R&D TAX INCENTIVES: A REAPPRAISAL

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Abstract

This paper examines R&D tax incentives in oligopolistic markets. We characterize the conditions under which tax incentives reach the socially desirable level of firm-financed R&D spending. The outcome of the market depends not only on the level of technological spillover in the industry but also on the degree of strategic interaction between the firms. One major result emerges from the model: The socially desirable level of R&D investment is not necessarily reached by subsidizing R&D. When the sector spillover is sufficiently low, the government might want to tax R&D investments, and this result does not necessarily arise because firms are overinvesting in R&D. There are also cases in which an R&D tax is desirable even though firms are underinvesting in R&D compared with the first-best optimum. In practice, this theoretical finding calls for a lower sales tax combined with an R&D subsidy in oligopolistic industries with high technological spillovers, and a lower sales tax combined with an R&D tax in oligopolistic industries with low technological spillovers.

Keywords: oligopoly, public policy, R&D tax incentive, spillover, strategic interaction

JEL Classification: H25; L13; O31

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1 Introduction

Despite the debate in the empirical literature over the effectiveness of R&D tax incentives, public policy regarding R&D investment has not received much attention from applied microeconomists. This paper, among a few others, is an attempt to fill this gap in the literature. We set up a fairly standard model of R&D investment in the presence of strategic interaction and technological spillovers among firms and obtain a folk theorem. We show that the widely held idea that subsidizing R&D is beneficial may not in fact hold.

The usual story in support of subsidies is that R&D investment has some of the characteristics of a public good. It is nonrivalrous and *partially* nonexclusive in the Samuelsonian sense, and hence, there are positive externalities that require government subsidization to achieve Pareto optimality. In fact, many countries have chosen to subsidize R&D investment over the past twenty to thirty years.[1] It is certain that R&D has something of the nature of a public good, but this is not its only property. Markets are usually oligopolistic in industries that engage in large amounts of R&D investment. In such industries, the strategic interaction between firms also plays a crucial role in firms' decisions about their output and R&D investment levels.

In analyzing R&D, one needs to take into account not only the spillover effects that give R&D the characteristics of a public good but also the strategic interaction effects that come from the oligopolistic nature of the industry. As is well known by now, technological spillovers between firms leads them to free-ride on each other when making their R&D investment decisions. This would seem to lead firms to underinvest in R&D. However, at the same time, the oligopolistic nature of the industry gives firms the power to strategically act against each other. It is well known in game theory that, in a strategic substitutes (strategic complements) case, the strategic interaction between the players can lead them to overinvest (underinvest) in the strategic variable. Therefore, in an oligopolistic industry, the strategic interaction between the firms, *in and of itself,* leads firms to overinvest in R&D when the R&D investments of the firms are strategic substitutes. Hence, it is *ex ante* less clear whether oligopolistic markets would result in too little R&D investment, and thus, the very widely

[1] An increasing number of countries are offering special fiscal incentives to businesses to induce spending on R&D and increase their level of innovation. A survey of the European Commission (1995) on state aid reports that its members spent over \$1 billion per annum on R&D tax incentives during the early 1990's. In Canada, because of R&D tax incentives, the after-tax cost of R&D expenditure ranges between 35 and 50 cents per dollar spent depending on the type of firm and the province in which the R&D activity is conducted. In many other major countries, governments are trying to stimulate and encourage the creation of new technical knowledge. See Hall (1993, 1995) for a review of the history, regulations, and methodology related to tax credits in the US and other OECD countries.

applied R&D subsidies may not be justified in those cases. Indeed, this paper shows that the socially desirable level of firm-financed R&D investment may not be reached by imposing a subsidy. To the contrary, it is in society's interest to tax R&D investment when the technological spillovers are sufficiently low. Second, and more importantly, the paper shows that taxation does not necessarily result in a lower level of R&D investment in equilibrium.

More specifically, the outcome of the market depends not only on the level of technological spillover in the industry but also on the degree of strategic interaction between the firms. The degree of strategic interaction in the model presented here is simply the number of firms in the industry, which in turn determines the firms' market power. Thus, whether the government should subsidize R&D investments must at least be contingent on the technological spillovers and the number of firms in the industry. In particular, the model shows that it is desirable to subsidize R&D investments only when the technological spillover exceeds a threshold which is dependent on the number of firms in the industry (which is formally shown in Proposition 1). Otherwise, it is in society's best interest to tax R&D investments.

The less intuitive result of the paper is that the government might want to tax R&D investments even if the firms are underinvesting in R&D. That is, the first-best level of R&D investment can be reached by imposing a tax on R&D. This tax decreases R&D investment for every given level of output for sure, but this does not necessarily mean that the equilibrium level of R&D will be lower than the equilibrium level without the tax. This seemingly contradictory result is resolved when one focuses on research intensity (R&D investment per unit of output) rather than on the equilibrium level of R&D, since, as noted above, oligopolistic power distorts not only levels of R&D but also levels of output. In particular, one should note that firms tend to produce too little output in an oligopolistic industry. A tax always discourages research intensity regardless of whether there is equilibrium overinvestment or underinvestment in R&D *ex ante*. In a theoretical world without taxes for revenue purposes but with corrective taxes only, optimality requires the subsidization of output (which is formally shown in Proposition 2). Of course, in reality, governments do not subsidize output to correct the output problem in oligopolistic industries.[2] Indeed, in a realistic general equilibrium, the result of subsidization of output should be interpreted as a lower level of sales tax on output since governments usually collect sales tax for revenue purposes. Combining the corrective taxes and subsidies on R&D with this intuition, this paper concludes that, given the number of firms in an industry,

- when technological spillover is sufficiently high, the government should impose a lower

[2] Note also that Appendix A.8 derives the second-best outcome in which we show that there might be an R&D tax even in the absence of a corrective tax (or subsidy) on output.

sales tax combined with an R&D subsidy;

- when technological spillover is sufficiently low, the government should impose a lower sales tax combined with an R&D tax.

The analysis below focuses on the n firm case of the canonical R&D investment model first introduced by D'Aspremont and Jacquemin (1988). The model derives only the symmetric equilibrium, assumes quantity competition à la Cournot, and does not consider the possibility of R&D cooperation. I do not consider a location choice either. Some of these variations, which have been studied in depth in the literature (for example, see Atallah (2005, forthcoming) for a recent analysis of asymmetric spillovers with R&D cooperation, and Devereux and Griffith (1998, 2003) for the analysis of the interaction of location of production and tax policy), may alter the results here, and the expected changes in such cases are well known. However, the model below is sufficient to make our point that there might be an R&D tax in the optimum.

This paper is also closely related to Leahy and Neary (1997), which focuses on the separate influences of strategic behavior and R&D cooperation in both Cournot and Bertrand games. Leahy and Neary compare industry profits in different equilibria and describe the optimal government intervention but they hardly discuss the possibility of an R&D tax. We believe that there are industries with small enough spillovers to call for a tax. Moreover, we show that the tax result becomes more likely when the industry is highly concentrated. Some sectors have both low spillovers and high concentration.

The paper is organized as follows. Section 2 defines the model. Section 3 characterizes the R&D investment for a regulated oligopoly. Section 4 discusses the welfare issues in the decentralized equilibrium. The direct optimum is also given here. Section 5 concludes and summarizes the paper. An appendix contains some detailed derivations and proofs.

2 The Model

We set up a three-stage game theoretic model in which we have a representative consumer, n firms, and a social utility-maximizing government. We assume an oligopolistic market in which firms engage in output competition à la Cournot. Therefore, we immediately expect there will be underproduction due to the market power.

In the first stage of the game, the government commits to its optimal tax/subsidy policy on output and R&D investment. In the second stage, firms decide how much to spend on R&D.

In the final stage, they decide how much to sell in an oligopolistic market. We allow for the possibility of imperfect appropriability in the form of inter-firm spillovers.

2.1 Consumers

There is a representative consumer whose preferences are given by a money metric utility function which is additively separable and linear in the numeraire good m:

$$U(Q, m) = u(Q) + m \quad , \tag{1}$$

where Q is the total demand of the representative consumer for output. With this functional form, there are no income effects, and therefore, the discussion boils down to a partial equilibrium analysis. Suppose the utility from the total output is quadratic and takes the following form:

$$u(Q) = aQ - b\frac{Q^2}{2} \qquad a, b > 0 \quad . \tag{2}$$

From the maximization problem of the representative consumer we get the linear inverse demand function:

$$p(Q) = a - bQ \qquad \frac{a}{b} \geq Q \quad , \tag{3}$$

where p is the price of the good.[3]

2.2 Firms

Consider an oligopolistic industry that consists of n symmetric firms indexed by i. Firms produce a single homogeneous good. When they do not engage in R&D activities the marginal cost of production is constant and equal to c. We call this the *initial marginal cost.* When they engage in R&D activities, the marginal cost of production decreases. In accord with D'Aspremont and Jacquemin (1988), we assume that R&D investment results in cost reductions. That it enhances quality can also be assumed in a product differentiation model. In market economies, the level of cost-reducing R&D is determined by profits. Since profits may understate/overstate the social benefits at the margin, there is no reason to believe ex ante that the market outcome is optimal.

For various reasons,[4] we assume that the rival firms' R&D expenditures also reduce a firm's

[3] In the general form of the demand function, we need $p(Q)$ to be twice continuously differentiable and to satisfy $\partial p(Q)/\partial Q < 0$ for all $Q \geq 0$ such that $p(Q) \geq 0$.

[4] An example might be the imperfect protection of intellectual property rights.

marginal cost, but by less than the firm's own R&D investment. In the model, this effect is captured by a technical knowledge spillover α within the industry. On one hand, spillovers reduce the costs of production of one firm. On the other hand, they decrease incentives for new R&D investment since they help competitors to decrease their costs. We assume that the marginal cost function is the same for all firms, and takes the following form for $firm\ i$:

$$\Phi_i = \Phi_i(I_i, \boldsymbol{I}_{-i}; c, \alpha) \qquad \forall i \in \{1, 2, ..., n\} \text{ and } \forall \alpha \in [0, 1] \quad , \tag{4}$$

where I_i represents the R&D expenditure level of the $firm\ i$ and $\boldsymbol{I}_{-i}$ for all other firms. Φ_i is assumed to be convex in I_i and twice continuously differentiable in I_i and $\boldsymbol{I}_{-i}$. Since we have spillovers within the sector, any industrial knowledge rapidly becomes a public good. This prevents firms from enjoying all the benefits of their own R&D investments. One functional form which captures this effect for $firm\ i$ is[5]

$$\Phi_i = c - [(1 - \alpha)I_i + \alpha \sum_{k=1}^{n} I_k] \quad . \tag{5}$$

With n firms, the inverse demand function can be rewritten as follows:

$$p(q_i, \boldsymbol{q}_{-i}) = a - b \sum_{k=1}^{n} q_k \quad , \tag{6}$$

which is a linear, and therefore, a *weakly* concave demand function. The convex form for marginal cost and concave form for the demand function assures the concavity of the profit function, and hence, the existence of the equilibrium. Here, q_i is the output of $firm\ i$, and $\boldsymbol{q}_{-i}$ is the output of all other firms. We assume that R&D represents a cost that depends on the efficiency of the R&D activities. This has an effect on marginal cost and allocative efficiency.[6] Some amount of subsidy is given for every dollar spent on R&D and output is taxed.[7] The profit function for $firm\ i$ is, then, given by

$$\Pi_i(q_i, \boldsymbol{q}_{-i}, I_i, \boldsymbol{I}_{-i}) = [p(q_i, \boldsymbol{q}_{-i}) - t]q_i - q_i\Phi_i - (1 - s)\Psi(I_i) \qquad \forall i \in \{1, 2, ..., n\} \quad , \tag{7}$$

[5] This is also the average cost of production. For future purposes, marginal cost can also be written as $\Phi_i = c - [I_i + \alpha \sum_{k \neq i}^{n} I_k]$.

[6] Allocative inefficiency occurs when pricing deviates from marginal cost. It is easy to show that $p = \frac{\varepsilon}{n\varepsilon+1}(\Phi_i + t)$ where ε is defined to be price elasticity ($\varepsilon = -\frac{d \log p(Q)}{d \log Q}$). Therefore, the price is a markup over after-tax marginal cost. This is similar to the mark-up that can be derived from a Dixit-Stiglitz (1977) type product differentiation model with one difference: in the Dixit-Stiglitz framework the mark-up is not dependent on the number of firms in the industry, whereas here it is.

[7] Initially, we define $0 < t < p(q_i, \boldsymbol{q}_{-i})$ and $s \in [0, 1]$, but we shall relax these assumptions without any loss of generality whenever an optimum occurs with a negative tax (*a subsidy on output*) or with a negative subsidy (*a tax on R&D*).

where t is the tax on output, s is the subsidy on R&D, and $\Psi(I_i)$ is the cost of R&D activities which is a function of R&D investment.

R&D investment might be considered the most important source of product differentiation in determining whether firms can earn excess profits. Moreover, since R&D investment is costly, it is going to be a very good way to deter entry. If a new entrant wants to compete effectively with the incumbents, it has to bear high R&D costs, making it difficult to enter the market. Therefore, in the R&D-driven sectors we expect to see higher concentration ratios. Our model is, by no means, a product differentiation model, but it captures the latter effect. Moreover, we look at those sectors in which a substantial amount of production cost is affected by R&D investments. Markets are usually oligopolistic in such sectors, which is why we prefer to use an oligopoly model here.

2.3 The Government

We treat the government as an active player with full commitment powers to set the available policy tools. The government chooses the optimal policy mix (t^*, s^*) so as to maximize its social welfare function, which is to be defined later. Optimal policy comprises an output tax t to deal with market power in the output market and an R&D subsidy s to deal with R&D market failure.

As with any investment decision, R&D investment is not undertaken by firms unless it is profitable. Firms sometimes overinvest in R&D as a result of strategic reasons. This makes government intervention crucial for optimality. We shall show later that a decentralized equilibrium with tax/subsidy policies by the government exactly matches the optimum of a welfare-maximizing social planner. By changing the relative cost of R&D for every given level of output to the cost of any other investment through the optimal tax/subsidy policy on R&D investments, the government can influence the generation of technical knowledge in the second best sense. It is an effective tool to internalize spillover effects. Moreover, *if possible*, the government can use another tax/subsidy policy to regulate the market power. These two policies addressing two market failures are sufficient to bring the economy to its first best frontier.

2.4 The Game

We consider a three-stage game in which the firms and the government interact strategically. There are infinite continuous choice spaces in all stages of the game.

In the first stage, the government decides on the linear tax/subsidy rate on the output and the tax/subsidy for R&D investment. Both together form the optimal policy mix (t^*, s^*).

In the second stage, firms decide their R&D investment levels. When firms act strategically by choosing R&D before they set their output level, they have a stronger incentive to invest more in R&D.[8] By doing so they gain a stronger competitive advantage over their rivals. However, externalities associated with technological spillovers mitigates this incentive. When spillovers are high enough, we expect firms to underinvest in R&D in an unregulated market equilibrium since spillovers will decrease the marginal costs of all firms.

In the final stage, firms decide how much to sell in the oligopolistic market by engaging in Cournot competition. Strategic interaction between the firms tends to reduce output, which makes subsidies justifiable. In the absence of an optimal policy mix (t^*, s^*), there will be underproduction relative to the direct optimum due to imperfect competition.

We assume that all firms and the government know how their actions will affect the actions of all the others in the next stages of the game, which requires subgame perfection. Moreover, for the sake of completeness, it is worth mentioning that our solution concept is backward induction. Assume n is fixed, for example, because of entry and exit barriers. Therefore, we can treat the number of firms as a parameter from now on.

3 Characterization of R&D Investment in a Regulated Oligopoly

The maximization problem of $firm\ i$ yields the reaction functions for the Cournot outputs:[9]

$$q_i = \frac{a - b\sum_{k\neq i}^{n} q_k - \Phi_i - t}{2b} \qquad \forall i \in \{1, 2, ..., n\} \quad . \tag{8}$$

We can get the Cournot-Nash equilibrium of the stage-game by solving these reaction functions simultaneously:

$$q_i = \frac{a - t - (n+1)\Phi_i + \sum_{k=1}^{n} \Phi_k}{b(n+1)} \quad . \tag{9}$$

[8] We assume output levels can be adjusted more quickly than R&D investment levels. Therefore, the output game is played at a later stage than the R&D game.

[9] All the conditions guaranteeing nonnegativity are assumed from now on. In particular $a > \Phi_i > 0$.

Details are given in Appendix A.1. By substituting (9) into the profit function (7) we get the profit function in the compact format:

$$\hat{\Pi}_i(q_i(I_i, \boldsymbol{I}_{-i}), I_i) = bq_i^2 - (1-s)\Psi(I_i) \quad . \tag{10}$$

The first-order conditions for the second stage of the game are, then,

$$\frac{\partial \hat{\Pi}_i}{\partial I_i} = 2b\frac{\partial q_i}{\partial I_i}q_i - (1-s)\Psi'(I_i) \qquad \forall i \in \{1, 2, ..., n\} \quad . \tag{11}$$

Therefore, the marginal benefit of R&D for $firm\ i$ is $2b\frac{\partial q_i}{\partial I_i}q_i$.

Observation 1 *In an oligopolistic market with technology spillovers, a firm's marginal benefit of R&D investment can be decomposed as follows:*

$$(M)(TTC)(\frac{1}{RI}) = \underbrace{\frac{2}{(n+1)}}_{mark\text{-}up} \underbrace{\left(\overbrace{\sum_{k \neq i}^{n} (\eta_{\Phi_k:I_i} \Phi_k)}^{inter\text{-}firm\ effect} - n \overbrace{(\eta_{\Phi_i:I_i} \Phi_i)}^{internal\ effect} \right)}_{total\ technology\ change} \underbrace{\frac{q_i}{I_i}}_{inverse\ of\ research\ intensity} \quad . \tag{12}$$

The derivation of Observation 1 is given in the Appendix A.3. Here, M is a mark-up and $\eta_{\Phi_i:I_j}$ is the elasticity of the marginal cost of $firm\ i$ with respect to the R&D investment of $firm\ j$. I_i/q_i is the R&D investment per unit of output which is defined to be level of *the research intensity* (RI). The importance of research intensity will become clear when we discuss the optimal policies on R&D investment. The change in the marginal cost of production can be interpreted as the change in the production technology of the firm (Total Technology Change, TTC). As shown in (12), this technology change comes from two sources. First, the firm's investment in R&D decreases its own cost of production. This *internal effect* is captured by the second term in the large parentheses. Second, there is an external effect on the firm coming from its rivals. It is impossible for a firm to contribute only to its own technological accumulation without also contributing to the others' because of the spillover involved. Remember that in a Cournot setting, relative technology change is more important than absolute technology change since all the firms are interacting strategically. Any technology investment creates a change in the technology of the other firms, which is captured by the first term in the large parentheses. We cannot say anything about the sign of this *inter-firm effect* unless we know the nature of the sector, especially the spillovers associated with technological improvements. Therefore, the positive externality for the other firms might have a negative feedback effect on the firm itself. However, intuitively, the positive externality is more likely to have a positive effect on the firm itself when there is high technological transmission in the sector. We shall pin down this result more in Observation 2.

To go further, assume that the cost of R&D investment is given by

$$\Psi(I_i) = \gamma \frac{I_i^2}{2} \quad , \tag{13}$$

which means that there are diminishing returns to R&D. The parameter γ captures the efficiency of R&D activities, and $1/\gamma$ can be interpreted as the *cost effectiveness of R&D*. The greater the cost effectiveness, the better. Since there are n symmetric firms with the same cost effectiveness parameter, n/γ is an indicator of sectoral cost effectiveness. After the steps given in Appendix A.2, we can get the R&D spending of the firm given the R&D spending of the other firms. In other words, the reaction function for the R&D expenditure is

$$I_i = \frac{2[n - \alpha(n-1)][a - t - c + (2\alpha - 1)\sum_{k \neq i}^{n} I_k]}{(1-s)b(n+1)^2\gamma - 2[n - \alpha(n-1)]^2} \quad . \tag{14}$$

Observation 2 *i) If $\alpha > 0.5$, R&D investments are strategic complements. ii) If $\alpha < 0.5$, R&D investments are strategic substitutes.*

The proof is obvious. Therefore, if there is high technological transmission (*meaning $\alpha > 0.5$; e.g., there is no strong protection on intellectual property rights*) within the industry, firms enjoy the R&D investments of the other firms. In other words, the R&D investments of the other firms are supporting the equilibrium level of R&D investment of the firm in consideration. If there are low spillovers (*meaning $\alpha < 0.5$*), the technical information diffuses less. Then, the R&D investments of the other firms have a negative effect on the firm's strategic choice of R&D investments. Since firms are all the same, the stage Nash equilibrium of the second stage of the game is given by

$$I_i^* = \frac{2[n - \alpha(n-1)](a - t - c)}{(1-s)b(n+1)^2\gamma - 2[n - \alpha(n-1)][1 + \alpha(n-1)]} \quad . \tag{15}$$

This is the locally stable symmetric R&D profile. Here, the denominator is positive because of a local stability condition.[10] One immediate implication of (15) is that whether the equilibrium level of R&D in the regulated oligopoly is less or greater than its unregulated

[10] A short discussion on the derivation of the stability condition is given in Appendix A.7.

market equilibrium (*meaning* $t = 0$ *and* $s = 0$) depends *mainly* on the number of firms in the industry.

4 Welfare Implications

In this section, we present the welfare implications not only of the decentralized equilibrium but also of a direct optimum in which a benevolent social planner can plan both the production and R&D levels directly. We show that, with the two policy tools, the decentralized equilibrium achieves the first-best equilibrium levels of output and R&D of the direct optimum. Therefore, in our model, the government's policy is nothing but a strategic commitment to ensure that the decentralized equilibrium achieves the direct optimum.

4.1 Decentralized Equilibrium

In the decentralized case, the government can change the equilibrium levels indirectly via its tax/subsidy policies. Assume that the government maximizes a social welfare function of the standard type in which the tax revenue is redistributed.[11]

$$W(t,s) = u(Q) - \frac{\gamma}{2}\sum_{k=1}^{n} I_k^2 - \sum_{k=1}^{n} q_k \Phi_k \quad . \tag{16}$$

So, the social welfare function is the sum of the surplus of the representative consumer and industry profits (*which is the total revenue from output sold minus the total cost of R&D and total cost of production*). After substituting for the utility function and imposing symmetry across the firms, social welfare is

$$W(t,s) = anq_i - b\frac{n^2 q_i^2}{2} - \frac{\gamma}{2} n I_i^2 - nq_i\Phi_i \quad . \tag{17}$$

The maximization of (17) gives us the following optimal policy mix for the government.[12]

$$(t^*, s^*) = \left(\frac{b\gamma(a-c)}{[1+(n-1)\alpha]^2 - bn\gamma}, \frac{3+n}{1+n} - \frac{2}{1+\alpha(n-1)}\right) \quad . \tag{18}$$

[11] Remember we have an additively separable, money metric utility function.

[12] Remember that we take the number of firms as given, and restrict our attention only to the symmetric equilibrium. Therefore, we implicitly constrain the policy space of the government to a symmetric one. That is, we rule out the differential treatment of the firms in which the government prefers closing down some firms rather than taxing/subsidizing all.

The optimal tax/subsidy for R&D equates the marginal social return to R&D to its marginal cost, and the marginal social return to output to its marginal cost. Therefore, a committed government would announce the (t^*, s^*) social policy scheme. It is worth mentioning that the optimal subsidy to R&D investment is independent of a, b, c, γ. That is, the optimal subsidy is independent of demand characteristics, the initial marginal cost, and the cost effectiveness of R&D in our model. It depends only on the spillover parameter and number of firms in the industry. As can be seen from (18), given the number of firms in the sector, the degree of spillover dictates the degree of government intervention via tax and subsidy policies.

By substituting the optimal policy values into the social welfare function we get

$$W^* = \frac{1}{2}\left(\frac{n\gamma(a-c)^2}{bn\gamma - [1+(n-1)\alpha]^2}\right) \quad . \tag{19}$$

Proposition 1 *The government subsidizes R&D investment iff the sector spillover is greater than $\frac{1}{3+n}$. If the sector spillover is less than $\frac{1}{3+n}$, taxing R&D investment is the optimum.*

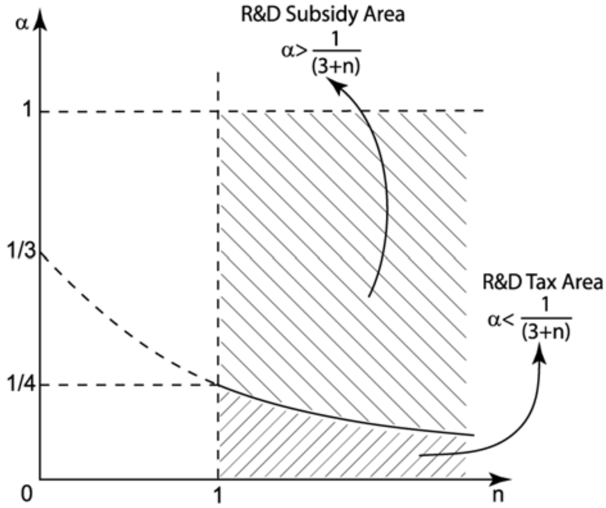


Figure 1: Optimal Subsidy and Tax Subspaces

The proof is given in Appendix A.4. Subsidizing R&D is the more typical result. As was mentioned before, the government's motivation for subsidizing R&D is to make use of the positive externality of spillover. If spillover is sufficiently high, it is optimal for firms to underinvest in R&D due to the well known externality story. It is this story that leads many governments to introduce R&D tax credits. However, another interesting result emerges from the model. If spillover is sufficiently low, given a fixed number of firms in the industry, the government might want to tax R&D in the decentralized equilibrium.[13] The intuition is the following. We have two effects acting in the model, namely a strategic effect and a spillover effect.

The strategic effect comes from the nature of the market, which is a Cournot market in which firms act strategically. They always want to increase their competitive advantage over the others. One way of doing that is to overinvest in R&D for every given level of output. From the perspective of society, this means choosing a higher research intensity than the desired level. To focus only on this strategic effect, assume for the moment that there is no spillover. In that case, as shown in Figure 4.1, it is optimal for government to tax R&D investment for any number of firms in the industry. The motivation for a firm is to decrease its production cost more than any other firm by investing more in R&D, and therefore increase its competitiveness. This results in an undesirably high level of research intensity for the society. This situation is very similar to that of the defense strategies of two hostile countries such as India and Pakistan. It is individually rational for both countries to overinvest in military forces. One country's resources devoted to defense make it a greater danger for the other, which creates an incentive in that country for overinvestment in the army. This is, nevertheless, not a collectively rational behavior.

We also have the spillover effect acting in the other way. The more R&D a single firm is engaged in, the more the sector benefits via spillover effects. In the absence of public policies, the spillover effect decreases incentives for investing in R&D-intensive technologies because of the externalities involved. If spillover is sufficiently low, the strategic effect dominates the spillover effect, and as a result firms overinvest in R&D for every given level of output. However, if it is high enough, the spillover effect dictates the outcome, and as a result firms underinvest in R&D for every given level of output.

Figure 4.1 indicates that if the number of firms increases in the market, the taxation of R&D is at the optimum when there are low spillover levels. Combining Figure 4.1 and Figure 4.1, we conclude that an R&D tax is more likely to be put in effect when there are fewer firms in the industry. High performance of the market occurs when there are optimal subsidies associated with high spillovers or when there are optimal taxes associated with low spillovers.

Proposition 2 *The government subsidizes output* $\forall \alpha \in [0, 1]$.

The proof is in Appendix A.5, and it directly follows from the comparison of the decentralized equilibrium with the socially desirable level of output. Output is subsidized in our model to

[13] Even in a second-best world in which output cannot be subsidized an R&D tax result still persists. A brief discussion of the second-best is given in Appendix A.8.

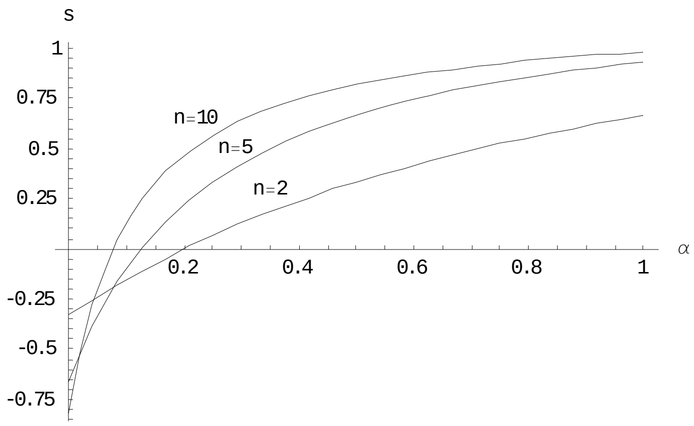


Figure 2: Contour Plot of Optimal Tax/Subsidy

offset the underproduction due to market power. Since firms are acting in an oligopolistic market, they have market power, which lowers social welfare both at the margin and in the aggregate. Firms are producing too little output, and this is bad for the well-being of the representative consumer. By subsidizing output, the government not only makes the firms better off, or at least no worse off, but it also increases the output provided to the consumer, thereby generating a welfare improvement. It is, indeed, always socially optimal to subsidize output to close the gap between the marginal cost of production and the price of output. However, this partial equilibrium result would mean a lower sales tax on output in a general equilibrium since many governments impose sales tax on output for revenue purposes.

The subgame perfect Nash equilibrium of the game is the following R&D and output profiles:[14]

$$I_i^* = \frac{(a-c)[1+(n-1)\alpha]}{bn\gamma - [1+(n-1)\alpha]^2} \tag{20}$$

$$q_i^* = \frac{(a-c)\gamma}{bn\gamma - [1+(n-1)\alpha]^2} \quad . \tag{21}$$

4.2 Direct Optimum[15]

As a benchmark, consider the direct optimum, the optimum of a welfare-maximizing social planner. That is, assume there is a benevolent social planner who has direct control over both the output produced by and the R&D investment in a given market. The social welfare

[14] By looking at the signs of the leading principal minors it is easy to show that second-order condition is given by $bn\gamma - [1+(n-1)\alpha]^2 > 0$. In the worst case $\alpha = 1$. When it is the case, $n < b\gamma$.

[15] Some details are provided in Appendix A.6.

function of the planner takes the same form as before, but since he has direct control over output and R&D, his choice variables are now the R&D investment profile $\boldsymbol{I} = (I_1, I_2, ..., I_n)$, and the output profile $\boldsymbol{q} = (q_1, q_2, ..., q_n)$.

$$\hat{W}(\boldsymbol{q}, \boldsymbol{I}) = u(Q) - \frac{\gamma}{2}\sum_{k=1}^{n} I_k^2 - \sum_{k=1}^{n} q_k \Phi_k \quad . \tag{22}$$

Therefore,

$$\hat{W}(\boldsymbol{q}, \boldsymbol{I}) = a\sum_{k=1}^{n} q_k - b\frac{(\sum_{k=1}^{n} q_k)^2}{2} - \frac{\gamma}{2}\sum_{k=1}^{n} I_k^2 - \sum_{k=1}^{n} q_k \Phi_k \quad . \tag{23}$$

There is still some possibility for imperfect spillover due to idiosyncratic effects, imperfect communication, and so forth. The first-orders conditions (FOC) for the social planner are

$$\frac{\partial \hat{W}(\boldsymbol{q}, \boldsymbol{I})}{\partial q_i} = a - b\sum_{k=1}^{n} q_k - \Phi_i = 0 \quad , \tag{24}$$

and

$$\frac{\partial \hat{W}(\boldsymbol{q}, \boldsymbol{I})}{\partial I_i} = -\gamma I_i + q_i + \alpha\sum_{k\neq i}^{n} q_k = 0 \quad . \tag{25}$$

The FOC with respect to R&D indicates that the social marginal cost of R&D investment has to equal to the total output from the firms resulting from the marginal R&D investment. After some manipulation the output and R&D levels chosen by the social planner are:

$$I_i^S = \frac{(a-c)[1+(n-1)\alpha]}{bn\gamma - [1+(n-1)\alpha]^2} \tag{26}$$

$$q_i^S = \frac{(a-c)\gamma}{bn\gamma - [1+(n-1)\alpha]^2} \quad , \tag{27}$$

where $b\gamma > n$. These are the same equilibria we get from the decentralized economy with the government intervention via tax/subsidy policies (eqs. (20) and (21)). Therefore, the decentralized economy achieves the first-best optimum. In the next section, we provide the comparison of the $FOCs$ of the direct optimum with the $FOCs$ of the market equilibrium to give more insight into how the mechanism works.

4.3 Comparison of the Direct Optimum with the Market Equilibrium

We define the market equilibrium as the outcome in which there is no government intervention via tax and subsidy policies. In an unregulated market R&D incentives are inhibited by the existence of spillover. The $FOCs$ for the market equilibrium are as follows:

$$2bq_i + b\sum_{k\neq i}^{n} q_k = a - \Phi_i \tag{28}$$

$$2b\frac{\partial q_i}{\partial I_i}q_i = \gamma I_i \quad , \tag{29}$$

where $\frac{\partial q_i}{\partial I_i} = \frac{n-\alpha(n-1)}{b(n+1)}$. From (28), which is the FOC with respect to output, we get

$$b(n+1)q_i = (a-c) + [1+\alpha(n-1)]I_i \quad . \tag{30}$$

From (29), which is the FOC with respect to R&D investment:

$$2[n-\alpha(n-1)]q_i = (n+1)\gamma I_i \quad . \tag{31}$$

Equations (30) and (31) characterize the market equilibrium in $I-q$ space. The $FOCs$ of the direct optimum are

$$nbq_i = (a-c) + [1+\alpha(n-1)]I_i \tag{32}$$

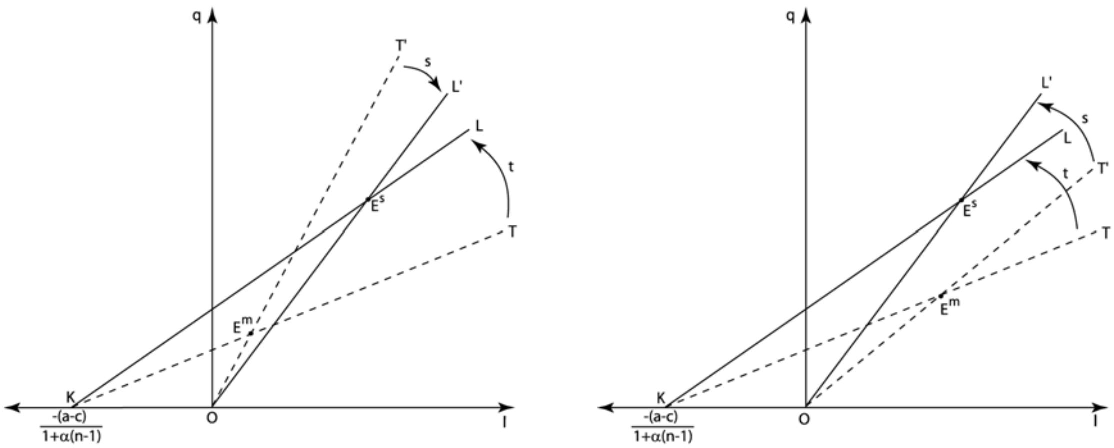
$$[1+\alpha(n-1)]q_i = \gamma I_i \quad . \tag{33}$$

Equation (32), which is the FOC with respect to output, and equation (33), which is the FOC with respect to R&D investment, characterize the direct optimum in the $I-q$ space. It is worth mentioning that equations (31) and (33) define the research intensity in the market equilibrium and the direct optimum, respectively.

Market Equilibrium	$b(n+1)q_i = (a-c) + [1+\alpha(n-1)]I_i$ $2[n-\alpha(n-1)]q_i = (n+1)\gamma I_i$
Direct Optimum	$nbq_i = (a-c) + [1+\alpha(n-1)]I_i$ $[1+\alpha(n-1)]q_i = \gamma I_i$

Table 1: The First-order Conditions

The $FOCs$ for the market equilibrium and direct optimum are summarized in Table 1. By making use of these two pairs of $FOCs$ one can get the optimal tax/subsidy policy which we derived earlier. More insight can be gleaned from the graphs of these equations in the $I - q$ space. Graphs of equations (30), (31), (32), and (33) are given in Figure 4.3. In both of the cases the market fails to provide the optimal amount of output and R&D investment in the absence of regulation, and either an R&D subsidy or an R&D tax carries the industry to its direct optimum with the optimal research intensity. Market equilibrium is characterized by lines KT and OT', and the direct optimum is characterized by lines KL and OL'. The slope of KL is always greater than the slope of KT. That means the optimal policy regarding market power is always to subsidize output (*Proposition 1*). However, the slope of OL' is smaller than the slope of OT' only if $\alpha > 1/(3+n)$. In this situation, it is socially optimal to subsidize R&D as shown in Figure 4.3a. However, if $\alpha > 1/(3+n)$, the slope of OL' is greater than the slope of OT', meaning that taxing R&D investment is the optimum (*Proposition 2*). This situation is shown in Figure 4.3b.



(a) R&D Subsidy:
Market Equilibrium and Direct Optimum
when $\alpha > 1/(3+n)$

(b) R&D Tax:
Market Equilibrium and Direct Optimum
when $\alpha < 1/(3+n)$

Figure 3: Characterization of the Social Policy

When a firm engages in R&D investment, it is affected twice. First, the production cost decreases due to the *cost-reducing behavior* of R&D investments. Second, the production cost of the other firms decreases due to the spillover effect, which in turn decreases the competitive advantage of the firm. This second effect is the source of the strategic effect. Therefore, the investment decision in R&D depends purely on which effect dominates the other.

Whenever the spillover effect is high enough, firms collectively choose too low a level of R&D conditional on output compared to the direct optimum. The system is then characterized by

Figure 4.3a, and the government should provide an R&D subsidy to hit the first-best frontier. However, if the spillovers level is low, it is in firms' best interest to choose collectively too high a level of R&D conditional on output. In this case, we essentially have the strategic effect dominating the spillover effect. Thus, firms are mostly dependent on their own level of R&D investment and do not have much incentive to free-ride. This results in overinvestment in R&D conditional on output level. Such a situation is shown in Figure 4.3b.

Notice that we prefer to focus on R&D per unit of output rather than the equilibrium level of R&D. Previous literature on R&D investment games tends to focus only on the equilibrium levels of R&D investment and output. We believe this might be misleading because of the following reason. As is shown in Figure 4.3, in an R&D tax case, equilibrium levels of both R&D investment and output can either be higher or lower than those of the direct optimum, whereas research intensity is always higher in the decentralized equilibrium. A tax on R&D creates disincentives for R&D investment for every given level of output for sure, but this does not necessarily result in a lower level of the equilibrium. That is why we use "*R&D for every given level of output*" rather than equilibrium level in discussing over- and underinvestment phenomena. More clearly, in some cases, a tax on R&D might be required for efficiency even though there is equilibrium underinvestment in R&D. Figure 4.3b illustrates such a case in which both R&D and output profiles of the direct optimum are larger than those of the market equilibrium (e.g., $I_i^S > I_i^M$ and $q_i^S > q_i^M$) and it is still beneficial to tax R&D investments. This requires $\frac{n(n-2)-1}{(n-1)[n(n+4)+1]} < \alpha < \frac{1}{3+n}$. However, if $\alpha < \frac{n(n-2)-1}{(n-1)[n(n+4)+1]}$, then $I_i^S < I_i^M$ and $q_i^S < q_i^M$.

An important point is that we can reach the first-best optimum since we have two policies, namely output subsidies and R&D subsidies (or taxes), addressing the two sources of problems in the industry, namely market power and the incentive-reducing effect of spillover. Therefore, a committed government can achieve Pareto optimality within the standard decentralized competitive market simply by levying proper taxes or subsidies that direct competitive behavior to the Pareto-optimal conditions.

5 Conclusion

In this paper we build a simple framework which is quite flexible and useful for analyzing many kinds of public policies regarding R&D investment. There is a well-known externality associated with R&D investment that results from the spillover effect which is a source of market failure. Such failures generally cause firms to underinvest in research. On the other hand, our analysis shows that in an oligopolistic market firms might want to overinvest in R&D in response to interaction with other firms. Therefore, the market will fail to provide the right amount of R&D. The most direct way to correct these problems is to subsidize (or tax) R&D investment to control for under- (or over-) investment. Thus, we need to determine how these two effects interact in order to discover the conditions under which subsidies (or taxes) are appropriate.

To solve this problem, this paper sets up a simple game-theoretical model to identify the conditions under which tax and subsidy policies serve as the means to reach the socially desirable level of privately financed R&D in a regulated oligopolistic industry. We have shown that there are cases in which the government might want to tax R&D, contrary to widespread assumptions in favor of subsidies. Here the tax/subsidy policy can change the vector of relative prices in the economy from their values in the market equilibrium to the socially optimal ones so as to reach Pareto efficiency. However, this is more likely to happen when there are relatively few firms in the industry.

We have also shown that when considering a tax-subsidy policy, we should focus on not only the equilibrium level of R&D or output but also the research intensity. A tax on R&D lowers the level of R&D investment for every given level of output, but this does not necessarily lead to an equilibrium with a lower level of investment. These surprising results come to light when we consider the realistic play between the spillover effect and the strategic effect.

The model we provide here is a very simple one consisting of linear demand functions and quadratic costs for R&D investment. This does oversimplify the result, however. Even if a general demand function were used, the R&D tax result would still persist since a linear demand function is always going to be a subset of a general demand function. The focus of the reader, therefore, should not be the optimal values of subsidies and taxes in this model but the existence of an R&D tax result even in a very general framework.

One might argue that the model presented here is not suitable for the R&D games under consideration. But then, one should also criticize quite a large number of papers using the same D'Aspremont and Jacquemin (1988) framework to model R&D games in the same respect.

One possible reaction to an R&D tax is that there is no empirical evidence showing that there is overinvestment in R&D. This is correct, but one should also note neither is there empirical evidence in support of underinvestment. The contemporary techniques are not capable of analyzing the *social optimum* empirically. Given the tools, one can at most find some indirect evidence about over- or underinvestment in R&D. Moreover, as is shown before, an R&D

tax might be necessary even when there is underinvestment in R&D.

When we consider tax incentives in the real world one practical problem emerges immediately. We assume that the government can clearly define what constitutes an investment in R&D, and that the firms do not misreport their R&D investments. In reality, however, this is not the case. Governments are not able to define R&D activities perfectly, and firms have an incentive to report some other activities as if they were R&D expenditures. Both have been important problems in practice. Moreover, in an ideal world, the government should determine which sectors have the highest social rate of return after an extensive cost-benefit analysis. In reality, there is a tendency to reward lobbyists and other influential groups instead of decisions on the basis of cost-benefit analysis.

A Appendix

A.1 Third stage

Firm i's profit is given by

$$\Pi_i(q_i, \boldsymbol{q}_{-i}, I_i, \boldsymbol{I}_{-i}) = [p(q_i, \boldsymbol{q}_i) - t]q_i - q_i\Phi_i - (1-s)\Psi(I_i) \qquad \forall i \in \{1, 2, ..., n\} \quad . \tag{7}$$

Therefore, the *FOC* is

$$\frac{\partial \Pi_i}{\partial q_i} = p(q_i, \boldsymbol{q}_{-i}) - t - \Phi_i + \frac{\partial p(q_i, \boldsymbol{q}_{-i})}{\partial q_i} q_i = 0 \quad . \tag{A-1}$$

With a linear demand function of the form $p(q_i, \boldsymbol{q}_{-i}) = a - b\sum_{k=1}^{n} q_k$, we have

$$q_i = \frac{a - b\sum_{k \neq i}^{n} q_k - \Phi_i - t}{2b} \qquad \forall i \in \{1, 2, ..., n\} \quad . \tag{8}$$

Thus

$$\begin{aligned}
2bq_1 &= a - b(0 + q_2 + q_3 + ... + q_n) - \Phi_1 - t \\
2bq_2 &= a - b(q_1 + 0 + q_3 + ... + q_n) - \Phi_2 - t \\
&\vdots \\
2bq_n &= a - b(q_1 + q_2 + ... + q_{n-1} + 0) - \Phi_n - t \quad .
\end{aligned}$$

These equations form a system of n equations in n unknowns. Therefore,

$$2b\sum_{k=1}^{n} q_k = na - b(n-1)\sum_{k=1}^{n} q_k - \sum_{k=1}^{n}\Phi_k - nt \tag{A-2}$$

$$\iff \sum_{k=1}^{n} q_k = \frac{n(a-t) - \sum_{k=1}^{n}\Phi_k}{b(n+1)} \quad . \tag{A-3}$$

For future reference note that this can be rearranged to get

$$\sum_{k\neq i}^{n} q_k \quad = \quad \frac{n(a-t) - \sum_{k=1}^{n}\Phi_k}{b(n+1)} - q_i \tag{A-4}$$

$$\implies \quad q_i = \frac{a - b\frac{n(a-t)-\sum_{k=1}^{n}\Phi_k}{b(n+1)} + bq_i - \Phi_i - t}{2b} \tag{A-5}$$

$$\implies q_i = \frac{a - t - (n+1)\Phi_i + \sum_{k=1}^{n}\Phi_k}{b(n+1)} \quad . \tag{9}$$

A.2 Second Stage

$$\begin{aligned}\Pi_i(q_i, \boldsymbol{q}_{-i}, I_i, \boldsymbol{I}_{-i}) &= [p(q_i, \boldsymbol{q}_{-i}) - t]q_i - q_i\Phi_i - (1-s)\Psi(I_i) \qquad \forall i \in \{1,2,...,n\} \qquad (7)\\ \Pi_i(q_i, \boldsymbol{q}_{-i}, I_i, \boldsymbol{I}_{-i}) &= [a - b\sum_{k=1}^{n} q_k - t - \Phi_i]q_i - (1-s)\Psi(I_i) \quad .\end{aligned}$$

By substituting for $\sum_{k=1}^{n} q_k$ and q_i

$$\hat{\Pi}_i(I_i, \boldsymbol{I}_{-i}) = [a - b\left(\frac{n(a-t) - \sum_{k=1}^{n}\Phi_k}{b(n+1)}\right) - t - \Phi_i]\left(\frac{a - t - (n+1)\Phi_i + \sum_{k=1}^{n}\Phi_k}{b(n+1)}\right) - (1-s)\Psi(I_i)$$

$$\implies \hat{\Pi}_i(I_i, \boldsymbol{I}_{-i}) = b\left(\frac{a - t - (n+1)\Phi_i + \sum_{k=1}^{n}\Phi_k}{b(n+1)}\right)^2 - (1-s)\Psi(I_i) \quad ,$$

which boils down to

$$\hat{\Pi}_i(q_i(I_i, \boldsymbol{I}_{-i}), I_i) = bq_i^2 - (1-s)\Psi(I_i) \quad . \tag{10}$$

First-order conditions for the second stage of the game are then,

$$\frac{\partial\hat{\Pi}_i}{\partial I_i} = 2b\frac{\partial q_i}{\partial I_i}q_i - (1-s)\Psi'(I_i) = 0 \qquad \forall i \in \{1,2,...,n\} \quad . \tag{11}$$

The marginal cost of R&D is $(1-s)\Psi'(I_i)$, which is equal to marginal benefit $2b\frac{\partial q_i}{\partial I_i}q_i$. Moreover,

$$\begin{aligned}
\Phi_i(I_i, \boldsymbol{I}_{-i}) &= c - [(1-\alpha)I_i + \alpha \sum_{k=1}^{n} I_k] \\
&= c - (1-\alpha)I_i - \alpha(I_1 + I_2 + ... + I_n) \qquad \text{(A-7)} \\
\sum_{k=1}^{n} \Phi_k &= nc - (1-\alpha)\sum_{k=1}^{n} I_k - n\alpha \sum_{k=1}^{n} I_k \\
&= nc - [1 + \alpha(n-1)] \sum_{k=1}^{n} I_k \quad . \qquad \text{(A-8)}
\end{aligned}$$

Hence,

$$(1-s)\Psi'(I_i) = 2\frac{\partial q_i}{\partial I_i}\left(\frac{a - t - (n+1)\Phi_i + \sum_{k=1}^{n}\Phi_k}{n+1}\right) \quad . \qquad \text{(A-9)}$$

By using $\frac{\partial q_i}{\partial I_i} = \frac{(n+1)-1-\alpha(n-1)}{b(n+1)}$

$$(1-s)\Psi'(I_i) = 2\left(\frac{(n+1)-1-\alpha(n-1)}{b(n+1)}\right)\left(\frac{a - t - (n+1)\Phi_i + \sum_{k=1}^{n}\Phi_k}{n+1}\right) \quad . \qquad \text{(A-10)}$$

Define the cost of R&D investment as

$$\Psi(I_i) = \gamma\frac{I_i^2}{2} \quad . \qquad (13)$$

After some rearrangement, the reaction function in terms of R&D investments for *firm i* is given by

$$I_i = \frac{2[n - \alpha(n-1)][a - t - c + (2\alpha - 1)\sum_{k \neq i}^{n} I_k}{(1-s)b(n+1)^2\gamma - 2[n - \alpha(n-1)]^2} \quad . \qquad (14)$$

Here, the denominator is positive by local stability condition. Since the firms are all the same the stage Nash equilibrium of the second stage of the game is given by

$$I_i^* = \frac{2[n - \alpha(n-1)](a - t - c)}{(1-s)b(n+1)^2\gamma - 2[n - \alpha(n-1)][1 + \alpha(n-1)]} \quad . \qquad (15)$$

A.3 Derivation of Observation 1:

From (11) we know that $(1-s)\Psi'(I_i) = 2b\frac{\partial q_i}{\partial I_i}q_i$. Here, the differentiation of output with respect to R&D investment can be decomposed as follows:

$$\frac{\partial q_i}{\partial I_i} = \frac{\frac{\partial\left(\sum_{k=1}^{n}\Phi_k\right)}{\partial I_i}}{b(n+1)} - \frac{\frac{\partial \Phi_i}{\partial I_i}}{b} \quad . \qquad \text{(A-11)}$$

Hence,

$$
\begin{aligned}
(1-s)\Psi'(I_i) &= 2bq_i \left(\frac{\frac{\partial(\sum_{k=1}^{n}\Phi_k)}{\partial I_i}}{b(n+1)} - \frac{\frac{\partial \Phi_i}{\partial I_i}}{b} \right) \\
&= \frac{2q_i}{n+1} \left(\sum_{k=1}^{n} \left(\frac{\partial \Phi_k}{\partial I_i} - \frac{\partial \Phi_i}{\partial I_i} \right) - \frac{\partial \Phi_i}{\partial I_i} \right) \quad . \qquad \text{(A-12)}
\end{aligned}
$$

This can also be written in terms of elasticities. Let $\frac{\partial \Phi_i}{\partial I_j}$ be the sensitivity of marginal cost of $firm$ i with respect to R&D expenditure of $firm\ j$. Therefore, elasticity of marginal cost of $firm\ i$ with respect to the R&D investment of $firm\ j$ is defined as

$$
\eta_{\Phi_i:I_j} = \frac{I_j}{\Phi_i} \frac{\partial \Phi_i}{\partial I_j} \quad . \qquad \text{(A-13)}
$$

Thus,

$$
(1-s)\Psi'(I_i) = \underbrace{\frac{2}{(n+1)}}_{\text{mark-up}} \underbrace{\left(\overbrace{\sum_{k \neq i}^{n} (\eta_{\Phi_k:I_i}\Phi_k)}^{\text{inter-firm effect}} - n\overbrace{(\eta_{\Phi_i:I_i}\Phi_i)}^{\text{own effect}} \right)}_{\text{total technology change}} \underbrace{\frac{q_i}{I_i}}_{\text{inverse of research intensity}} \quad . \qquad (12)
$$

A.4 Proof of Proposition 1:

We have already found that $s = \frac{3+n}{1+n} - \frac{2}{1+\alpha(n-1)}$. If $s > 0$, then $\frac{3+n}{1+n} - \frac{2}{1+\alpha(n-1)} > 0$. With n greater than or equal to 1 this reduces $\alpha > \frac{1}{3+n}$.

A.5 Proof of Proposition 2:

Simple comparison of the decentralized equilibrium's $FOCs$ with the direct optimum $FOCs$ shows that $t^* = -bq_i^*$ where $q_i^* > 0$. Therefore, the government subsidizes output.

A.6 Direct Optimum

$$
\hat{W}(\boldsymbol{q}, \boldsymbol{I}) = u(Q) - \frac{\gamma}{2} \sum_{k=1}^{n} I_k^2 - \sum_{k=1}^{n} q_k \Phi_k \qquad (22)
$$

$$
\hat{W}(\boldsymbol{q}, \boldsymbol{I}) = a \sum_{k=1}^{n} q_k - b \frac{\left(\sum_{k=1}^{n} q_k\right)^2}{2} - \frac{\gamma}{2} \sum_{k=1}^{n} I_k^2 - \sum_{k=1}^{n} q_k \Phi_k \quad . \qquad (23)
$$

The marginal cost function is $\Phi_i(I_i, \boldsymbol{I}_{-i}) = c - (1-\alpha)I_i - \alpha \sum_{k=1}^{n} I_k$. The first order conditions for the social planner are

$$\frac{\partial \hat{W}(\boldsymbol{q}, \boldsymbol{I})}{\partial q_i} = a - b\sum_{k=1}^{n} q_k - \Phi_i = 0 \tag{24}$$

$$\frac{\partial \hat{W}(\boldsymbol{q}, \boldsymbol{I})}{\partial I_i} = -\gamma I_i + q_i + \alpha \sum_{k \neq i}^{n} q_k = 0 \quad . \tag{25}$$

From (24) $a - b\sum_{k=1}^{n} q_k - c + (1-\alpha)I_i + \alpha\sum_{k=1}^{n} I_k = 0$. From (25) $[1+\alpha(n-1)]\sum_{k=1}^{n} q_k = \gamma \sum_{k=1}^{n} I_k$. Solving both together we get

$$na - nb\sum_{k=1}^{n} q_k = nc - \frac{[1+\alpha(n-1)]^2}{\gamma}\sum_{k=1}^{n} q_k \quad . \tag{A-14}$$

By imposing symmetry $I_i^S = \frac{(a-c)[1+(n-1)\alpha]}{bn\gamma - [1+(n-1)\alpha]^2}$ and $q_i^S = \frac{(a-c)\gamma}{bn\gamma - [1+(n-1)\alpha]^2}$ where $b\gamma > n$.

A.7 Stability Condition

Here, we do not go into details of the issue of stability in Cournot markets, since it is out of the scope of this paper.[16] Instead, we will derive the local stability condition that we need.

The stage Nash equilibrium of the game is locally stable iff

$$\frac{\partial^2 \hat{\Pi}_i^*}{\partial I_i^2} + (n-1)\frac{\partial^2 \hat{\Pi}_i^*}{\partial I_i \partial I_j} < 0 \qquad \forall n; \quad i \neq j; \quad i,j \in \{1,2,...n\} \quad . \tag{A-15}$$

where $\hat{\Pi}_i^*$ represents the equilibrium level of profits and

$$\frac{\partial^2 \hat{\Pi}_i^*}{\partial I_i^2} = 2b\left(\left(\frac{\partial q_i}{\partial I_i}\right)^2 + q_i \frac{\partial^2 q_i}{\partial I_i^2}\right) - (1-s)\gamma \tag{A-16a}$$

$$= \frac{2[n-\alpha(n-1)]^2}{b(n+1)^2} - (1-s)\gamma \tag{A-16b}$$

$$\frac{\partial^2 \hat{\Pi}_i^*}{\partial I_i \partial I_j} = 2b\left(\frac{\partial^2 q_i}{\partial I_i \partial I_j} q_i + \frac{\partial q_i}{\partial I_i}\frac{\partial q_i}{\partial I_j}\right) \tag{A-17a}$$

$$= \frac{2[n-\alpha(n-1)](2\alpha-1)}{b(n+1)^2} \quad . \tag{A-17b}$$

Therefore, the stability condition is

$$(1-s)b(n+1)^2\gamma - 2[n-\alpha(n-1)][1+\alpha(n-1)] > 0 \quad . \tag{A-18}$$

which is exactly the same as the denominator of the equilibrium level of R&D spending.

[16] For a discussion of stability in oligopolies see Leahy and Neary (1997).

A.8 Second-Best Optimum

In this case we cannot reach the direct optimum, but the R&D tax result is still preserved. Suppose it is not possible to subsidize output. This can be obtained from our calculations for the first-best simply by setting $t = 0$ in the social welfare function. Then, the optimal subsidy in the second-best is as follows:

$$s^{SB} = \frac{2 - n + (n-1)(n+4)\alpha}{2 + n + (n+2)(n-1)\alpha} \quad . \tag{A-19}$$

where SB stands for second-best. It is optimal to tax R&D whenever $\alpha < \frac{n-2}{(n-1)(n+4)}$.

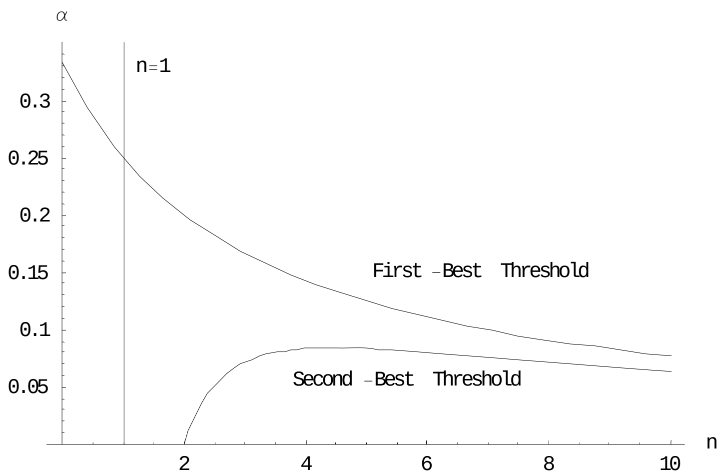


Figure 4: First-best and Second-best Thresholds

Figure A.8 shows the first-best and second-best thresholds at the same graph. First-best threshold is the same curve given in Figure 4.1 and the second-best threshold is the counterpart of it in the second best environment. In a second-best environment, taxing R&D is optimal anywhere below that threshold.

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