A LOCATION-ROUTING PROBLEM WITH MULTIPLE TRIPS ARISING IN E-COMMERCE DELIVERY

by
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ABSTRACT

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The classical location routing problem (LRP) is a well studied combinatorial optimization problem that aims to identify optimal depot location(s) and the routing decisions. In this study, we consider a two-echelon location routing problem with multiple trips under constrained distances as a generalization of the traditional LRP. Given the location of the single distribution center, we determine the locations of the regional depots among a set of candidate locations and decide how to serve the customers - from which regional depot and via which route. We consider a setting where vehicles can perform multiple trips originated from their regional depots as long as the total distance traveled does not exceed a predetermined level. We develop different mathematical models for the problem and strengthen them with simple valid inequalities. We also propose a heuristic solution method that gives feasible solutions in reasonable times even for very large problem instances. The computational experiments are designed and conducted to observe the performance of all formulations and enhancements for solving different problem sizes and parameter settings. The results of the computational experiments show that using valid inequalities improves the solution performance and different modeling approaches perform differently in terms of their run-time and solution quality in small and large instances.

ÖZET

E-TİCARET TESLİMATINDA ORTAYA ÇIKAN ÇOKLU TURLU YER SEÇİMİ ARAÇ ROTALAMA PROBLEMİ

ÇAĞRI DOĞUŞ İYICAN

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Anahtar Kelimeler: lokasyon rotalama, e-ticaret, çoklu tur, araç rotalama problemi, karma tamsayılı doğrusal programlama

Klasik yer seçimi rotalama problemi (YSRP), optimal depo konumlarını ve rota kararlarını belirlemeyi amaçlayan yaygın bir şekilde çalışılmış bir kombinatoriyel optimizasyon problemdir. Bu çalışmada, geleneksel YSRP'nin daha genel bir hali olarak, kısıtlı mesafe kısıtı altında çoklu seferlere izin verilen iki aşamalı bir lokasyon rotalama problemi ele alınmıştır. Konumu önceden bilinen tek bir dağıtım merkezine göre, aday lokasyonlar arasından bölgesel depoların konumları belirlenmiş ve müşterilere nasıl hizmet edileceğine - hangi bölgesel depodan ve hangi rota ile hizmet verileceğine karar verilmiştir. Araçların, toplam seyahat ettikleri mesafe önceden belirlenmiş bir düzeyi aşmadığı sürece, bölgesel depolarına birkaç kez uğrayarak birden fazla tur yapmalarına izin verilmiştir. Problem için farklı matematiksel modeller geliştirilmiş ve modeller basit geçerli eşitsizliklerle güçlendirilmiştir. Ayrıca, çok büyük problem örnekleri için bile makul sürelerde iyi kalitede çözümler sunan bir sezgisel yöntem önerilmiştir. Formülasyonların ve iyileştirmelerin farklı problem boyutlarını ve parametre ayarlarını çözmekteki performansını gözlemlemek amacıyla bir sayısal çalışma tasarlanmış ve gerçekleştirilmiştir. Deneyler sonucunda, geçerli eşitsizliklerin modellerin performansını iyileştirdiği ve farklı modelleme yaklaşımlarının farklı problem boyutlarında farklı sürelerde ve kalitelerde sonuç verebildiği görülmüştür.

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To my beloved Tuba
To my loving siblings Cansu and Cavit
To my loving parents Mustafa and Gülşen

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1. INTRODUCTION

It is widely acknowledged that the expenses associated with the logistics operations constitute a significant portion of the budget of the most of the companies nowadays. These costs can be reduced by designing the network of the related supply chain carefully, i.e. by deciding the locations of the depots and determining the vehicle routes accordingly (Prodhon & Prins, 2014). Location and routing decisions are studied separately and extensively in the literature for a long time, and it is proven that the integration of these two decisions (when it is possible) might yield a more efficient network in terms of the cost (Salhi & Rand, 1989). With the recent developments in the optimization techniques, we are able to address these two hard problems simultaneously (Prodhon & Prins, 2014). The decisions of both locating the depots and determining the vehicle routes are studied under the name of Location Routing Problem (LRP).

The two-echelon location routing problem (2E-LRP) is a generalization of the classical LRP in which the goods are transported from a main distribution center (DC) to the customers through regional depots. In other words, the goods are first sent from the DC to the regional depots, and from the regional depots to the customers in 2E-LRP. In this supply chain design problem, locations of the regional depots and the routes of the vehicles are the decided simultaneously Cuda, Guastaroba & Speranza (2015). Many of these studies on 2E-LRP focus on mainly heuristic methods such as hybrid genetic algorithm Moon, Salhi & Feng (2020), tabu search heuristic Boccia, Crainic, Sforza & Sterle (2010), etc.

The location routing problem has several real-life applications, such as last-mile delivery and city logistics. In last-mile deliveries, the goods intended for customers may face challenges due to issues related to truck sizes and inner-city infrastructures. Specifically, in the e-commerce sector, companies may need to design their network taking into account the complexities of cities' traffic and distribution areas. To clarify further, without an intermediate decision (such as opening regional depots), direct transportation from distribution centers would suffer from time constraints and distribution inefficiencies. Due to the complexities of city traffic and distribution areas, in real life applications, e-commerce companies might design their network so as to enable direct transportation from a single facility to regional depots. This approach of direct transportation, without considering routes between regional depots, aims to reduce the time complexity of deliveries. Additionally, since the vehicles used for this transportation are larger compared to those utilized for customer delivery services, their mobility range is limited within the inner city. Establishing a multi-trip environment between regional depots and customer locations could be crucial in reducing the number of vehicle usages for planning. Moreover, making decisions about both the locations of regional depots and the routes collectively results in a more cost-effective logistical operation for both short-term and long-term planning.

In this study, we address a location-routing problem with multiple trips (LRPMT) which arises in e-commerce delivery and where the vehicles are allowed to perform multiple trips. In the inbound transportation, the goods are transported to the regional depots from a single distribution center with a direct transportation (no routing decision is required). However, this operation has a cost component which is effected by both the load of the vehicle and distance between depots. In the outbound transportation, the goods are transported from the regional depots to the customers with the vehicles that can do multiple trips starting and ending at the same regional depot. Each customer should be served by exactly one regional depot and one truck. The goal is to determine the locations of regional depots as well as the routes of the vehicle to minimize the overall inbound, outbound and depot opening costs. The motivation behind deciding both routes and regional depot locations, as depicted by Salhi & Rand (1989), leads to a logistics environment with lower costs. Furthermore, this approach contributes to a potential reduction in number of vehicle usage. To the best of our knowledge, there exists no study on LRPMT in the context of e-commerce delivery applications in the literature.

In this thesis, we develop different mathematical models as well as a heuristic method for solving LRPMT, and test them through an extensive computational study. Our contributions can be summarized as follows.

- LRPMT is defined and studied for the first time in the literature.
- Three mixed integer linear programming formulations (Two indexed/Three indexed/Route Based) are proposed.
- The models are improved through different valid inequalities and tested computationally.
- The affect of important problem parameters on the structure of the optimal

solutions of the problem is discussed.

The remainder of this thesis is organized as follows. In Chapter 2, we present the relevant literature review. We describe our problem and develop our mathematical models in Chapter 3. Chapter 4 consists of the experimental data design and the results of the computational studies. We conclude our thesis and discuss the future research problems for LRPMT in Chapter 5.

2. LITERATURE REVIEW

In this chapter, we present a summary of the related literature in order to give the general overview of the work. As our study relies on the location routing problem (LRP), two-echelon location routing problem (2E-LRP) and the multi-trip vehicle routing problem (VRPMT), below we discuss the related studies on these three main problems.

LRP have been studied since the 1970s and involves the combined decisions of locating depots and determining optimal routes for the vehicles. One of the first ideas of simultaneously determining the depot location and the vehicle route potentially dates back to the study by Watson-Gandy & Dohrn (1973) where a depot location and travelling salesman problem with a "sales function" in which sales decline with the distance from the depot is considered. The effect of deciding depot locations and vehicle route together is depicted first in Salhi & Rand (1989). The authors demonstrate that solving the depot location problem and the vehicle routing problem separately might result in sub-optimal results and there might be potential gains of combining these two problems in terms of logistics cost. A survey published by Nagy & Salhi (2007) revealed that heuristic approaches for solving LRP are more prevalent than exact methods which is an expected result since LRP is a combination of two hard problems, namely the facility location problem and the vehicle routing problem. The authors also categorize the studies in the literature with respect to their operational structures, objective functions, number of depots and the solution methodology. One of the first heuristic approaches which is based on a tabu search is due to Tuzun & Burke (1999) where the problem is solved in two stages.

The first exact solution method for LRP is proposed by Laporte & Nobert (1981) where the authors decide the location of a single depot by determining the optimal route for each possible depot location and selecting the best one. With the latest advances on computation power, development of exact solution algorithms has also emerged such as Belenguer, Benavent, Prins, Prodhon & Wolfler Calvo (2011) and Contardo, Cordeau & Gendron (2014) where the capacitated version of LRP (CLRP) with the capacitated depots is studied. To the best of our knowledge, the latest survey on LRP is by Prodhon & Prins (2014), and they address the first studies, heuristic approaches, solution methods, benchmark instances and different variants of LRP. The authors also emphasize that heuristic approaches are more studied in the LRP literature compared to the exact methods. The authors also propose valuable recommendations for future research directions, such as developing precise approaches, addressing realistic scenarios where not all customers are served, and exploring other relevant aspects.

The number of studies on LRP variants has been increasing recently. Moon et al. (2020) studied the LRP with multi-trip and multiple commodities. The authors consider a single echelon structure with multiple commodities where the vehicles perform multiple trips, if possible. They present a mathematical formulation and a heuristic solution approach consisting of a hybrid genetic algorithm, and compare them. They observe that the mathematical formulation solves the problem to optimality in small problem instances and use the heuristic approach for finding solutions for larger instances. Another variant of LRP which uses a distance constraint with electrical vehicles is studied by Almouhanna, Quintero-Araujo, Panadero, Juan, Khosravi & Ouelhadj (2020). Although a mathematical formulation for the problem is proposed, the problem is solved with heuristic methods such as multi-start biased-randomization and a biased randomized variable neighborhood search. The authors report that the proposed heuristic gives better results compared to the previous studies in terms of the depot opening and transportation costs. Furthermore, another related problem is the inventory-location-routing problem (ILRP) in e-commerce which is studied by Liu, Chen, Li, Liu & others (2015) and Deng, Li, Guo, Liu & others (2016) on a network where a direct transportation from the plant to merchandise centers and then routing from the centers to the retailers is considered. Both studies work with the same network considering return operation of the goods, but use different solution methodologies. The main difference of these studies from ours is that in ILRP, the decision maker also decides the inventory level at the retailers while in our problem the demand of customers is fixed. Besides, their problem has a stochastic environment as the demand at the retailers is assumed as uncertain, and this affects the whole modeling scheme.

Another problem that is in the scope of our study is the two-echelon location routing problem (2E-LRP) which is first studied by Jacobsen & Madsen (1980) based on a newspaper distribution case. In their problem description, there exists a single distribution center (DC) which is the newspaper printing office, and there are regional depots that will be used as transfer points for the printed newspapers. The problem's objective is to minimize the total depot locating and distance traveling cost. A two-phase method is proposed in their study. In first phase, they decide on which depots (transfer points) to open to satisfy the overall demand at a given service level. In second phase they use the savings algorithm to generate routes from their pre-decided opened depots. Prodhon & Prins (2014) point out in their LRP survey that no other study was conducted on 2E-LRP in the literature until Lin & Lei (2009). They proposed a genetic algorithm to identify uncapacitated DCs (satellite depots) while considering a collection of plants, big and small customers. The difference between big and small customers is their way of transportation of goods. The big clients can be served directly from the DC while small customers can not. The goal is to select a group of big customers to serve in the initial routing level and to create routes for both levels. Later, Contardo, Hemmelmayr & Crainic (2012) propose a branch-and-cut algorithm to solve 2E-CLRP instances to optimality including 50 customers and 10 satellites. Two-echelon vehicle routing problem (2E-VRP) can be seen as a special case of 2E-LRP where the locations of satellites/regional depots are already given - see the survey of Cuda et al. (2015) for the studies on this topic. Even if our problem seems like a two-echelon setting, our problem does not contain routing decisions from distribution centers to regional depot locations due to the e-commerce application we consider.

Vehicle routing problem with multiple trips (VRPMT) is an area which is studied immensely. In VRPMT, there exist a set of capacitated vehicles that can start and end their tours on a given depot. The sum of duration of the trips are limited and the vehicles can visit the depot multiple times for replenishment which is the main idea of the multi-trip mentality. Fleischmann (1990) considers a distribution system that consists of heterogeneous fleet with time window restrictions, and presents a constructive heuristic approach. However, there were no benchmark instances for VRPMT at that time. The first benchmark instance set for VRPMT is due to Taillard, Laporte & Gendreau (1996) where a tabu search algorithm with a parallel search structure is proposed for solving VRPMT. In their computational experiments, the authors consider the famous benchmark instances of Christofides et al. (1979) with different number of vehicles and distance restrictions, and generate the first benchmark instance set. In the following years, many heuristics and mathematical formulations have been proposed for VRPMT. The natural formulation of VRPMT requires four indices on the decision variables: two indices for the nodes, one index for the vehicle and another for the trip. Aghezzaf, Raa & Van Landeghem (2006) develops a three-index mathematical formulation that does not use the trip indexes while Azi, Gendreau & Potvin (2010) proposes another three-index mathematical formulation that does not use vehicle indices.

Koc & Karaoglan (2011) presents the first two-index mathematical model without trip and vehicle indices. Similar to Azi et al. (2010), the authors define additional binary variables to determine the last and the first nodes of the two consecutive trips of a vehicle, and propose a branch-and-cut algorithm that makes use of valid inequalities. Another two-index formulation for VRPMT is developed by Rivera, Afsar & Prins (2014) where auxiliary arcs and decision variables are introduced to model two consecutive trips performed by the same vehicle. Cattaruzza, Absi & Feillet (2016) provide a survey of VRPMTs including mathematical formulations and heuristic methods, the variants of the problem and the benchmark instances.

In general, LRP has an important application area on medical waste distribution. Recently, Tirkolaee, Abbasian & Weber (2021) study a combination of LRP with VRPMT where a case study on medical waste management on a network including hospitals, infirmaries and disposal sites, during Covid-19 is considered. The authors determine the locations of the disposal sites that will be opened given that the vehicles start their routes from a single parking site and visit hospitals and infirmaries for their medical waste in a given time window. They use chance constraints and consider multiple objectives. Similarly, Cheng, Zhu, Costa, Thompson & Huang (2022) study a disaster waste management problem on a system of landfills and recycling facilities. Another important application area of LRP is e-commerce. Pichka, Bajgiran, Petering, Jang & Yue (2018) mentions the potential economic earnings of using 2E-LRP structure in e-commerce logistics where a main factory could send products to smaller depots which will be closer to the customers.

To sum up, in this study we consider a location-routing problem that is observed in an e-commerce delivery network, wherein vehicles can make multiple trips. The inbound transportation cost between the main depot and the regional depots that will be opened is affected by both the demand and the distance between them. But, in this part of the network we consider direct transportation based on our observations in the real life applications in e-commerce. Furthermore, different from the literature, we permit multiple trips in the outbound transportation for utilizing relatively compact vehicles that are well-suited for urban transportation. We observe that there are few studies based on exact methods on the related topics, and our contribution lies in proposing several mathematical formulations for the problem we introduced.

3. PROBLEM DEFINITION AND MODEL FORMULATION

We consider a network that involves inbound and outbound operations, as well as the related decisions. This network comprises a single distribution center (DC), a set of possible regional depot locations, and a set of customers to be served. The transport of goods from the DC to the regional depots is referred as the inbound transportation, while the transportation from the regional depots to the customers is referred as the outbound transportation. In this problem, we mainly focus on three key decisions:

- Given a set of possible locations for regional depots, which of them will be opened to serve the customers?
- Which customer will be served by which opened regional depot?
- How the customers will be served from the regional depots (outbound transportation routes)?

We remark that the inbound transportation decisions are directly affected by the assignment of the customers to the regional depots that will be opened. Hence, though we do not indicate the inbound transportation decisions separately, they are determined based on the last two decisions stated above.

We consider a setting where each customer should be served by a single regional depot. Besides, the fleet of identical vehicles should be also allocated to the opened regional depots. Each customer should be visited exactly once by a truck that is routed from the customer's regional depot. Different from the traditional routing problems, we allow trucks used for outbound transportation operations to perform multiple trips. Each truck has to start and end its route in the same depot whether it has a single or multi-trip.

There is a fixed cost for opening a regional depot. There are no capacity restrictions for the inbound/outbound transportation volumes that can be operated from the regional depots. This assumption is based on the expectation that the depots will be opened/deployed according to the planned amounts from the solution we provide.

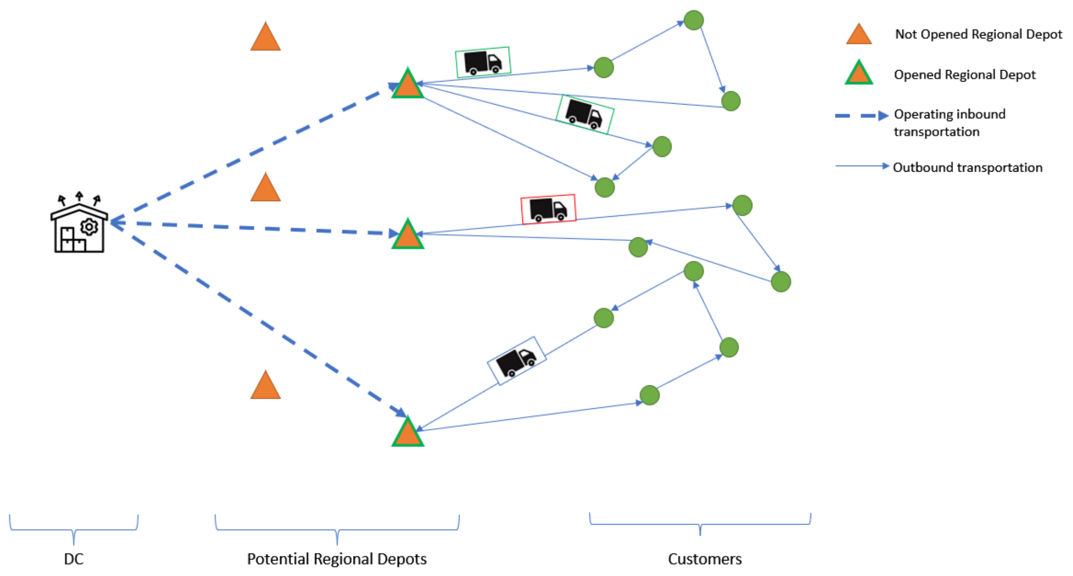
The transportation of goods from the DC to regional depots is a direct transportation mode. On the other hand, the transportation from the regional depots to the customers is performed like a multi-trip VRP (VRPMT) with truck usage costs. We call this problem as LRPMT.

3.1 Problem Definition

Consider a directed graph $G = (N, A)$ with the set of nodes $N = I \cup J$ where I and J represent the set of possible locations for the regional depots and the customers, respectively. We consider a partially complete graph by $A = ((N \times N)/(I \times I))$ where only the arcs connecting the regional depots to each other are ignored, i.e. transportation between regional depots is not allowed.

We assume that the DC is the main source of the goods that will be sent to the customers. But, instead of directly serving the customers from the DC, the goods are first sent to the regional depots opened by direct transportation, and then distributed to the customers through routes of the trucks originated from the regional depots. The cost of operating regional depot $i \in I$ is given by f_i. The demand of customer $j \in J$ is denoted by D_j.

Figure 3.1 Example of the Network Described



We assume that different types of vehicles are available for inbound and outbound transportation. For the vehicles dedicated to the inbound transportation, b_i represents the unit cost of moving goods from the DC to regional depot $i \in I$. For the vehicles that will be used for outbound transportation, we assume that there is a fixed cost c of using each vehicle. The reason for considering a fixed cost of c is to analyze and observe the effect of deciding on routes and depot locations simultaneously. In other words, the fixed cost of using a vehicle can be seen as the scaling factor between the two different objective components: the total fixed cost of opening a regional depot and the number of vehicles used. Due to the cost structure considered for the inbound transportation, the capacities of the vehicles are not important and ignored. The capacity of the vehicles used for the outbound transportation is given by Q, and we assume that there exist $|K|$ homogeneous vehicles available for outbound transportation. To make it easier to follow, we give the list of parameters of LRPMT in Table 3.1. Figure 3.1 represents an example network of LRPMT.

Table 3.1 The list of parameters of LRPMT

Parameter	Definition
I	Set of candidate regional depot locations
J	Set of customers to be served
N	Set of all regional depot and customer locations, $N = I \cup J$
K	Set of available vehicles
C_{max}	Maximum distance a vehicle can travel during the outbound transportation
D_j	Demand of customer $j \in J$
Q	Capacity of vehicles used for outbound transportation
d_{ij}	Distance between nodes $i \in N$ and $j \in N$
f_i	Fixed cost of opening regional depot $i \in I$
b_i	Unit transportation cost from the distribution center to regional depot $i \in I$
c	Fixed cost of using a truck for outbound transportation

The objective of LRPMT is to minimize the overall operation cost which includes the fixed cost of opening regional depots, the inbound transportation cost from the DC to the regional depots, and the cost of using vehicles for the outbound transportation. The inbound transportation cost consists of the total demand and distance between DC and any regional depot and we do not consider a distance based cost in outbound transportation.

3.2 Mathematical Models

In this section, we present three different mathematical models for LRPMT. The models basically differ from each other based on the number of indices of the decisions variables.

3.2.1 3IM: Three-Index Model

3IM is a three-index model where vehicle related variables are defined for each vehicle separately, but no indices are defined for the trips. Inspiring from the three index model of Aghezzaf et al. (2006) where a mathematical model for an inventory routing problem is proposed, in 3IM we allow vehicles to visit their regional depots multiple times to perform multiple trips.

Let y_i be equal to 1 if the regional depot $i \in I$ is opened; 0 otherwise. The binary decision variable ϕ_{ij} equals to 1 if the demand of customer $j \in J$ is satisfied from regional depot $i \in I$, 0 otherwise. To allocate the vehicles to the regional depots and the customers, we define the binary variable z_{ik} which will be equal to 1 if truck $k \in K$ is assigned to regional depot $i \in I$ or customer $i \in J$; 0 otherwise. The binary variable z_{ik} is introduced also for customers due to the presence of symmetry-breaking constraints involving both customer and truck indexes. These constraints are elaborated upon in the subsequent sections. The binary decision variable x_{ijk} equals to 1 if truck $k \in K$ traverses arc $(i, j) \in A$; 0 otherwise. Finally, we define the decision variable q_{ijk} to determine the total load of truck $k \in K$ while traversing the arc $(i, j) \in A$.

3IM is given below:

$$\text{(3.1a)} \qquad \min \quad \sum_{i \in I} f_i y_i + \sum_{i \in I} b_i \sum_{j \in J} D_j \phi_{ij} + c \sum_{k \in K} \sum_{i \in I} z_{ik}$$

$$\text{(3.1b)} \qquad \text{s.t.} \quad \sum_{j:(i,j) \in A} x_{ijk} = \sum_{j:(j,i) \in A} x_{jik} = z_{ik} \qquad i \in J, k \in K$$

$$\text{(3.1c)} \qquad \sum_{j:(i,j) \in A} x_{ijk} = \sum_{j:(j,i) \in A} x_{jik} \leq |J| z_{ik} \qquad i \in I, k \in K$$

$$\text{(3.1d)} \qquad \sum_{k \in K} z_{ik} = 1 \qquad i \in J$$

$$\text{(3.1e)} \qquad \sum_{i \in I} z_{ik} \leq 1 \qquad k \in K$$

$$\text{(3.1f)} \qquad z_{ik} \leq y_i \qquad i \in I, k \in K$$

$$\text{(3.1g)} \qquad \sum_{(i,j) \in A} x_{ijk} d_{ij} \leq C_{max} \qquad k \in K$$

$$\sum_{j:(j,i)\in A} q_{jik} - \sum_{j:(i,j)\in A} q_{ijk} = D_i z_{ik} \qquad i \in J, k \in K \tag{3.1h}$$

$$q_{ijk} \leq Q x_{ijk} \qquad (i,j) \in A, k \in K \tag{3.1i}$$

$$z_{ik} + z_{jk} - 1 \leq \phi_{ij} \qquad i \in I, j \in J, k \in K \tag{3.1j}$$

$$\sum_{i\in I} \phi_{ij} = 1 \qquad j \in J \tag{3.1k}$$

$$\phi_{ij} \leq y_i \qquad i \in I, j \in J \tag{3.1l}$$

$$y_i \in \{0,1\} \qquad i \in I \tag{3.1m}$$

$$x_{ijk} \in \{0,1\} \qquad (i,j) \in A, k \in K \tag{3.1n}$$

$$z_{ik} \in \{0,1\} \qquad i \in N, k \in K \tag{3.1o}$$

$$\phi_{ij} \in \{0,1\} \qquad i \in I, j \in J \tag{3.1p}$$

$$q_{ijk} \geq 0 \qquad (i,j) \in A, k \in K \tag{3.1q}$$

In the objective function (3.1a), we minimize the total cost of opening regional depots, and the inbound and outbound transportation. Constraints (3.1b) ensure the inflow-outflow balance of the routing decision variables for the customers. In other words, if vehicle $k \in K$ arrives customer $i \in J$, then it should also leave that customer. Moreover, this is only possible if $z_{ik} = 1$, i.e. customer $i \in J$ is served by vehicle $k \in K$. Constraints (3.1c) are the modifications of constraints (3.1b) for the regional depot locations. If vehicle k is assigned to regional depot $i \in I$, then the number of times vehicle k leaves depot i should be equal to the number of its arrival times. Note that in the traditional VRP models, this number can be at most one. But, since we allow multiple trips, it can be greater than one in our model. But, it can be at most $|J|$ since each customer can be visited at most once. Constraints (3.1d) ensure that each customer can be visited by a single truck, and constraints (3.1e) ensure that a truck is assigned to at most one regional depot. Constraints (3.1f) relate the binary variables z and y: truck k might serve to regional depot i only if depot i is opened. Constraints (3.1g) guarantee the total distance that is traveled by truck k does not exceed the maximum distance allowed per truck, denoted by C_{max}. Constraints (3.1h) ensure that the difference between the total load of vehicle k while arriving and leaving customer i is equal to the demand of customer i if vehicle k visits customer i, and zero otherwise. Constraints (3.1i) make sure that the total load of vehicle k while traversing arc (i,j) does not exceed the capacity Q. Constraints (3.1j) relate the binary variables z and ϕ: if truck k visits both the regional depot i and the customer j, then j should be assigned to i. Constraints (3.1k) ensure that each customer j is served by exactly one regional depot, and (3.1l) relate the binary variables ϕ and y by ensuring that ϕ_{ij} is zero if y_i in zero, i.e. if a regional depot at i is not opened, then i cannot serve to any customer. Finally, constraints (3.1m)-(3.1p) define the ranges and bounds of the decision variables.

3.2.1.1 Symmetry Breaking Inequalities for 3IM

3IM suffers from the solution symmetry since the same solution can be observed under different assignments of the journeys to the vehicles as the vehicles are identical. Based on the symmetry breaking techniques available in the literature Darvish, Coelho & Jans (2020), we consider the following symmetry breaking inequalities for 3IM:

$$\sum_{i \in I} z_{ik} \leq \sum_{i \in I} z_{ik-1} \qquad k \in K \setminus \{1\} \tag{3.2a}$$

$$z_{ik} \leq \sum_{j \in J : j \leq i-1} z_{jk-1} \qquad i \in J, k \in K \setminus \{1\} \tag{3.2b}$$

$$z_{ik} \leq \sum_{j \in J : j \leq i-1} z_{jt} \qquad i \in J, t \in \{1,, ..., k-1\}, k \in K \setminus \{1\} \tag{3.2c}$$

$$(k-1) z_{ik} \leq \sum_{j \in J : j \leq i-1} \sum_{t=1}^{k-1} z_{jt} \qquad i \in J \setminus \{1\}, k \in K \setminus \{1\} \tag{3.2d}$$

Inequalities (3.2a) assure that vehicle k can be used only if vehicle $k-1$ is used. Inequalities (3.2b) ensure that if customer i is visited by vehicle k then at least one customer with a smaller index than i must be visited by truck $k-1$. Similarly, due to inequalities (3.2c) if customer i is visited by vehicle k, then all vehicles smaller than k should visit at least one customer with a smaller index than i. Finally, inequalities (3.2d) aggregate (3.2c) over the vehicles with smaller indices.

3.2.2 2IM: Two-Index Model

2IM is a two-index model that does not include any vehicle or trip indices, and it is inspired from Koc & Karaoglan (2011) where additional decisions variables are defined for relating the last and the first customers visited in two consecutive trips of a vehicle.

The binary variables y and ϕ are the same with 3IM: y_i will be equal to 1 if the regional depot $i \in I$ is opened, and ϕ_{ij} will be equal to 1 if $j \in J$ is served by the regional depot $i \in I$. We define the binary decision variable x_{ij} which is equal to 1 if there is a vehicle traversing from node i to node j; and 0 otherwise. The binary decision variable w_{ij} equals to 1 if a trip that ends at customer i and another trip that starts with customer j are performed by the same vehicle. Finally, we define two types of continuous decision variables: q_{ij} denotes the load of the vehicle while traversing the arc (i,j), and L_j represents the total distance traveled by a vehicle while visiting customer $j \in J$.

2IM is as follows:

(3.3a) $\min c(\sum_{j\in J}\sum_{a\in I} x_{aj} - \sum_{i\in J}\sum_{j\in J} w_{ij}) + \sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{i\in J} b_i D_j \phi_{ij}$

(3.3b) s.t. $\sum_{j:(i,j)\in A} x_{ij} = \sum_{j:(j,i)\in A} x_{ji} = 1 \qquad i \in J$

(3.3c) $\sum_{j:(i,j)\in A} x_{ij} = \sum_{j:(j,i)\in A} x_{ji} \le |J| y_i \qquad i \in I$

(3.3d) $\phi_{ij_2} \ge \phi_{ij_1} + x_{j_1 j_2} - 1 \qquad j_1, j_2 \in J,\ i \in I$

(3.3e) $x_{ij} \le \phi_{ij} \qquad j \in J,\ i \in I$

(3.3f) $x_{ji} \le \phi_{ij} \qquad j \in J,\ i \in I$

(3.3g) $\sum_{i\in I} \phi_{ij} = 1 \qquad j \in J$

(3.3h) $\sum_{j\in J} w_{ij} \le \sum_{a\in I} x_{ia} \qquad i \in J$

(3.3i) $\sum_{i\in J} w_{ij} \le \sum_{a\in I} x_{aj} \qquad j \in J$

(3.3j) $w_{ij} \le 2 - \phi_{a1,j} - \sum_{a2\in I/a1} \phi_{a2,i} \qquad a1 \in I,\ i,j \in J$

(3.3k) $\sum_{i\in J}\sum_{a\in I} x_{aj} - \sum_{i\in J}\sum_{j\in J} w_{ij} \le |K|$

(3.3l) $x_{ij} \le y_i \qquad i \in I,\ J \in J$

(3.3m) $\phi_{ij} \le y_i \qquad i \in I,\ J \in J$

(3.3n) $\sum_{j:(j,i)\in A} q_{ji} - \sum_{j:(i,j)\in A} q_{ij} = D_i \qquad i \in J$

(3.3o) $q_{ij} \le Q x_{ij} \qquad (i,j) \in A$

(3.3p) $L_i + d_{ij} x_{ij} \le L_j + C_{max}(1 - x_{ij}) \qquad i \in J,\ j \in J,\ i \neq j$

(3.3q) $L_i + \sum_{a\in I} (d_{ia}\phi_{ai} + d_{aj}\phi_{aj}) \le 2C_{max}(1 - w_{ij}) + L_j \qquad i \in J,\ j \in J,\ i \neq j$

(3.3r) $\sum_{a\in I} d_{aj}\phi_{aj} \le L_j \le C_{max} - \sum_{a\in I} d_{ja}\phi_{aj} \qquad j \in J$

(3.3s) $y_i \in \{0,1\} \qquad i \in I$

$$x_{ij} \in \{0,1\} \qquad (i,j) \in A \tag{3.3t}$$

$$w_{ij} \in \{0,1\} \qquad i,j \in J \tag{3.3u}$$

$$\phi_{ij} \in \{0,1\} \qquad i \in I, j \in J \tag{3.3v}$$

$$q_{ij} \geq 0 \qquad (i,j) \in A \tag{3.3w}$$

$$L_i \geq 0 \qquad i \in J \tag{3.3x}$$

The objective function (3.3a) minimizes the total truck usage cost for the outbound transportation, the regional depot opening cost and the inbound transportation cost. Note that the number of vehicles used in the outbound transportation is given by $\sum_{j\in J}\sum_{a\in I} x_{0j} - \sum_{i\in J}\sum_{j\in J} w_{ij}$ since we allow multiple trips. Constraints (3.3b) ensure that every customer is visited exactly once. Constraints (3.3c) guarantee that the number of arrivals and the departures are equal to each other for each regional depot i and can be positive only if i is opened. Constraints (3.3d) relate the binary variables ϕ and x for the customers visited in the middle positions of the tours: if customer j_1 is assigned to the regional depot i and if arc (j_1, j_2) is traversed in the solution, then customer j_2 should be also served by the regional depot i. Similarly, constraints (3.3e) and (3.3f) relate the binary variables ϕ and x for the customers visited in the first and the last positions of the tours: if the arcs (i,j) or (j,i) are traversed for a regional depot i and a customer j, then j should be served by the regional depot i. The constraints (3.3g) ensure that each customer j is served from a single regional depot. The constraints (3.3h) and (3.3i) relate the binary variables w and x: w_{ij} can be 1 for two customers $i, j \in J$ if i is the last node of a tour, i.e. $\sum_{a\in I} x_{ia} = 1$, and j is the first node of a tour, i.e. $\sum_{a\in I} x_{aj} = 1$. Besides, w_{ij} can be 1 for two customers $i, j \in J$, if both of them are assigned to the same service region, and this is achieved by (3.3j). Constraints (3.3k) limit the number of vehicles that can be used in the outbound transportation by $|K|$. The constraints (3.3l) and (3.3m) relate the binary variables x, y and ϕ: if the regional depot i is not opened, then x_{ij} and ϕ_{ij} can not be 1 for any customer j. Constraints (3.3n) and (3.3o) are demand satisfaction and vehicle capacity constraints, respectively. Constraints (3.3p) and (3.3q) determine the total distance traveled by a vehicle while visiting customer j for the cases where j is a customer that is visited in the same tour with the previous node and j is the first customer visited in a new tour of a vehicle, respectively. Finally, the constraints (3.3r) ensure the maximum distance allowance for each vehicle, and the remaining constraints (3.3s) - (3.3x) define the ranges and boundaries of the variables.

3.2.2.1 Valid Inequalities for 2IM

We add the following valid inequalities to 2IM for relating binary decision variables x and ϕ which is inspired from Koc & Karaoglan (2011):

$$\sum_{j\in J} x_{ij} \geq \frac{1}{Q}\sum_{j\in J} D_j \phi_{ij} \qquad i \in I \tag{3.4}$$

Inequalities (3.4) ensure that the number of departures from a regional depot i is larger the lower bound on the number of vehicles required to satisfy the demand of the customers assigned to i which is given by the right-hand-side of the inequality.

3.2.2.2 Symmetry Breaking Inequalities for 2IM

2IM also suffers from the solution symmetry due to the decision variables x and w. Note that two tours performed by a vehicle can be merged in two ways (symmetry with respect to w). For instance, if there exist two tours performed by the same vehicle originating from the regional depot 0 as $0-1-4-5-0$ and $0-3-2-0$, setting $w_{53}=1$ or $w_{21}=1$ result in two different but equivalent solutions. Similarly, the tours can be reversed without changing the cost and the feasibility: the tours $0-1-4-5-0$ and $0-5-4-1-0$ result in two different but equivalent solutions (symmetry with respect to x).

We consider the following symmetry breaking constraints for 2IM:

$$w_{ij}=0 \qquad i,j \in J, \quad i>j \tag{3.5}$$

In constraints (3.5), we allow w_{ij} to be 1 if customer j's index is bigger than i: $j>i$.

3.2.3 A-2IM: An Alternative Two-Index Model

A-2IM is an alternative two-index model that is inspired from Rivera et al. (2014) where auxiliary arcs, called as replenishment arcs, are defined between the customers to represent two consecutive trips performed by the same vehicle. So, in A-2IM we use the same decision variables with 2IM, but different from 2IM, here we ensure that x_{ij} is one for at most one customer $j \in J$ for each regional depot $i \in I$, and w_{jl} can be one without requiring x_{ji} and x_{il} to be one for some regional depot i. But, $w_{jl}=1$ still represents the case where the last and the first customers visited by two consecutive trips of a vehicle are j and l, respectively. Note that this can happen if both of the customers are assigned to the same regional depot.

Our alternative two-index model A-2IM is given as follows:

$$\text{(3.6a)} \qquad \min \quad c\sum_{i\in I}\sum_{j\in J} x_{ij} + \sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{i\in J} b_i D_j \phi_{ij}$$

$$\text{(3.6b)} \qquad \text{s.t.} \quad \sum_{j:(i,j)\in A} x_{ij} + \sum_{j\in J} w_{ij} = \sum_{j:(j,i)\in A} x_{ji} + \sum_{j\in J} w_{ji} = 1 \qquad i \in J$$

$$\text{(3.6c)} \qquad \sum_{j:(i,j)\in A} x_{ij} = \sum_{j:(j,i)\in A} x_{ji} = y_i \qquad i \in I$$

$$\text{(3.6d)} \qquad \sum_{i\in J}\sum_{i\in I} x_{ij} \le |K|$$

$$\text{(3.6e)} \qquad q_{ij} \le Q x_{ij} \qquad i \in J,\ j \in N$$

$$\text{(3.6f)} \qquad q_{ij} \le Q(x_{ij} + \sum_{k\in J} w_{kj}) \qquad i \in I,\ j \in J$$

$$\text{(3.6g)} \qquad q_{ij} \le Q\phi_{ij} \qquad i \in I,\ j \in J$$

$$(3.3\text{d}) - (3.3\text{g}), (3.3\text{j}), (3.3\text{l}) - (3.3\text{n}), (3.3\text{p}) - (3.3\text{x}).$$

In A-2IM, since $x_{ij} = 1$ only for the first customer j that is visited in the first tour of a vehicle originating from the regional depot i, $\sum_{i\in I}\sum_{j\in J} x_{ij}$ gives the number of vehicles that are used in the outbound transportation. We make the necessary changes in the objective function (3.6a) and (3.6d) according to this observation. Similarly, the flow conservation constraints (3.6b) and (3.6c), and the capacity constraints (3.6e) - (3.6g) are updated based on the new representation of the variables x and w.

Note that the valid inequalities (3.4) and the symmetry breaking constraints (3.5) can be also used with A-2IM.

3.2.4 RBM: Route-Based Model

In this section, we present our route based model (RBM) that gives a feasible solution for the problem in shorter times. Assume that we have a pre-generated set of feasible routes (trips, indeed) denoted by $R = \{1, \ldots.., r\}$ where each route $r \in R$ starts and ends at the same regional depot. Assume that we know the values of the parameters for each route $r \in R$ given in the following table. We will determine a feasible solution for LRPMT by considering only these routes which is inspired from Ercan (2019).

We again define the binary decision variable y_i which is equal to 1 if the regional

Table 3.2 Route-based model additional parameters

Parameter	Definition
R	Set of routes for outbound transportation considering all candidate regional depot locations
C_r	Total distance of the route $r \in R$
R_i	Set of routes that are originating from regional depot $i \in I$
R_j	Set of routes that contain customer $j \in J$
J_r	Customers covered in route $r \in R$

depot i is opened; 0 otherwise. Let v_{rk} be equal to 1 if route r is used by vehicle k; 0 otherwise, and z_{ik} be equal to 1 if truck k is used and assigned to depot i; 0 otherwise. In our model, we allow the vehicles to visit a subset of the customers that belong to a route. Hence, determining the customers that will be visited from a route is also a decision. The binary decision variable x_{jrk} will be equal to 1 if truck k which uses route r visits customer j; and 0 otherwise.

The following MIP determines the best possible solution for LRPMT given the pre-generated route set R:

$$
\begin{aligned}
&(3.7a) && \min && c\sum_{i\in I}\sum_{k\in K} z_{ik} + \sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{r\in R_i}\sum_{J\in J_r}\sum_{k\in K} b_i D_j x_{jrk} \\
&(3.7b) && \text{s.t.} && \sum_{r\in R_j}\sum_{k\in K} x_{jrk} = 1 \qquad j \in J \\
&(3.7c) && && \sum_{j\in J_r} D_j x_{jrk} \leq Q v_{rk} \qquad r \in R, k \in K \\
&(3.7d) && && \sum_{k\in K} v_{rk} \leq 1 \qquad r \in R \\
&(3.7e) && && v_{rk} \leq z_{ik} \qquad k \in K, i \in I, r \in R_i \\
&(3.7f) && && v_{rk} \leq y_i \qquad k \in K, i \in I, r \in R_i \\
&(3.7g) && && \sum_{r\in R} C_r v_{rk} \leq C_{max} \qquad k \in K \\
&(3.7h) && && \sum_{i\in I} z_{ik} \leq 1 \qquad k \in K \\
&(3.7i) && && y_i \in \{0,1\} \qquad i \in I \\
&(3.7j) && && x_{jrk} \in \{0,1\} \qquad j \in J, r \in R, k \in K \\
&(3.7k) && && z_k \in \{0,1\} \qquad k \in K \\
&(3.7l) && && v_{rk} \in \{0,1\} \qquad r \in R, k \in K
\end{aligned}
$$

The objective function (3.7a) again minimizes the total truck usage cost for the outbound transportation, regional depot opening cost and the inbound transportation cost. Constraints (3.7b) ensure that each customer is visited exactly once. Constraints (3.7c) are the capacity constraints that are guaranteeing the total demand of the customers visited by vehicle k throughout the route r does not exceed the truck capacity Q. Constraints (3.7d) ensure that each route r is used by at most one vehicle. Constraints (3.7e) ensure that if a route r originated from depot i is used by vehicle k then the same vehicle k must work with that depot. Due to constraints (3.7f) route r can be used by vehicle k if the depot that route r originates from is opened. Constraints (3.7g) limit the total distance that can be traveled by any vehicle over all routes it performs. Constraints (3.7h) ensures that each truck can operate at most 1 regional depot. Finally, constraints (3.7i) - (3.7k) define the ranges and boundaries of the variables.

Note that when R includes all possible tours, our route-based model (3.7) turns into an exact method with exponential number of decision variables. However, since this is not possible in practice, (3.7) is a heuristic approach in the sense that a subset of feasible tours are generated in R, and the model gives the best solution among the possible solutions that can be obtained from the routes of R.

For our route based model (RBM), we generate the route set R using the following algorithm which is called Expanded Nearest Neighborhood Search (ENNS) (Ercan, 2019). For each customer $j \in J$, we determine the shortest distance between j and any regional depot location, and then set the routing diameter R_d to the maximum of these distances among all customers. Then, for every regional depot location $i \in I$, if the distance between i and a customer $j \in J$ does not exceed R_d, we include j to J_i which is the set of customers that can be served from i, and initialize the ENNS algorithm. For each $i \in I$ and $j \in J_i$, we initialize a route from i to j and construct the remaining part of the route according to the Nearest Neighborhood Algorithm, where the customer addition to the route stops when $2R_d$ distance limit has been reached. We apply this procedure for each regional depot location to construct the set of all routes R.

The following pseudo-code gives the ENNS algorithm flow for generation of route set R

Algorithm 1 Expanded Nearest Neighbor Heuristic

Require: Set of regional depots I, set of customers J, candidate sets of visitable customers J' for each regional depot, maximum route distance parameter $2R_d$

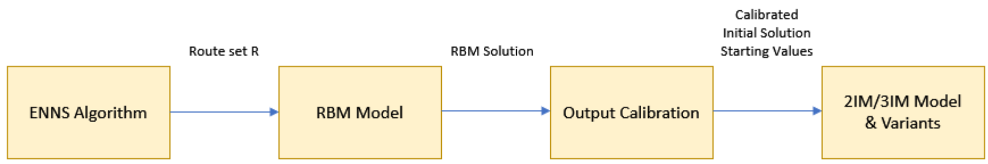
Ensure: Routes covering all visitable customers

for each regional depot $i \in I$ **do**
    for each visitable customer set J_i for regional depot i **do**
        Initialize an empty route R
        Initialize a set *visited* to keep track of visited customers
        for each customer j in J_i **do**
            Add j to R and *visited*
            while $J_i' \setminus visited \neq \emptyset$ **do**
                Find nearest neighbor $j \in J_i \setminus visited$ to the last customer in R
                if Distance between last customer in R and $j' > 2R_d$ **then**
                    Add current route R to the list of routes
                    Clear R and *visited*
                    Add j to R and *visited*
                    break ▷ Terminate the inner loop
                else
                    Add j to R and *visited*
                end if
            end while
            Add current route R to the list of routes
            Clear R and *visited*
        end for
    end for
end for

We make use the route-based model (3.7) for determining a feasible solution for LRPMT in short computation times. After a feasible solution for LRPMT is obtained from the route-based model (3.7), we provide it to the models presented in the previous sections as an initial solution, and run them further to improve the initial solution as much as possible within the time limit. Figure 3.2 illustrates the flow of our warm start approach. For models 2IM and A-2IM, since we do not have vehicle indices, we convert the solution obtained from (3.7) to the appropriate form (calibration step).

After generating our route set R, we will use of RBM model as an initial solution giver to our 2IM and 3IM models and their variants. The following image shows the overall input-output flow of how RBM model is used as a warm starting(initial solution) method in general.

Figure 3.2 Warm start strategy using RBM Model



4. COMPUTATIONAL RESULTS

In this chapter, we present the computational results for the models presented in the previous chapter. First, we describe our data generation approach, and then, we discuss the computational performance of the methods of previous chapter. Finally, we present our findings for business and modeling insights.

Throughout the experiments, we use GUROBI 10.0 with PYTHON 3.7 on an HPC system with Intel(R) Xeon(R) Gold 5122 CPU 8 core processor with 3.60 GHz speed, 128 GB RAM, and 64-bit Linux operating system. All the coding for data reading, model preparation, and output generation are implemented in Python 3.7 with Anaconda Spyder. The time limit is set to 4 hours (14400 seconds) unless otherwise is stated, and TL is used to represent it in the tables when the solver terminates due to the time limit. Furthermore, in RBM we set the time limit to 600 seconds for the route-based model (3.7).

4.1 Experimental Data Design

LRPMT has no benchmark instances in literature. In order to generate different problem instances in different sizes, we make use of CMT4 problem instance of Christofides (1979) which contains a single depot and the locations and the demand rates for 150 customers. Location of the DC in our instances is set to the location of the depot in CMT4 instance. To test our methods with small to large scale problem instances, we generate problem instances with:

- 2 possible regional depot locations, 10 customers: $|I| = 2, |J| = 10$
- 5 possible regional depot locations, 15 customers: $|I| = 5, |J| = 15$
- 10 possible regional depot locations, 30 customers: $|I| = 10, |J| = 30$

- 15 possible regional depot locations, 50 customers: $|I| = 15$, $|J| = 50$

Customers of these instances are selected randomly from the CMT4 instance along with their coordinates and the demand data. Possible locations for the regional depots are determined using the k-means algorithm, in sense of the distribution of customer locations might have an effect on pre-decided candidate regional depots (Hartigan & Wong, 1979). The k-means algorithm involves iteratively assigning data points to the clusters based on their distance from cluster centers and updating the centers to minimize the distance within each cluster.

Regional depot opening cost, denoted by f_i for $i \in I$, is generated randomly from U[20000, 40000]. The additional parameters for our problem including the inbound and outbound transportation costs, vehicle capacity, and the maximum route length restriction are decided as follows. We determine 3 different levels for the outbound vehicle capacity Q by scaling the average demand of each instance with 3, 4 and 5, i.e. $Q = \beta\bar{d}$ where $\bar{d}$ is the average customer demand and $\beta \in \{3, 4, 5\}$. For example, if the total demand for an instance that has 10 customers is 180, then the average demand for this instance is $\bar{d} = 18$, and the vehicle capacity Q is set to 54, 72 and 90 for $\beta = 3, 4, 5$, respectively.

We assume that the unit vehicle usage cost for the outbound transportation depends on the vehicle capacity. The unit vehicle usage cost c for the smallest vehicle is 2500, and it increases proportionally with the capacity of the truck by the coefficient 0.8 for representing the economies of scale. We assume that the unit inbound transportation cost b_i for regional depot location $i \in I$ is proportional to the distance between the DC and i, and given by $b_i = t_i$ where t_i represents the distance between the DC and the regional depot location i. For the maximum distance limitation C_{max}, we consider three different levels by $C_{max} = \gamma\bar{C}$ where $\bar{C}$ represents the average distance traveled in the optimal solution of the VRP model, i.e. total distance traveled is divided by the number of vehicles used in the optimal solution, and $\gamma = 1, 2, 3$.

Consequently, for each I and J setting, we consider 9 different parameter values - 3 different β values, 3 different γ values. We also generate 3 random problem instances for each setting. Since there are 4 different I and J levels, we generate 108 problem instances in total.

4.2 Computational Results for 3IM

In this section, we present the results for the three-index model (3IM) given in the previous chapter, the three-index model with valid inequalities (3IM-V), and 3IM-V with a route-based model used as a warm start (3IM-V-RBM).

This part aims to observe the performance of 3IM in solving problem instances with different sizes, and the effect of enhancements in the solution time and the optimality gaps.

4.2.1 3IM and The Symmetry Breaking Constraints

In this section, we analyze the effect of using the symmetry breaking constraints (3.2) in 3IM. Here, we consider all of the 9 different parameter settings for the 3 random problem instances with 2 regional depots and 10 customers, denoted by D10-1, D10-2, D10-3, and with 5 regional depots 15 customers, denoted by D15-1, D15-2, D15-3, and we report the average results over all parameter setting. More specifically, in Table 4.1 we report the average run time (in seconds) for the instances that are solved to optimality within the time limit, average percentage gap for the instances that cannot be solved within the time limit and the number of instances where an optimal solution is found.

Table 4.1 Effect of symmetry breaking inequalities in 3IM

	3IM			**3IM-V**		
Data Set	Avg. Run Time (s)	Avg. Gap (%)	Optimal #	Avg. Run Time (s)	Avg. Gap (%)	Optimal #
D10-1	40	0.00	9	5	0.00	9
D10-2	38	0.00	9	4	0.00	9
D10-3	230	0.00	9	4	0.00	9
D15-1	2157	10.98	3	433	0.00	9
D15-2	6061	5.42	2	414	0.00	9
D15-3	1125	13.75	4	527	0.00	9

Table 4.1 shows the efficiency of symmetry breaking inequalities in 3IM by reducing the solution times. We see that all instances of D10-1 and D10-2 are solved to optimality by both 3IM and 3IM-V in very short times. Considering the data set D15, the number of instances solved to optimality by 3IM within the time limit decreases drastically. However, 3IM-V again finds optimal solutions for this data set in relatively shorter times. Hence, we can conclude that 3IM with symmetry breaking constraints performs definitely better in terms of the solution times and the optimality gaps reported.

Table 4.2 The worst 3 results of 3IM for the problem instances with 15 customers

Data Set	β	γ	Gap (%)
D15-1	5	1	23.66
	4	1	12.60
	3	1	12.02
D15-2	5	1	15.38
	3	2	6.85
	4	2	5.02
D15-3	5	1	32.81
	3	1	21.18
	4	2	7.44

Table 4.2 shows the worst 3 results of 3IM for the problem instances with 15 customers (D15). Recall that β and γ represent different levels of the vehicle capacity Q, and the maximum allowed distance per vehicle C_{max}, respectively. Note that the worst optimality gaps within the time limit are observed when the truck capacity is large and the max route length is small, i.e. $\beta = 5$, $\gamma = 1$. One potential explanation for this situation is that finding a feasible solution might be also hard in this case. Hence, improving the solution and decreasing the optimality gap is also harder for 3IM without symmetry breaking inequalities. From Table 4.2, we also see that the results for the last random instance (D15-3) is worse than the other two random instances, in general, and this shows the affect of the data in the problem difficulty.

As a result of these observations, in the remainder of the thesis we ignore 3IM and continue our tests considering 3IM-V.

4.2.2 3IM-V with a Warm Start

In this section, we discuss the results of 3IM-V and 3IM-V with a warm start based on our route-based model (3IM-V-RBM) that are given in Table 4.3.

We see that the performance of 3IM-V is affected significantly by the size of the problem instance. Although 3IM-V solves all problem instances with 10 and 15 customers to optimality, its performance declines considerably in the problem instances with 30 and 50 customers. From Table 4.3, we observe that 3IM-V terminates with optimality gaps between 25% and 35% in all D30-1 instances and in one D30-3 instance within the time limit. The results get worse when the number of customers increases to 50. Note that 3IM-V cannot find a feasible solution within the time limit - 5, 7 and 6 out of 9 for D50-1, D50-2 and D50-3, respectively.

Table 4.3 Three-index model with symmetry breaking constraints and with warm start

	3IM-V			3IM-V-RBM			
Data Set	Avg Run Time (s)	Avg Gap (%)	Opt # (Feas. #)	Avg Run Time (s)	Avg Initial Gap %	Avg Gap (%)	Opt # (Feas #)
D10-1	5	0.00	9 (9)	4	47.23	0.00	9 (9)
D10-2	4	0.00	9 (9)	4	25.12	0.00	9 (9)
D10-3	4	0.00	9 (9)	3	38.46	0.00	9 (9)
D15-1	433	0.00	9 (9)	529	56.63	0.00	9 (9)
D15-2	414	0.00	9 (9)	342	54.33	0.00	9 (9)
D15-3	527	0.00	9 (9)	803	67.43	0.00	9 (9)
D30-1	TL	29.23	0 (9)	TL	84.78	30.08	0 (9)
D30-2	7251	41.06	3 (6)	TL	74.58	29.37	0 (9)
D30-3	3683	33.39	2 (7)	3658	83.72	28.35	1 (8)
D50-1	TL	50.19	0 (4)	TL	81.45	62.15	0 (9)
D50-2	TL	58.06	0 (2)	TL	84.03	67.39	0 (9)
D50-3	TL	50.75	0 (3)	TL	87.92	68.27	0 (9)

The results of 3IM-V-RBM can be seen in the right part of Table 4.3. As problem instances with 10 and 15 customers are already solved by 3IM-V very efficiently, we observe no significant improvement with the addition of the warm start in these data sets. Furthermore, average solution times slightly increase in some setting due to the warm start. Besides, we observe that including the warm start reduces the number of instances that are solved to optimality in some D30 instances which is an undesired case. In other words, 3IM-V solves 3 instances (out of 18 instances) of D30-2 to optimality while 3IM-V-RBM can not solve any. Hence, no improvement is obtained, and additionally some decline is obtained with the addition of warm start in D30 instances. But, for the instances with 50 customers, since our route-based model model gives an initial feasible solution in very short times, 3IM-V-RBM reports a feasible solution for all instances while 3IM-V cannot find a feasible solution for more than half of the instances of D50. We also report the average percentage gap of the initial solution (reported by the solver in the log file) that is provided by the route based model under the column Avg Initial Gap. Note that 3IM-V-RBM improves the initial solution in all settings.

From the results presented above, we conclude that 3IM-V works better without a warm start when it can find a feasible solution for the problem. On the other hand, when the problem size is very large and when 3IM-V cannot find a feasible solution, it is better to provide a feasible solution to the model using a heuristic approach.

4.3 Computational Results for Two-Index Model

In this section, we present the results for the two-index model (2IM) given in the previous chapter, the two-index model with valid inequalities (2IM-V), and 2IM-V with a route-based model used as a warm start (2IM-V-RBM).

Similar to the previous part, our main aim in this part is to observe the performance of 2IM in solving problem instances with different sizes, and the effect of enhancements in the solution time and the optimality gaps.

4.3.1 2IM and The Valid Inequalities

In this section, we analyze the effect of adding the symmetry breaking constraints (3.5), and the valid inequalities (3.3) to 2IM. Similar to the previous section, we run 2IM and 2IM-V, which denotes 2IM with the additional inequalities, for all parameter settings (9 different settings) of the 3 random problem instances with 10 customers, denoted by D10-1, D10-2, D10-3, and report the average results in Table 4.4.

Table 4.4 Effect of additional inequalities in 2IM

	2IM			**2IM-V**			
Data Set	Avg. Run Time (s)	Avg. Gap (%)	Optimal #	Avg. Run Time (s)	Avg. Gap (%)	Optimal #	# of instances 2IM-V is better
D10-1	1732	0.00	9	237	0.00	9	8
D10-2	1694	0.00	9	115	0.00	9	9
D10-3	242	0.00	9	43	0.00	9	8

From Table 4.4, we see that both models find optimal solutions in all settings. However, the average solution time of 2IM decreases drastically with the addition of the valid inequalities as it can be observed from the running times of 2IM-V. From the last column of the table, we also observe that 2IM-V is better again in terms of the solution times in almost all of the settings considered. The settings where 2IM-V performs worse have $\beta = 3$ and $\gamma = 1$, i.e. when the maximum distance allowed and the truck capacity are small. One potential explanation for this situation is that because of the potential multi-trip occurrence is significantly limited in this setting, there might not be any feasible solution in which symmetry breaking constraints are useful. Hence, we can conclude that 2IM-V performs definitely better in terms of the solution times in general.

4.3.2 2IM-V with a Warm Start

In this section, we discuss the affect of using a warm start, which is explained in figure 3.2, with 2IM-V based on our route-based model. In Table 4.5, we report the average run time (in seconds) for the instances that are solved to optimality within the time limit, average percentage gap for the instances that cannot be solved within the time limit and the number of instances where an optimal solution is found for 2IM-V and 2IM-V-RBM. From Table 4.5, we see that 2IM-V solves all D10 instances in very short times, solves 3 out 9 instances of all D15 instances, and terminates between 20% and 25% optimality gap in the other instances of D15. None of the D30 and D50 instances can be solved by 2IM-V. The parameter settings where performance of 2IM-V is relatively better (the optimality gap is smaller) is when the maximum distance allowed per vehicle C_{max} is small, i.e. $\gamma = 1$. Especially, all of the problem instances solved to optimality in D15 have small C_{max} values, i.e. $\gamma = 1$, a similar behavior is observed in the other data sets. Note that providing a feasible solution to 2IM-V as a warm start using our route-based model based approach has negligible effect on the overall performance of 2IM-V.

Table 4.5 Two-index model with the additional constraints and with warm start

	2IM-V			2IM-V-RBM		
Data Set	Avg Run Time (s)	Avg Gap (%)	Optimal #	Avg Run Time (s)	Avg Gap (%)	Optimal #
D10-1	253	0.00	9	236	0.00	9
D10-2	207	0.00	9	206	0.00	9
D10-3	30	0.00	9	30	0.00	9
D15-1	365	24,91	3	371	24.62	3
D15-2	15	21.06	3	15	21.06	3
D15-3	81	19.05	3	69	18.79	3
D30-1	TL	32.18	0	TL	32.02	0
D30-2	TL	28.81	0	TL	29.04	0
D30-3	TL	27.96	0	TL	27.60	0
D50-1	TL	40.26	0	TL	39.65	0
D50-2	TL	44.17	0	TL	44.46	0
D50-3	TL	40.88	0	TL	39.84	0

4.4 Overall Model Performance Comparison

In this section, we compare the best implementations of the two-index and three-index models, namely 3IM-V, 3IM-V-RBM and 2IM-V in terms of their ability to solve the problem instances, optimality gaps, and solution times. Recall that, β and γ represent different levels of the vehicle capacity Q, and the maximum allowed distance per vehicle C_{max}, respectively.

Tables 4.6 - 4.9 present the average results of all parameter settings for the D10 - D50 data sets over all randomly generated instances .

Table 4.6 Results of 2IM-V, 3IM-V and 3IM-V-RBM for D10 instances

		2IM-V		3IM-V		3IM-V-RBM	
β	γ	Avg Run Time (s)	Avg Gap (%)	Avg Run Time (s)	Avg Gap (%)	Avg Run Time (s)	Avg Gap (%)
3	1	2	0	3	0	2	0
	2	19	0	7	0	7	0
	3	151	0	1	0	1	0
4	1	0	0	3	0	3	0
	2	85	0	8	0	6	0
	3	418	0	1	0	0	0
5	1	0	0	2	0	2	0
	2	90	0	10	0	8	0
	3	711	0	3	0	1	0
Average		164	0	4	0	3	0

Table 4.6 shows that all problem instances can be solved in short durations by all models, but the solution times are shorter for the models 3IM-V and 3IM-V-RBM compared to 2IM. Besides, there is no significant change in the performance of 3IM with the usage of the warm start. We also observe that the solution time of 2IM increases with the max route length parameter γ.

Table 4.7 Results of 2IM-V, 3IM-V and 3IM-V-RBM for D15 instances

		2IM-V			3IM-V			3IM-V-RBM		
β	γ	Avg Run Time (s)	Avg Gap (%)	Solved #	Avg Run Time (s)	Avg Gap (%)	Solved #	Avg Run Time (s)	Avg Gap (%)	Solved #
3	1	94	0.00	3	535	0.00	3	753	0.00	3
	2	TL	17.30	0	401	0.00	3	317	0.00	3
	3	TL	22.53	0	106	0.00	3	140	0.00	3
4	1	119	0.00	3	765	0.00	3	810	0.00	3
	2	TL	22.73	0	394	0.00	3	317	0.00	3
	3	TL	23.67	0	166	0.00	3	180	0.00	3
5	1	258	0.00	3	1196	0.00	3	1052	0.00	3
	2	TL	21.77	0	363	0.00	3	241	0.00	3
	3	TL	21.20	0	239	0.00	3	178	0.00	3
Average		157	21.53	1	462	0.00	3	443	0.00	3

The superiority of 3IM-V over 2IM becomes clearer with the results given in Table 4.7. Note that all D15 instances are solved to optimality by both 3IM-V and 3IM-V-RBM while only 9 (out of 27) instances can be solved by 2IM-V within the time limit. Indeed, the solution times of 2IM-V are better than 3IM-V for these 9 instances.

However, 2IM-V terminates with optimality gaps larger than 20% on the average for the other parameter settings. From Table 4.7, we also observe an interesting result. While the performance of 2IM-V is the best for smaller C_{max} values ($\gamma = 1$) compared to the other γ values, a reverse result is observed for 3IM-V and 3IM-V-RBM since the largest solution times are seen when C_{max} is smaller ($\gamma = 1$). Comparing 3IM-V and 3IM-V-RBM, again, we do not observe any significant improvement which is consistent with our previous observation.

Table 4.8 Results of 2IM-V, 3IM-V and 3IM-V-RBM for D30 instances

		2IM-V			3IM-V			3IM-V-RBM		
β	γ	Avg Run Time (s)	Avg Gap (%)	Solved #	Avg Run Time (s)	Avg Gap (%)	Solved #	Avg Run Time (s)	Avg Gap (%)	Solved #
3	1	TL	12.07	0	TL	63.43	0	TL	63.70	0
	2	TL	39.40	0	TL	9.43	0	TL	7.90	0
	3	TL	33.40	0	3770	6.80	2	TL	6.67	0
4	1	TL	11.93	0	TL	70.43	0	TL	68.87	0
	2	TL	40.33	0	TL	10.40	0	TL	8.57	0
	3	TL	34.87	0	4886	7.80	1	3685	7.80	1
5	1	TL	13.73	0	TL	73.40	0	TL	71.70	0
	2	TL	44.53	0	6931	11.30	1	TL	12.50	0
	3	TL	36.60	0	9762	8.50	1	TL	8.83	0
Average		TL	29.65	0	5823	29.05	0.5	3685	28.05	0.1

Similar results can be observed from Table (4.8). 2IM-V performs better when $\gamma = 1$, and for the other settings 3IM-V provides better results.

Table 4.9 Results of 2IM-V, 3IM-V and 3IM-V-RBM for D50 instances

		2IM-V			3IM-V			3IM-V-RBM		
β	γ	Avg Run Time (s)	Avg Gap (%)	Solved #	Avg Run Time (s)	Avg Gap (%)	Solved (Feas) #	Avg Run Time (s)	Avg Gap (%)	Solved (Feas) #
3	1	TL	25.23	0	TL	NA	0 (0)	TL	82.83	0 (3)
	2	TL	52.07	0	TL	NA	0 (0)	TL	63.37	0 (3)
	3	TL	47.57	0	TL	49.85	0 (2)	TL	52.60	0 (3)
4	1	TL	23.67	0	TL	NA	0 (0)	TL	81.93	0 (3)
	2	TL	49.47	0	TL	NA	0 (0)	TL	60.67	0 (3)
	3	TL	47.43	0	TL	50.10	0 (3)	TL	49.37	0 (3)
5	1	TL	26.97	0	TL	NA	0 (0)	TL	83.70	0 (3)
	2	TL	53.50	0	TL	56.50	0 (1)	TL	66.00	0 (3)
	3	TL	50.03	0	TL	54.20	0 (3)	TL	52.93	0 (3)
Average		TL	41.77	0	TL	52.66	0 (1)	TL	65.93	0(3)

The results for our largest data set D50 are shown in Table 4.9 where NA is used for the cases where a feasible solution cannot be obtained by the solver within the time limit. First, note that none of these problem instances can be solved to optimality by any of our models. Since 3IM-V cannot find any feasible solution in most of the parameter settings, this time the results of 3IM-V-RBM are better than 3IM-V. From the previous tables for D10, D15 and D30 instances, we know that the performance of 2IM is worse than the other models. However, interestingly, for D50 instances, the optimality gaps reported by 2IM-V are better than that of the other models in all parameter settings. One possible explanation for this behavior is that the number of variables in the 3IM and 3IM-V models is increasing much more compared to the 2IM-V model. As a result, this increase in variables leads to an increase in model complexity. Note that providing an initial solution to 3IM-V does not help it to report a better optimaltiy gap than 2IM-V.

To sum up, while all models can solve small problem instances (D10) efficiently, our three-index formulation with symmetry breaking constraints 3IM-V performs better in medium sized instances (D15 and D30), and the two-index model with the symmetry breaking constraints and valid inequalities reports better optimality gaps in large problem instances (D50).

In this section, we consider our data set with 15 customers (D15) and analyze the structure of the optimal solutions to provide some business insights. We again consider 9 parameter settings for the 3 random instances of D15, and additionally introduce a new parameter setting $\gamma = 0.75$ to investigate the structure of the optimal solutions.

We explore the advantages of the multi-trip environment and examine the relationship between different parameter settings for each data set instance.

We first investigate the effect of allowing vehicles to perform multiple trips throughout the day. To observe it, we solve the same problem instances under the traditional restriction that each vehicle can perform at most one trip, called as LRP, and compare the cost components with our problem LRPMT in Table 4.10 where I.C, D.C., O.C. and T.C. represent the inbound cost, regional depot opening cost, the outbound cost, and the total cost respectively. We also report the percentage reduction in the total cost due to the usage of multiple trips under the last column of the table. Also, Avg # V. and Avg # D. represent the average number of vehicles used and the regional depots opened, respectively.

Table 4.10 The effect of allowing multiple trips

Data Set	LRP						LRPMT						Cost Improvement
	Avg I.C.	Avg D.C.	Avg O.C.	Avg T.C.	Avg # V.	Avg # D.	Avg I.C.	Avg D.C.	Avg O.C.	Avg T.C.	Avg # V.	Avg # D.	
D15-1	4038	63218	15724	82980	5.58	2.5	4251	63212	12960	80422	4.58	2.5	**3.2%**
D15-2	3731	55099	14446	73276	5.17	2.0	3184	55761	12682	71627	4.50	2.0	**2.3%**
D15-3	3941	64893	17862	86696	5.92	2.25	3938	64893	16140	84970	5.33	2.25	**2.0%**

As it can be seen from Table 4.10, the average inbound cost and the depot opening cost do not change so much with the inclusion of multiple trips. However, as expected, the outbound cost and consequently the total cost decrease when multiple trips are allowed. Note that LRPMT uses less vehicles in all instances, and this is the main reason of the decrease in the total cost. Hence, we could suggest a business stakeholder to allow multiple trips as a general mentality in their supply chain operations.

In Tables 4.11 and 4.12 the detailed results of LRP and LRPMT for the instances D15-1 and D15-2 are given. We should remark that the parameter γ for the maximum distance allowed per vehicle is still available among the restrictions of LRP to observe only the effect of multiple trip option. We first note that the number of regional depots opened is the same for both models in all settings. However, we can

Table 4.11 Multi-trip effect on the cost components for data set D15-1.

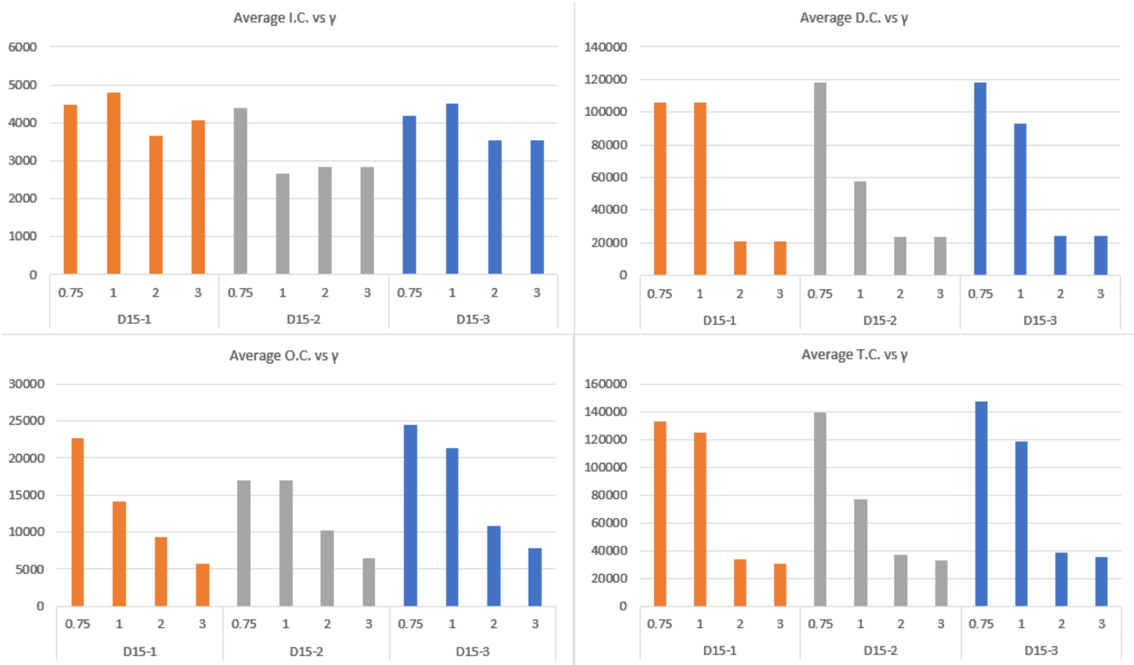
		LRP						LRPMT						T.C Improvement
β	γ	I.C	D.C	O.C	T.C	# of V.	# of D.	I.C	D.C	O.C	T.C	# of V.	# of D.	%
3	0.75	4240	105570	20000	129810	8	4	4240	105570	20000	129810	8	4	0.0
	1	4896	105780	15000	125676	6	4	5125	105780	12500	123405	5	4	1.8
	2	4042	20590	12500	37132	5	1	3173	20880	10000	34053	4	1	8.3
	3	3173	20880	12500	36553	5	1	5225	20800	5000	31025	2	1	15.1
4	0.75	4240	105570	21336	131146	8	4	4240	105570	21336	131146	8	4	0.0
	1	4191	105570	16002	125763	6	4	4330	105570	13335	123235	5	4	2.0
	2	3173	20880	10668	34721	4	1	4042	20590	8001	32633	3	1	6.0
	3	3173	20880	10668	34721	4	1	3173	20880	5334	29387	2	1	18.2
5	0.75	4240	105570	26672	136482	8	4	4240	105570	26672	136482	8	4	0.0
	1	4191	105570	16670	126431	5	4	4191	105570	16670	126431	5	4	0.0
	2	3173	20880	13336	37389	4	1	3173	20880	10002	34055	3	1	8.9
	3	3173	20880	13336	37389	4	1	3173	20880	6668	30721	2	1	17.8

Table 4.12 Multi-trip effect on the cost components for data set D15-2.

		LRP						LRPMT						T.C Improvement
β	γ	I.C	D.C	O.C	T.C	# of V.	# of D.	I.C	D.C	O.C	T.C	# of V.	# of D.	%
3	0.75	4153	118060	17500	139713	7	4	4153	118060	15000	137213	6	4	1.7
	1	2418	57290	17500	77208	7	2	2593	57290	15000	74883	6	2	3.0
	2	1642	25170	12500	39312	5	1	1642	25170	10000	36812	4	1	6.3
	3	4784	21200	12500	38484	5	1	4784	21200	7500	33484	3	1	12.9
4	0.75	4153	118060	16002	138215	6	4	4153	118060	16002	138215	6	4	0.0
	1	2696	57290	16002	75988	6	2	2593	57290	16002	75885	6	2	0.1
	2	4784	21200	10668	36652	4	1	4784	21200	10668	36652	4	1	0.0
	3	4784	21200	10668	36652	4	1	1642	25170	5334	32146	2	1	12.3
5	0.75	4153	118060	20004	142217	6	4	4153	118060	20004	142217	6	4	0.0
	1	2418	57290	20004	79712	6	2	2418	57290	20004	79712	6	2	0.0
	2	1642	25170	10002	36814	3	1	1642	25170	10002	36814	3	1	0.0
	3	4784	21200	10002	35986	3	1	1642	25170	6668	33480	2	1	6.9

see that number of vehicles used is decreasing when γ is increasing. i.e. when the maximum distance allowed per vehicle C_{max} is increasing, since the trucks can visit more customers and perform more trips during their routes in this case. Besides, when C_{max} is very small, i.e. $\gamma = 0.75$, there is no difference between the solutions of the models. Furthermore, the number of vehicles used difference between LRP and LRPMT is increasing when β parameter level is increasing and γ levels kept the same. The results are consistent with the D15-3 data set which can be found in Appendix. Hence, our main observation from these tables is that multiple trip option is more valuable (provides more benefit) when the maximum distance allowed per vehicle is large but the vehicle capacities are small or medium sized.

Figure 4.1 γ parameter level effect on objective components



Next, we investigate the effect of max route length parameter C_{max} (γ) on the cost components in Figure 4.1. By intuition, we expect to see the outbound cost to decrease while the max route length parameter C_{max} (γ) increases since the problem becomes more relaxed and it might be possible to fulfill the customer demand with less vehicles. This can be observed from the figure. Furthermore, the regional depot opening cost seems to decrease while the max route length parameter increases. One possible explanation is that, when the traveling range of the vehicles increases the customer points that are further away could be satisfied without having opening a depot that is closer to that customer point. Finally, for the inbound cost levels, the inbound cost seems to decrease when max route length parameter level increases. Because of the trucks can travel farther away, which decreases the number of depots that must be opened again farther away from DC, its effect decreases the inbound cost levels as well.

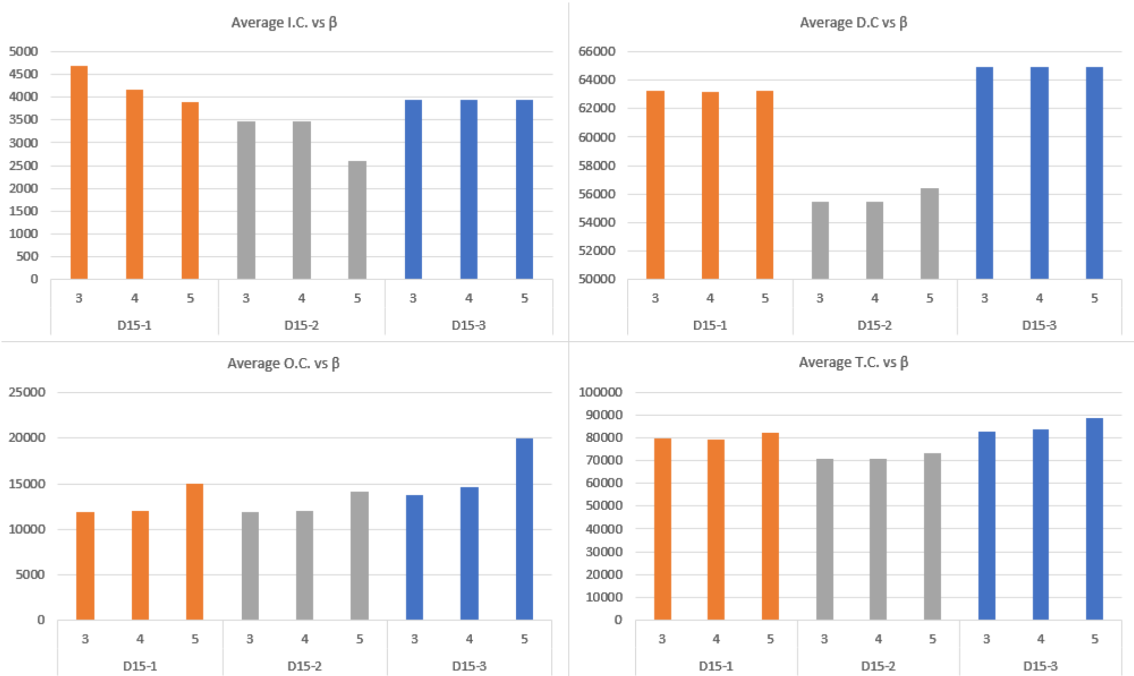
Table 4.13 γ parameter effect on relative KPIs

Data set	γ	Average Vehicle Count	Average # of Depots	Average trip per Truck
D15-1	0.75	8.0	4.0	1.1
	1	5.0	4.0	1.4
	2	3.3	1.0	1.5
	3	2.0	1.0	2.8
D15-2	0.75	6.0	4.0	1.2
	1	6.0	2.0	1.1
	2	3.7	1.0	1.4
	3	2.3	1.0	2.1
D15-3	0.75	8.0	4.0	1.2
	1	7.0	3.0	1.1
	2	3.7	1.0	1.5
	3	2.7	1.0	2.1

Table 4.13 indicates the effect of the max route length parameter γ on the number of vehicles used, number of opened depots and the average trip per truck. The results are consistent with Figure 4.1. When max route length parameter level increases, in general, the number of vehicles used and number of depots opened decrease. Furthermore, the average trip per truck also increases because a truck can perform more trips with the relaxation of the total distance traveled. Therefore, as a business point of view, we could indicate that having a more range allowance for each truck could yield a less number of depots, vehicles and in general less cost for their operations.

Next, Figure 4.2 displays the effect of truck capacity levels (β) on the objective components. As explained in the experimental data design section, the unit cost of the truck increases proportional to the capacity of the truck. Hence, if the number of vehicles used in the solutions for two different β values are the same, then the outbound cost for larger β value will be larger. This can be observed in Figure 4.2 (the graph for O.C.) when β increases from 4 to 5. Also note that the outbound cost stays the same when β increases from 3 to 4. This means that using larger but more expensive vehicles is not beneficial for the inbound transportation when multiple trip option is available. Moreover, the inbound cost seems to be staying the same or decreasing while increasing the truck cost and capacity levels. Depot cost seems to be stay at the same level majority of the time and finally as expected the truck cost component of the objective is increasing.

Figure 4.2 β level effect on objective components.



One discussion point is that, whenever we increase the capacity and therefore the unit cost of the truck, we would expect to use less trucks, but this does not guarantee a decrease in the total transportation cost. Therefore, in Table 4.14 we report the number of vehicles used, number of opened depots and average trip per truck values for different β values.

Table 4.14 β parameter effect on relative KPIs

Data set	β	Average Vehicle Count	Average # of Depots	Average trip per Truck
	3	4,8	2,5	2,0
D15-1	4	4,5	2,5	1,7
	5	4,5	2,5	1,5
	3	4,8	2,0	1,5
D15-2	4	4,5	2,0	1,5
	5	4,3	2,0	1,4
	3	5,5	2,3	1,6
D15-3	4	5,5	2,3	1,4
	5	5,0	2,3	1,4

The average vehicle count is decreasing while the β parameter level is increasing. However, the overall outbound cost is increasing in this case as it can be seen from Figure 4.2 because of the economies of scale that we assumed in our parameter setting. Hence, we can conclude that though using larger vehicles require less vehicles in total, if larger vehicles are expensive, then it does not make sense to use them. Furthermore, the average number of depots that is opened is not changing for any of the parameter levels given. Finally, we see a slight decrease on the average trip per truck with increasing β. One possible explanation is that, model seems to utilize vehicles more when β is increasing, and therefore the multi-tripping option becomes unnecessary.

Table 4.15 Inbound cost parameter effect on relative KPIs

Data Set	Inbound Cost	Average Vehicle Count	Average # of Depots	Average Trip per Truck	Average I.C.	Average D.C.	Average O.C.
D15-1	1.0 Scaled	4.58	2.5	1.65	4027	63212	12960
	2.0 Scaled	4.67	2.5	1.66	7626	63218	13168
	3.0 Scaled	4.75	2.5	1.59	11220	63201	13377
D15-2	1.0 Scaled	4.50	2	1.46	3017	55761	12682
	2.0 Scaled	4.50	2	1.55	4543	56423	12682
	3.0 Scaled	4.50	2	1.60	6815	56423	12682
D15-3	1.0 Scaled	5.33	2.25	1.53	4136	64893	16140
	2.0 Scaled	5.33	2.25	1.48	8933	64893	16140
	3.0 Scaled	5.33	2.25	1.45	13400	64893	16140

Next, we investigate the effect of the inbound cost b_i. We scale the unit inbound cost by multiplying b_i with 2.0 and 3.0, and the results can be found in Table 4.15. Considering D15-1 data set and comparing the scaling factors 1.0 and 2.0, we observe that the difference occurs due to the selection of different depot locations with higher fixed costs, using more trucks but saving from the increasing (with the power of scale) inbound cost. The same behaviour is observed between the scales of 2.0 and 3.0 of D15-1. Furthermore, in the D15-2 data set when the inbound cost is scaled by 2, the average depot opening cost increases as a the result of the selection of different depot locations. Hence, the value of the inbound cost affects the locations of the regional depots that will be opened, and this also affects the inbound transportation cost.

Finally, to analyze the impact of depot opening cost, we scale the depot opening cost by 0.5 and 2, and report the results in Table 4.16. The number of vehicles used in the outbound transportation and the number of regional depots opened do not change with the depot opening cost. Furthermore, average multi trip per truck seems to change with the scale. However, this difference has no trend in any data-set. One potential explanation is that there might be multiple optimal solutions which consists the same number of trucks and multi-trip number could vary within the max route length (γ) limitations which creates the variation.

There are slight differences occurred in D15-2 data set instance. Considering the

Table 4.16 Depot cost parameter effect on relative KPIs

Data Set	Depot Cost	Average Vehicle Count	Average # of Depots	Average Trip per Truck	Average I.C.	Average D.C.	Average O.C.
D15-1	0.5 Scaled	4.58	2.50	1.66	4027	31606	12960
	1.0 Scaled	4.58	2.50	1.65	4027	63212	12960
	2.0 Scaled	4.58	2.50	1.76	4027	126423	12960
D15-2	0.5 Scaled	4.50	2.00	1.52	2493	28211	12682
	1.0 Scaled	4.50	2.00	1.46	3017	55761	12682
	2.0 Scaled	4.83	2.00	1.37	4064	108875	13668
D15-3	0.5 Scaled	5.33	2.25	1.46	4136	32446	16140
	1.0 Scaled	5.33	2.25	1.53	4136	64893	16140
	2.0 Scaled	5.33	2.25	1.58	4136	129785	16140

scales between 0.5 and 1.0, the main cause of the difference is model's selection of different depots which has less cost but higher inbound cost. Furthermore, considering scales between 1.0 and 2.0 the same behavior occurred with an additional more vehicle usage. Because of the power of scale, depot opening costs increases rapidly and model is tend to select the depots with less fixed cost and using the depots with higher inbound cost and even using extra vehicles. Thus, model's behaviour for minimization of the total cost is consistent with the relative actions that is taken which is explained above. Hence, we can way that the number of depots opened is not affected but the locations might be affected with the changes in the regional depot opening cost.

We summarize our findings as follows:

- The maximum distance allowed per vehicle should be larger to get the most benefit from the multiple trip option.
- When the maximum distance allowed per vehicle is larger, the company might need less number of regional depots to reduce the total cost.
- If larger vehicles are expensive, then it is better to use smaller (or medium sized) vehicles with longer ranges to minimize the total cost.
- The changes in the depot opening cost do not change the structure of the solutions so much.

5. CONCLUSION AND FUTURE RESEARCH

In this thesis, we study a location-routing problem with multiple trips arising in e-commerce delivery option under the max route length per vehicle constraints. To the best of our knowledge, this problem is not studied in the literature before. We present two main models (3IM and 2IM) and strengthen them by using valid inequalities (3IM-V and 2IM-V). These valid inequalities mainly focus on breaking the symmetry due to the truck usages and trip merging. Also, we present a route based model (RBM) to determine a feasible solution for the problem in very short times, and we further use it as a warm-start model for 3IM-V and 2IM-V to improve the performance of our exact methods.

Since we introduce a new problem to the literature, there exists no benchmark instances for the problem. Using the CMT4 instance of Christofides (1979), we generate different problem instances and test our solution methods. We first observe that the addition of the valid inequalities definitely improve the mathematical models 2IM and 3IM. Besides, while 3IM-V can solve small to medium sized instances in shorter times, it struggles to find even a feasible solution for larger problem instances. On the other hand, 2IM-V performs better in larger problem instances though its performance is inferior in the small and medium sized instances compared to 3IM-V. Hence, we can state that these two model types are complements to each other for solving problem instances in different sizes. As a business point of view, we observe that allowing trucks to have more range of mobility, in other words relaxing the distance constraint parameter, allows us to use less trucks and utilizes the multi-tripping option more. Also, the number of regional depots opened decreases in this case.

There exist studies on VRPMT where the time windows constraints are considered in place of the maximum distance allowed per vehicle, and this can be considered for our problem in a future research. Besides, we assume that there are no capacity limitations for the regional depots that will be opened, and this might be relaxed in a future work. Besides, there might be multiple distribution centers. The pool of routes used in our route based model RBM could be extended with other heuristic algorithms such as Clarke and Wright savings algorithm (Clarke & Wright, 1964). Besides, alternative and more sophisticated heuristic methods such as ALNS and GRASP might be used to provide initial solution to the models to obtain better results for larger instances.

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APPENDIX A

Table A.1 Multi-trip effect on the cost components for data set D15-3.

		2E-LRP						2E-LRPMT						T.C Improvement
Beta	Gamma	I.C	D.C	O.C	T.C	# of V.	# of D.	I.C	D.C	O.C	T.C	# of V.	# of D.	%
3	0.75	4428	118470	22500	145398	9	4	4427	118470	20000	142897	8	4	1.7
	1	4764	92860	17500	115124	7	3	4718	92860	17500	115078	7	3	0.0
	2	3719	24120	12500	40339	5	1	3719	24120	10000	37839	4	1	6.2
	3	3719	24120	12500	40339	5	1	3719	24120	7500	35339	3	1	12.4
4	0.75	4367	118470	21336	144173	8	4	4367	118470	21336	144173	8	4	0.0
	1	4718	92860	18669	116247	7	3	4718	92860	18669	116247	7	3	0.0
	2	3719	24120	10668	38507	4	1	3719	24120	10668	38507	4	1	0.0
	3	3719	24120	10668	38507	4	1	3719	24120	8001	35840	3	1	6.9
5	0.75	4367	118470	32000	154837	8	4	4367	118470	32000	154837	8	4	0.0
	1	4718	92860	28000	125578	7	3	4718	92860	28000	125578	7	3	0.0
	2	3719	24120	16000	43839	4	1	3719	24120	12000	39839	3	1	9.1
	3	3719	24120	12000	39839	3	1	3719	24120	8000	35839	2	1	10.0