A COMPARATIVE ANALYSIS ON UNDIRECTED CUT-BASED FORMULATIONS OF PERIODIC VEHICLE ROUTING PROBLEM

by
OĞULCAN DOĞAN

Submitted to the Graduate School of Engineering and Natural Sciences
in partial fulfilment of
the requirements for the degree of Master of Science

Sabancı University
July 2022

ABSTRACT

A COMPARATIVE ANALYSIS ON UNDIRECTED CUT-BASED FORMULATIONS OF PERIODIC VEHICLE ROUTING PROBLEM

OĞULCAN DOĞAN

INDUSTRIAL ENGINEERING M.S. THESIS, JULY 2022

Thesis Supervisor: Asst. Prof. Amine Gizem Tiniç

Keywords: vehicle routing and scheduling, periodic vehicle routing problem, integer programming, branch-and-cut

The classical Vehicle Routing Problem (VRP) is a well-studied combinatorial optimization problem whose aim is to identify optimal routes for a fleet of homogeneous vehicles to satisfy the demands of a geographically dispersed set of customers, considering a single planning period. The Periodic Vehicle Routing Problem (PRVP) is a generalization of the classical VRP in which the planning horizon consists of multiple periods and each customer has a set of associated possible visit schedules. In the literature, there are many solution approaches proposed such as exact solution methods, heuristic algorithms, and metaheuristics. However, exact solution methods are much fewer in number compared to heuristic methods, and a comprehensive study focused on cut-based formulations of the problem is not available in the literature. In this thesis, we propose and study several cut-based formulations of the PVRP defined on an undirected network. Different versions of connectivity constraints and schedule selection constraints are used to develop alternative formulations. Moreover, new cut-based formulations which eliminates the use of vehicle indices are also introduced. Since the proposed formulations contain exponentially many connectivity constraints, we devise and implement branch-and-cut procedures to solve them exactly. The computational experiments are prepared and conducted for all of the formulations. The results of the experiments indicate that some of the alternative formulations have the potential to improve computational performance. The model without vehicle indices provides improvement in terms of reaching optimality and computation times, while the models with alternative schedule selection constraints provide promising results in terms of solution quality and average computation times.

ÖZET

PERİYODİK ARAÇ ROTALAMA PROBLEMİNİN KESİ TEMELLİ FORMÜLASYONLARI ÜZERİNE KARŞILAŞTIRMALI BİR İNCELEME

OĞULCAN DOĞAN

ENDÜSTRİ MÜHENDİSLİĞİ YÜKSEK LİSANS TEZİ, TEMMUZ 2022

Tez Danışmanı: Dr. Öğr. Üyesi Amine Gizem Tiniç

Anahtar Kelimeler: araç rotalama ve çizelgeleme, periyodik araç rotalama problemi, tamsayılı programlama, dal-ve-kesi

Klasik Araç Rotalama Problemi (ARP), tek bir planlama periyodu içerisinde bir müşteri grubunun taleplerini karşılamak amacıyla türdeş araçlardan oluşan bir araç filosu için en düşük maliyetli rotaları belirlemeye çalışan bir kombinatoryal eniyileme problemidir. Periyodik Araç Rotalama Problemi (PARP), planlama döneminin birden çok periyottan oluştuğu ve her müşterinin bir dizi olası ziyaret çizelgesine sahip olduğu klasik ARP'nin bir genellemesidir. Literatürde bu problemi çözmek amacıyla önerilmiş kesin çözüm yöntemleri, sezgisel algoritmalar ve metasezgisel çözüm yaklaşımları bulunmaktadır. Bununla birlikte PARP literatüründe, kesi temelli formülasyonlara odaklanan kapsamlı çalışma eksikliği bulunmaktadır. Bu tezde, yönsüz bir çizge üzerinde tanımlanan bir PARP için, bir temel modeli ve beş alternatif kesi temelli formülasyon öneriyor ve bunları inceliyoruz. Alternatif formülasyonlar geliştirmek için bağlantı kısıtlarının ve çizelgeleme seçimi kısıtlarının farklı türevleri kullanılmaktadır. Ayrıca, PARP için araç indislerinin kullanımını ortadan kaldıran bir yönsüz ve kesi temelli formülasyon da ortaya çıkarılmıştır. Tüm formülasyonlar sayıca üstel olarak büyüyen bağlantı kısıtları içerdiğinden, önerilen formülasyonların kesin çözümüne ulaşabilmek amacıyla dal-ve-kesi algoritması tasarlanmış ve kullanılmıştır. Formülasyon ve çözüm yöntemlerinin belirlenmesinin ardından, tüm formülasyonlar için hesaplamalı deneyler hazırlanmış ve uygulanmıştır. Deneylerin sonuçları, bazı alternatif formülasyonların çözüm performansını iyileştirme potansiyeline sahip olduğunu göstermektedir. Araç indisleri olmayan model, en iyi sonuca ulaşma ve çözüm süreleri açısından iyileştirme sağlarken, alternatif çizelge seçim kısıtlarına sahip modeller, çözüm kalitesi ve ortalama çözüm süreleri açısından umut verici sonuçlar vermektedir.

ACKNOWLEDGEMENTS

First, I would like to thank Asst. Prof. Gizem Tiniç for helping me with her expertise on the subject. I feel privileged to have written this thesis under her supervision. I am grateful for her invaluable guidance and support.

I would like to thank the jury members, Prof. Bülent Çatay and Asst. Prof. Duygu Taş Küten for taking part in the thesis jury and, their constructive comments and suggestions.

I would like to express my deepest gratitude to my parents Nefise and Necdet Doğan, and my beloved İdil Yörük for their constant love, support, patience and understanding.

I am thankful to my friends I have met during my time at Sabancı University. I would like to thank Yunus Emre Üslü for his invaluable support and friendship, especially during the difficult times of COVID. I am thankful to Aras Atmaca, Erdi Uzun and Emre Beledin for their invaluable friendship and support.

This research is conducted as part of a TÜBİTAK project, with the project number 120M358. That's why, I would also like to thank to TÜBİTAK for their support, and also thank again Asst. Prof. Gizem Tiniç for accepting me to this project.

To my beloved İdil
To my loving parents Nefise and Necdet

TABLE OF CONTENTS

LIST OF TABLES

1. INTRODUCTION

Vehicle Routing Problems (VRPs) are among the most challenging and well-studied combinatorial optimization problems. The VRP and its variants have attracted plenty of attention within the scientific community since its introduction by Dantzig & Ramser (1959). In the classical VRP, the aim is to identify optimal routes for a fleet of homogeneous vehicles in order to satisfy the (given) demands of a geographically dispersed set of customers, considering a single planning period. All routes must begin and end at the same depot, each customer must be visited exactly once, and the total demand assigned to any vehicle's route should not exceed the vehicle capacity. The objective is to minimize the total routing cost while respecting the aforementioned constraints. To be able to model and solve VRPs encountered in practice, numerous variants of the problem have been addressed by considering different objectives, including additional constraints and/or relaxing some constraints of the classical VRP. Among some examples are the VRP with Time Windows, the VRP with Pickup and Delivery, the Split Delivery VRP, and the Periodic VRP. This thesis focuses on the Period VRP.

The Periodic Vehicle Routing Problem (PRVP) is a generalization of the classical VRP in which the planning horizon consists of multiple periods and each customer has a set of associated possible visit schedules. For each customer, one of these schedules must be selected and the demand of the customer must be satisfied in accordance with the selected schedule. Hence, customers may have to be visited multiple times throughout the planning horizon depending on their schedules, but the number of visits to a given customer at a given period is at most one. The objective of the PVRP is to identify a set of vehicle routes for each period of the planning horizon in a way to serve the demand at minimum total cost.

The PVRP has been a subject of research since 1974. The municipal waste collection study of Beltrami and Bodin (1974) is considered as the introduction of PVRP although the name PVRP did not appear explicitly in this study where varying visit schedules are used for regions that require different amount of waste collection service in a week. The problem can be found in the literature with different names such as

Periodic VRP, Period VRP or Multi-Period VRP. First, the PVRP is named "the Assignment Routing Problem" in the study of Russell and Igo (1979) which allows visits in each period of the planning horizon with a restriction on the total number of visits to each customer. Additionally, the instances proposed by Russell and Igo are used as PVRP benchmarks in early studies. The paper of Christofides and Beasley (1984) is the first to label the problem as "the Period Routing Problem" based on the periodic structure of the PVRP. As the VRP, the PVRP has practical relevance in a wide variety of applications such as the collection of waste and recycled goods (Beltrami & Bodin, 1974; Russell & Igo, 1979), distribution of soft drinks (Golden & Wasil, 1987) and more generally fast-moving consumer goods, delivery of hospital linens (Banerjea-Brodeur, Cordeau, Laporte & Lasry, 1998) and procurement of blood products (Hemmelmayr, Doerner, Hartl & Savelsbergh, 2009).

For the PVRP, several modeling approaches are available in the literature such as flow-based formulations of Archetti et al. (2015), load-based formulations of Archetti et al. (2017) and Larrain et al. (2019), and cut-based formulation of Rodriguez-Martin et al. (2019). In the case of symmetric distances, cut-based formulations have the potential to be more efficient as they allow reducing (halving) the number of variables by eliminating directions on the arcs of a given network.

In the PVRP literature, exact solution methods are very limited compared to heuristics and a comprehensive study on cut-based formulations has not been conducted before. With this motivation, we propose and study several cut-based formulations of the PVRP in this thesis. In particular, a modified version of the formulation proposed by Rodriguez-Martin et al. (2019) on an undirected graph is adopted as the base model. The modifications made on the original formulation are: (1) removal of the driver consistency restrictions and (2) relaxation of the unit demand assumption to allow arbitrary demands. The alternative formulations are developed by using (i) different versions of connectivity constraints, (ii) schedule selection constraints, and/or by eliminating (iii) vehicle indices from the base model. Performances of the formulations are investigated through computational tests conducted on a set of instances taken from the benchmark data set introduced by Rodriguez-Martin et al. (2019). Due to the modifications made on the original model on Rodriguez-Martin et al., the selected benchmark instances are also modified in terms of vehicle capacity.

The contributions of this thesis are mainly twofold: (i) we introduce new cut-based formulations for the PVRP with the goal of reducing the numbers of variables and constraints as well as eliminating symmetry in the solution space compared to an existing cut-based formulation modified and adopted as a basis, and (ii) we provide comparative analyses of the performances of the proposed formulations based on the results of a detailed computational study.

The remainder of this thesis is organized as follows. The related literature on the PVRP is reviewed in Chapter 2. In Chapter 3, the problem is defined formally and the alternative cut-based formulations are presented. Branch-and-cut procedures devised for solving the proposed formulations and the cut separation routine are also described in this chapter. In Chapter 4, the results of the computational experiments are provided along with the analyses. Chapter 5 concludes the thesis with a summary and a discussion of our findings.

2. LITERATURE REVIEW

The PVRP has been studied since the 1970s, and heuristics and exact approaches have been proposed to solve the problem by many researchers. Due to the computational complexity of the PVRP, many heuristic methods are proposed instead of exact solution approaches that require longer processing times especially in early years. The municipal waste collection study Beltrami and Bodin (1974) is considered by many researchers as the introduction of PVRP. At the time, 80% of the budget of New York City Department of Sanitation is related to the costs came from waste collection and sanitation services. The aim of the paper is to create an efficient method to solve the problem, and to result in cost savings derived from the efficiency of the routes created. For this purpose, Beltrami and Bodin proposed two different heuristic methods and days of the week excluding Sunday as the planning horizon in this study. While some sites have to be visited three times, while others have to be visited on every day of the planning horizon. For the sites visited three times a week, there are two options which are service on Monday, Wednesday and Friday, or service on Tuesday, Thursday and Saturday. The first heuristic method is to solve routing problem by using Clarke and Wright (1964)'s saving procedure and then assigning routes to the day of the planning horizon based on visiting options of the sites. The second method is to assign sites to the visiting options and then solving the routing problem by using Clarke and Wright (1964)'s algorithm.

Five years after Beltrami and Bodin's study, Russell and Igo (1979) published their own study on waste collection and named the problem as "The Assignment Routing Problem". In the problem, more flexible visiting options are introduced. Each site can be visited a number of times from 1 to 6 in a planning horizon. Additionally, consecutive visits may be considered undesirable, if the number of visits are suitable. The problem in study of Russel and Igo consists of 490 sites located in a large city and the sites are served by four trucks. The sites that need to be visited are clustered into 126 clusters based on their proximity between them and their visit schedules. The dataset which consists of these 126 serviceable clusters is used by many researchers as a benchmark dataset. After the clustering, a feasible solution is obtained by solving a VRP for each day and the feasible solution is sequentially processed by three different heuristic methods to gain improvements on total routing cost.

In the history of PVRP, Ryan and Foster (1976)'s integer programming formulation is considered as one of the earliest mathematical models of the PVRP. The IP formulation is mainly created for the VRP, but it is extended to include periodicity of demand. The extensions discussed in the paper are specification on which days each customer needs to be visited, interval between consecutive visits, revision of visit schedules and workload of each vehicle over the planning horizon. Although the IP formulation is introduced, heuristic algorithms are utilized to solve the problem with respect to the constraints of the model. In addition, any computational results of the PVRP formulation are not included in the paper.

Christofides and Beasley (1984) labels the problem as "the Period Routing Problem" and suggests a mathematical model to solve the PVRP. This model is widely considered as the first IP formulation of the PVRP. In the model, a predetermined number of visits and a set of possible visit schedules are provided for each customer, and the scheduling constraints are written based on these sets. The objective of the model is to minimize total routing cost over the planning horizon while satisfying the scheduling, capacity, and sub-tour elimination constraints. These constraints and the objective are used in majority of the PVRP formulations. Nowadays, there are several different mathematical modeling formulations available in the literature to solve the PVRP. The flow-based formulation of Archetti et al. (2015), load-based formulations of Archetti et al. (2017) and Larrain et al. (2019), and cut-based formulation of Rodriguez-Martin et al. (2019) can be mentioned as recent formulation examples.

Despite the introduction of the IP formulation, Christofides and Beasley (1984) choose heuristic methods to solve the problem. Firstly, a visit schedule is selected for each customer and the customers are assigned to days based on the selected visit schedules. Then, a VRP is solved for each day, and the improvement to the solution is searched via the interchanges between visit schedules. Considering the utilization of both mathematical modelling and heuristic methods, the hybrid optimization algorithm proposed by Cacchiani et al. (2014) is a decent approach which provides good quality solutions for well-known benchmark instances of the literature. In their study, the authors proposed an algorithm which benefits from the set-covering formulation of the PVRP and an iterated local search algorithm. Firstly, a column generation approach is applied to solve the LP relaxation of the set covering formulation. The column generation process is carried out with the utilization of an iterated local search algorithm. Then, an additional heuristic procedure which utilizes from a tabu list is used to obtain feasible solutions to the PVRP. The computational results of the proposed algorithm provide several best-known solutions for well-known benchmark instances of the literature. As the studies benefited from both mathematical modelling and heuristic methods, the studies of Tan and Beasley (1984), Russell and Gribbin (1991), Chao et al. (1995) can also be mentioned.

Exact solution methods for the PVRP are rare compared to heuristic methods due to the computational complexity and exponential growth in size. Among the exact solution methods, the algorithm proposed by Baldacci et al. (2011) is considered as state-of-the-art algorithm up to the present. The study of Baldacci et al. (2011) provides an exact solution method using a set-partitioning formulation for the PVRP. The method utilizes from an IP formulation, and LP-relaxations of the formulation. Firstly, an optimal solution is derived from the dual of a LP-relaxation of the model. All LP-relaxations are used to calculate stronger lower bounds for the problem. It is proven that these lower bounds do not eliminate any optimal solution of the IP formulation. Finally, the information gained from the dual solution and the lower bounds are benefited to solve the IP formulation via commercial IP solver such as CPLEX or Gurobi. As examples of the exact solution approach, there are several other studies such as the studies of Francis et al. (2006), Francis and Smilowitz (2006), Mourgaya and Vanderbeck (2007), Kang et al. (2005), Huerta-Muñoz (2018) and Rothenbächer (2019).

The PVRPs can be modified with different constraints and objectives with regard to the purpose of applications. This leads to many variants and extensions of the PVRP. In 1992, the study of Gaudioso and Paletta (1992) was the first paper to name the problem as "Periodic Vehicle Routing Problem". This study can be seen as a variant of the PVRP, because the objective of the mathematical model proposed by the study is to minimize required number of vehicles to serve the demand instead of minimizing the total routing cost over the planning horizon. The traveling times are used in the constraints to restrict total traveling time of each vehicle. Additionally, the proposed model of Gaudioso and Paletta (1992) allows vehicles to be assigned to more than one route. With these modifications, the optimization model is solved by using a heuristic algorithm.

One of the PVRP variants is proposed by Rodriguez-Martin et al. (2019). In their study, each customer must be visited by the same vehicle across the planning horizon. The motivation for their study comes from a soft drinks company which makes regular visits to its customers. To solve this problem, the exact solution approach is used. A MILP formulation is provided and an exact branch-and-cut algorithm is benefited by the authors. Another contribution of this study is the benchmark instances randomly generated for this problem. The authors generate 240 different benchmark instances which have varying number of nodes, fleet size of vehicles, length of planning horizon, frequency of visit and visit schedules for customers. The studies of Baptista et al. (2002), Francis et al. (2006), Mourgaya and Vanderbeck (2007), Schedl and Strauss (2011), Hemmelmayr et al. (2013), Fauske et al. (2020) and Huerta-Muñoz et al. (2022) provide different variations of the PVRP with different objective functions, constraints and solution methods based on the motivation of their problem.

3. PROBLEM DEFINITION AND FORMULATIONS

Consider a complete undirected graph $G = (N, A)$ with node set N and arc set A. The set of nodes N can be defined as $N = \{0, 1, \ldots, n\}$, where node 0 corresponds to the depot, and the other nodes correspond to customer locations. Let $A = \{(i, j) : i, j \in N, i < j\}$ be the set of arcs, and let c_{ij} represent a non-negative cost incurred by the traversal of arc $(i, j) \in A$. The set of time periods is given by $T = \{1, \ldots, t\}$. A homogeneous fleet of k vehicles, denoted as $K = \{1, \ldots, k\}$, is based at the depot node, each having a capacity of Q units. Each customer $i \in N \setminus \{0\}$ has (1) an associated demand denoted by d_i, which is the quantity that must be delivered per each visit to the customer, and (2) a set of possible visit schedules denoted by P_i. A possible visit schedule $p \in P_i$ prescribes the periods in which customer i is to be visited, should this schedule be selected for the customer. For example, if a customer has to be visited twice in a planning horizon of five periods, a possible visit schedule might require visits in periods 1 and 3, and prohibit visits on other periods. In practice, alternative visit schedules that have different combinations of allowable periods, are typically possible for each customer, which implies that the set P_i usually contains multiple schedules regarding a given customer i.

The PVRP is the problem of identifying a least cost set of vehicle routes to satisfy the demands of a given set of customers over a finite planning horizon that spans multiple periods. A feasible solution to the PVRP must satisfy the following conditions: (i) for each customer, a possible visit schedule must be selected, (ii) the total demand that is assigned to a vehicle route should not exceed the vehicle capacity, (iii) all routes must start and end at the depot, and (iv) the number of routes in a given period cannot exceed the fleet size.

In this thesis, a modified version of the undirected cut-based model proposed by Rodriguez-Martin et al. (2019) originally for the PVRP with driver consistency constraints is adopted as a base model and alternative formulations are derived by using (i) different versions of connectivity constraints, (ii) schedule selection constraints, and/or by eliminating (iii) vehicle indices from the base model. The base model and alternative formulations are compared against each other in terms of objective function value, computation time and optimality gap. Since all formulations involve exponentially many connectivity constraints, branch-and-cut procedures are devised and implemented to solve them.

We define and use the following decision variables in our formulations:

$$\mathrm{x}_{ijtk} = \begin{cases} 1, & \text{if arc } (i,j) \in A \text{ is traversed in period } t \in T \text{ by vehicle } k \in K \\ 0, & \text{otherwise} \end{cases}$$

$$\mathrm{x}_{ijt} = \begin{cases} 1, & \text{if arc } (i,j) \in A \text{ is traversed in period } t \in T \\ 0, & \text{otherwise} \end{cases}$$

$$\mathrm{y}_{it} = \begin{cases} 1, & \text{if customer } i \in N \setminus \{0\} \text{ is visited in period } t \in T \\ 0, & \text{otherwise} \end{cases}$$

$$\mathrm{z}_{itk} = \begin{cases} 1, & \text{if node } i \in N \text{ is visited at period } t \in T \text{ by vehicle } k \in K \\ 0, & \text{otherwise} \end{cases}$$

$$\mathrm{s}_{ip} = \begin{cases} 1, & \text{if schedule } p \in P \text{ is selected as visit schedule for customer } i \in N \setminus \{0\} \\ 0, & \text{otherwise} \end{cases}$$

Moreover, we let $\delta(S) = \{(i,j) \in A : |S \cap (i,j)| = 1\}$ for $S \subseteq N$. If $S = \{i\}$, we simply use $\delta(i)$ instead of $\delta(\{i\})$. Finally, we write $x^{tk}(A') = \sum_{(i,j) \in A'} x_{ijtk}$ for $A' \subseteq A$.

3.1 Cut-based formulations of the PVRP

In this section, six alternative cut-based formulations of the PVRP are presented. First, the modified version of the undirected cut-based model proposed by Rodriguez-Martin et al. (2019) is provided in Section 3.1.1 as the base model. In Section 3.1.2, an alternative formulation is derived by expressing the connectivity constraints in a different way, i.e., through the binary variables indicating whether a customer is visited in a given period or not, regardless of the particular vehicle visiting the customer (unlike the base model). Combining the connectivity constraints in the first two formulations yields another model for the PVRP which is presented in Section 3.1.3. Then, a new formulation is developed by eliminating vehicle indices from the base model as given in Section 3.1.4 where the complexity of the base model is reduced in terms of the number of variables and constraints. Modifying the schedule selection constraints of the base model, another formulation is obtained as illustrated in Section 3.1.5. Lastly, in Section 3.1.6, a combination of the models from Sections 3.1.4 and 3.1.5 is proposed as an alternative formulation.

3.1.1 Base Model

As mentioned earlier, the undirected cut-based model proposed by Rodriguez-Martin et al. (2019) is considered as a base model with some modifications. There are mainly two differences between the formulation proposed by Rodriguez-Martin et al. (2019) and our base formulation: (1) the driver consistency restrictions are removed, and (2) customers are allowed to have arbitrary demands instead of unit demands.

Model 1 (M1) is written as follows:

$$\text{minimize} \sum_{(i,j)\in A}\sum_{t\in T}\sum_{k\in K} c_{ij}x_{ijtk} \tag{3.1}$$

$$\text{s.t.} \sum_{p\in P_i} s_{ip} = 1 \qquad i \in N\setminus\{0\} \tag{3.2}$$

$$\sum_{p\in P_i : t\in p} s_{ip} = y_{it} \qquad i \in N\setminus\{0\}, t\in T \tag{3.3}$$

$$\sum_{k\in K} z_{itk} = y_{it} \qquad i \in N\setminus\{0\}, t\in T \tag{3.4}$$

$$\sum_{i\in N\setminus\{0\}} d_i z_{itk} \le Q z_{0tk} \qquad k\in K, t\in T \tag{3.5}$$

$$x^{tk}(\delta(i)) = 2z_{itk} \qquad i\in N, t\in T, k\in K \tag{3.6}$$

$$x^{tk}(\delta(S)) \ge 2z_{itk} \qquad i\in S, S\subseteq N\setminus\{0\}, k\in K, t\in T \tag{3.7}$$

$$x_{ijtk} \in \{0,1\} \qquad (i,j)\in A, t\in T, k\in K \tag{3.8}$$

$$s_{ip} \in \{0,1\} \qquad i\in N\setminus\{0\}, p\in P_i \tag{3.9}$$

$$y_{it} \in \{0,1\} \qquad i\in N\setminus\{0\}, t\in T \tag{3.10}$$

$$z_{itk} \in \{0,1\} \qquad i\in N, t\in T, k\in K \tag{3.11}$$

The objection function (3.1) is to minimize the total cost of routing. Constraints (3.2) and (3.3) ensure that one of the possible schedules is selected for each customer and that each customer is visited in accordance with the selected schedule, respectively. Constraints (3.4) guarantee that each customer is served by a vehicle, and they establish the connection between the variables y and z. Constraints (3.5) state the capacity restrictions for each vehicle in each period. Constraints (3.6) are the degree constraints for the customers and the depot. Constraints (3.7) prevent the formation of subtours, hence ensuring the connectivity of the solution. They enforce that if a customer $i \in S$ is serviced in period t by vehicle k, then at least two arcs in the set $\delta(S)$ must be traversed in the same period. Constraints (3.8) - (3.11) define the variable domain restrictions.

3.1.2 Model with an alternative set of connectivity constraints

Model 2 (M2) is given as:

$$\text{minimize } (3.1)$$

$$\text{s.t. } (3.2)-(3.6), (3.8)-(3.11)$$

$$\sum_{k \in K} x^{tk}(\delta(S)) \geq 2y_{it} \qquad i \in S, S \subseteq N \setminus \{0\}, t \in T. \tag{3.12}$$

In this model, constraints (3.7) are replaced with constraints (3.12). This alternative set of connectivity constraints is formulated based on the relationship $\sum_{k \in K} z_{itk} = y_{it}$ for all $i \in N \setminus \{0\}$ and $t \in T$. This relationship ensures that each customer is serviced by at most one vehicle in period $t \in T$. Therefore, constraints (3.12) specify that if a customer $i \in S$ is visited in period t, then at least two arcs in the set $\delta(S)$ must be traversed in the same period.

3.1.3 Model with both versions of connectivity constraints

Model 3 (M3) is provided below:

$$\text{minimize } (3.1)$$

$$\text{s.t. } (3.2)-(3.12).$$

In this model, constraints (3.7) and (3.12) are included in the formulation simultaneously, with the goal of exploring its computational performance in the presence of both sets of connectivity constraints.

3.1.4 Model without the vehicle indices

In this model, we use the variables x_{ijt} instead of their vehicle-indexed versions, and we eliminate the z_{itk} variables.

Model 4 (M4) is presented in the following:

$$\text{minimize} \sum_{(i,j)\in A}\sum_{t\in T} c_{ij}x_{ijt} \tag{3.13}$$

$$\text{s.t. } (3.2), (3.3), (3.9), (3.10)$$

$$\sum_{j\in N\setminus\{0\}} x_{0jt} \geq 2 \qquad t \in \mathcal{T} \tag{3.14}$$

$$\sum_{j\in N\setminus\{0\}} x_{0jt} \leq 2K \qquad t \in T \tag{3.15}$$

$$x^t(\delta(i)) = 2y_{it} \qquad i \in N\setminus\{0\}, t \in T \tag{3.16}$$

$$\sum_{i\in S}\sum_{j\in S} x_{ijt} \leq |S| - r(S) \qquad S \subset N\setminus\{0\}, t \in T \tag{3.17}$$

$$x_{ijt} \in \{0,1\} \qquad (i,j) \in A, t \in T. \tag{3.18}$$

Our aim is to reduce the complexity of the model (in terms of the numbers of variables and constraints) and eliminate symmetry by removing the vehicle index k. To the best of our knowledge, there is no other undirected cut-based formulation, which do not use vehicle-indexed variables, in the PVRP literature. When the vehicle indices are omitted, the variables z and y become identical. Therefore, z variables can be eliminated as well. As a result, constraints (3.4) – (3.7), (3.8) are replaced with constraints (3.14) – (3.18). Objective function is now expressed using the x_{ijt} variables, which again is to minimize the total routing cost. Constraints (3.14) guarantee that arcs connected to the depot are traversed at least twice in each period. Constraints (3.15) set an upper bound of K on the number of vehicle routes per period to respect the fleet size restriction. Constraints (3.16) are the degree constraints for the depot and the customers. In the absence of the vehicle indices, we use a similarly defined function x^t instead of x^{tk}. In particular, for a given subset $A' \subseteq A$, the function x^t is defined as $x^t(A') = \sum_{(i,j)\in A'} x_{ijt}$. Constraints (3.17) are the subtour elimination constraints proposed by Laporte et al. (1985), where $r(S)$

is a lower bound on the number of vehicles needed to serve the customers in the subset S and it is calculated as $r(S) = \sum_{i \in S} \frac{d_i}{Q}$. Constraints (3.18) impose domain restrictions on the x_{ijt} variables.

3.1.5 Model with a different schedule selection approach

Model 5 (M5) is defined as:

$$\text{minimize } (3.1)$$

$$\text{s.t. } (3.4) - (3.8), (3.10) - (3.11)$$

$$\sum_{t=t'}^{t'+f-1} y_{it} \leq 1 \qquad i \in S_f, t' \in \{1, ..., T-f+1\}, f \in T \tag{3.19}$$

$$\sum_{t \in T} y_{it} = v_i \qquad i \in N \setminus \{0\}. \tag{3.20}$$

In this model, schedule selection constraints (3.2) and (3.3) are replaced with (3.19) and (3.20). Constraints (3.19) limit the number of times a customer $i \in N \setminus \{0\}$ is visited within a given number of consecutive time periods. Equivalently, these constraints ensure that at least a predetermined number of periods is inserted between two consecutive visits of a customer. Here, S_f is defined as the set of nodes that have a visit frequency of f over the planning horizon T. Constraints (3.20) guarantee that each customer $i \in N \setminus \{0\}$ is visited exactly v_i times, which represents the frequency of visit (i.e., the required number of visits) for customer $i \in N \setminus \{0\}$ within the planning horizon. In this way, instead of making a selection from a set of predetermined schedules, the model is allowed to decide on the periods in which each customer will be visited with respect to the constraints (3.19) and (3.20). Different from Model (1), this model enforces that the visits to any customer are equally spaced in time within the planning horizon as much as possible depending on the number of visits required. For instance, if a customer must be visited three times in a planning horizon of five periods, only allowable visit schedule requires visits in periods 1, 3 and 5, and prohibit visits on other periods.

3.1.6 Model without vehicle indices with a different schedule selection approach

Combining Model 4 with the schedule selection approach presented in Model 5, Model 6 (M6) can be written as:

$$\text{minimize } (3.13)$$
$$\text{s.t. } (3.10), (3.14) - (3.20).$$

This formulation contains the objective function (3.13) and constraints (3.14) - (3.18) of Model 4. The schedule selection constraints (3.2) and (3.3) are replaced with constraints (3.19) and (3.20) of Model 5. Hence, just as in Model 5, periods to visit each customer $i \in N \setminus \{0\}$ is selected by the model under the constraints (3.19) and (3.20), without supplying predetermined visit schedules to the model.

3.2 Branch-and-cut procedures

All of the formulations presented above involve a number of connectivity constraints that is exponential in the size of the problem. Since it is impractical to enumerate all customer subsets and include the connectivity constraint associated with each subset in the model, we solve the proposed formulations using a branch-and-cut approach. In particular, we relax the connectivity constraints and separate them on the fly. For a given formulation, we initialize the solve procedure with the corresponding relaxation (no connectivity constraints at the beginning). Whenever an integer solution (with an objective function value at least as good as that of the incumbent solution) encountered during the search, we check for violation of the connectivity constraints. If no violations are found, the current solution can be considered as the new incumbent. Otherwise, we add the constraints violated by the current solution (if any) to the model, which is then solved again. Detecting violations at integer solutions is fairly easy: it can be done simply by inspecting the solution to identify whether there are vehicle routes that are disconnected from the depot. For each formulation, the branch-and-cut procedure outlined above is implemented using the callback feature of Gurobi Optimizer 8.1.1.

3.2.1 Separation of connectivity constraints for fractional solutions

The constraints (3.7) and (3.17) are separated by solving min-cut problems on properly defined support graphs. To separate constraints (3.7), LP-relaxation of Model 1 is solved without the constraints (3.7). For each $i \in N \setminus \{0\}$, $k \in K$, and $t \in T$, an undirected support graph $G' = (N', A')$ is defined with the set of nodes $N' = N$ and the set of arcs $A' = \{(i,j) \in A : x_{ijtk} > 0\}$. The capacity of each arc $(i,j) \in A'$ is set to x_{ijtk}. Afterwards, a min-cut problem is solved to find minimum capacity cutset $S \subset N'$ that separates i and depot node 0 where $i \in S$ and $0 \notin S$. Finally, the capacity of the min-cut induced by S is checked for the violation of the constraints (3.7). If a violation is detected, the constraints (3.7) are added to the formulation. The constraints (3.17) are separated via the same procedure with some differences in the definition of the support graph. For each $i \in N \setminus \{0\}$, and $t \in T$, an undirected support graph $G'' = (N'', A'')$ is defined with a set of nodes N'' and set of arcs A''. Let $A'' = \{(i,j) \in A : x_{ijt} > 0\}$ and $N'' = N$. The capacity of each arc $(i,j) \in A''$ is set to x_{ijt}. The remaining steps are the same as the separation of the constraints (3.7). The separation procedure adopted here is similar to the one presented in Rodriguez-Martin et al. (2019).

4. COMPUTATIONAL RESULTS

In this chapter, we report the results of our computational study which is conducted in order to investigate the performances of the base model (M1) and the alternative models (M2-M6) described in the previous chapter, and we discuss our findings in detail. In particular, a total of four sets of experiments were performed. The first one focuses on the evaluation of the models M1-M3, which are identical except for the connectivity constraints they involve. In the second set of experiments, the model exhibiting relatively the best performance among M1-M3 is then compared with M4. Based on the results of the first two sets of experiments, we also consider separating the connectivity constraints for fractional solutions found at the root node of the branch-and-cut tree for M1 and M4. The impact of applying the separation routine outlined in Section 3.2.1 at fractional solutions is explored in a third set of experiments. The fourth and final set of experiments is designed to analyze the effect of adopting alternative schedule selection approaches presented in the previous chapter on the computational performance of the proposed PVRP formulations.

All branch-and-cut procedures were implemented in Python 3.7.4 using the commercial solver Gurobi Optimizer 8.1.1 under its default settings and a time limit of 7200 seconds. The experiments were carried out on a virtual machine equipped with Intel Xeon CPU E5-2640 v3 processor with 4 cores and 2.60 GHz speed, 16 GB RAM, and 64-bit operating system.

4.1 Benchmark Instances

We performed our tests on a subset of the instances in the benchmark data set proposed by Rodriguez-Martin et al. (2019). This data set includes three versions of each problem instance labeled as *a*, *b*, and *c*, which differ based on the geographical locations of the nodes in the network. For each version, there are a total of 80 instances with number of nodes $n \in \{11, 21, \ldots, 71\}$, number of periods within a planning horizon $t \in \{2, 3, 4, 5\}$, and homogeneous vehicle fleet size of $k \in \{2, 3, 4\}$. Considering the size of the instances that can be tackled by our branch-and-cut procedures, a subset of this data set is used in the experiments. The selected subset contains 32 instances with $|N| \in \{11, 21, 31\}$, $t \in \{2, 3, 4, 5\}$, and $k \in \{2, 3, 4\}$ for each of the three versions a, b, and c. Hence, a total of 96 instances was selected for our computational study. The spatial distribution and the possible visit schedules of the nodes are generated randomly by Rodriguez-Martin et al. (2019) in all instances of the data set. The visit frequency of each node is considered during the generation of the visit schedules.

4.2 Computational Results

The results of the first set of experiments, where the performances of M1-M3 are evaluated, are provided in Tables A.1, A.2 and A.3. Second, M1 and M4 are compared against each other based on the results in Tables A.4, A.5 and A.6. For the third set of experiments, where the separation routine described earlier concerning the fractional solutions is applied to M1 and M4, and the results are reported in Tables A.7, A.8 and A.9. The versions of M1 and M4 with separation at fractional solutions are denoted as M1-S and M4-S, respectively. We would like to note here that a similar separation procedure is also applied to M3 due to its promising performance, but the results are not reported here as the improvements achieved are not significant with respect to M1-S and M4-S.

The last set of experiments is aimed at exploring the effect of the adopted schedule selection constraints on the solution process. To this end, the models that are identical except for these constraints, namely, M1, M4, M5, and M6, are compared against each other. As described in Sections 3.1.5 and 3.1.6, M5 and M6 employ an alternative set of constraints to determine a visit schedule for each customer based on the given visit frequencies. These constraints guarantee that the visits to any customer are equally spaced in time within the planning horizon (as much as possible depending on the number of visits required). On the other hand, the sets of possible schedules supplied to the model in M1 and M4 do not necessarily satisfy this condition. Hence, for a fair comparison, the set of possible visit schedules given to M1 and M4 for each customer are altered in a way to ensure consistency among the instances solved by all formulations. In other words, the original visit schedules of the customers are modified so that each customer has the same set of possible schedules for each model. The results are shown in Tables A.10, A.11 and A.12.

In the tables where detailed computational results are presented, first five columns contain information about the benchmark instances used in the tests. The numbers under the column “Ins” correspond to the (unique) id numbers of the instances and range from 1 to 96. The columns 2-5 show the characteristics of the benchmark instances, in particular, the number of nodes (n), the number of periods within the planning horizon (t), the fleet size (k), and the version (a, b, or c). On the right side of the instance information columns, the results obtained with each model are provided in a block of three columns. For a given model, the “OBJ” column shows the objective function value of the best solution found within the time limit, the “Gap” column indicates the relative difference between the best upper bound and the best lower bound at termination of the branch-and-cut procedure. When an instance is solved to proven optimality, the gap is 0%; otherwise, it is equal to the final gap reported by Gurobi. Under the column “Time”, the solution times are displayed in terms of seconds.

Three abbreviations are used in the tables. When the solver concludes that no feasible solution exists for a given instance, “INF” is used to state the infeasibility of the problem instance. When there is no conclusion of infeasibility and no feasible solution can be found within the specified time limit, the term “NFS” is used to express that no feasible solution could be identified. Lastly, the term “TLR” indicates that the predetermined time limit is reached before a proof of optimality or infeasibility is obtained.

We also provide summary tables which highlights our key findings. The abbreviations used in these tables are listed and described below:

- nOpt: Number of instances solved to optimality by a model within the time limit.
- nOpt-Best: Number of instances solved to optimality by a model with the shortest computation time.
- nOpt-All: Number of instances solved to optimality by all models within the time limit.
- nFeas: Number of instances for which a feasible solution was found, but optimality of the solution could not be proven by a model within the time limit.
- nFeas-Best: Number of instances for which a best feasible solution was found, but its optimality could not be proven by a model within the time limit.

- nFeas-All: Number of instances for which a feasible solution was found by all models within the time limit, but its optimality could not be proven by any of the models in an experiment.
- nInf: Number of instances certified as infeasible by a model.
- nUns: Number of instances for which a model could neither find a feasible solution nor prove infeasibility.
- nIns: Number of instances examined in a table.
- %-gap: Average gap between the best objective function value and the best lower bound available at termination.
- Avg. Time: Average computation time of a model given a selected group of instances.

In our comparisons, a model is considered to have returned "the best solution" if it satisfies either one of the following conditions:

(I) Among the models that found a (certified) optimal solution, it takes the shortest amount of computation time.

(II) Among the models that found a best feasible solution (without a proof of optimality) when the time limit is reached, it has the smallest gap.

In Tables A.1, A.2 and A.3, the detailed results obtained with M1, M2 and M3 are reported. These results are analyzed to determine the best one among the three models, and to evaluate if M2 and M3 are able to achieve significant improvements over M1 which is the base model. All models returned either an optimal or a feasible solution to 94 instances, and labeled the instances (41) and (44) as infeasible. As can be seen in Table 4.1, M1 is able to solve 49 instances to optimality, which is the highest number of optimal solutions obtained across all three models, followed by 46 optimal solutions found by M3. The instance (69) is solved to optimality by M1 and M2 within the same amount of computation time, and thus, M1 and M2 are both considered to have found the best solution in this particular case. Hence, the total number of the best solutions (nOpt-Best and nFeas-Best) in Table 4.1 is 95 instead of 94. Examining the results further, we observe that 29 of these instances are solved fastest by M1. In a similar manner, when the instances for which the models can find a feasible solution are examined, M1 has the best solution for 21 instances in terms of solution quality and optimality gap. Consequently, it can be concluded that M1 is overall the best performer among these three models taking the results for all instances into account.

Table 4.1 Summary table for the results of Tables A.1, A.2 and A.3

nIns = 96	**M1**	**M2**	**M3**
nOpt	49	43	46
nFeas	45	51	48
nInf	2	2	2
nUns	0	0	0
nOpt-Best	29	10	12
nFeas-Best	21	7	16

The average computation times of the optimal solutions and the average percentage gaps of the feasible solutions without a certified optimality are also analyzed. To make a fair comparison with respect to these two performance measures among the three models, the instances that are not solved to optimality by all models, and the ones for which a feasible solution could not be identified by all models are excluded from the analysis. These cases are listed below:

- Only M3 could not find optimal solutions for the instance (17) within the time limit.
- The instances (18), (45) and (53) cannot be solved to optimality by M2 unlike the other two models.
- M1 is the only model that is able to find optimal solutions for the instances (43), (48) and (85).
- The optimality of the solution for the benchmark instance (80) is only proven by M3.
- For all of above instances, the same best objective function values are attained by all models, but the optimality of the solutions cannot be proven by all three models within the time limit.

When we exclude these instances, M1, M2 and M3 are able to identify an optimal solution for 42 instances and all of them can find a feasible solution (without proven optimality) for 44 instances within the time limit. The results of this analysis are summarized in Table 4.2. Accordingly, M3 is the best performing model in terms of average computation time, while M1 is the best performing in terms of average percentage gap although the difference between the two models with regard to these quantities and the solution quality is quite small. On the other hand, M2 seems to have significantly poor performance compared to M1 and M3.

Table 4.2 Comparison of M1, M2 and M3 based on the selected benchmark instances

nIns = 86	M1	M2	M3
nOpt-All	42	42	42
Avg. Time	81.31	342.57	76.47
nFeas-All	44	44	44
%-Gap	5.21%	6.38%	5.59%

Taking all of the results in Tables 4.1 and 4.2 into consideration, M1 can be regarded as the best performing model among the three models despite the improvements achieved by M3 in terms of average computation time. M3 seems to be the second best, while M2 is the worst performing model. Consequently, M1 is selected to compare against M4 in the second set of experiments.

The results obtained with M1 and M4 are shown in Tables A.4, A.5 and A.6. M1 returns either an optimal or a feasible solution for a total of 94 instances, while this is true for 86 instances in the case of M4. The instances (41) and (44) are certified as infeasible by both models. M4 is not able produce a feasible solution or prove infeasibility for eight instances, all of which have 31 nodes and four or five vehicles, whereas M1 yields a feasible solution for each of these instances. Considering the ability to identify optimal solutions, M1 and M4 achieve optimality in 49 and 64 instances, respectively. These results can be found in Table 4.3. Of the 64 instances solved to optimality by M4, the best solutions of 58 instances are attributed to this model in terms of computation time under the conditions discussed earlier. Considering the instances for which an optimal solution could not be attained but a feasible solution is discovered within the time limit, M1 has a better solution than M4 for 24 instances. M1 is better in reaching solutions with smaller percentage gap between obtained lower bound and best objective function value while providing equal or better solution quality.

Table 4.3 Summary table for the results of Tables A.4, A.5 and A.6

nIns = 96	M1	M4
nOpt	49	64
nFeas	45	22
nInf	2	2
nUns	0	8
nOpt-Best	7	58
nFeas-Best	24	5

Just like we did in the first set of experiments, here we compare M1 and M4 with respect to the average computation times regarding the instances solved to optimality by both models, and the average percentage gaps regarding the instances for which a feasible solution is returned by both models at the end of the time limit. The instances excluded from this comparison are outlined below:

- There are eight instances for which M4 cannot produce a feasible solution or establish infeasibility within the given time limit. As mentioned earlier, M1 finds feasible solutions to all of these instances.
- 16 instances are solved to optimality only by M4 within the time limit.
- M1 identifies an optimal solution for instance (18), while M4 cannot.

The results of this comparison are reported in Table 4.4. There are 48 instances which are solved to optimality, and 21 instances for which a feasible solution is found, by both models. The average computation time of M1 is nearly twice as long as that of M4, whereas the corresponding average gap of M1 is almost half of that associated with M4.

Table 4.4 Comparison of M1 and M4 based on the selected benchmark instances

nIns = 69	**M1**	**M4**
nOpt-All	48	48
Avg. Time	340.39	182.86
nFeas-All	21	21
%-Gap	5.89%	11.26%

Consequently, M4 is seemingly better (and faster) at attaining optimal solutions for the benchmark instances used in our tests. In cases where optimality is achieved by both models, M4 has a significantly smaller average computation time. However, considering the instances for which only a feasible solution is discovered, M1 is mostly superior to M4 in terms of solution quality as well as the average gap. In addition, M4 fails to produce a feasible solution for eight instances that are relatively more difficult to solve. Overall, the choice of the best performing model between M1 and M4 is not quite obvious based on the results of Tables A.4, A.5, A.6, 4.3, 4.4.

After the second set of experiments with M1 and M4, the versions of these two models including the separation routine for fractional solutions at the root node of the search tree (explained in Section 3.2.1), i.e., M1-S and M4-S are also added to the comparative analyses. The main goal here is to strengthen the linear relaxation bounds of the formulations, thereby, enhancing the computational performance of the branch-and-cut procedures. The results obtained with M1, M1-S, M4 and M4-S are reported in Tables A.7, A.8 and A.9.

First, we provide comparisons between M1 and M1-S, and between M4 and M4-S. M1 and M1-S find an optimal solution for 49 and 50 instances, respectively. While M1 returns the best solution for 32 instances under the condition (I), M1-S produces the best solution for 24 instances under the condition (II). These results are summarized in Table 4.5. The instance (53) is solved to optimality only by M1, whereas only M1-S identifies an optimal solution for the instances (80) and (82). Excluding these three instances, both models achieve optimality for 48 instances, and discover a feasible solution for 43 instances as shown in Table 4.6. M1-S has approximately 14% smaller average computation time than M1. It also has a lower average percentage gap. Taking everything into account (both the aforementioned averages and the results at an individual instance level), we conclude that M1-S outperforms M1 in a majority of the instances.

Table 4.5 Summary table for the results of M1 and M1-S from Tables A.7, A.8 and A.9

nIns = 96	**M1**	**M1-S**
nOpt	49	50
nFeas	45	44
nInf	2	2
nUns	0	0
nOpt-Best	32	19
nFeas-Best	19	24

Table 4.6 Comparison of M1 and M1-S based on the selected benchmark instances

nIns = 91	**M1**	**M1-S**
nOpt-All	48	48
Avg. Time	251.07	217.93
nFeas-All	43	43
%-Gap	5.28%	4.95%

A summary of the results obtained with M4 and M4-S are presented in Tables 4.7 and 4.8. Accordingly, M4 fails to produce a feasible solution or establish infeasibility for eight instances. Same applies to nine instances in the case of M4-S. M4 manages to reach an optimal solution for 64 instances, while M4-S solves 59 instances to optimality. For 46 out of these 64 instances, M4 has the best solution under condition (I). M4-S is able to return the best solution for 15 instances under condition (II). These results are reported in Table 4.7. Only M4 finds a feasible solution to the instances (24), (26), (30), (31) and (32) within the time limit. On the other hand, M4-S is the only model to detect a feasible solution for the instances (27) and (63). Focusing only on the 59 instances solved optimally by both models and the 19 instances for which both models yield a feasible solution Table 4.8 displays the average computation times and the average gaps for M4 and M4-S. Based on the given results, M4 has a smaller computation time than M4-S on the average, yet the opposite is true regarding the average gap. Given that the quality of the solutions obtained by M4-S is at least as good as those obtained by M4 in a larger number of cases (as indicated by the nFeas-Best values in Table 4.7), having a lower average gap points to the fact that employing the separation routine at fractional solutions is mostly helpful in strengthening the linear relaxation bounds of the model. Despite the improvements achieved, M4-S is mostly inferior to M4 in terms of reaching optimality and the computation times.

Table 4.7 Summary table for the results of M4 and M4-S from Tables A.7, A.8 and A.9

nIns = 96	**M4**	**M4-S**
nOpt	64	59
nFeas	22	26
nInf	2	2
nUns	8	9
nOpt-Best	46	18
nFeas-Best	9	15

Table 4.8 Comparison of M4 and M4-S based on the selected benchmark instance

nIns = 78	**M4**	**M4-S**
nOpt-All	59	59
Avg. Time	183.66	218.81
nFeas-All	19	19
%-Gap	10.09%	9.29%

Next, we compare all of M1, M1-S, M4 and M4-S with each other based on the computational results shown in Tables A.7, A.8 and A.9. All models detected the infeasibility of the instances (41) and (44) within the time limit. In addition, M4 and M4-S fail to find a feasible solution or establish infeasibility for eight and nine instances, respectively. M4 has 43 best solutions among optimally solved instances under condition (I). Based on condition (II), M1 and M1-S are able to reach the best solutions for 13 and 14 instances, respectively. These results can be seen in Table 4.9.

Table 4.9 Summary table for the results of Tables A.7, A.8 and A.9

nIns = 96	**M1**	**M1-S**	**M4**	**M4-S**
nOpt	49	50	64	59
nFeas	45	44	22	26
nInf	2	2	2	2
nUns	0	0	8	9
nOpt-Best	3	5	43	15
nFeas-Best	13	14	1	0

There are 31 instances that cannot be solved by at least one of the models within the time limit. When these instances are excluded, the remaining 63 instances can be used to compare the average computation time and the average percentage gap of the models. The results of this comparison are provided in Table 4.10. Given the time limit, there are 46 instances that can be solved to optimality by all four models. For these instances, the shortest and the longest average computation times belong to M4 and M1, respectively, with M4 being at least five times faster than M1. Regarding the remaining 17 instances for which all models return a feasible solution at the end of the time limit, M1-S has the lowest average percentage gap and the largest number of best solutions. Despite being superior in terms of computation time, M4 is the worst performing model in terms of solution quality and average gap. In conclusion, M4 seems to be the most favorable option to solve the problems which has 31 or fewer nodes and less than 4 vehicles. For larger problem sizes, M1 and M1-S are more preferable in order to obtain solutions of higher quality.

Table 4.10 Comparison of M1, M1-S, M4 and M4-S based on the selected benchmark instance

nIns = 63	**M1**	**M1-S**	**M4**	**M4-S**
nOpt-All	46	46	46	46
Avg. Time	331.6	220.62	53.8	71.73
nFeas-All	17	17	17	17
%-Gap	5.39%	4.65%	10.14%	9.63%

In our fourth and final set of experiments, the effects of different schedule selection constraints are examined. In these experiments, the originally given possible visit schedules of the customers are modified to ensure consistency of the feasible solution spaces associated with different models. M1, M5, M4, and M6 are compared and the results are shown in Tables A.10, A.11 and A.12. We divide our analysis here into two parts. To investigate the impact of alternative schedule selection constraints on computational performance, the models that are identical except for the schedule selection constraints are considered separately.

In the first part, results obtained with M1 and M5 are used to compare these two models. Both models identify either an optimal or a feasible solution for a total of 94 instances. The instances (41) and (44) are certified as infeasible by both models. M1 and M5 are able to optimally solve 37 and 36 instances, respectively. M1 returns 22 best solutions with regard to condition (I), and M5 yields the best solution for 35 instances regarding condition (II). Eliminating the instance (18) that is solved to optimality only by M1, both models can find an optimal solution for 36 instances, and a feasible solution for 57 instances within the time limit. The comparison between M1 and M5 is summarized in Tables 4.11 and 4.12. Accordingly, the average computation time of M1 is 10.7% lower than that of M5, whereas the average gaps of the two models are close. It can be concluded that M1 is capable of obtaining optimal solutions faster, but when an instance cannot be solved to optimality within the time limit, M5 seems to be more promising in terms of solution quality based on the number of best solutions returned.

Table 4.11 Summary table for the results of M1 and M5 from Tables A.10, A.11 and A.12

nIns = 96	**M1**	**M5**
nOpt	37	36
nFeas	57	58
nInf	2	2
nUns	0	0
nOpt-Best	22	15
nFeas-Best	24	34

Table 4.12 Comparison of M1 and M5 based on the selected benchmark instances

nIns = 93	**M1**	**M5**
nOpt-All	36	36
Avg. Time	387.69	433.88
nFeas-All	57	57
%-Gap	6.62%	6.53%

In the second part of our analysis, M4 and M6 are compared against each other, and the results are provided in Tables 4.13 and 4.14. M4 fails to detect a feasible solution or prove infeasibility for 14 instances. Same applies to seven instances with M6. The instances (41) and (44) are certified as infeasible by both models. The numbers of instances for which an optimal solution is found by M4 and M6 are 48 and 50, respectively. The instances (82) and (84) are solved to proven optimality only by M6. Although M4 terminates with the same solutions for these two instances, it fails to prove their optimality within the time limit. M4 has a higher number of best solutions regarding each of the conditions (I) and (II). Focusing on the instances solved optimally by both models, M6 has a shorter average computation time. In cases where both models return a feasible solution at the end of the time limit, M4 has a slightly lower average gap. In conclusion, even though a majority of the best solutions is attributed to M4 with respect to conditions (I) and (II), M6 achieves considerable improvements over M4 by adopting a different schedule selection approach, as indicated by the quantities nOpt and nFeas as well as the average computation time.

Table 4.13 Summary table for the results of M4 and M6 from Tables A.10, A.11 and A.12

nIns = 96	**M4**	**M6**
nOpt	48	50
nFeas	32	37
nInf	2	2
nUns	14	7
nOpt-Best	30	20
nFeas-Best	19	18

Table 4.14 Comparison of M4 and M6 based on the selected benchmark instances

nIns = 78	**M4**	**M6**
nOpt-All	48	48
Avg. Time	550.07	491.4
nFeas-All	30	30
%-Gap	9.50%	9.78%

Considering the results of both parts of the analysis, it is not possible to draw a definite conclusion as to the improvements achieved by adopting an alternative schedule selection approach. According to the first part of the analysis, M1 outperforms M5 in terms of attaining optimal solutions faster, whereas M5 seems to yield more promising results in terms of solution quality. On the other hand, the second part of the analysis shows that M6 performs much better than M4 in several respects. Consequently, it can be inferred that the alternative schedule selection constraints have the potential to improve computational performance depending on the choice of model.

5. CONCLUSION

In this thesis, we propose and study cut-based formulations of the PVRP. In total, five alternative formulations are proposed and compared with a model designated as the base model (M1). Two of the alternative formulations (M2 and M3) are derived by using different constraints for connectivity of the routes. Another alternative formulation (M4) is proposed to reduce the complexity of the model with the elimination of the vehicle indices. To the best of our knowledge, this is the first cut-based PVRP formulation in the literature which do not involve vehicle-indexed variables. The other two formulations (M5 and M6) are derived by using different schedule selection constraints. Branch-and-cut procedures are devised and implemented to solve the formulations because of the exponential nature of the connectivity constraints. In addition, a cut separation routine for eliminating fractional solutions at the root node of the search tree is embedded within the branch-and-cut procedures devised to solve M1 and M4. The resulting procedures are also taken into consideration in our comparative analyses.

After the model development, the base model and alternative formulations are compared through four sets of experiments. The computational results are evaluated in terms of objective function value, optimality gap and computation time. As a result of the first three sets of experiments, M4 is the best model in terms of computation times, when instances have 31 or less nodes and less than 4 vehicles. The performance of the model deteriorates with increasing problem size. We observe that M4 has difficulty in discovering an initial feasible solution as the problem instances get larger. With the increasing complexity, M1 and M1-S become prominent as the best options to reduce optimality gap, thus M1-S remains ahead of M1 in terms of both average computational time and average percentage gap. When considering the results of the fourth set of experiments, a definite conclusion cannot be reached on the improvements gained from the alternative schedule selection constraints. While M5 and M6 can provide promising results in terms of solution quality and average computation times, they also have shortcomings compared to M1 and M4. As a result, we can say that the alternative schedule selection constraints have the potential to improve computational performance depending on the modeling choices made.

Future work may include models with driver consistency constraints and utilization of the valid inequalities from the study of Rodriguez-Martin et al. (2019). Developing a method to construct an initial feasible solution that can be supplied to the solver when solving M4 may be useful in the future, since the model seems to have the potential to solve more complex problems efficiently with its apparent computational advantages. Additionally, the computational performances of the models under unit demand assumption may be another subject that can be examined in the future instead of using arbitrary demands, and the comparative analysis can be extended using different benchmark instances from the literature.

BIBLIOGRAPHY

Archetti, C., Fernández, E., & Huerta-Muñoz, D. L. (2017). The flexible periodic vehicle routing problem. *Computers & Operations Research*, *85*, 58–70.

Archetti, C., Jabali, O., & Speranza, M. G. (2015). Multi-period vehicle routing problem with due dates. *Computers & Operations Research*, *61*, 122–134.

Baldacci, R., Bartolini, E., Mingozzi, A., & Valletta, A. (2011). An exact algorithm for the period routing problem. *Operations research*, *59*(1), 228–241.

Banerjea-Brodeur, M., Cordeau, J.-F., Laporte, G., & Lasry, A. (1998). Scheduling linen deliveries in a large hospital. *Journal of the Operational Research Society*, *49*(8), 777–780.

Baptista, S., Oliveira, R. C., & Zúquete, E. (2002). A period vehicle routing case study. *European Journal of Operational Research*, *139*(2), 220–229.

Beltrami, E. J. & Bodin, L. D. (1974). Networks and vehicle routing for municipal waste collection. *Networks*, *4*(1), 65–94.

Cacchiani, V., Hemmelmayr, V. C., & Tricoire, F. (2014). A set-covering based heuristic algorithm for the periodic vehicle routing problem. *Discrete Applied Mathematics*, *163*, 53–64.

Chao, I.-M., Golden, B. L., & Wasil, E. (1995). An improved heuristic for the period vehicle routing problem. *Networks*, *26*(1), 25–44.

Christofides, N. & Beasley, J. E. (1984). The period routing problem. *Networks*, *14*(2), 237–256.

Clarke, G. & Wright, J. W. (1964). Scheduling of vehicles from a central depot to a number of delivery points. *Operations Research*, *12*(4), 568–581.

Dantzig, G. B. & Ramser, J. H. (1959). The truck dispatching problem. *Management science*, *6*(1), 80–91.

Fauske, M. F., Mannino, C., & Ventura, P. (2020). Generalized periodic vehicle routing and maritime surveillance. *Transportation Science*, *54*(1), 164–183.

Foster, B. A. & Ryan, D. M. (1976). An integer programming approach to the vehicle scheduling problem. *Journal of the Operational Research Society*, *27*(2), 367–384.

Francis, P. & Smilowitz, K. (2006). Modeling techniques for periodic vehicle routing problems. *Transportation Research Part B: Methodological*, *40*(10), 872–884.

Francis, P., Smilowitz, K., & Tzur, M. (2006). The period vehicle routing problem with service choice. *Transportation science*, *40*(4), 439–454.

Gaudioso, M. & Paletta, G. (1992). A heuristic for the periodic vehicle routing problem. *Transportation Science*, *26*(2), 86–92.

Golden, B. L. & Wasil, E. A. (1987). Or practice—computerized vehicle routing in the soft drink industry. *Operations research*, *35*(1), 6–17.

Hemmelmayr, V., Doerner, K. F., Hartl, R. F., & Rath, S. (2013). A heuristic solution method for node routing based solid waste collection problems. *Journal of Heuristics*, *19*(2), 129–156.

Hemmelmayr, V., Doerner, K. F., Hartl, R. F., & Savelsbergh, M. W. (2009). Delivery strategies for blood products supplies. *OR spectrum*, *31*(4), 707–725.

Huerta Muñoz, D. L. (2018). The flexible periodic vehicle routing problem: modeling alternatives and solution techniques.

Huerta-Muñoz, D. L., Archetti, C., Fernández, E., & Perea, F. (2022). The heterogeneous flexible periodic vehicle routing problem: Mathematical formulations and solution algorithms. *Computers & Operations Research*, *141*, 105662.

Kang, K. H., Lee, Y. H., & Lee, B. K. (2005). An exact algorithm for multi depot and multi period vehicle scheduling problem. In *International conference on computational science and its applications*, (pp. 350–359). Springer.

Laporte, G., Nobert, Y., & Desrochers, M. (1985). Optimal routing under capacity and distance restrictions. *Operations research*, *33*(5), 1050–1073.

Larrain, H., Coelho, L. C., Archetti, C., & Speranza, M. G. (2019). Exact solution methods for the multi-period vehicle routing problem with due dates. *Computers & Operations Research*, *110*, 148–158.

Mourgaya, M. & Vanderbeck, F. (2007). Column generation based heuristic for tactical planning in multi-period vehicle routing. *European Journal of Operational Research*, *183*(3), 1028–1041.

Rodríguez-Martín, I., Salazar-González, J.-J., & Yaman, H. (2019). The periodic vehicle routing problem with driver consistency. *European Journal of Operational Research*, *273*(2), 575–584.

Rothenbächer, A.-K. (2019). Branch-and-price-and-cut for the periodic vehicle routing problem with flexible schedule structures. *Transportation Science*, *53*(3), 850–866.

Russell, R. & Igo, W. (1979). An assignment routing problem. *Networks*, *9*(1), 1–17.

Russell, R. A. & Gribbin, D. (1991). A multiphase approach to the period routing problem. *Networks*, *21*(7), 747–765.

Schedl, M. & Strauss, C. (2011). A periodic routing problem with stochastic demands. In *2011 International Conference on Complex, Intelligent, and Software Intensive Systems*, (pp. 350–357). IEEE.

Tan, C. & Beasley, J. (1984). A heuristic algorithm for the period vehicle routing problem. *Omega*, *12*(5), 497–504.

APPENDIX A - COMPUTATIONAL RESULT TABLES

Table A.1 Computational results of M1, M2 and M3 for the benchmark instances with version *a*

					M1			M2			M3		
Ins.	**n**	t	k	**Ver.**	**OBJ**	**Gap**	**Time**	**OBJ**	**Gap**	**Time**	**OBJ**	**Gap**	**Time**
1	**11**	**2**	**2**	*a*	620.48	0.00%	0.64	620.48	0.00%	0.50	620.48	0.00%	0.59
2	**11**	**2**	**3**	*a*	620.48	0.00%	1.20	620.48	0.00%	0.95	620.48	0.00%	1.00
3	**11**	**3**	**2**	*a*	834.96	0.00%	1.88	834.96	0.00%	0.95	834.96	0.00%	0.89
4	**11**	**3**	**3**	*a*	834.96	0.00%	2.42	834.96	0.00%	1.38	834.96	0.00%	10.69
5	**11**	**4**	**2**	*a*	1043.91	0.00%	1.45	1043.91	0.00%	1.21	1043.91	0.00%	1.20
6	**11**	**4**	**3**	*a*	1043.91	0.00%	3.95	1043.91	0.00%	4.95	1043.91	0.00%	2.86
7	**11**	**5**	**2**	*a*	1083.97	0.00%	5.82	1083.97	0.00%	7.88	1083.97	0.00%	9.34
8	**11**	**5**	**3**	*a*	1083.97	0.00%	49.21	1083.97	0.00%	71.48	1083.97	0.00%	55.58
9	**21**	**2**	**2**	*a*	828.98	0.00%	1.08	828.98	0.00%	0.97	828.98	0.00%	1.48
10	**21**	**2**	**3**	*a*	828.98	0.00%	1.66	828.98	0.00%	1.77	828.98	0.00%	2.88
11	**21**	**2**	**4**	*a*	828.98	0.00%	18.32	828.98	0.00%	21.55	828.98	0.00%	3.67
12	**21**	**3**	**2**	*a*	1088.36	0.00%	3.11	1088.36	0.00%	3.95	1088.36	0.00%	4.03
13	**21**	**3**	**3**	*a*	1088.36	0.00%	20.93	1088.36	0.00%	23.06	1088.36	0.00%	10.05
14	**21**	**3**	**4**	*a*	1088.36	0.00%	252.12	1088.36	0.00%	631.92	1088.36	0.00%	504.83
15	**21**	**4**	**2**	*a*	1253.47	0.00%	30.07	1253.47	0.00%	30.91	1253.47	0.00%	37.21
16	**21**	**4**	**3**	*a*	1253.47	0.00%	501.69	1253.47	0.00%	181.25	1253.47	0.00%	299.25
17	**21**	**4**	**4**	*a*	1253.47	0.00%	1199.81	1253.47	0.00%	1336.42	1253.47	1.29%	TLR
18	**21**	**5**	**2**	*a*	1729.00	0.00%	1168.47	1729.00	0.61%	TLR	1729.00	0.00%	1136.09
19	**21**	**5**	**3**	*a*	1729.00	1.44%	TLR	1729.00	2.26%	TLR	1729.00	1.41%	TLR
20	**21**	**5**	**4**	*a*	1729.00	4.92%	TLR	1738.54	6.64%	TLR	1731.58	5.00%	TLR
21	**31**	**2**	**2**	*a*	916.53	2.33%	TLR	916.53	4.38%	TLR	916.53	2.02%	TLR
22	**31**	**2**	**3**	*a*	916.53	3.20%	TLR	916.53	6.43%	TLR	916.53	4.23%	TLR
23	**31**	**2**	**4**	*a*	916.53	3.69%	TLR	916.53	5.57%	TLR	916.53	5.03%	TLR
24	**31**	**3**	**2**	*a*	1285.54	5.39%	TLR	1287.10	6.31%	TLR	1285.54	5.00%	TLR
25	**31**	**3**	**3**	*a*	1285.54	5.75%	TLR	1285.54	6.52%	TLR	1285.54	5.80%	TLR
26	**31**	**3**	**4**	*a*	1287.10	6.31%	TLR	1287.10	7.37%	TLR	1285.54	6.68%	TLR
27	**31**	**4**	**2**	*a*	1740.92	6.09%	TLR	1751.13	6.24%	TLR	1740.92	5.57%	TLR
28	**31**	**4**	**3**	*a*	1748.91	6.86%	TLR	1769.25	8.65%	TLR	1748.91	6.27%	TLR
29	**31**	**4**	**4**	*a*	1748.91	7.62%	TLR	1752.34	6.07%	TLR	1740.92	6.85%	TLR
30	**31**	**5**	**2**	*a*	1796.70	9.38%	TLR	1862.42	12.96%	TLR	1777.63	8.13%	TLR
31	**31**	**5**	**3**	*a*	1780.23	8.25%	TLR	1788.59	9.19%	TLR	1789.11	8.82%	TLR
32	**31**	**5**	**4**	*a*	1828.77	12.16%	TLR	1786.50	10.27%	TLR	1792.46	9.95%	TLR

Table A.2 Computational results of M1, M2 and M3 for the benchmark instances with version *b*

					M1			**M2**			**M3**		
Ins.	**n**	*t*	*k*	**Ver.**	**OBJ**	**Gap**	**Time**	**OBJ**	**Gap**	**Time**	**OBJ**	**Gap**	**Time**
33	**11**	**2**	**2**	*b*	555.05	0.00%	0.33	555.05	0.00%	0.41	555.05	0.00%	0.42
34	**11**	**2**	**3**	*b*	555.05	0.00%	0.50	555.05	0.00%	0.89	555.05	0.00%	0.58
35	**11**	**3**	**2**	*b*	933.64	0.00%	0.80	933.64	0.00%	1.05	933.64	0.00%	1.17
36	**11**	**3**	**3**	*b*	933.64	0.00%	1.69	933.64	0.00%	1.34	933.64	0.00%	1.48
37	**11**	**4**	**2**	*b*	1202.49	0.00%	1.53	1202.49	0.00%	1.00	1202.49	0.00%	1.11
38	**11**	**4**	**3**	*b*	1202.49	0.00%	1.41	1202.49	0.00%	1.53	1202.49	0.00%	2.27
39	**11**	**5**	**2**	*b*	1457.10	0.00%	2.09	1457.10	0.00%	2.55	1457.10	0.00%	3.36
40	**11**	**5**	**3**	*b*	1457.10	0.00%	11.86	1457.10	0.00%	6.98	1457.10	0.00%	17.58
41	**21**	**2**	**2**	*b*	INF	INF	INF	INF	INF	INF	INF	INF	INF
42	**21**	**2**	**3**	*b*	797.84	0.00%	408.08	797.84	0.00%	843.41	797.84	0.00%	262.73
43	**21**	**2**	**4**	*b*	797.84	0.00%	1266.40	797.84	3.24%	TLR	797.84	4.45%	TLR
44	**21**	**3**	**2**	*b*	INF	INF	INF	INF	INF	INF	INF	INF	INF
45	**21**	**3**	**3**	*b*	1216.80	0.00%	2340.51	1216.80	1.34%	TLR	1216.80	0.00%	2164.90
46	**21**	**3**	**4**	*b*	1216.80	4.59%	TLR	1216.80	4.96%	TLR	1216.80	4.81%	TLR
47	**21**	**4**	**2**	*b*	1310.34	0.00%	69.11	1310.34	0.00%	524.95	1310.34	0.00%	207.46
48	**21**	**4**	**3**	*b*	1310.34	0.00%	4259.67	1310.34	0.73%	TLR	1310.34	0.24%	TLR
49	**21**	**4**	**4**	*b*	1310.34	2.33%	TLR	1325.25	8.09%	TLR	1310.34	9.95%	TLR
50	**21**	**5**	**2**	*b*	1710.65	0.00%	429.31	1710.65	0.00%	2469.63	1710.65	0.00%	469.61
51	**21**	**5**	**3**	*b*	1710.65	1.12%	TLR	1710.65	2.67%	TLR	1710.65	1.10%	TLR
52	**21**	**5**	**4**	*b*	1734.87	4.67%	TLR	1710.65	4.06%	TLR	1710.65	3.76%	TLR
53	**31**	**2**	**2**	*b*	1026.38	0.00%	655.68	1026.38	0.94%	TLR	1026.38	0.00%	818.22
54	**31**	**2**	**3**	*b*	1026.38	3.84%	TLR	1026.38	4.89%	TLR	1026.38	4.69%	TLR
55	**31**	**2**	**4**	*b*	1026.38	5.99%	TLR	1026.38	4.78%	TLR	1026.38	4.83%	TLR
56	**31**	**3**	**2**	*b*	1310.01	2.53%	TLR	1316.85	4.80%	TLR	1310.01	3.35%	TLR
57	**31**	**3**	**3**	*b*	1312.00	3.93%	TLR	1312.67	5.57%	TLR	1310.01	4.37%	TLR
58	**31**	**3**	**4**	*b*	1316.49	4.57%	TLR	1310.01	4.12%	TLR	1310.01	4.53%	TLR
59	**31**	**4**	**2**	*b*	1528.74	2.44%	TLR	1528.74	2.10%	TLR	1528.74	2.33%	TLR
60	**31**	**4**	**3**	*b*	1528.74	2.56%	TLR	1528.74	2.67%	TLR	1533.55	2.92%	TLR
61	**31**	**4**	**4**	*b*	1528.74	2.64%	TLR	1528.74	2.75%	TLR	1528.74	2.75%	TLR
62	**31**	**5**	**2**	*b*	2141.50	2.94%	TLR	2180.76	4.87%	TLR	2165.20	3.86%	TLR
63	**31**	**5**	**3**	*b*	2140.22	3.04%	TLR	2140.22	2.88%	TLR	2190.73	5.43%	TLR
64	**31**	**5**	**4**	*b*	2151.46	3.61%	TLR	2216.22	6.53%	TLR	2158.98	4.10%	TLR

Table A.3 Computational results of M1, M2 and M3 for the benchmark instances with version *c*

					M1			M2			M3		
Ins.	**n**	***t***	***k***	**Ver.**	**OBJ**	**Gap**	**Time**	**OBJ**	**Gap**	**Time**	**OBJ**	**Gap**	**Time**
65	**11**	**2**	**2**	*c*	581.49	0.00%	0.39	581.49	0.00%	0.66	581.49	0.00%	0.45
66	**11**	**2**	**3**	*c*	581.49	0.00%	0.77	581.49	0.00%	1.00	581.49	0.00%	0.78
67	**11**	**3**	**2**	*c*	672.53	0.00%	0.27	672.53	0.00%	0.33	672.53	0.00%	0.31
68	**11**	**3**	**3**	*c*	672.53	0.00%	0.41	672.53	0.00%	0.55	672.53	0.00%	0.52
69	**11**	**4**	**2**	*c*	982.55	0.00%	1.45	982.55	0.00%	1.45	982.55	0.00%	1.67
70	**11**	**4**	**3**	*c*	982.55	0.00%	2.81	982.55	0.00%	2.22	982.55	0.00%	4.27
71	**11**	**5**	**2**	*c*	1106.05	0.00%	0.70	1106.05	0.00%	0.81	1106.05	0.00%	0.77
72	**11**	**5**	**3**	*c*	1106.05	0.00%	1.39	1106.05	0.00%	1.94	1106.05	0.00%	1.27
73	**21**	**2**	**2**	*c*	807.85	0.00%	4.48	807.85	0.00%	34.73	807.85	0.00%	15.49
74	**21**	**2**	**3**	*c*	807.85	0.00%	63.97	807.85	0.00%	719.81	807.85	0.00%	74.46
75	**21**	**2**	**4**	*c*	807.85	0.00%	357.26	807.85	0.00%	1771.83	807.85	0.00%	290.56
76	**21**	**3**	**2**	*c*	966.11	0.00%	16.44	966.11	0.00%	38.13	966.11	0.00%	6.81
77	**21**	**3**	**3**	*c*	966.11	0.00%	166.42	966.11	0.00%	1434.76	966.11	0.00%	178.55
78	**21**	**3**	**4**	*c*	966.11	0.00%	600.13	966.11	0.00%	2279.26	966.11	0.00%	451.56
79	**21**	**4**	**2**	*c*	1187.64	0.00%	375.99	1187.64	0.00%	3262.25	1187.64	0.00%	271.03
80	**21**	**4**	**3**	*c*	1187.64	2.35%	TLR	1187.64	4.83%	TLR	1187.64	0.00%	3116.62
81	**21**	**4**	**4**	*c*	1187.64	3.27%	TLR	1187.64	6.33%	TLR	1187.64	3.01%	TLR
82	**21**	**5**	**2**	*c*	1820.05	2.31%	TLR	1837.80	7.40%	TLR	1820.05	2.13%	TLR
83	**21**	**5**	**3**	*c*	1818.90	6.15%	TLR	1878.39	10.63%	TLR	1819.84	6.20%	TLR
84	**21**	**5**	**4**	*c*	1824.40	16.55%	TLR	1839.81	18.42%	TLR	1831.35	15.94%	TLR
85	**31**	**2**	**2**	*c*	913.59	0.00%	3201.56	913.59	3.19%	TLR	913.59	2.37%	TLR
86	**31**	**2**	**3**	*c*	913.59	4.58%	TLR	913.59	3.91%	TLR	913.59	3.40%	TLR
87	**31**	**2**	**4**	*c*	913.59	2.40%	TLR	913.59	4.23%	TLR	913.59	5.45%	TLR
88	**31**	**3**	**2**	*c*	1309.23	6.14%	TLR	1317.52	6.29%	TLR	1317.40	7.29%	TLR
89	**31**	**3**	**3**	*c*	1325.32	7.83%	TLR	1329.52	8.95%	TLR	1317.40	6.96%	TLR
90	**31**	**3**	**4**	*c*	1335.24	8.87%	TLR	1317.40	8.15%	TLR	1329.47	8.65%	TLR
91	**31**	**4**	**2**	*c*	1816.23	8.60%	TLR	1803.58	8.83%	TLR	1849.35	10.83%	TLR
92	**31**	**4**	**3**	*c*	1787.49	7.79%	TLR	1808.56	9.18%	TLR	1812.51	8.97%	TLR
93	**31**	**4**	**4**	*c*	1795.30	7.95%	TLR	1812.34	9.43%	TLR	1796.86	8.77%	TLR
94	**31**	**5**	**2**	*c*	1754.44	4.54%	TLR	1767.73	3.84%	TLR	1748.04	4.03%	TLR
95	**31**	**5**	**3**	*c*	1734.87	3.50%	TLR	1770.04	5.48%	TLR	1762.72	4.88%	TLR
96	**31**	**5**	**4**	*c*	1745.10	4.47%	TLR	1761.60	4.87%	TLR	1775.74	5.97%	TLR

Table A.4 Computational results of M1 and M4 for the benchmark instances with version a

					M1			M4		
Ins.	**n**	t	k	**Ver.**	**OBJ**	**Gap**	**Time**	**OBJ**	**Gap**	**Time**
1	**11**	**2**	**2**	a	620.48	0.00%	0.64	620.48	0.00%	0.32
2	**11**	**2**	**3**	a	620.48	0.00%	1.20	620.48	0.00%	0.25
3	**11**	**3**	**2**	a	834.96	0.00%	1.88	834.96	0.00%	0.65
4	**11**	**3**	**3**	a	834.96	0.00%	2.42	834.96	0.00%	0.74
5	**11**	**4**	**2**	a	1043.91	0.00%	1.45	1043.91	0.00%	0.59
6	**11**	**4**	**3**	a	1043.91	0.00%	3.95	1043.91	0.00%	0.50
7	**11**	**5**	**2**	a	1083.97	0.00%	5.82	1083.97	0.00%	2.14
8	**11**	**5**	**3**	a	1083.97	0.00%	49.21	1083.97	0.00%	3.05
9	**21**	**2**	**2**	a	828.98	0.00%	1.08	828.98	0.00%	0.45
10	**21**	**2**	**3**	a	828.98	0.00%	1.66	828.98	0.00%	0.50
11	**21**	**2**	**4**	a	828.98	0.00%	18.32	828.98	0.00%	0.44
12	**21**	**3**	**2**	a	1088.36	0.00%	3.11	1088.36	0.00%	5.95
13	**21**	**3**	**3**	a	1088.36	0.00%	20.93	1088.36	0.00%	6.33
14	**21**	**3**	**4**	a	1088.36	0.00%	252.12	1088.36	0.00%	4.47
15	**21**	**4**	**2**	a	1253.47	0.00%	30.07	1253.47	0.00%	39.86
16	**21**	**4**	**3**	a	1253.47	0.00%	501.69	1253.47	0.00%	43.40
17	**21**	**4**	**4**	a	1253.47	0.00%	1199.81	1253.47	0.00%	48.13
18	**21**	**5**	**2**	a	1729.00	0.00%	1168.47	1729.19	8.39%	TLR
19	**21**	**5**	**3**	a	1729.00	1.44%	TLR	1729.19	6.79%	TLR
20	**21**	**5**	**4**	a	1729.00	4.92%	TLR	1729.00	7.92%	TLR
21	**31**	**2**	**2**	a	916.53	2.33%	TLR	916.53	0.00%	507.12
22	**31**	**2**	**3**	a	916.53	3.20%	TLR	916.53	0.00%	258.35
23	**31**	**2**	**4**	a	916.53	3.69%	TLR	916.53	0.00%	404.10
24	**31**	**3**	**2**	a	1285.54	5.39%	TLR	1285.54	0.00%	3934.15
25	**31**	**3**	**3**	a	1285.54	5.75%	TLR	1285.54	0.00%	4886.66
26	**31**	**3**	**4**	a	1287.10	6.31%	TLR	1285.54	0.00%	6151.28
27	**31**	**4**	**2**	a	1740.92	6.09%	TLR	NFS	NFS	TLR
28	**31**	**4**	**3**	a	1748.91	6.86%	TLR	1746.66	11.15%	TLR
29	**31**	**4**	**4**	a	1748.91	7.62%	TLR	1752.03	10.84%	TLR
30	**31**	**5**	**2**	a	1796.70	9.38%	TLR	1812.03	16.20%	TLR
31	**31**	**5**	**3**	a	1780.23	8.25%	TLR	1866.34	18.37%	TLR
32	**31**	**5**	**4**	a	1828.77	12.16%	TLR	1863.94	18.63%	TLR

Table A.5 Computational results of M1 and M4 for the benchmark instances with version *b*

					M1			M4		
Ins.	**n**	***t***	***k***	**Ver.**	**OBJ**	**Gap**	**Time**	**OBJ**	**Gap**	**Time**
33	**11**	**2**	**2**	*b*	555.05	0.00%	0.33	555.05	0.00%	0.27
34	**11**	**2**	**3**	*b*	555.05	0.00%	0.50	555.05	0.00%	0.25
35	**11**	**3**	**2**	*b*	933.64	0.00%	0.80	933.64	0.00%	0.69
36	**11**	**3**	**3**	*b*	933.64	0.00%	1.69	933.64	0.00%	0.75
37	**11**	**4**	**2**	*b*	1202.49	0.00%	1.53	1202.49	0.00%	0.64
38	**11**	**4**	**3**	*b*	1202.49	0.00%	1.41	1202.49	0.00%	0.63
39	**11**	**5**	**2**	*b*	1457.10	0.00%	2.09	1457.10	0.00%	1.66
40	**11**	**5**	**3**	*b*	1457.10	0.00%	11.86	1457.10	0.00%	2.72
41	**21**	**2**	**2**	*b*	INF	INF	INF	INF	INF	INF
42	**21**	**2**	**3**	*b*	797.84	0.00%	408.08	797.84	0.00%	57.56
43	**21**	**2**	**4**	*b*	797.84	0.00%	1266.40	797.84	0.00%	67.06
44	**21**	**3**	**2**	*b*	INF	INF	INF	INF	INF	INF
45	**21**	**3**	**3**	*b*	1216.80	0.00%	2340.51	1216.80	0.00%	17.53
46	**21**	**3**	**4**	*b*	1216.80	4.59%	TLR	1216.80	0.00%	31.94
47	**21**	**4**	**2**	*b*	1310.34	0.00%	69.11	1310.34	0.00%	589.48
48	**21**	**4**	**3**	*b*	1310.34	0.00%	4259.67	1310.34	0.00%	1068.10
49	**21**	**4**	**4**	*b*	1310.34	2.33%	TLR	1310.34	0.00%	1102.30
50	**21**	**5**	**2**	*b*	1710.65	0.00%	429.31	1710.65	0.00%	6181.64
51	**21**	**5**	**3**	*b*	1710.65	1.12%	TLR	1710.65	0.00%	5332.16
52	**21**	**5**	**4**	*b*	1734.87	4.67%	TLR	1710.65	0.00%	5039.51
53	**31**	**2**	**2**	*b*	1026.38	0.00%	655.68	1026.38	0.00%	120.99
54	**31**	**2**	**3**	*b*	1026.38	3.84%	TLR	1026.38	0.00%	162.87
55	**31**	**2**	**4**	*b*	1026.38	5.99%	TLR	1026.38	0.00%	136.29
56	**31**	**3**	**2**	*b*	1310.01	2.53%	TLR	1310.44	7.14%	TLR
57	**31**	**3**	**3**	*b*	1312.00	3.93%	TLR	1310.01	7.47%	TLR
58	**31**	**3**	**4**	*b*	1316.49	4.57%	TLR	1310.44	7.46%	TLR
59	**31**	**4**	**2**	*b*	1528.74	2.44%	TLR	1549.48	10.44%	TLR
60	**31**	**4**	**3**	*b*	1528.74	2.56%	TLR	1548.59	10.23%	TLR
61	**31**	**4**	**4**	*b*	1528.74	2.64%	TLR	1557.72	11.07%	TLR
62	**31**	**5**	**2**	*b*	2141.50	2.94%	TLR	2235.42	14.87%	TLR
63	**31**	**5**	**3**	*b*	2140.22	3.04%	TLR	NFS	NFS	TLR
64	**31**	**5**	**4**	*b*	2151.46	3.61%	TLR	2211.16	13.87%	TLR

Table A.6 Computational results of M1 and M4 for the benchmark instances with version c

					M1			M4		
Ins.	n	p	m	Ver.	OBJ	Gap	Time	OBJ	Gap	Time
65	**11**	**2**	**2**	c	581.49	0.00%	0.39	581.49	0.00%	0.29
66	**11**	**2**	**3**	c	581.49	0.00%	0.77	581.49	0.00%	0.36
67	**11**	**3**	**2**	c	672.53	0.00%	0.27	672.53	0.00%	0.17
68	**11**	**3**	**3**	c	672.53	0.00%	0.41	672.53	0.00%	0.18
69	**11**	**4**	**2**	c	982.55	0.00%	1.45	982.55	0.00%	4.05
70	**11**	**4**	**3**	c	982.55	0.00%	2.81	982.55	0.00%	3.77
71	**11**	**5**	**2**	c	1106.05	0.00%	0.70	1106.05	0.00%	0.55
72	**11**	**5**	**3**	c	1106.05	0.00%	1.39	1106.05	0.00%	0.78
73	**21**	**2**	**2**	c	807.85	0.00%	4.48	807.85	0.00%	1.73
74	**21**	**2**	**3**	c	807.85	0.00%	63.97	807.85	0.00%	1.52
75	**21**	**2**	**4**	c	807.85	0.00%	357.26	807.85	0.00%	1.72
76	**21**	**3**	**2**	c	966.11	0.00%	16.44	966.11	0.00%	6.21
77	**21**	**3**	**3**	c	966.11	0.00%	166.42	966.11	0.00%	5.23
78	**21**	**3**	**4**	c	966.11	0.00%	600.13	966.11	0.00%	4.80
79	**21**	**4**	**2**	c	1187.64	0.00%	375.99	1187.64	0.00%	102.79
80	**21**	**4**	**3**	c	1187.64	2.35%	TLR	1187.64	0.00%	125.55
81	**21**	**4**	**4**	c	1187.64	3.27%	TLR	1187.64	0.00%	95.83
82	**21**	**5**	**2**	c	1820.05	2.31%	TLR	1834.91	10.88%	TLR
83	**21**	**5**	**3**	c	1818.90	6.15%	TLR	1832.45	7.54%	TLR
84	**21**	**5**	**4**	c	1824.40	16.55%	TLR	1830.25	7.27%	TLR
85	**31**	**2**	**2**	c	913.59	0.00%	3201.56	913.59	0.00%	375.37
86	**31**	**2**	**3**	c	913.59	4.58%	TLR	913.59	0.00%	285.83
87	**31**	**2**	**4**	c	913.59	2.40%	TLR	913.59	0.00%	243.23
88	**31**	**3**	**2**	c	1309.23	6.14%	TLR	1314.11	13.04%	TLR
89	**31**	**3**	**3**	c	1325.32	7.83%	TLR	1309.86	12.44%	TLR
90	**31**	**3**	**4**	c	1335.24	8.87%	TLR	1311.94	12.87%	TLR
91	**31**	**4**	**2**	c	1816.23	8.60%	TLR	NFS	NFS	TLR
92	**31**	**4**	**3**	c	1787.49	7.79%	TLR	NFS	NFS	TLR
93	**31**	**4**	**4**	c	1795.30	7.95%	TLR	NFS	NFS	TLR
94	**31**	**5**	**2**	c	1754.44	4.54%	TLR	NFS	NFS	TLR
95	**31**	**5**	**3**	c	1734.87	3.50%	TLR	NFS	NFS	TLR
96	**31**	**5**	**4**	c	1745.10	4.47%	TLR	NFS	NFS	TLR

Table A.7 Computational results of M1, M1-S, M4 and M4-S for the benchmark instances with version a

					M1			M1-S			M4			M4-S		
Ins.	**n**	t	k	**Ver.**	**OBJ**	**Gap**	**time**	**OBJ**	**Gap**	**time**	**OBJ**	**Gap**	**time**	**OBJ**	**Gap**	**time**
1	**11**	**2**	**2**	a	620.48	0.00%	0.64	620.48	0.00%	1.20	620.48	0.00%	0.32	620.48	0.00%	0.37
2	**11**	**2**	**3**	a	620.48	0.00%	1.20	620.48	0.00%	2.11	620.48	0.00%	0.25	620.48	0.00%	0.45
3	**11**	**3**	**2**	a	834.96	0.00%	1.88	834.96	0.00%	1.92	834.96	0.00%	0.65	834.96	0.00%	0.96
4	**11**	**3**	**3**	a	834.96	0.00%	2.42	834.96	0.00%	5.45	834.96	0.00%	0.74	834.96	0.00%	0.69
5	**11**	**4**	**2**	a	1043.91	0.00%	1.45	1043.91	0.00%	1.69	1043.91	0.00%	0.59	1043.91	0.00%	0.94
6	**11**	**4**	**3**	a	1043.91	0.00%	3.95	1043.91	0.00%	3.41	1043.91	0.00%	0.50	1043.91	0.00%	1.38
7	**11**	**5**	**2**	a	1083.97	0.00%	5.82	1083.97	0.00%	8.03	1083.97	0.00%	2.14	1083.97	0.00%	2.99
8	**11**	**5**	**3**	a	1083.97	0.00%	49.21	1083.97	0.00%	48.14	1083.97	0.00%	3.05	1083.97	0.00%	3.78
9	**21**	**2**	**2**	a	828.98	0.00%	1.08	828.98	0.00%	3.28	828.98	0.00%	0.45	828.98	0.00%	1.99
10	**21**	**2**	**3**	a	828.98	0.00%	1.66	828.98	0.00%	6.67	828.98	0.00%	0.50	828.98	0.00%	2.45
11	**21**	**2**	**4**	a	828.98	0.00%	18.32	828.98	0.00%	8.73	828.98	0.00%	0.44	828.98	0.00%	1.75
12	**21**	**3**	**2**	a	1088.36	0.00%	3.11	1088.36	0.00%	3.77	1088.36	0.00%	5.95	1088.36	0.00%	5.94
13	**21**	**3**	**3**	a	1088.36	0.00%	20.93	1088.36	0.00%	31.77	1088.36	0.00%	6.33	1088.36	0.00%	7.19
14	**21**	**3**	**4**	a	1088.36	0.00%	252.12	1088.36	0.00%	237.56	1088.36	0.00%	4.47	1088.36	0.00%	5.23
15	**21**	**4**	**2**	a	1253.47	0.00%	30.07	1253.47	0.00%	28.50	1253.47	0.00%	39.86	1253.47	0.00%	53.90
16	**21**	**4**	**3**	a	1253.47	0.00%	501.69	1253.47	0.00%	250.41	1253.47	0.00%	43.40	1253.47	0.00%	53.52
17	**21**	**4**	**4**	a	1253.47	0.00%	1199.81	1253.47	0.00%	839.97	1253.47	0.00%	48.13	1253.47	0.00%	22.74
18	**21**	**5**	**2**	a	1729.00	0.00%	1168.47	1729.00	0.00%	191.20	1729.19	8.39%	TLR	1729.19	5.18%	TLR
19	**21**	**5**	**3**	a	1729.00	1.44%	TLR	1729.00	0.90%	TLR	1729.19	6.79%	TLR	1729.19	5.29%	TLR
20	**21**	**5**	**4**	a	1729.00	4.92%	TLR	1729.00	3.36%	TLR	1729.00	7.92%	TLR	1729.00	4.95%	TLR
21	**31**	**2**	**2**	a	916.53	2.33%	TLR	916.53	2.25%	TLR	916.53	0.00%	507.12	916.53	0.00%	1041.04
22	**31**	**2**	**3**	a	916.53	3.20%	TLR	916.53	3.00%	TLR	916.53	0.00%	258.35	916.53	0.00%	446.23
23	**31**	**2**	**4**	a	916.53	3.69%	TLR	916.53	3.80%	TLR	916.53	0.00%	404.10	916.53	0.00%	538.51
24	**31**	**3**	**2**	a	1285.54	5.39%	TLR	1287.10	5.54%	TLR	1285.54	0.00%	3934.15	1287.90	2.47%	TLR
25	**31**	**3**	**3**	a	1285.54	5.75%	TLR	1285.54	5.73%	TLR	1285.54	0.00%	4886.66	1285.54	0.00%	5161.57
26	**31**	**3**	**4**	a	1287.10	6.31%	TLR	1287.10	6.30%	TLR	1285.54	0.00%	6151.28	1287.10	2.42%	TLR
27	**31**	**4**	**2**	a	1740.92	6.09%	TLR	1743.31	6.33%	TLR	NFS	NFS	TLR	1775.10	12.49%	TLR
28	**31**	**4**	**3**	a	1748.91	6.86%	TLR	1757.65	7.19%	TLR	1746.66	11.15%	TLR	1769.11	11.00%	TLR
29	**31**	**4**	**4**	a	1748.91	7.62%	TLR	1761.96	8.23%	TLR	1752.03	10.84%	TLR	1756.39	11.32%	TLR
30	**31**	**5**	**2**	a	1796.70	9.38%	TLR	1767.50	7.75%	TLR	1812.03	16.20%	TLR	NFS	NFS	TLR
31	**31**	**5**	**3**	a	1780.23	8.25%	TLR	1781.72	9.89%	TLR	1866.34	18.37%	TLR	NFS	NFS	TLR
32	**31**	**5**	**4**	a	1828.77	12.16%	TLR	1816.21	10.70%	TLR	1863.94	18.63%	TLR	NFS	NFS	TLR

Table A.8 Computational results of M1, M1-S, M4 and M4-S for the benchmark instances with version b

					M1			M1-S			M4			M4-S		
Ins.	**n**	t	k	**Ver.**	**OBJ**	**Gap**	**time**	**OBJ**	**Gap**	**time**	**OBJ**	**Gap**	**time**	**OBJ**	**Gap**	**time**
33	**11**	**2**	**2**	b	555.05	0.00%	0.33	555.05	0.00%	1.13	555.05	0.00%	0.27	555.05	0.00%	0.33
34	**11**	**2**	**3**	b	555.05	0.00%	0.50	555.05	0.00%	1.05	555.05	0.00%	0.25	555.05	0.00%	0.38
35	**11**	**3**	**2**	b	933.64	0.00%	0.80	933.64	0.00%	2.44	933.64	0.00%	0.69	933.64	0.00%	1.72
36	**11**	**3**	**3**	b	933.64	0.00%	1.69	933.64	0.00%	3.12	933.64	0.00%	0.75	933.64	0.00%	0.86
37	**11**	**4**	**2**	b	1202.49	0.00%	1.53	1202.49	0.00%	2.46	1202.49	0.00%	0.64	1202.49	0.00%	0.81
38	**11**	**4**	**3**	b	1202.49	0.00%	1.41	1202.49	0.00%	3.63	1202.49	0.00%	0.63	1202.49	0.00%	0.80
39	**11**	**5**	**2**	b	1457.10	0.00%	2.09	1457.10	0.00%	4.57	1457.10	0.00%	1.66	1457.10	0.00%	1.61
40	**11**	**5**	**3**	b	1457.10	0.00%	11.86	1457.10	0.00%	13.24	1457.10	0.00%	2.72	1457.10	0.00%	1.56
41	**21**	**2**	**2**	b	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF
42	**21**	**2**	**3**	b	797.84	0.00%	408.08	797.84	0.00%	820.86	797.84	0.00%	57.56	797.84	0.00%	65.85
43	**21**	**2**	**4**	b	797.84	0.00%	1266.40	797.84	0.00%	2446.90	797.84	0.00%	67.06	797.84	0.00%	93.51
44	**21**	**3**	**2**	b	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF
45	**21**	**3**	**3**	b	1216.80	0.00%	2340.51	1216.80	0.00%	897.12	1216.80	0.00%	17.53	1216.80	0.00%	24.33
46	**21**	**3**	**4**	b	1216.80	4.59%	TLR	1216.80	3.22%	TLR	1216.80	0.00%	31.94	1216.80	0.00%	25.42
47	**21**	**4**	**2**	b	1310.34	0.00%	69.11	1310.34	0.00%	101.03	1310.34	0.00%	589.48	1310.34	0.00%	1487.74
48	**21**	**4**	**3**	b	1310.34	0.00%	4259.67	1310.34	0.00%	1605.88	1310.34	0.00%	1068.10	1310.34	0.00%	1023.09
49	**21**	**4**	**4**	b	1310.34	2.33%	TLR	1310.34	2.92%	TLR	1310.34	0.00%	1102.30	1310.34	0.00%	1303.56
50	**21**	**5**	**2**	b	1710.65	0.00%	429.31	1710.65	0.00%	121.17	1710.65	0.00%	6181.64	1710.65	2.26%	TLR
51	**21**	**5**	**3**	b	1710.65	1.12%	TLR	1710.65	1.73%	TLR	1710.65	0.00%	5332.16	1710.65	2.45%	TLR
52	**21**	**5**	**4**	b	1734.87	4.67%	TLR	1720.76	3.40%	TLR	1710.65	0.00%	5039.51	1710.65	1.20%	TLR
53	**31**	**2**	**2**	b	1026.38	0.00%	655.68	1026.38	1.57%	TLR	1026.38	0.00%	120.99	1026.38	0.00%	54.52
54	**31**	**2**	**3**	b	1026.38	3.84%	TLR	1026.38	3.51%	TLR	1026.38	0.00%	162.87	1026.38	0.00%	44.75
55	**31**	**2**	**4**	b	1026.38	5.99%	TLR	1026.38	5.53%	TLR	1026.38	0.00%	136.29	1026.38	0.00%	78.01
56	**31**	**3**	**2**	b	1310.01	2.53%	TLR	1310.01	3.71%	TLR	1310.44	7.14%	TLR	1310.01	7.29%	TLR
57	**31**	**3**	**3**	b	1312.00	3.93%	TLR	1310.01	4.05%	TLR	1310.01	7.47%	TLR	1310.01	7.08%	TLR
58	**31**	**3**	**4**	b	1316.49	4.57%	TLR	1310.01	4.17%	TLR	1310.44	7.46%	TLR	1310.01	7.11%	TLR
59	**31**	**4**	**2**	b	1528.74	2.44%	TLR	1528.74	1.75%	TLR	1549.48	10.44%	TLR	1557.61	10.56%	TLR
60	**31**	**4**	**3**	b	1528.74	2.56%	TLR	1528.74	2.52%	TLR	1548.59	10.23%	TLR	1547.83	9.79%	TLR
61	**31**	**4**	**4**	b	1528.74	2.64%	TLR	1528.74	2.79%	TLR	1557.72	11.07%	TLR	1550.62	9.89%	TLR
62	**31**	**5**	**2**	b	2141.50	2.94%	TLR	2145.59	2.78%	TLR	2235.42	14.87%	TLR	2202.93	13.20%	TLR
63	**31**	**5**	**3**	b	2140.22	3.04%	TLR	2164.29	3.88%	TLR	NFS	NFS	TLR	2226.67	14.23%	TLR
64	**31**	**5**	**4**	b	2151.46	3.61%	TLR	2175.03	4.87%	TLR	2211.16	13.87%	TLR	2219.21	13.88%	TLR

Table A.9 Computational results of M1, M1-S, M4 and M4-S for the benchmark instances with version c

					M1			M1-S			M4			M4-S		
Ins.	**n**	t	k	**Ver.**	**OBJ**	**Gap**	**time**	**OBJ**	**Gap**	**time**	**OBJ**	**Gap**	**time**	**OBJ**	**Gap**	**time**
65	**11**	**2**	**2**	c	581.49	0.00%	0.39	581.49	0.00%	0.54	581.49	0.00%	0.29	581.49	0.00%	0.23
66	**11**	**2**	**3**	c	581.49	0.00%	0.77	581.49	0.00%	1.34	581.49	0.00%	0.36	581.49	0.00%	0.20
67	**11**	**3**	**2**	c	672.53	0.00%	0.27	672.53	0.00%	0.54	672.53	0.00%	0.17	672.53	0.00%	0.23
68	**11**	**3**	**3**	c	672.53	0.00%	0.41	672.53	0.00%	0.98	672.53	0.00%	0.18	672.53	0.00%	0.23
69	**11**	**4**	**2**	c	982.55	0.00%	1.45	982.55	0.00%	5.70	982.55	0.00%	4.05	982.55	0.00%	1.84
70	**11**	**4**	**3**	c	982.55	0.00%	2.81	982.55	0.00%	6.17	982.55	0.00%	3.77	982.55	0.00%	2.09
71	**11**	**5**	**2**	c	1106.05	0.00%	0.70	1106.05	0.00%	2.13	1106.05	0.00%	0.55	1106.05	0.00%	0.97
72	**11**	**5**	**3**	c	1106.05	0.00%	1.39	1106.05	0.00%	3.02	1106.05	0.00%	0.78	1106.05	0.00%	1.17
73	**21**	**2**	**2**	c	807.85	0.00%	4.48	807.85	0.00%	15.12	807.85	0.00%	1.73	807.85	0.00%	2.48
74	**21**	**2**	**3**	c	807.85	0.00%	63.97	807.85	0.00%	43.64	807.85	0.00%	1.52	807.85	0.00%	2.94
75	**21**	**2**	**4**	c	807.85	0.00%	357.26	807.85	0.00%	216.59	807.85	0.00%	1.72	807.85	0.00%	2.98
76	**21**	**3**	**2**	c	966.11	0.00%	16.44	966.11	0.00%	13.14	966.11	0.00%	6.21	966.11	0.00%	4.89
77	**21**	**3**	**3**	c	966.11	0.00%	166.42	966.11	0.00%	36.17	966.11	0.00%	5.23	966.11	0.00%	7.11
78	**21**	**3**	**4**	c	966.11	0.00%	600.13	966.11	0.00%	735.65	966.11	0.00%	4.80	966.11	0.00%	6.75
79	**21**	**4**	**2**	c	1187.64	0.00%	375.99	1187.64	0.00%	94.20	1187.64	0.00%	102.79	1187.64	0.00%	115.47
80	**21**	**4**	**3**	c	1187.64	2.35%	TLR	1187.64	0.00%	1709.26	1187.64	0.00%	125.55	1187.64	0.00%	83.31
81	**21**	**4**	**4**	c	1187.64	3.27%	TLR	1187.64	1.64%	TLR	1187.64	0.00%	95.83	1187.64	0.00%	89.48
82	**21**	**5**	**2**	c	1820.05	2.31%	TLR	1820.05	0.00%	4677.73	1834.91	10.88%	TLR	1837.69	7.69%	TLR
83	**21**	**5**	**3**	c	1818.90	6.15%	TLR	1818.87	4.18%	TLR	1832.45	7.54%	TLR	1826.76	6.99%	TLR
84	**21**	**5**	**4**	c	1824.40	16.55%	TLR	1819.84	7.46%	TLR	1830.25	7.27%	TLR	1824.40	7.52%	TLR
85	**31**	**2**	**2**	c	913.59	0.00%	3201.56	913.59	0.00%	1587.37	913.59	0.00%	375.37	913.59	0.00%	281.13
86	**31**	**2**	**3**	c	913.59	4.58%	TLR	913.59	4.17%	TLR	913.59	0.00%	285.83	913.59	0.00%	425.65
87	**31**	**2**	**4**	c	913.59	2.40%	TLR	913.59	4.98%	TLR	913.59	0.00%	243.23	913.59	0.00%	318.02
88	**31**	**3**	**2**	c	1309.23	6.14%	TLR	1317.40	7.05%	TLR	1314.11	13.04%	TLR	1310.28	12.22%	TLR
89	**31**	**3**	**3**	c	1325.32	7.83%	TLR	1309.86	6.90%	TLR	1309.86	12.44%	TLR	1310.28	12.08%	TLR
90	**31**	**3**	**4**	c	1335.24	8.87%	TLR	1309.86	7.16%	TLR	1311.94	12.87%	TLR	1310.28	13.47%	TLR
91	**31**	**4**	**2**	c	1816.23	8.60%	TLR	1812.29	8.44%	TLR	NFS	NFS	TLR	NFS	NFS	TLR
92	**31**	**4**	**3**	c	1787.49	7.79%	TLR	1804.77	8.88%	TLR	NFS	NFS	TLR	NFS	NFS	TLR
93	**31**	**4**	**4**	c	1795.30	7.95%	TLR	1799.69	8.82%	TLR	NFS	NFS	TLR	NFS	NFS	TLR
94	**31**	**5**	**2**	c	1754.44	4.54%	TLR	1740.64	2.66%	TLR	NFS	NFS	TLR	NFS	NFS	TLR
95	**31**	**5**	**3**	c	1734.87	3.50%	TLR	1734.87	3.55%	TLR	NFS	NFS	TLR	NFS	NFS	TLR
96	**31**	**5**	**4**	c	1745.10	4.47%	TLR	1767.73	5.20%	TLR	NFS	NFS	TLR	NFS	NFS	TLR

Table A.10 Computational results of M1, M5, M4 and M6 for the benchmark instances with version *a*

					M1			M5			M4			M6		
Ins.	**n**	*t*	*k*	**Ver.**	**OBJ**	**Gap**	**Time**	**OBJ**	**Gap**	**Time**	**OBJ**	**Gap**	**Time**	**OBJ**	**Gap**	**Time**
1	**11**	**2**	**2**	*a*	620.48	0.00%	0.72	620.48	0.00%	1.43	620.48	0.00%	1.55	620.48	0.00%	1.82
2	**11**	**2**	**3**	*a*	620.48	0.00%	2.02	620.48	0.00%	2.69	620.48	0.00%	0.96	620.48	0.00%	2.19
3	**11**	**3**	**2**	*a*	838.32	0.00%	0.59	838.32	0.00%	0.94	838.32	0.00%	0.75	838.32	0.00%	1.50
4	**11**	**3**	**3**	*a*	838.32	0.00%	1.28	838.32	0.00%	1.65	838.32	0.00%	0.69	838.32	0.00%	1.94
5	**11**	**4**	**2**	*a*	1027.52	0.00%	6.27	1027.52	0.00%	7.85	1027.52	0.00%	8.58	1027.52	0.00%	9.19
6	**11**	**4**	**3**	*a*	1027.52	0.00%	129.58	1027.52	0.00%	177.59	1027.52	0.00%	12.80	1027.52	0.00%	9.95
7	**11**	**5**	**2**	*a*	1064.49	0.00%	16.49	1064.49	0.00%	12.30	1064.49	0.00%	7.96	1064.49	0.00%	6.77
8	**11**	**5**	**3**	*a*	1064.49	0.00%	39.46	1064.49	0.00%	47.94	1064.49	0.00%	8.74	1064.49	0.00%	6.46
9	**21**	**2**	**2**	*a*	827.88	0.00%	13.17	827.88	0.00%	21.97	827.88	0.00%	41.29	827.88	0.00%	39.98
10	**21**	**2**	**3**	*a*	827.88	0.00%	109.35	827.88	0.00%	154.64	827.88	0.00%	37.36	827.88	0.00%	33.98
11	**21**	**2**	**4**	*a*	827.88	0.00%	1063.72	827.88	0.00%	600.78	827.88	0.00%	35.88	827.88	0.00%	45.99
12	**21**	**3**	**2**	*a*	1063.53	0.00%	36.41	1063.53	0.00%	35.88	1063.53	0.00%	108.42	1063.53	0.00%	169.71
13	**21**	**3**	**3**	*a*	1063.53	0.00%	401.15	1063.53	0.00%	504.75	1063.53	0.00%	67.61	1063.53	0.00%	73.82
14	**21**	**3**	**4**	*a*	1063.53	1.31%	TLR	1063.53	1.31%	TLR	1063.53	0.00%	199.54	1063.53	0.00%	62.31
15	**21**	**4**	**2**	*a*	1219.16	0.00%	742.85	1219.16	0.00%	379.67	1219.16	3.05%	TLR	1219.16	2.98%	TLR
16	**21**	**4**	**3**	*a*	1219.16	0.77%	TLR	1219.16	0.84%	TLR	1219.16	2.90%	TLR	1221.03	3.24%	TLR
17	**21**	**4**	**4**	*a*	1219.16	2.13%	TLR	1219.16	1.54%	TLR	1221.03	3.00%	TLR	1220.28	2.92%	TLR
18	**21**	**5**	**2**	*a*	1768.06	0.00%	TLR	1768.06	1.70%	TLR	1768.06	3.88%	TLR	1768.06	5.43%	TLR
19	**21**	**5**	**3**	*a*	1768.98	3.62%	TLR	1769.13	3.59%	TLR	1768.06	3.92%	TLR	1768.06	4.76%	TLR
20	**21**	**5**	**4**	*a*	1771.23	9.18%	TLR	1771.08	7.90%	TLR	1768.06	5.56%	TLR	1768.06	4.22%	TLR
21	**31**	**2**	**2**	*a*	884.92	7.19%	TLR	883.13	7.25%	TLR	900.66	14.39%	TLR	915.25	16.59%	TLR
22	**31**	**2**	**3**	*a*	883.13	8.15%	TLR	884.92	7.87%	TLR	896.31	14.70%	TLR	896.98	14.42%	TLR
23	**31**	**2**	**4**	*a*	917.06	11.59%	TLR	891.67	9.11%	TLR	905.76	15.58%	TLR	907.02	15.44%	TLR
24	**31**	**3**	**2**	*a*	1310.75	6.82%	TLR	1310.62	6.63%	TLR	1294.91	0.00%	1373.96	1294.91	0.00%	1361.66
25	**31**	**3**	**3**	*a*	1301.27	7.00%	TLR	1294.91	6.71%	TLR	1294.91	0.00%	1528.90	1294.91	0.00%	1428.25
26	**31**	**3**	**4**	*a*	1294.91	6.61%	TLR	1297.03	6.80%	TLR	1294.91	0.00%	1940.53	1294.91	0.00%	2002.14
27	**31**	**4**	**2**	*a*	1753.35	6.60%	TLR	1759.47	6.54%	TLR	NFS	NFS	TLR	NFS	NFS	TLR
28	**31**	**4**	**3**	*a*	1740.52	6.27%	TLR	1739.26	7.82%	TLR	NFS	NFS	TLR	NFS	NFS	TLR
29	**31**	**4**	**4**	*a*	1740.38	6.69%	TLR	1848.24	13.26%	TLR	NFS	NFS	TLR	1829.34	17.26%	TLR
30	**31**	**5**	**2**	*a*	1846.06	12.31%	TLR	1827.62	9.92%	TLR	NFS	NFS	TLR	1956.54	20.37%	TLR
31	**31**	**5**	**3**	*a*	1821.41	10.19%	TLR	1798.53	8.93%	TLR	NFS	NFS	TLR	1888.00	17.78%	TLR
32	**31**	**5**	**4**	*a*	1821.31	13.52%	TLR	1799.48	9.98%	TLR	NFS	NFS	TLR	1858.93	16.10%	TLR

Table A.11 Computational results of M1, M5, M4 and M6 for the benchmark instances with version b

					M1			M5			M4			M6		
Ins.	**n**	t	k	**Ver.**	**OBJ**	**Gap**	**Time**	**OBJ**	**Gap**	**Time**	**OBJ**	**Gap**	**Time**	**OBJ**	**Gap**	**Time**
33	**11**	**2**	**2**	b	555.05	0.00%	0.86	555.05	0.00%	0.61	555.05	0.00%	0.55	555.05	0.00%	0.92
34	**11**	**2**	**3**	b	555.05	0.00%	0.94	555.05	0.00%	0.80	555.05	0.00%	0.83	555.05	0.00%	1.58
35	**11**	**3**	**2**	b	954.87	0.00%	0.69	954.87	0.00%	1.05	954.87	0.00%	0.61	954.87	0.00%	0.87
36	**11**	**3**	**3**	b	954.87	0.00%	2.84	954.87	0.00%	4.48	954.87	0.00%	0.64	954.87	0.00%	1.45
37	**11**	**4**	**2**	b	1193.36	0.00%	1.64	1193.36	0.00%	1.86	1193.36	0.00%	1.95	1193.36	0.00%	2.34
38	**11**	**4**	**3**	b	1193.36	0.00%	2.86	1193.36	0.00%	3.52	1193.36	0.00%	2.41	1193.36	0.00%	2.73
39	**11**	**5**	**2**	b	1512.26	0.00%	0.75	1512.26	0.00%	0.59	1512.26	0.00%	0.78	1512.26	0.00%	1.56
40	**11**	**5**	**3**	b	1512.26	0.00%	12.63	1512.26	0.00%	8.59	1512.26	0.00%	0.70	1512.26	0.00%	1.33
41	**21**	**2**	**2**	b	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF
42	**21**	**2**	**3**	b	797.84	0.00%	3547.25	797.84	0.00%	2279.54	797.84	0.00%	126.88	797.84	0.00%	189.75
43	**21**	**2**	**4**	b	797.84	3.13%	TLR	797.84	3.75%	TLR	797.84	0.00%	138.22	797.84	0.00%	146.66
44	**21**	**3**	**2**	b	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF
45	**21**	**3**	**3**	b	1215.79	1.91%	TLR	1215.79	1.43%	TLR	1215.79	0.00%	317.97	1215.79	0.00%	288.02
46	**21**	**3**	**4**	b	1215.79	4.67%	TLR	1215.79	4.73%	TLR	1215.79	0.00%	177.32	1215.79	0.00%	341.84
47	**21**	**4**	**2**	b	1300.01	2.01%	TLR	1300.01	1.93%	TLR	1326.57	13.11%	TLR	1313.63	12.46%	TLR
48	**21**	**4**	**3**	b	1306.41	4.89%	TLR	1305.18	10.25%	TLR	1318.3	12.20%	TLR	1325.39	12.41%	TLR
49	**21**	**4**	**4**	b	1322.60	14.95%	TLR	1306.41	6.37%	TLR	1304.25	11.10%	TLR	1328.58	13.17%	TLR
50	**21**	**5**	**2**	b	1832.80	0.00%	2174.04	1832.80	0.00%	3857.83	NFS	NFS	TLR	NFS	NFS	TLR
51	**21**	**5**	**3**	b	1782.17	4.58%	TLR	1783.39	5.74%	TLR	1772.8	3.16%	TLR	1772.80	4.00%	TLR
52	**21**	**5**	**4**	b	1805.49	8.06%	TLR	1785.61	6.69%	TLR	1772.8	2.08%	TLR	1786.23	5.88%	TLR
53	**31**	**2**	**2**	b	995.26	3.68%	TLR	995.26	2.41%	TLR	995.26	0.00%	2074.67	995.26	0.00%	2915.75
54	**31**	**2**	**3**	b	995.51	3.41%	TLR	995.26	3.08%	TLR	995.26	0.00%	2142.59	995.26	0.00%	2476.46
55	**31**	**2**	**4**	b	995.26	3.81%	TLR	995.26	3.77%	TLR	995.26	0.00%	2395.96	995.26	0.00%	2668.25
56	**31**	**3**	**2**	b	1304.14	3.84%	TLR	1304.14	1.71%	TLR	1304.14	7.46%	TLR	1304.14	7.87%	TLR
57	**31**	**3**	**3**	b	1304.14	4.88%	TLR	1304.14	4.50%	TLR	1306.82	7.93%	TLR	1305.92	7.75%	TLR
58	**31**	**3**	**4**	b	1304.14	3.97%	TLR	1304.14	1.42%	TLR	1304.14	7.53%	TLR	1307.25	7.94%	TLR
59	**31**	**4**	**2**	b	1521.71	1.14%	TLR	1521.71	0.09%	TLR	NFS	NFS	TLR	1588.42	14.42%	TLR
60	**31**	**4**	**3**	b	1521.71	2.39%	TLR	1540.90	4.91%	TLR	1555.13	12.37%	TLR	1640.94	16.43%	TLR
61	**31**	**4**	**4**	b	1521.71	2.25%	TLR	1553.82	5.56%	TLR	1614.2	15.99%	TLR	1588.95	14.18%	TLR
62	**31**	**5**	**2**	b	2274.21	6.68%	TLR	2298.95	7.17%	TLR	2292.51	14.14%	TLR	2284.33	13.60%	TLR
63	**31**	**5**	**3**	b	2278.74	7.86%	TLR	2267.65	7.61%	TLR	2281.84	13.69%	TLR	2288.82	14.15%	TLR
64	**31**	**5**	**4**	b	2277.51	8.35%	TLR	2274.21	7.76%	TLR	2280.9	13.65%	TLR	2275.38	13.43%	TLR

Table A.12 Computational results of M1, M5, M4 and M6 for the benchmark instances with version c

					M1			M5			M4			M6		
Ins.	**n**	t	k	**Ver.**	**OBJ**	**Gap**	**Time**	**OBJ**	**Gap**	**Time**	**OBJ**	**Gap**	**Time**	**OBJ**	**Gap**	**Time**
65	**11**	**2**	**2**	c	536.92	0.00%	1.52	536.92	0.00%	1.31	536.92	0.00%	1.23	536.92	0.00%	1.06
66	**11**	**2**	**3**	c	536.92	0.00%	1.58	536.92	0.00%	3.59	536.92	0.00%	0.78	536.92	0.00%	1.09
67	**11**	**3**	**2**	c	627.69	0.00%	0.83	627.69	0.00%	0.69	627.69	0.00%	0.31	627.69	0.00%	0.34
68	**11**	**3**	**3**	c	627.69	0.00%	0.73	627.69	0.00%	0.98	627.69	0.00%	0.30	627.69	0.00%	0.31
69	**11**	**4**	**2**	c	982.55	0.00%	2.22	982.55	0.00%	2.37	982.55	0.00%	12.34	982.55	0.00%	6.20
70	**11**	**4**	**3**	c	982.55	0.00%	3.31	982.55	0.00%	3.27	982.55	0.00%	15.50	982.55	0.00%	8.98
71	**11**	**5**	**2**	c	1101.06	0.00%	1.36	1101.06	0.00%	1.20	1101.06	0.00%	1.39	1101.06	0.00%	1.22
72	**11**	**5**	**3**	c	1101.06	0.00%	2.50	1101.06	0.00%	4.23	1101.06	0.00%	1.42	1101.06	0.00%	1.14
73	**21**	**2**	**2**	c	796.71	0.00%	347.73	796.71	0.00%	144.60	796.71	0.00%	71.38	796.71	0.00%	40.64
74	**21**	**2**	**3**	c	796.71	0.00%	3229.03	796.71	0.00%	3453.99	796.71	0.00%	42.94	796.71	0.00%	47.47
75	**21**	**2**	**4**	c	796.71	3.69%	TLR	796.71	2.62%	TLR	796.71	0.00%	44.57	796.71	0.00%	47.46
76	**21**	**3**	**2**	c	933.39	0.00%	126.25	933.39	0.00%	126.50	933.39	0.00%	39.24	933.39	0.00%	25.34
77	**21**	**3**	**3**	c	933.39	0.00%	1932.33	933.39	0.00%	3767.98	933.39	0.00%	18.35	933.39	0.00%	23.49
78	**21**	**3**	**4**	c	933.39	1.71%	TLR	933.39	2.11%	TLR	933.39	0.00%	20.58	933.39	0.00%	24.17
79	**21**	**4**	**2**	c	1185.19	6.80%	TLR	1188.18	6.49%	TLR	1183.61	9.69%	TLR	1183.58	9.51%	TLR
80	**21**	**4**	**3**	c	1194.82	15.16%	TLR	1199.46	17.25%	TLR	1183.58	10.00%	TLR	1187.34	8.87%	TLR
81	**21**	**4**	**4**	c	1183.58	15.27%	TLR	1201.22	10.90%	TLR	1188.26	10.03%	TLR	1189.53	9.92%	TLR
82	**21**	**5**	**2**	c	1871.20	7.29%	TLR	1851.72	6.43%	TLR	1848.16	2.34%	TLR	1848.16	0.00%	7004.61
83	**21**	**5**	**3**	c	1840.16	7.11%	TLR	1836.02	7.28%	TLR	1827.84	0.61%	TLR	1827.84	0.65%	TLR
84	**21**	**5**	**4**	c	1842.04	8.59%	TLR	1841.17	9.08%	TLR	1827.84	1.99%	TLR	1827.84	0.00%	2204.15
85	**31**	**2**	**2**	c	899.61	3.80%	TLR	899.61	3.98%	TLR	909.41	16.34%	TLR	902.05	15.22%	TLR
86	**31**	**2**	**3**	c	899.61	4.08%	TLR	899.61	4.08%	TLR	905.07	16.04%	TLR	899.64	14.87%	TLR
87	**31**	**2**	**4**	c	907.46	4.94%	TLR	899.61	4.39%	TLR	906.16	15.50%	TLR	908.55	17.09%	TLR
88	**31**	**3**	**2**	c	1332.07	8.67%	TLR	1347.96	9.10%	TLR	1318.01	0.00%	7115.11	1318.01	0.00%	4014.70
89	**31**	**3**	**3**	c	1347.96	10.05%	TLR	1359.52	10.21%	TLR	1318.01	5.29%	TLR	1318.26	3.94%	TLR
90	**31**	**3**	**4**	c	1347.96	10.23%	TLR	1318.01	6.91%	TLR	1318.01	0.00%	6260.45	1318.01	0.00%	5046.15
91	**31**	**4**	**2**	c	1757.73	8.27%	TLR	1766.81	7.95%	TLR	NFS	NFS	TLR	NFS	NFS	TLR
92	**31**	**4**	**3**	c	1803.81	10.75%	TLR	1792.55	9.03%	TLR	NFS	NFS	TLR	NFS	NFS	TLR
93	**31**	**4**	**4**	c	1802.77	10.69%	TLR	2112.47	23.46%	TLR	NFS	NFS	TLR	NFS	NFS	TLR
94	**31**	**5**	**2**	c	1798.69	8.07%	TLR	1788.67	7.40%	TLR	NFS	NFS	TLR	1863.78	19.97%	TLR
95	**31**	**5**	**3**	c	1799.49	7.93%	TLR	1835.51	9.50%	TLR	NFS	NFS	TLR	1865.07	19.60%	TLR
96	**31**	**5**	**4**	c	1791.22	7.64%	TLR	1798.77	7.37%	TLR	NFS	NFS	TLR	NFS	NFS	TLR