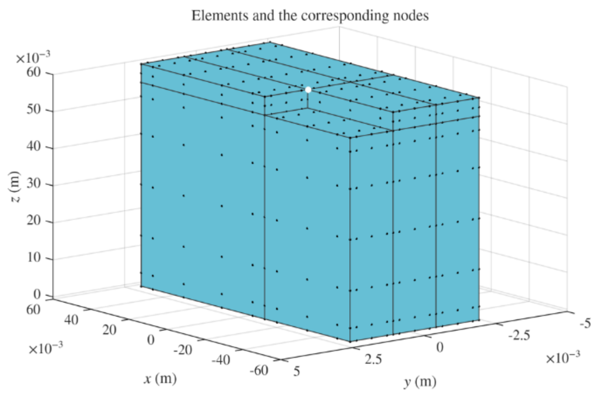

Fig. 1. Discretization of the thin-walled workpiece.

Second, the mass and stiffness matrices for each element are computed via the 3D spectral Chebyshev method [9]. Subsequently, the element matrices are systematically assembled to construct global mass, $\mathbf{M_{global}}$, and stiffness, $\mathbf{K_{global}}$, matrices for the entire thin-walled workpiece. In the final step, the following eigenvalue problem is solved to obtain the natural frequencies and corresponding mode shapes of the thin-walled workpiece.

$$(\mathbf{K}^*_{\mathbf{global}} - \omega^2 \mathbf{M}^*_{\mathbf{global}})\boldsymbol{\phi} e^{i\omega t} = 0 \quad (1)$$

Here, $\mathbf{M}^*_{\mathbf{global}}$ and $\mathbf{K}^*_{\mathbf{global}}$ are the global system matrices including boundary conditions, $\boldsymbol{\phi}'$s are the mode shapes, and ω's are the natural frequencies. Compared to conventional finite element analysis (FEA), the SEM provides high accuracy with significantly fewer elements by employing higher-order polynomials, thereby reducing the computational cost while maintaining precision. Furthermore, for chatter stability analysis, the frequency response functions (FRFs) of the thin-walled workpiece in both the *x* and *y*-directions along the tool path are required. Accordingly, they are computed as follows:

$$\boldsymbol{\alpha}(\omega) = \sum_{n=1}^{N_{modes}} \frac{\boldsymbol{\phi}_\mathbf{n} \boldsymbol{\phi}_\mathbf{n}^\mathsf{T}}{(\omega_n^2 - \omega^2) + i(2\zeta\omega\omega_n)} \quad (2)$$

where $\boldsymbol{\alpha}(\omega)$ is the FRF matrix, n is the mode number, N_{modes} is the total number of modes used in the computation, $\boldsymbol{\phi}_\mathbf{n}$ is the n^{th} mass-normalized mode shape, ω_n is the corresponding natural frequency, ω is the excitations frequency, and ζ is the damping ratio.

3. Stability Analysis

Stability analysis for milling of a thin-walled workpiece was performed using stability lobe diagrams generated with a Fourier series-based approach [10]. The stability lobes were computed for each material removal step along the tool path. In these computations, machine-spindle-holder-tool assembly was assumed to be rigid, while the FRFs of the thin-walled workpiece in both *x* and *y*-directions were predicted along the tool path with 5 mm material removal steps on the top edge of the thin-walled workpiece (e.g., white point shown in Fig. 1.) using 3D SEM.

For peripheral down milling operation: the thin-walled workpiece material was 6013-T6 aluminum, the cutting tool was 12 mm diameter square uncoated carbide end mill with two flutes, and the cutting force coefficients for tool-aluminum workpiece pair were $K_t = 677.48$ N/mm^2 and $K_n = 0.1844$, which were determined experimentally, as described in detail in [11].

4. Results and Discussion

Fig. 2 illustrates the prediction procedure for the thin-walled workpiece along the tool path. The initial workpiece, with the dimensions of 110×4.5×60 mm in *x*, *y* and *z*-directions respectively (see Fig. 2a), was discretized into 6 bricks elements, and then milled until the completion of the first pass, as indicated in Fig. 2b. During this milling process, the dept of cut was taken to be 1 mm as an initial guess, and the FRFs of the thin-walled workpiece for both *x* and *y*-directions were computed step-by-step with 5 mm increments along *x*-direction (i.e. along the tool path). The FRF computation point is located at the top edge of tool-workpiece interface. As such, Figs. 2c and 2d illustrate 3D waterfall plots for FRFs of the thin-walled workpiece in *x* and *y*-directions as a function of the tool position. In total, 23 FRFs were computed for each direction. As seen in Figs. 2c and 2d, the workpiece dynamics vary significantly with both tool position and material removal, and these variations are successfully and efficiently captured by the 3D SEM. It is important to note here that the computation times of the 3D SEM in eigenvalue and harmonic analyses are remarkably lower, by a fraction of around 24 and 11 respectively, compared to the those for the FEM.

To further investigate the effect of varying thin-walled workpiece dynamics on chatter stability, a 3D stability lobe diagram was generated using the predicted FRFs for the first milling pass (pass-1), as shown in Fig. 3. In this plot, the *x*-direction represents the tool position, corresponding FRFs plots in Fig. 2, while *y*- and *z*-directions denote spindle speed and axial depth of cut, respectively.

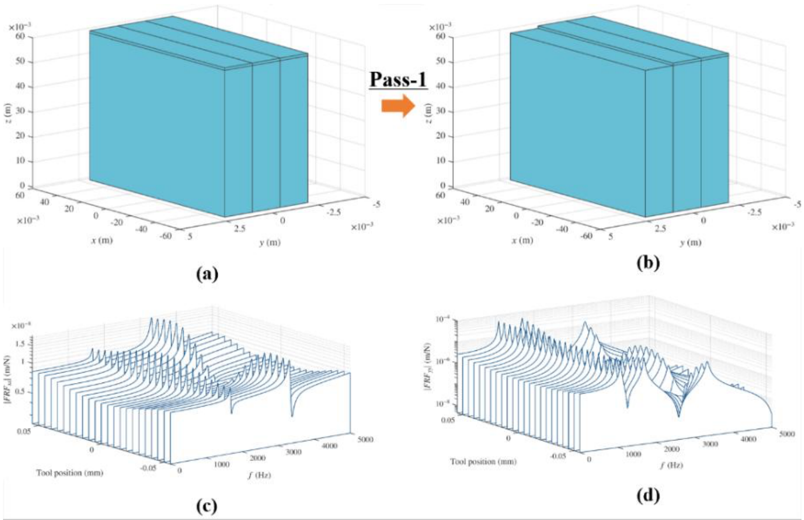

Fig. 2. Prediction procedure of thin-walled workpiece dynamics along the tool path: (a) Discretizaiton of thin-walled workpiece, (b) Discretizaiton of thin-walled workpiece after first material removal pass, (c) FRFs for *x*-direction, (d) FRFs for *y*-direction.

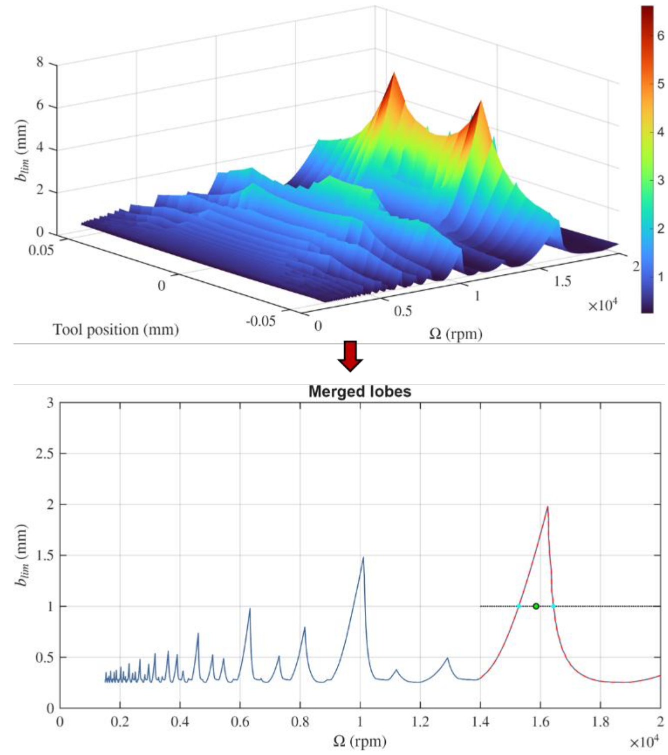

Fig. 3. Stability lobe diagrams along the tool path for pass-1.

Correspondingly, the 3D stability lobe diagram consists of 23 2D stability lobe diagrams, each representing a particular material removal step. To facilitate the selection of stable cutting parameters (i.e., stable spindle speed–axial depth of cut combinations), these 23 2D stability lobe diagrams were merged into a single 2D stability lobe diagram by taking the minimum axial depth of cut for each spindle speed, as illustrated in the bottom plot of Fig. 3. In this plot, a horizontal line at axial depth of 1 mm was drawn to determine whether stable spindle speeds exist at that depth. Stable spindle speeds were indeed observed around 16000 rpm. To select an appropriate value from this range, the spindle speed points (i.e., cyan points in the bottom plot of Fig. 3) at which the line intersects the stability boundary were identified, and their average was taken. Accordingly, for the first pass, a stable milling operation can be achieved at the cutting parameters Ω=15854 rpm and $b_{lim} = 1$ mm (i.e., green point in the bottom plot of Fig. 3). To verify this stable point, a time domain simulation [12] was conducted, and the resulting cutting forces and workpiece vibrations for this milling operation are shown in Fig. 4. The results confirm that the milling operation was stable as the cutting force signals exhibit no signs of chatter-induced fluctuations.

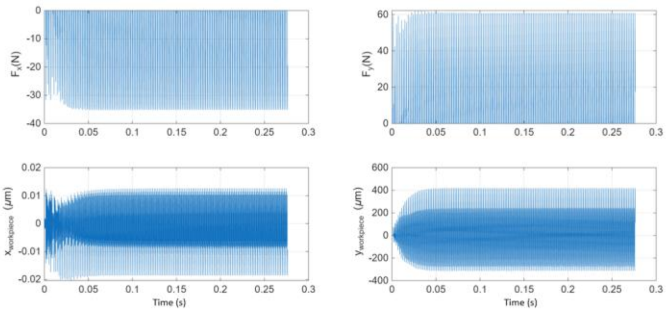

Fig. 4. Time domain simulation results for pass-1.

To further investigate the chatter stability of thin-wall milling, nine more passes were performed, and the final geometry of the thin-walled workpiece is shown in Fig. 5. The stable cutting parameters for each pass, presented in Fig. 6, were determined using the merged stability-lobe diagrams generated for each pass in the same manner as for the first pass. As evident from Fig. 6, the stable cutting parameters vary in accordance with the changes in the thin-walled workpiece dynamics. It is important to note that all of these stable cutting parameters were verified through time-domain simulations.

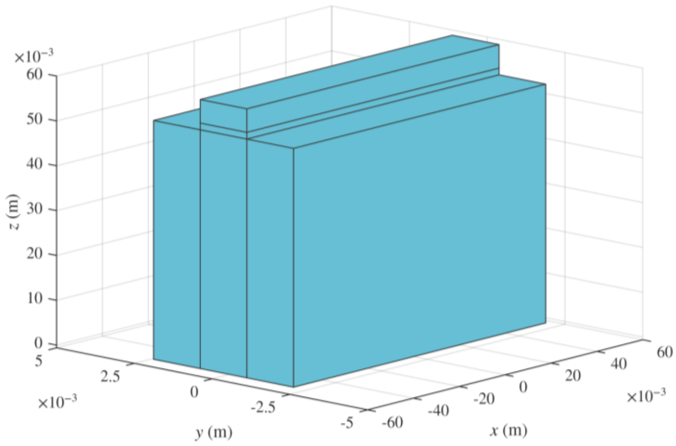

Fig. 5. Thin-walled workpiece geometry after ten passes.

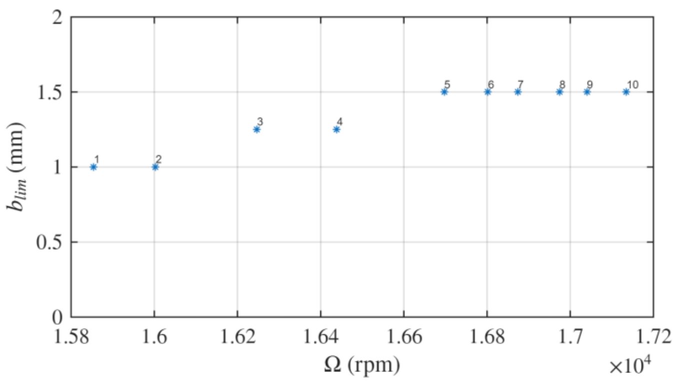

Fig. 6. Stable cutting parameters for passes-1 to 10.

5. Conclusions

This paper presented a 3D spectral element-based study to investigate influence of the thin-walled workpiece dynamics on milling chatter stability. The frequency-response functions (FRFs) of the workpiece were accurately and efficiently predicted using the 3D spectral element method (SEM) in a step-by-step manner along the tool path, yielding 3D FRF data in both the *x*- and *y*-directions as functions of tool position and material removal. These data were then used to generate 3D stability-lobe diagrams for each milling pass, from which stable cutting parameters were determined. The predicted parameters were verified through time-domain simulations, which showed excellent agreement with the model predictions.

The study demonstrated that the thin-walled workpiece dynamics varies significantly with both tool position and material removal. The 3D SEM approach provides an accurate and computationally efficient means of capturing these variations. Its application to chatter stability predictions enables fast and reliable identification of stable cutting parameters, thereby contributing to improved productivity in milling operations.

Future work will focus on extending the proposed framework to the chatter-stability analysis of complex geometries such as rotor blades and airframe ribs, since the present study is limited to simple prismatic geometries. To achieve this, the 3D-SEM will be enhanced by incorporating a master-element strategy to enable the discretization of complex shapes. In addition, model-reduction techniques and structural-modification methods will be integrated into the SEM to further improve computational efficiency. Finally, efforts will be directed toward developing an automated algorithm to efficiently determine stable cutting parameters based on the predicted dynamics of thin-walled workpieces.

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