AN ESSAY ON URBAN ECONOMICS

by

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Submitted to the Graduate School of Social Sciences
in partial fulfillment of
the requirements for the degree of Master of Arts

Sabanci University

July 2022

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Date of Approval: July 1, 2022

ABSTRACT

AN ESSAY ON URBAN ECONOMICS

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ECONOMICS M.A. THESIS, JULY 2022

Thesis Supervisor: Prof. Eren İnci

Keywords: Park-and-ride, Location, Public transportation, Mode choice, Equilibrium

P&R facilities are widely adopted by many countries in order to mitigate traffic congestion and traffic-related pollution. This thesis investigates whether there might be disadvantages of P&R provision depending on the location choice. The main contribution is to provide a simple and tractable deterministic model with adopting a single public transportation in the form of a metro station and a P&R facility. Mode choice equilibria are characterized with and without P&R. Two numerical examples are documented to illustrate the results and their implications in which providing a P&R facility is not preferable at any location. By deriving the total cost and total distance driven functions, we interpret the results from both cost and environmental perspectives, and document the discrepancy between the location choices from the two perspectives.

ÖZET

KENT EKONOMİSİ ÜZERİNE BİR MAKALE

GÖRKEM AKCAN

EKONOMİ YÜKSEK LİSANS TEZİ, TEMMUZ 2022

Tez Danışmanı: Prof. Dr. Eren İnci

Anahtar Kelimeler: Park et devam et, Konum, Toplu taşıma, Türel seçim, Denge

"Park et devam et" tesisleri birçok ülke tarafından trafik sıkışıklığını ve trafik kaynaklı kirliliği azaltmak için kullanılıyor. Bu tez park et devam et tesisi sağlamanın konum seçimine bağlı olarak dezavantaj yaratıp yaratmayacağını araştırıyor. Çalışmanın temel katkısı metro formunda bir toplu taşıma imkanı ve bir park et devam et tesisi sunarak yalın ve işlenebilir deterministik bir model sağlamaktır. Denge vasıta seçimleri hem park et devam et tesisi sağlandığı hem de sağlanmadığı durumlar için tanımlanmıştır. Park et devam et tesisi sağlamanın her durumda tercih edilebilir olmağı sonuçlar ve anlamları iki sayısal örnek ile açıklanmıştır. Toplam maliyet ve toplam araba ile gidilen mesafe fonksiyonları elde edilerek sonuçlar hem maliyet hem çevresel bakış açısı ile yorumlanmış, iki bakış açısındaki konum seçimleri arasındaki farklılıklar gösterilmiştir.

ACKNOWLEDGEMENTS

I would like to begin with expressing my genuine gratitude to my thesis advisor Prof. Eren Inci for his supervision through my Master's studies. I am thankful for his constant support and guidance. I feel extremely lucky to have him enlightening the road especially when I thought I was facing a challenging problem.

I am thankful to all of the faculty members and Sabanci University family for providing me the resources that allowed me to be the person I am today.

I would also like to thank my family and friends for always being there with me along the road.

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1 INTRODUCTION

Park-and-ride (P&R) facilities provide individuals a place to park their private cars and take public transport conveniently, usually in the form of transferring to bus or rail services. P&R has been widely adopted by many countries since the emergence of early examples in 1930s (Noel 1988). Authorities are interested in building and operating P&R due to its potential in reducing traffic congestion and traffic-related pollution.

The location choice of where to provide P&R is an important question for the provider, because the benefits and demand for P&R might vary based on location. In this paper, we investigate the advantages and disadvantages of a P&R facility across different locations in a theoretical monocentric city model à la Hotelling (1929). While utilizing a deterministic model, given the unit costs and metro location, we can solve for all individuals mode choices assuming that everyone chooses the mode associated with the lowest cost. Available transport modes in our model are walking directly to the center, walking to and riding metro, driving directly to the center, and P&R.

The location of a P&R facility is usually determined given the existing location of public transport. In other words, the location of public transport is not decided together with the location of a possible P&R facility. As documented by Wang, Yang, and Lindsey (2004), a P&R is able to benefit the society in terms of welfare and the provider in terms of profits, if the provider is for-profit. However, we also consider the location choice of a public transport station as a constraint unlike them. It is our concern to investigate whether a P&R facility is provided at a suboptimal location, i.e., more costly than the case where it is not provided. Next, we investigate whether there is a difference between the optimal location choices with or without P&R provision. Moreover, we present the implications of our model documenting two numerical examples.

The rest of the thesis is organized as follows. Section 2 reviews the related literature on parking, public transportation and P&R as a combination of the first two. Section 3 presents the model, assumptions and give equilibrium definitions. Section 4 documents two numerical examples. Section 5 concludes. Lastly, an appendix continues with the Mathematica codes for the numerical example.

2 LITERATURE REVIEW

Transportation Economics literature focuses on a variety of tools in mitigating congestion-related externalities. Pricing and cruising for parking are viewed as important aspects that contribute to congestion in the parking literature. Incentivizing the use of public transportation is another strategy which is commonly adopted by many authorities. In this section, we review some of the papers from parking and transportation literature, particularly focusing on park-and-ride as a combination of parking and public transportation.

Shoup (2006) documents that cruising for parking is likely to lead to an increase in fuel consumption and negative externalities such as pollution and traffic congestion as a result of inaccurate pricing strategies.

Arnott and Rowse (1999) develop a simple stochastic model of spatial parking in which they analyze the search behavior for an available parking space leading to congestion. The environment in their model is a circular city while the available parking spaces are endogenously determined. They examine multiple equilibria with and without a parking fee. They suggest that the parking fee should be set equal to the value of congestion externality.

Arnott and Inci (2006) establish a model where they incorporate saturated on-street parking and traffic congestion which is caused by the cars looking for a parking space making their study the first to analyze cruising for parking through the lens of economics. They document that the efficient pricing strategy is to set the parking charge such that it eliminates the cruising activity while the parking spaces are fully occupied.

In economic theory, whether in relation to transportation or not, enhancing efficiency is usually the issue when there are externalities. As Santos et al. (2010) suggests, corrective instruments, for instance, taxes and cap-and-trade systems are implemented along with the complementary policies such as land use planning and public transportation. All of these instruments are studied to an extent by urban planners, civil engineers and economists in order to achieve a more efficient outcome. We are going to focus on some of the papers from public transport literature. As discussed by Parkhurst (1995), there might be potential advantages and disadvantages of providing park-and-ride facilities.

For decades, P&R facilities has been studied as an instrument that can potentially alleviate traffic congestion. While road pricing has been documented to be effective in that respect, few cities are adopting road pricing in practice due to political considerations (Inci 2015). In addition to pricing-related problems as in the parking literature, the location choice of public transport provision is another important factor in how many people are actually going to use it and whether it will be efficient to implement.

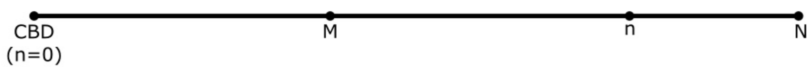
Wang, Yang, and Lindsey (2004) develop a deterministic model of a linear city to analyze the optimal location and pricing for a single P&R facility in order to investigate the optimal choice of location and parking charge. Assuming that the residents are uniformly distributed in between the central business district and the city boundary, each resident makes a one-way trip from where they reside to the city center. Available mode choices are driving through a congestible highway and a railway as a form of congestion-free public transport, both being continuously accessible at all points through the city boundaries. They find that the location of a P&R facility should not be too close to the city boundary or center, because then the demand for P&R is low such that it is unprofitable to compensate the high investment costs. Rather they find that P&R is optimally located at around the 70 to 80th percentile in their numerical example given the costs. In our setting, we also look for the optimal location of the public transport station without P&R. We also adopt an environmental perspective and calculate the total driven distances with and without P&R.

Basso, Navarro, and Silva (2021) analyze the impact of public transportation in shaping the urban structure. In a monocentric city framework, they allow for traveling via private car and public transportation with a finite number of stations. Furthermore, they allow for heterogeneity in income. Their untolled city results indicate that the share of car is around 60% and that of public transport is around 31%. Their tolled city results document that the share of car and of public transport are quite similar around 44-45% which indicates that toll allows more people to switch for public transport. Clearly, the demand for public transport is higher in a tolled city which results in a longer network of public transport. In a tolled city, the distance driven is also shorter leading pollution costs to decrease. In this paper, we do not look for tolls in particular, but provision of a P&R facility also leads costs and distances driven to decrease.

3 THE MODEL

The urban area has one Central Business District (CBD) at $n = 0$ with uniformly distributed individuals who reside at some $n \in (0, N)$ between the CBD and the boundary at $n = N$. Everyone makes a one-way trip from where they reside to the CBD. There is a highway which is accessible at all points along the city, and a single metro station which is accessible only at $n = M$. Everyone is assumed to own a car and all cars are identical. Parking at the CBD is available and of free-of-charge. Without the provision of the P&R facility, each individual is able to choose either walking to the CBD, driving to the CBD, or riding metro. If P&R facility is provided, an additional transportation mode becomes available which is driving to the metro station. We include costs of modes per unit distance as an approximation to the actual monetary and time costs. All individuals have homogeneous preferences over transportation modes such that they prefer the mode with the lowest cost. Figure 1 below presents a generic framework of monocentric city model where n is a location for a random individual.

Figure 1 A generic framework of monocentric city model



3.1 Travel Cost Functions

We now derive the cost functions of each travel mode one by one.

3.1.1 Travel cost of walking to the CBD

Walking to the CBD is available everywhere along the urban area and it is congestion free. Cost of walking per unit distance is fixed and it is equal to k_w for everyone. Therefore, the travel cost of walking to the CBD for an individual located at $n \in [0,N]$ is

$$C_w(n) = k_w n, \tag{1}$$

which yields an increasing function for the cost of walking as $\frac{\partial C_w(n)}{\partial n} = k_w > 0$.

3.1.2 Travel cost of riding metro

Riding metro is available everywhere along the urban area and is congestion free. This case can be considered as a special case of walking to the CBD as the travelers are walking from their residence to the metro station at $n = M$. In this case, travelers have to walk a distance of $|n - M|$ to the metro station from either side. There is also a cost of taking metro per unit distance, which is equal to k_m. Therefore, the travel cost of riding metro for an individual located at $n \in [0,N]$ is

$$C_{wm}(n) = k_w|n - M| + k_m M. \tag{2}$$

3.1.3 Travel cost of driving to the CBD

Driving to the CBD is available everywhere along the urban area, but it is congestible. Cost of driving per unit distance is fixed and equal to k_d. In addition, there is a congestion cost depending on the number of drivers at the equilibrium. Therefore, the travel cost of driving to the CBD for an individual located at $n \in [0,N]$ is

$$C_d(n) = k_d n + d(N_c), \tag{3}$$

where $d(N_c)$ is the congestion cost function resulting from the mass of drivers which is reflected by the term N_c such that

$$d(N_c) = \alpha N_c^{\beta}. \tag{4}$$

3.1.4 Travel cost of P&R

P&R is equivalent to driving to the metro station. If a P&R facility is provided, because driving is available everywhere, P&R is also available everywhere. In this case, travelers must drive a distance of $|n - M|$ to the metro station from either side. The congestion cost from driving is included as in Equation 3. In addition, the travel cost of taking metro is included, that is $k_m M$. Therefore, the travel cost of driving to the metro station, or P&R, for an individual located at $n \in [0, N]$ is

$$C_{P\&R}(n) = k_d|n - M| + d(N_c) + k_m M. \tag{5}$$

3.2 Assumptions on Unit Costs

Suppose that we are given an environment where the public transportation is not available at all, i.e., there is no metro station. In such an environment, only walking and driving to the CBD are available such that the set of transportation modes is $T = \{w, d\}$. Given a reasonable k_w, if the cost of driving per unit distance is greater than that of walking, we have $k_d > k_w$. Then, in the equilibrium of this two-mode model, everyone chooses to walk and not to drive to the CBD. This kind of mode choices in the equilibrium is not realistic as there should be individuals who choose to drive along the urban area, because driving is simply faster than walking if nothing else. This leads us to the following assumption:

Assumption 3.1. The cost of driving per unit distance is less than that of walking, i.e., $k_d < k_w$.

The equilibrium with this two-mode urban area includes individuals who are either walking or driving to the CBD. Then, there should be a point, N_1 where the modes are different at $N_1 - \epsilon$ and $N_1 + \epsilon$ for a small $\epsilon > 0$.

Now, suppose that the metro station is available but P&R is not provided so that the metro station is only accessible by walking. In this case, the set of transportation modes is $T = \{w, d, wm\}$. Given k_w and k_d with $k_w > k_d$ (A.3.1), if the cost of metro per unit distance is greater than that of walking, we need to have $k_m > k_w$. Then, in the equilibrium of this three-mode model, all individuals choose to either walk or drive to the CBD. This kind of mode choices in the equilibrium implies that no one takes the metro, and the investment into a public transportation is meaningless. Therefore, in order to incentivize some amount of public transportation use, the cost of metro per unit distance should be less than that of walking. This leads us to the following assumption:

Assumption 3.2. The cost of metro per unit distance is less than that of walking, i.e., $k_m < k_w$.

Suppose that walking to the CBD is not available and the set of transportation modes is $T = \{wm, d\}$. In such an environment, if $k_m > k_d$, everyone chooses to drive to the CBD and the investment into the public transportation is a waste of resources. Therefore, in order to incentivize some amount of riding metro, the cost of metro per unit distance should be less than that of driving. This leads us to the following assumption:

Assumption 3.3. The travel time per unit distance of metro is less than that of driving, i.e., $k_m < k_d$.

Based on Assumptions 3.1, 3.2, and 3.3, we need to have that nonnegative unit costs to satisfy $k_w > k_d > k_m$ in the context of the model presented here. Overall, the costs of modes per unit distance, k_w, k_d, and k_m represent compound costs consisting of time costs, fuel costs for driving, ticket costs for metro.

Given the parameter values for k_w, k_d and k_m, the transportation mode choices in the equilibrium differ depending on the location choice for the metro station (M). We investigate different layouts of mode choices along the urban area. Overall, there are three available mode choices in the case where P&R is not provided which are walking to the CBD, riding metro, and driving to the CBD. In the case where P&R is provided, in addition to these three modes available, travelers can also choose P&R.

Figure 2 describes the possible equilibrium layouts of mode choices within the urban area $[0, N]$ without P&R. In panel (a) of Figure 2, the metro station is located at $0 < M < M_1$ where M_1 is the first critical metro location such that driving is not observed for those $n \in [0, M]$. In panel (b) of Figure 2, the metro station is located at $M_1 \leq M < M_2$ where M_2 is the second critical metro location such that driving is observed for those $n \in [0, M]$. This is the case in which all possible mode choices are observed in the equilibrium both with and without P&R. In panel (c) of Figure 2, the metro station is located at $M_2 \leq M \leq N$ such that driving is not observed for those $n \in [M, N]$.

Table 1 Parameter definitions

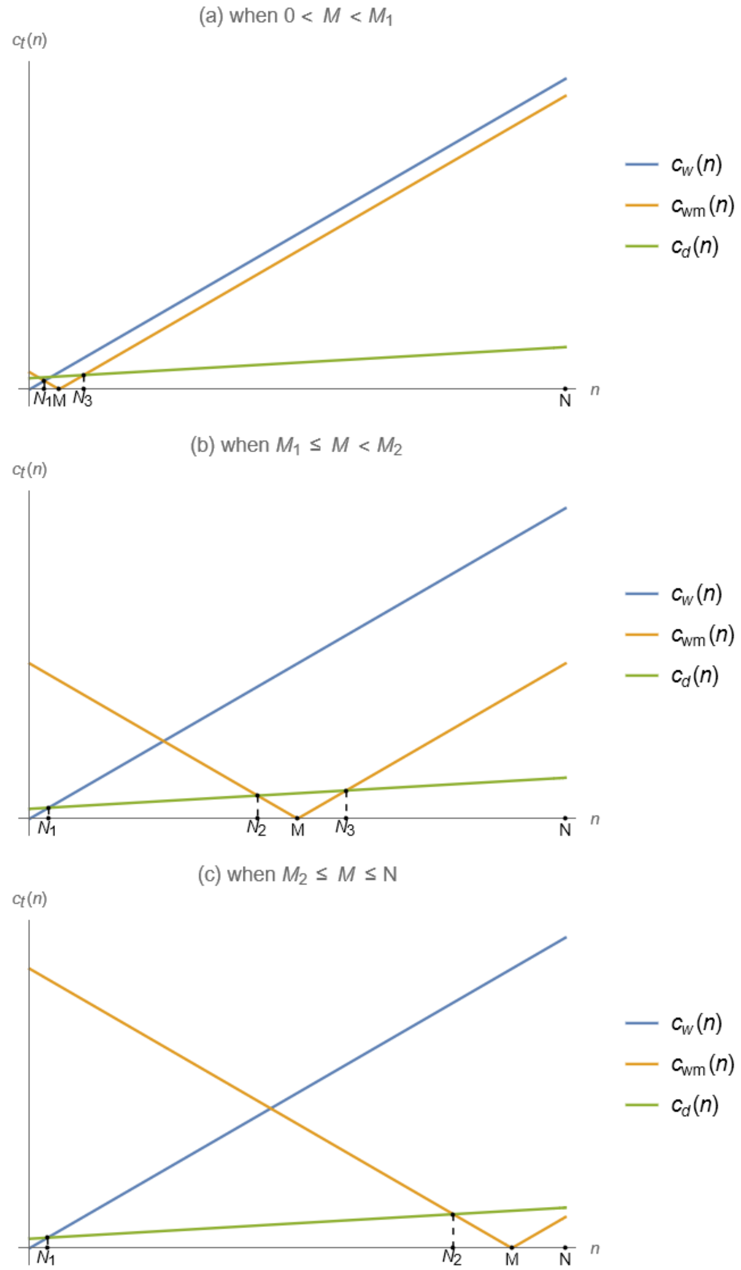
Symbol	Definition
k_w	Cost of walking per unit distance
k_d	Cost of driving per unit distance
k_m	Cost of taking metro per unit distance
M	Location of the metro station
M_1	Location of the first critical metro station
M_{12}	Location of the intermediate critical metro station
M_2	Location of the second critical metro station
N_1	Location of the individual who chooses to walk to CBD
N_2	Location of indifference between driving to CBD and walking to M, to the left of M
N_3	Location of indifference between driving to CBD and walking to M, to the right of M
N_4	Location of indifference between driving to CBD and doing P&R, to the left of M
N_5	Location of indifference between doing P&R and walking to M, to the left of M
N_6	Location of indifference between doing P&R and walking to M, to the right of M
N_c	Total percentage of drivers
N	Boundary of the urban area

Investigating Figure 2 on a deeper perspective, we now clarify who is choosing what transport mode. Across all panels in Figure 2, travelers who reside in $n \in (0, N_1)$ choose to walk to the CBD. In panel (a) of Figure 2, travelers who reside in $n \in [N_1, N_3]$ choose to ride metro at M, and those whose reside in $n \in [N_3, N]$ choose to drive to the CBD which results in a mass of drivers equal to $N_c = N - N_3$. In panel (b) of Figure 2, travelers who reside in $n \in [N_1, N_2]$ choose to drive to the CBD, those who reside in $n \in [N_2, N_3]$ choose to ride metro, and those who reside in $n \in [N_3, N]$ choose to drive to the CBD which gives $N_c = N - N_3 + N_2 - N_1$. In panel (c) of Figure 2, travelers who reside in $n \in [N_1, N_2]$ choose to drive to the CBD, and those who reside in $n \in [N_2, N]$ choose to ride metro, which gives $N_c = N_2 - N_1$.

Figure 3 illustrates the possible equilibrium layouts of mode choices within the urban area $[0, N]$ with P&R. In panel (a) of Figure 3, the metro station is located at $0 < M < M_1$ where M_1 is the first critical metro location such that driving is not observed for those $n \in (0, M)$ neither to the CBD nor to the metro station. In panel (b) of Figure 3, the metro station is located at $M_1 \leq M < M_{12}$ where M_{12} is the intermediate metro location such that driving to the CBD is observed but to the metro station is not for those $n \in (0, M)$. In panel (c) of Figure 3, the metro station is located at $M_{12} \leq M < M_2$ where M_2 is the second critical metro location such that driving both to the CBD and the metro station is observed for those $n \in [0, M]$. In panel (d) of Figure 3, the metro station is located at $M_2 \leq M < N$ such that driving is not observed for those $n \in (M, N)$.

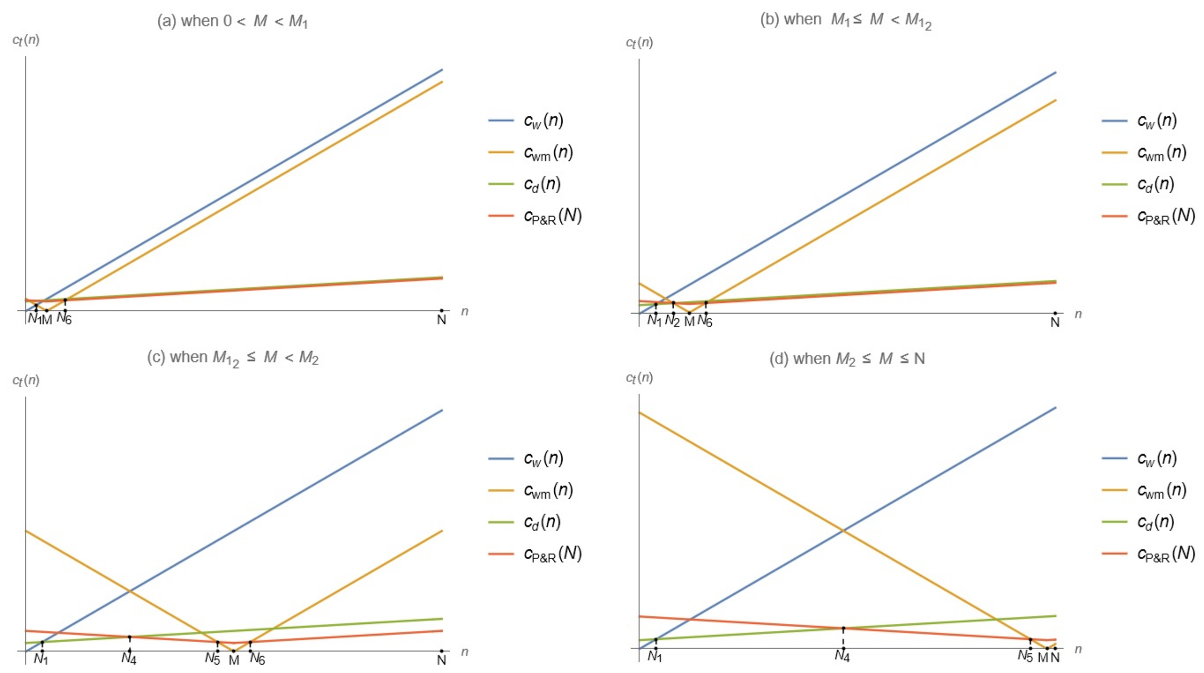
Let us also clarify who is choosing what transport mode in Figure 3. Across all the

Figure 2 Possible equilibrium layouts of mode choices without P&R



panels in Figure 3, travelers who reside in $n \in [0, N_1]$ choose to walk to the CBD. In panel (a) of Figure 3, travelers who reside in $n \in [N_1, N_6]$ choose to ride metro at M, and those who reside in $n \in [N_6, N]$ choose P&R, which gives $N_c = N - N_6$. In panel (b) of Figure 3, travelers who reside in $n \in [N_1, N_2]$ choose to drive to the CBD, those who reside in $n \in [N_2, N_6]$ choose to ride metro, and those who reside in $n \in [N_6, N]$ choose P&R, which results in $N_c = N - N_6 + N_2 - N_1$. In panel (c) of Figure 3, travelers who reside in $n \in [N_1, N_4]$ choose to drive to the CBD, those who reside in $n \in [N_4, N_5]$ choose P&R, those who reside in $n \in [N_5, N_6]$ choose to ride metro, and those who reside in

Figure 3 Possible equilibrium layouts of mode choices with P&R



$n \in [N_6, N]$ choose P&R, which results in $N_c = N - N_6 + N_5 - N_1$. In panel (d) of Figure 3, travelers who reside in $n \in [N_1, N_4]$ choose to drive to the CBD, those who reside in $n \in [N_4, N_5]$ choose P&R, those who reside in $n \in [N_5, N]$ choose to ride metro, which yields $N_c = N_5 - N_1$.

Both with and without P&R, the first critical metro location, M_1 is found by setting equations (1), (2) and (3) equal given the values for k_w, k_d, k_m. Without P&R, the second critical metro location, M_2, is found by setting Equations (2) and (3) equal at $n = N$ given k_w, k_d, k_m. With P&R, the intermediate critical metro location, M_{12}, is found by setting equations (2), (3), and (5). With P&R, the second critical metro location, M_2, is found by setting equations (2) and (5) equal at $n = N$ given k_w, k_d, k_m.

3.3 Equilibrium

We now give definitions of equilibrium mode layouts with and without P&R. Further, we present the conditions for which mode choice layout is observed as discussed by Figures 2 and 3.

Definition .1 (Equilibrium without P&R)**.** *Given k_w, k_{wm}, k_d satisfying Assumptions* 3.1 - 3.3*, and the metro location $M \in (0, N)$, any individual located at $n \in (0, N)$ chooses to* *travel to the CBD via the mode* $t^* \in T := \{w, wm, d\}$ *such that*

$$t^* = \begin{cases} w & min\{C_w, C_{wm}, C_d\} = C_w \\ wm & min\{C_w, C_{wm}, C_d\} = C_{wm} \\ d & min\{C_w, C_{wm}, C_d\} = C_d. \end{cases} \tag{6}$$

Given k_w, k_{wm}, k_d such that Assumptions 3.1 - 3.3 are satisfied, and the metro location $M \in (0, N)$, the equilibrium is characterized by the panel (a) of Figure 2 if we have that $0 < M < M_1$ and at the location of indifference N_1, driving needs to be more costly than walking to CBD and riding metro such that $C_d(N_1) > C_w(N_1) = C_{wm}(N_1) \iff \frac{d(N_c)}{k_w - k_d} > N_1$, and at the boundary N, riding metro needs to be more costly than driving such that $C_{wm}(N) > C_d(N) \iff N > N_3$ where $N_1 = \frac{M(k_w+k_m)}{2k_w}$, $N_3 = \frac{d(N_c)+M(k_w-k_m)}{k_w-k_d}$, $N_c = N - N_3$.

Given k_w, k_{wm}, k_d such that Assumptions 3.1 - 3.3 are satisfied, and the metro location $M \in (0, N)$, the equilibrium is characterized by the panel (b) of Figure 2 if we have that $M_1 \le M < M_2$, and at the location of indifference N_1, riding metro needs to be more costly than walking and driving to CBD such that $C_{wm}(N_1) > C_w(N_1) = C_d(N_1) \iff \frac{M(k_w+k_m)}{2k_w} > N_1$, and at the location of indifference N_2, walking to CBD needs to be more costly than riding metro and driving to CBD such that $C_w(N_2) > C_{wm}(N_2) = C_d(N_2) \iff N_2 > \frac{M(k_w+k_m)}{2k_w}$, and at the boundary N, riding metro needs to be more costly than driving such that $C_{wm}(N) > C_d(N) \iff N > N_3$ where $N_1 = \frac{d(N_c)}{k_w-k_d}$, $N_2 = M - \frac{d(N_c)}{k_w+k_d}$, $N_3 = \frac{d(N_c)+M(k_w-k_m)}{k_w-k_d}$, $N_c = N - N_3 + N_2 - N_1$.

Given k_w, k_{wm}, k_d such that Assumptions 3.1 - 3.3 are satisfied, and the metro location $M \in (0, N)$, the equilibrium is characterized by the panel (c) of Figure 2 if we have that $M_2 \le M < N$, and at the location of indifference N_1, riding metro needs to be more costly than walking and driving to CBD such that $C_{wm}(N_1) > C_w(N_1) = C_d(N_1) \iff \frac{M(k_w+k_m)}{2k_w} > N_1$, and at the location of indifference N_2, walking to CBD needs to be more costly than riding metro and driving to CBD such that $C_w(N_2) > C_{wm}(N_2) = C_d(N_2) \iff N_2 > \frac{M(k_w+k_m)}{2k_w}$, and at the boundary N, driving to CBD needs to be more costly than riding metro such that $C_d(N) > C_{wm}(N) \iff N_3 > N$ where $N_1 = \frac{d(N_c)}{k_w-k_d}$, $N_2 = M - \frac{d(N_c)}{k_w+k_d}$, $N_3 = \frac{d(N_c)+M(k_w-k_m)}{k_w-k_d}$, $N_c = N_2 - N_1$.

Definition .2 (Equilibrium with P&R)**.** *Given* k_w, k_{wm}, k_d *satisfying Assumptions* 3.1 - 3.3*, and the metro location* $M \in (0, N)$*, any individual located at* $n \in (0, N)$ *chooses to* *travel to the CBD via the mode* $t^* \in T := \{w, wm, d, P\&R\}$ *such that*

$$t^* = \begin{cases} w & min\{C_w, C_{wm}, C_d, C_{P\&R}\} = C_w \\ wm & min\{C_w, C_{wm}, C_d, C_{P\&R}\} = C_{wm} \\ d & min\{C_w, C_{wm}, C_d, C_{P\&R}\} = C_d \\ P\&R & min\{C_w, C_{wm}, C_d, C_{P\&R}\} = C_{P\&R}. \end{cases} \tag{7}$$

Given k_w, k_{wm}, k_d such that Assumptions 3.1 - 3.3 are satisfied, and the metro location $M \in (0, N)$, the equilibrium is characterized by the panel (a) of Figure 3 if we have that $0 < M < M_1$, and at the location of indifference N_1, driving to CBD needs to be more costly than walking to CBD and M such that $C_d(N_1) > C_w(N_1) = C_{wm}(N_1) \iff d(N_c) > N_1(k_w - k_d)$, and at the same location, doing P&R needs to be more costly than walking to CBD and M such that $C_{P\&R}(N_1) > C_w(N_1) = C_{wm}(N_1) \iff N_1 > \frac{M(k_w-k_d)-d(N_c)}{k_w-k_d}$ where $N_1 = \frac{M(k_w+k_m)}{2k_w}$, $N_6 = \frac{M(k_w-k_d)+d(N_c)}{k_w-k_d}$, and $N_c = N - N_6$.

Given k_w, k_{wm}, k_d such that Assumptions 3.1 - 3.3 are satisfied, and the metro location $M \in (0, N)$, the equilibrium is characterized by the panel (b) of Figure 3 if we have that $M_1 \le M < M_{12}$, and at the location of indifference N_1, riding metro needs to be more costly than walking and driving to CBD such that $C_{wm}(N_1) > C_w(N_1) = C_d(N_1) \iff \frac{M(k_w+k_m)}{2k_w} > N_1$, and at the same location, doing P&R needs to be more costly that walking and driving to CBD such that $C_{P\&R}(N_1) > C_w(N_1) = C_d(N_1) \iff \frac{M(k_d+k_m)}{2k_d} > N_1$, and at the location of indifference N_2, doing P&R needs to be more costly than riding metro and driving to CBD such that $C_{P\&R}(N_2) > C_{wm}(N_2) = C_d(N_2) \iff \frac{M(k_d+k_m)}{2k_d} > N_2$, and at the boundary, riding metro needs to be more costly than P&R such that $C_{wm}(N) > C_{P\&R}(N) \iff N > M + \frac{d(N_c)}{k_w-k_d}$ where $N_1 = \frac{d(N_c)}{k_w-k_d}$, $N_2 = \frac{M(k_w+k_m)-d(N_c)}{k_w+k_d}$, and $N_c = N - N_6 + N_2 - N_1$.

Given k_w, k_{wm}, k_d such that Assumptions 3.1 - 3.3 are satisfied, and the metro location $M \in (0, N)$, the equilibrium is characterized by the panel (c) of Figure 3 if we have that $M_{12} \le M < M_2$, and at the location of indifference N_1, riding metro needs to be more costly than walking and driving to CBD such that $C_{wm}(N_1) > C_w(N_1) = C_d(N_1) \iff \frac{M(k_w+k_m)}{2k_w} > N_1$, and at the same location, P&R needs to be more costly than walking and driving to CBD such that $C_{P\&R}(N_1) > C_w(N_1) = C_d(N_1) \iff \frac{M(k_d+k_m)}{2k_d} > N_1$, and at the location of indifference N_4, riding metro needs to be more costly than driving to CBD and P&R such that $C_{wm}(N_4) > C_d(N_4) = C_{P\&R}(N_4) \iff M - \frac{d(N_c)}{k_w-k_d} > N_4$, and at the same location, walking to CBD needs to be more costly than driving to CBD and P&R such that $C_w(N_4) > C_d(N_4) = C_{P\&R}(N_4) \iff N_4 > \frac{d(N_c)}{k_w-k_d}$, and at the location of indifference N_5, driving to CBD needs to be more costly than riding metro and doing P&R such that $C_d(N_5) > C_{wm}(N_5) = C_{P\&R}(N_5) \iff N_5 > \frac{M(k_d+k_m)}{2k_d}$, and at the location of indifference N_6, driving to CBD needs to be more costly than riding metro and doing P&R such that $C_d(N_6) > C_{wm}(N_6) = C_{P\&R}(N_6) \iff M + \frac{d(N_c)}{k_w - k_d} > N_6$, and at the boundary, riding metro needs to be more costly than doing P&R such that $C_{wm}(N) > C_{P\&R}(N) \iff N > N_6$ where $N_1 = \frac{d(N_c)}{k_w - k_d}$, $N_4 = \frac{M(k_d + k_m)}{2k_d}$, $N_5 = M - \frac{d(N_c)}{k_w - k_d}$, $N_6 = M + \frac{d(N_c)}{k_w - k_d}$, $N_c = N - N_6 + N_5 - N_1$.

Given k_w, k_{wm}, k_d such that Assumptions 3.1 - 3.3 are satisfied, and the metro location $M \in (0, N)$, the equilibrium is characterized by the panel (d) of Figure 3 if we have that $M_2 \leq M < N$, and at the location of indifference N_1, riding metro needs to be more costly than walking and driving to CBD such that $C_{wm}(N_1) > C_w(N_1) = C_d(N_1) \iff \frac{M(k_w + k_m)}{2k_w} > N_1$, and at the same location, P&R needs to be more costly than walking and driving to CBD such that $C_{P\&R}(N_1) > C_w(N_1) = C_d(N_1) \iff \frac{M(k_d + k_m)}{2k_d} > N_1$, and at the location of indifference N_4, riding metro needs to be more costly than driving to CBD and P&R such that $C_{wm}(N_4) > C_d(N_4) = C_{P\&R}(N_4) \iff M - \frac{d(N_c)}{k_w - k_d} > N_4$, and at the same location, walking to CBD needs to be more costly than driving to CBD and P&R such that $C_w(N_4) > C_d(N_4) = C_{P\&R}(N_4) \iff N_4 > \frac{d(N_c)}{k_w - k_d}$, and at the location of indifference N_5, driving to CBD needs to be more costly than riding metro and P&R such that $C_d(N_5) > C_{wm}(N_5) = C_{P\&R}(N_5) \iff N_5 > \frac{M(k_d + k_m)}{2k_d}$, and at the boundary, P&R needs to be more costly than riding metro such that $C_{P\&R}(N) > C_{wm}(N) \iff N_6 > N$ where $N_1 = \frac{d(N_c)}{k_w - k_d}$, $N_4 = \frac{M(k_d + k_m)}{2k_d}$, $N_5 = M - \frac{d(N_c)}{k_w - k_d}$, $N_6 = M + \frac{d(N_c)}{k_w - k_d}$, $N_c = N_5 - N_1$.

4 NUMERICAL EXAMPLES

In this section, we provide two numerical examples of how the model works and present the results and their implications. Starting with a generic solution of the model, we investigate the location choice of the metro station with and without P&R. Then, we choose a particular location for the metro station, and present the mode choices in detail along with a comparison of their associated costs.

4.1 First Example

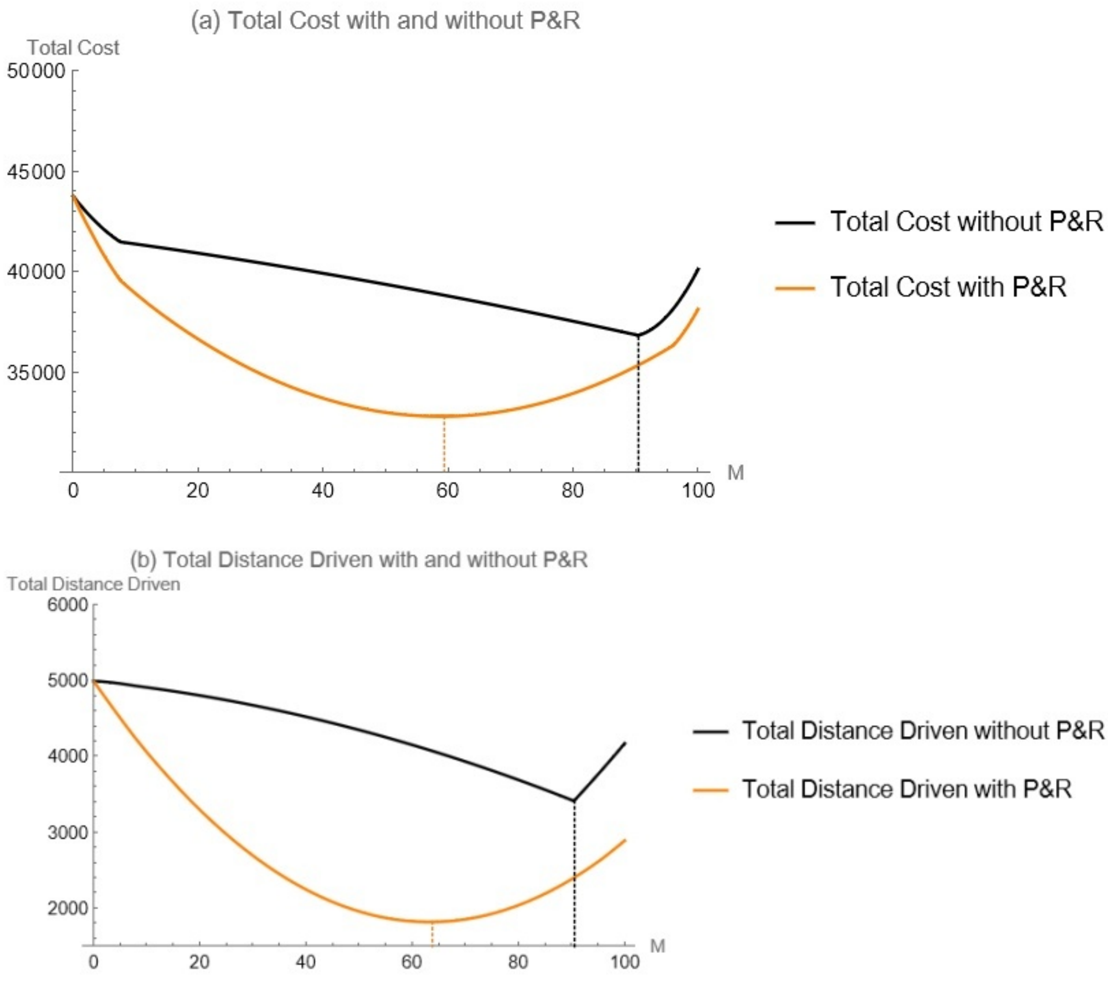
For the first numerical example, Table 2 presents the parameter values that are assigned to solve the model for optimal location of metro station both with and without P&R as a transport mode. α is assumed to be 2 in order to reflect the disutility from each driving individual. β is assumed to be 1 for avoiding complexities and turning the solution into a linear system of equations. While noticing that Assumptions 3.1 - 3.3 are satisfied, we assign $k_w = 50$, $k_d = 5$ and $k_m = 2$. The intuition is that walking is the transport mode with the highest cost per unit distance as it is much slower than the other modes.

Table 2 Parameter values for the first example

Parameter	Value
k_w	50 units of time per unit distance
k_d	5 units of time per unit distance
k_m	2 units of time per unit distance
α	2
β	1
N	100

Given the parameter values in Table 2, the total cost and total distance driven with P&R is always less than those without P&R for $M > 0$, where CBD is located at the origin as it is documented in Figure 4. One should also notice that there exist breaking points at the critical metro locations. These breaks occur due to the changes in the function such that a transport mode becomes available or disappears. For example, in the panel (a) of Figure 4, the left-most jump point in the total cost function without P&R occurs due to driving to CBD becomes available for those at $n \in (0, M)$. In the cases without P&R, total cost and total distance driven curves are non-convex. As non-convexities are known to be stemming from oligopolies and monopolies, and market power in general, we are likely to observe non-convex curves here due to the fact that we are allowing for a single public transportation option.

Figure 4 Total Cost and Total Distance Driven graphs for the first example



From the cost perspective, the optimal location for the metro station is found at around 90th percentile without P&R provision whereas it is found at around 58th percentile with P&R in the panel (a) of Figure 4. More specifically, the total cost is minimized at $M = 90.45$ without P&R and at $M = 58.8235$ with P&R. There are a number of ways to interpret the discrepancy between the two results. Suppose that the metro station is located optimally without P&R at 90th percentile and the planner considers providing P&R there. Then, the total cost will be lower when P&R is provided.

However, the percentage gains are small where it might not compensate the investment costs of P&R which are expected to be large. Now instead, suppose that the metro station is suboptimally located in the urban area and there is a possibility for large percentage gains from P&R provision there which might compensate the investment costs such as at $M = 50$.

From an environmental perspective, the total distance driven is also documented both with and without P&R in the panel (b) of Figure 4. The difference between the two cases occurs here as well. Suppose that the metro station is located without P&R at the environmentally optimal point around 90th percentile. Providing P&R at that point would bring less gains than it would have bring around the 65th percentile. More specifically, the total distance driven is minimized at $M = 90.45$ without P&R and at $M = 63.29$ with P&R.

Another important point is that optimal points are different from cost and environmental perspectives. Notice that the minimum cost with P&R is achieved at around 55-60th percentile, whereas the minimum distance driven is achieved further away from the city center at around 60-65th percentile. One could argue that the social planner's attitude toward the urban policies will determine at which optimal point is achieved.

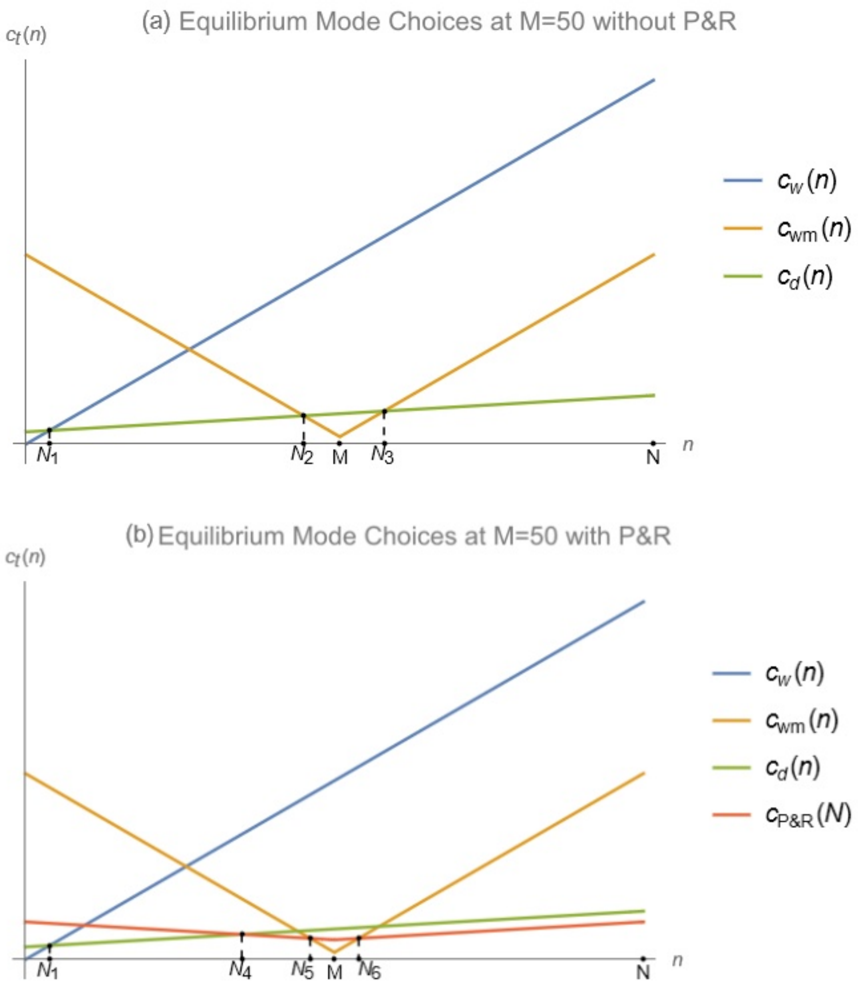
Now, suppose that metro station is located at $M = 50$. Figure 5 presents the equilibrium mode layouts for all $n \in (0, N)$. The panel (a) of Figure 5 documents the modes with associated costs for the case where P&R is not provided. It must be interpreted in the following manner: the transport mode with the lowest cost for an individual at $n \in (0, N_1)$ is walking to the CBD. Similarly the mode with the lowest cost for an individual at $n \in (N_1, N_2)$ is driving to the CBD. The mode with the lowest cost for an individual at $n \in (N_2, N_3)$. Lastly, the mode with the lowest cost for an individual at $n \in (N_3, N)$ is driving to the CBD. The panel (b) of Figure 5 documents the modes and their costs for the case where P&R is provided. Unlike the panel (a) of Figure 5, the mode with the lowest cost for an individual at $n \in (N_4, N_5)$ is doing P&R. Also P&R is the mode with the lowest cost for those at $n \in (N_6, N)$.

Table 3 Percentage distribution of mode choices in the first example

	Transport Mode			
Case	Walking	Driving	Riding metro	P&R
Without P&R	3.71%	83.48%	12.8%	
With P&R	3.92%	31.07%	7.84%	57.15%

Table 3 presents the percentages of mode choices given the parameter values in Table 2 at $M = 50$. According to our solutions, without P&R, 3.71% of indiviuals choose to

Figure 5 Equilibrium Mode Layouts at $M = 50$ for the first example



walk to the CBD, 83.48% choose to drive to the CBD, and 12.48% choose to ride metro. With P&R, 3.92% of individuals choose to walk to the CBD, 7.84% choose to ride metro, 31.07% choose to drive to the CBD, and 57.15% choose to do P&R which makes 88.22% of individuals drivers. Overall, the share of drivers increases when the P&R is provided, but the total distance driven decreases as it can be seen in the panel (b) of Figure 4 due to the availability of driving also to the metro station.

4.2 Second Example

For the second numerical example, Table 4 presents the parameter values that are assigned to solve the model for optimal location of metro station with and without P&R.

Reasoning for α and β follows from the first example above. We assign $k_w = 50$, $k_d = 5$, $k_m = 4$, and Assumptions 3.1 - 3.3 are satisfied.

Table 4 Parameter values for the second example

Parameter	Value
k_w	50 units of time per unit distance
k_d	5 units of time per unit distance
k_m	4 units of time per unit distance
α	2
β	1
N	100

Given the parameter values in Table 4, the total cost curves intersect while the total distance driven with P&R is always less than that without P&R for $M > 0$, as documented in Figure 6. The jump points at the critical metro locations and non-convexity of the curves without P&R exist in this case as well. While the environmentally optimal metro locations differ depending on the P&R provision, we do not observe an intersection here.

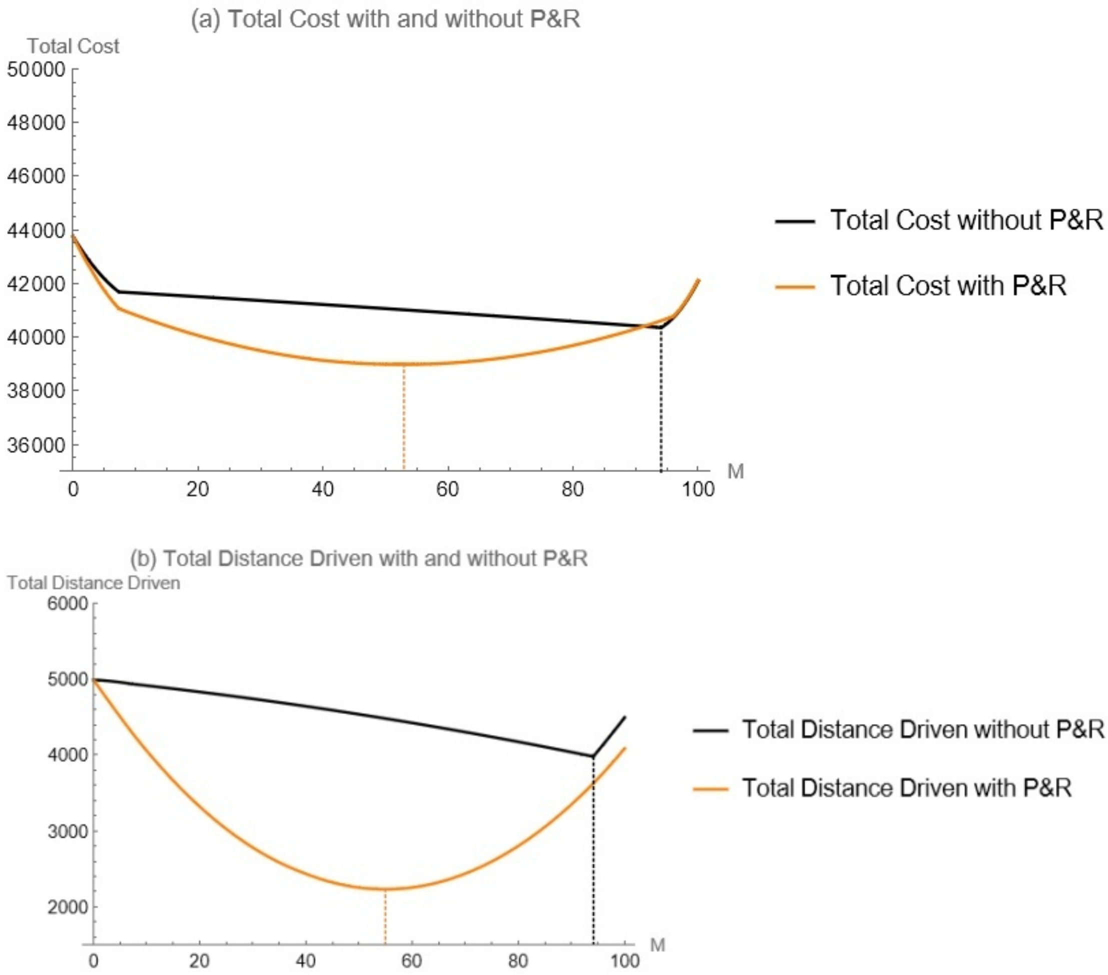
From the cost perspective, the optimal location for metro station without P&R is found at around the 95th percentile whereas it is at around 50th percentile with P&R provision in the panel (a) of Figure 6. More specifically, the total cost is minimized at $M = 94.11$ without P&R and at $M = 52.63$ with P&R. Suppose that the metro station without P&R is located optimally at 95th percentile and the planner is deciding whether to provide P&R there. If provided, the total cost goes up. However, there might be room for reducing the total cost whenever the metro without P&R is not located optimally such as at $M = 50$. The total costs curves intersect at some location choice for M, which means that providing P&R makes no difference at that point. Furthermore, to the right of that point, not providing P&R should be preferred as it is the less costly option.

From an environmental perspective, the total distance driven with and without P&R is documented in the panel (b) of Figure 6. Suppose that the metro station without P&R is located at the environmentally optimal point around the 95th percentile. Providing P&R at that point would bring smaller percentage gains than it would have brought around the 50th percentile. More specifically, the total distance driven is minimized at $M = 94.11$ without P&R and at $M = 54.94$ with P&R.

The difference between optimal points from the two perspectives persists here as well. While the minimum of total cost with P&R is achieved around the 50th percentile, the minimum of total distance driven with P&R is achieved around the 55th percentile. As it is argued in the first numerical example, the attitude of social planner will be the determining factor toward which point is achieved.

Now, suppose that metro station is located at $M = 50$. Figure 7 presents the

Figure 6 Total Cost and Total Distance Driven graphs for the second example



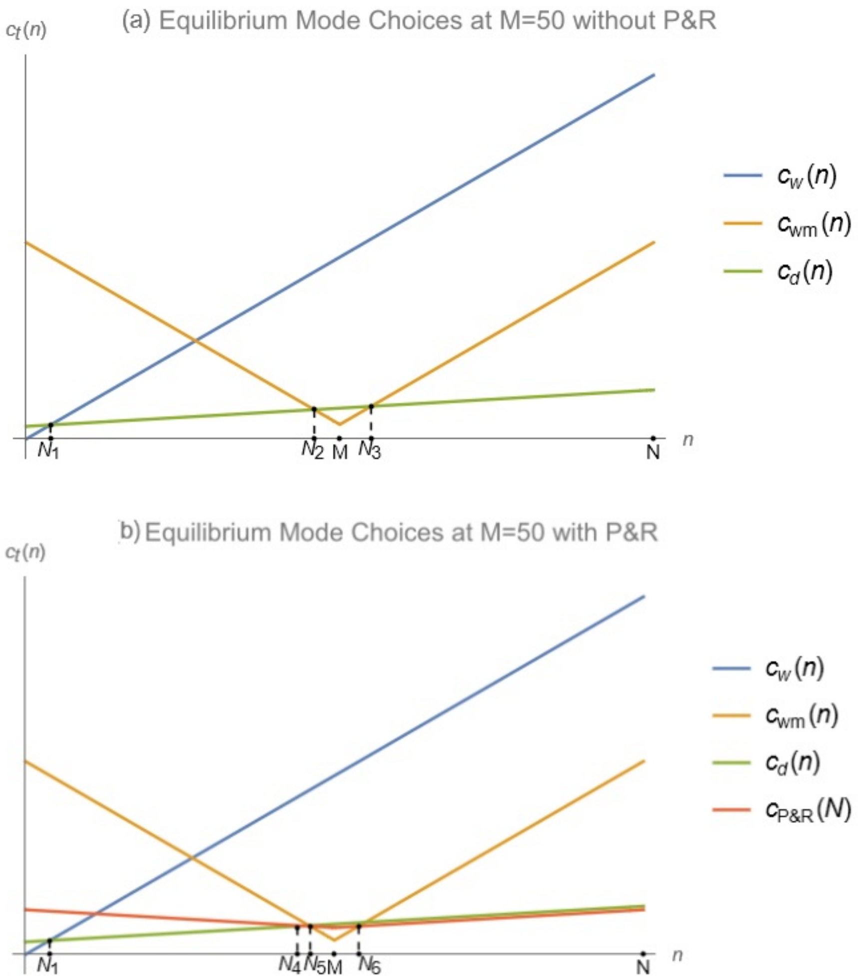
equilibrium mode layouts for all $n \in (0, N)$. The panel (a) of Figure 7 documents the modes with associated costs for the case where P&R is not provided. The interpretation of the chosen transport modes are as in the previous example. Th panel (b) of Figure 7 documents the modes and their costs for the case where P&R is provided.

Table 5 Percentage distribution of mode choices in the second example

	Transport Mode			
Case	Walking	Driving	Riding metro	P&R
Without P&R	3.87%	87.07%	9.06%	
With P&R	3.92%	41.08%	7.84%	47.15%

Table 5 presents the percentages of mode choices given the parameter values in Table 4 at $M = 50$. Without P&R, 3.87% of indiviuals choose to walk to the CBD, 87.07% choose to drive to the CBD, and 9.06% choose to ride metro. With P&R, 3.92% of individuals choose to walk to the CBD, 7.84% choose to ride metro, 41.07% choose to

Figure 7 Equilibrium Mode Layouts at $M = 50$ for the second example



drive to the CBD, and 47.15% choose P&R which makes 88.23% of individuals drivers. Overall, the share of drivers increases when the P&R is provided, but the total distance driven decreases as it can be seen in the panel (b) of Figure 6 due to the availability of driving also to the metro station.

As Wang, Yang, and Lindsey (2004) present the most similar model to ours, it would be useful to compare our results with theirs. While their results indicate that the optimal location of a P&R facility should be around 80th percentile, we have documented that it should be around 60th and 50th percentiles, respectively in our first and second numerical examples from a cost minimizing perspective as discussed above. The discrepancy between our results and Wang, Yang, and Lindsey (2004) might be arising from the main difference between the two models, that is the number of public transport stations provided along the urban area. We allow for a single public transport station for the ease of tractability, whereas they adopt a continuum of railway stations.

5 CONCLUSION

This article presents an investigation into the rationale behind the location choice of a P&R facility integrated into the public transportation in the form of a metro station. The model is presented as a linear monocentric urban area à la Hotelling (1929), where uniformly distributed individuals are able to choose among the available transportation modes: walking to the CBD, driving to the CBD, riding metro, and driving to the metro station which depends on the provision of P&R. We assume a ranking of the unit travel costs, $k_w > k_d > k_m$, as discussed by Assumptions 3.1 - 3.3. Given these parameters, the equilibrium mode-choices are investigated both with and without P&R. Two numerical examples are presented to illustrate how the model works. One additional assumption we make here is to assume a linear disutility cost (N_c) in order to abstain from complexities. The optimal locations from a total cost minimizing and total distance driven minimizing perspectives are presented and compared. We also gave insights into the cases where a P&R is integrated into an existing metro station, and both metro and P&R are decided to be built together. Furthermore, we assume that the metro station is located at the middle point ($M = 50$) in both examples, and compare the equilibrium mode choices.

We presented a simple tractable theoretical model of a monocentric urban area. There are several ways to extend our model. Similar to Basso, Navarro, and Silva (2021), one might consider different distributions of individuals which would bring the model closer to the reality as the population density will be higher in areas close to the CBD. Qualitatively interpreting, this would move the optimal P&R location closer to the CBD. Another way to extend our model would be allowing for multiple finite metro stations to make it more realistic. The model we presented in this paper includes a single metro station for the ease of simplicity, but allowing for more stations would clearly bring the model closer to what we observe in the real world.

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A Appendix: Mathematica Code for the First Numerical Example

In this appendix, we present the codes for the first numerical example that is discussed in section 4. Second example will follow the same code when the parameter values are adjusted as in Table 4.

```
 ClearAll["Global`*"];

(*Without P&R*)

 Clear[varscase1, dncc1, n1c1, ncc1]
Clear[varscase2, dncc2, n1c2, n3c2, ncc2]
Clear[varscase3, dncc3, n1c3, n2c3, n3c3, ncc3]
Clear[varscase4, dncc4, n1c4, n2c4, ncc4]
Clear[m, m1, m2, n, tcwoutpr, tcwoutprcase1, tcwoutprcase2, \
tcwoutprcase3, tcwoutprcase4, cw, cwm, cdcase1, cdcase2, cdcase3, \
cdcase4, kw, kc, km, m1vars, nm1, m2vars, nm2, tdwoutpr, \
tdwoutprcase1, tdwoutprcase2, tdwoutprcase3, tdwoutprcase4, dncc3m1, \
dncc4m2]
Clear [solscase1, solscase2, solscase3, solscase4, m1sols, m2sols, \
tcwoutprPLOT, tcwprPLOT, tdwoutprPLOT, tdwprPLOT]

kw := 50
kc := 5
km := 2

varscase1 = {dncc1, n1c1, ncc1};
solscase1 =
Solve[{dncc1 == 2 ncc1, n1c1 == dncc1/(kw - kc),
        ncc1 == 100 - n1c1}, varscase1][[1]];
MapThread[Set, {varscase1, varscase1 /. solscase1}];

varscase2 = {dncc2, n1c2, n3c2, ncc2};
solscase2 =
Solve[{dncc2 == 2 ncc2 , kw n1c2 == kw Abs[m - n1c2] + km m,
```

```
        kw Abs[m - n3c2] + km m == kc n3c2 + dncc2, ncc2 == 100 - n3c2},
varscase2][[1]];
MapThread[Set, {varscase2, varscase2 /. solscase2}];

 varscase3 = {dncc3, n1c3, n2c3, n3c3, ncc3};
solscase3 =
Solve[{dncc3 == 2 ncc3 , n1c3 == dncc3/(kw - kc),
        n2c3 == (m (kw + km) - dncc3)/(kw + kc),
        n3c3 == (m (kw - km) + dncc3)/(kw - kc),
        ncc3 == n2c3 - n1c3 + 100 - n3c3}, varscase3][[1]];
MapThread[Set, {varscase3, varscase3 /. solscase3}];

varscase4 = {dncc4, n1c4, n2c4, ncc4};
solscase4 =
Solve[{ dncc4 == 2 ncc4, n1c4 == dncc4/(kw - kc),
        n2c4 == (m (kw + km) - dncc4)/(kw + kc), ncc4 == n2c4 - n1c4},
varscase4][[1]];
MapThread[Set, {varscase4, varscase4 /. solscase4}];

dncc3m1 = dncc3 /. m -> m1;

(*Critical Values for m*)
Clear[m1vars, m1sols, m1, nm1]
m1vars = {m1, nm1};
m1sols = Solve[{kw nm1 == kw (m1 - nm1) + km m1 ==
        kc nm1 + (dncc3m1), m1 > 0}, m1vars][[1]];
MapThread[Set, {m1vars, m1vars /. m1sols}];

dncc4m2 = dncc4 /. m -> m2;

Clear[m2vars, m2sols, m2];
m2vars = {m2, nm2};
m2sols = Solve[{kw (100 - m2) + km m2 == kc 100 + (dncc4m2)},
m2vars][[1]];
MapThread[Set, {m2vars, m2vars /. m2sols}];
```

```
cw := kw n
cwm := kw Abs[m - n] + km m
cdcase1[n_] := kc n + dncc1;
cdcase2[n_] := kc n + dncc2;
cdcase3[n_] := kc n + dncc3;
cdcase4[n_] := kc n + dncc4;

tcwoutprcase1[m_] :=
Integrate[cw, {n, 0, n1c1}] + Integrate[cdcase1[n], {n, n1c1, 100}]
tcwoutprcase2[m_] :=
Integrate[cw, {n, 0, n1c2}] + Integrate[cwm, {n, n1c2, n3c2}] +
Integrate[cdcase2[n], {n, n3c2, 100}]
tcwoutprcase3[m_] :=
Integrate[cw, {n, 0, n1c3}] +
Integrate[cdcase3[n], {n, n1c3, n2c3}] +
Integrate[cwm, {n, n2c3, n3c3}] +
Integrate[cdcase3[n], {n, n3c3, 100}]
tcwoutprcase4[m_] :=
Integrate[cw, {n, 0, n1c4}] +
Integrate[cdcase4[n], {n, n1c4, n2c4}] +
Integrate[cwm, {n, n2c4, 100}]


tcwoutpr[m_] :=
Piecewise[{{tcwoutprcase1[m], m == 0}, {tcwoutprcase2[m],
                    0 < m < m1}, {tcwoutprcase3[m], m1 <= m < m2},
          {tcwoutprcase4[m], m2 <= m <= 100}}]

 Plot[tcwoutpr[m], {m, 0, 100}, Exclusions -> None, PlotRange -> Full]

tdwoutprcase1 := Integrate [n, {n, n1c1, 100}]
tdwoutprcase2 := Integrate[n, {n, n3c2, 100}]
tdwoutprcase3 :=
Integrate[n, {n, n1c3, n2c3}] + Integrate[n, {n, n3c3, 100}]
tdwoutprcase4 := Integrate[n, {n, n1c4, n2c4}]
tdwoutpr := Which[
m == 0, tdwoutprcase1,
```

```
0 < m < m1, tdwoutprcase2,
m1 <= m < m2, tdwoutprcase3,
m2 <= m <= 100, tdwoutprcase4]
Plot[tdwoutpr, {m, 0, 100}]


(*With P&R*)
Clear[varscase1p, varscase2p, varscase23p, varscase3p, varscase4p,
solscase1p, solscase2p, solscase3p, solscase4p];
Clear[dncpc1, n1pc1, ncpc1, dncpc2, n1pc2, n6c2, ncpc2, dncpc3, n1pc3,
n4c3, n5c3, n6c3, ncpc3, dncpc4, n1pc4, n4c4, n5c4, ncpc4, m1pvars,
m1psols, m2pvars, m2psols, m1p, m2p, nm2p, dncpc23, n1pc23, n2pc23,
n6c23, ncpc23];
Clear[cwp, cwmp, cdpcase1, cdpcase2, cdpcase23, cdpcase3, cdpcase4,
cdmpcase2, cdmpcase23, cdmpcase3, cdmpcase4, tcwprcase1, tcwprcase2,
tcwprcase23, tcwprcase3, tcwprcase4, tcwpr, tdwpr, tdwprcase1,
tdwprcase2, tdwprcase23, tdwprcase3, tdwprcase4, dncpc4m2, dncpc3m1,
dncpc3m12, m12pvars];

varscase1p = {dncpc1, n1pc1, ncpc1};
solscase1p =
Solve[{dncpc1 == 2 ncpc1, n1pc1 == dncpc1/(kw - kc),
        ncpc1 == 100 - n1pc1}, varscase1p][[1]];
MapThread[Set, {varscase1p, varscase1p /. solscase1p}];

varscase2p = {dncpc2, n1pc2, n6c2, ncpc2};
solscase2p =
Solve[{dncpc2 == 2 ncpc2, kw n1pc2 == kw Abs[m - n1pc2] + km m,
        n6c2 == (m (kw - kc) + dncpc2)/(kw - kc), ncpc2 == 100 - n6c2},
varscase2p][[1]];
MapThread[Set, {varscase2p, varscase2p /. solscase2p}];



varscase23p = {dncpc23, n1pc23, n2pc23, n6c23, ncpc23};
solscase23p =
Solve[{dncpc23 == 2 ncpc23, n1pc23 == dncpc23/(kw - kc),
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        n2pc23 == (m (kw + km) - dncpc23)/(kw + kc),
        n6c23 == (m (kw - kc) + dncpc23)/(kw - kc),
        ncpc23 == 100 - n6c23 + n2pc23 - n1pc23}, varscase23p][[1]];
MapThread[Set, {varscase23p, varscase23p /. solscase23p}];

varscase3p = {dncpc3, n1pc3, n4c3, n5c3, n6c3, ncpc3};
solscase3p =
Solve[{dncpc3 == 2 ncpc3, n1pc3 == dncpc3/(kw - kc),
        kc (m - n4c3) + km m == kc n4c3,
        kc (m - n5c3) + dncpc3 == kw (m - n5c3),
        n6c3 == (m (kw - kc) + dncpc3)/(kw - kc),
        ncpc3 == 100 - n6c3 - n1pc3 + n5c3}, varscase3p][[1]];
MapThread[Set, {varscase3p, varscase3p /. solscase3p}];

varscase4p = {dncpc4, n1pc4, n4c4, n5c4, ncpc4};
solscase4p =
Solve[{dncpc4 == 2 ncpc4, n1pc4 == dncpc4/(kw - kc),
        kc (m - n4c4) + km m == kc n4c4,
        kc (m - n5c4) + dncpc4 == kw (m - n5c4), ncpc4 == n5c4 - n1pc4},
varscase4p][[1]];
MapThread[Set, {varscase4p, varscase4p /. solscase4p}];

dncpc3m1 = dncpc3 /. m -> m1p;

Clear[m1pvars, m1psols, m1p, nm1p]
m1pvars = {m1p, nm1p};
m1psols =
Solve[{kw nm1p == kw Abs[m1p - nm1p] + km m1p ==
        kc nm1p + (dncpc3m1) && m1p > 0}, m1pvars][[1]];
MapThread[Set, {m1pvars, m1pvars /. m1psols}];

dncpc3m12 = dncpc3 /. m -> m12p;

 Clear[m12pvars, m12psols, m12p, nm12p]
m12pvars = {m12p, nm12p};
m12psols =
Solve[{kw Abs[m12p - nm12p] + km m12p == kc nm12p + dncpc3m12 ==
```

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        kc (m12p - nm12p) + (dncpc3m12) + km m12p && m12p > 0},
m12pvars][[1]];
MapThread[Set, {m12pvars, m12pvars /. m12psols}];

dncpc4m2 = dncpc4 /. m -> m2p;

Clear[m2pvars, m2psols, m2p, nm2p]
m2pvars = {m2p, nm2p};
m2psols =
Solve[{kw Abs[m2p - nm2p] + km m2p ==
        kc Abs[m2p - nm2p] + (dncpc4m2) + km m2p && nm2p == 100},
m2pvars][[1]];
MapThread[Set, {m2pvars, m2pvars /. m2psols}];

cwp := kw n
cwmp := kw Abs[m - n] + km m
cdpcase1[n_] := kc n + dncpc1
cdpcase2[n_] := kc n + dncpc2
cdpcase23[n_] := kc n + dncpc23
cdpcase3[n_] := kc n + dncpc3
cdpcase4[n_] := kc n + dncpc4
cdmpcase2[n_] := kc Abs[m - n] + dncpc2 + km m
cdmpcase23[n_] := kc Abs[m - n] + dncpc23 + km m
cdmpcase3[n_] := kc Abs[m - n] + dncpc3 + km m
cdmpcase4[n_] := kc Abs[m - n] + dncpc4 + km m



tcwprcase1[m_] :=
Integrate[cwp, {n, 0, n1pc1}] +
Integrate[cdpcase1[n], {n, n1pc1, 100}]
tcwprcase2[m_] :=
Integrate[cwp, {n, 0, n1pc2}] + Integrate[cwmp, {n, n1pc2, n6c2}] +
Integrate[cdmpcase2[n], {n, n6c2, 100}]
tcwprcase23[m_] :=
Integrate[cwp, {n, 0, n1pc23}] +
Integrate[cdpcase23[n], {n, n1pc23, n2pc23}] +
Integrate[cwmp, {n, n2pc23, n6c23}] +
```

```
Integrate[cdmpcase23[n], {n, n6c23, 100}]
tcwprcase3[m_] :=
Integrate[cwp, {n, 0, n1pc3}] +
Integrate[cdpcase3[n], {n, n1pc3, n4c3}] +
Integrate[cdmpcase3[n], {n, n4c3, n5c3}] +
Integrate[cwmp, {n, n5c3, n6c3}] +
Integrate[cdmpcase3[n], {n, n6c3, 100}]
tcwprcase4[m_] :=
Integrate[cwp, {n, 0, n1pc4}] +
Integrate[cdpcase4[n], {n, n1pc4, n4c4}] +
Integrate[cdmpcase4[n], {n, n4c3, n5c3}] +
Integrate[cwmp, {n, n5c3, 100}]

tcwpr[m_] :=
Piecewise[{{tcwprcase1[m], m == 0}, {tcwprcase2[m], 0 < m < m1p},
         {tcwprcase23[m], m1p <= m < m12p},
         {tcwprcase3[m], m12p <= m < m2p},
         {tcwprcase4[m], m2p <= m <= 100}}]

Plot[tcwpr[m], {m, 0, 100}]

tdwprcase1 := Integrate[n, {n, n1pc1, 100}]
tdwprcase2 := Integrate[n - m, {n, n6c2, 100}]
tdwprcase23 :=
Integrate[n, {n, n1pc23, n2pc23}] + Integrate [n - m, {n, n6c23, 100}]
tdwprcase3 :=
Integrate[n, {n, n1pc3, n4c3}] + Integrate[m - n, {n, n4c3, n5c3}] +
Integrate[n - m, {n, n6c3, 100}]
tdwprcase4 :=
Integrate[n, {n, n1pc4, n4c4}] + Integrate[m - n, {n, n4c3, n5c3}]
tdwpr := Which[
m == 0, tdwprcase1,
0 < m < m1p, tdwprcase2,
m1p <= m < m12p, tdwprcase23,
m12p <= m < m2p, tdwprcase3,
m2p <= m <= 100, tdwprcase4]
```

```
Plot[tdwpr, {m, 0, 100}]

FindArgMin[{tcwpr[m], 0 < m < 100}, {m, 50}]

FindArgMin[tdwprcase3, m]

tcwoutprPLOT =
Plot[tcwoutpr[m], {m, 0, 100},
PlotLegends -> LineLegend[{"Total Cost without P&R"}],
PlotStyle -> Black, PlotRange -> {30000, 50000}];
tcwprPLOT =
Plot[tcwpr[m], {m, 0, 100},
PlotLegends -> LineLegend[{"Total Cost with P&R"}],
PlotStyle -> Orange, PlotRange -> {30000, 50000}];
fig3panela =
Show[{tcwoutprPLOT, tcwprPLOT},
PlotLabel -> "(a) Total Cost with and without P&R" ,
AxesLabel -> {"M", "Total Cost"} ]


tdwoutprPLOT =
Plot[tdwoutpr, {m, 0, 100}, PlotStyle -> Black,
PlotLegends -> LineLegend[{"Total Distance Driven without P&R"}],
PlotRange -> {1500, 6000}];
tdwprPLOT =
Plot[tdwpr, {m, 0, 100}, PlotStyle -> Orange,
PlotLegends -> LineLegend[{"Total Distance Driven with P&R"}],
PlotRange -> {1500, 6000}];
fig3panelb =
Show[{tdwoutprPLOT, tdwprPLOT},
PlotLabel -> "(b) Total Distance Driven with and without P&R" ,
AxesLabel -> {"M", "Total Distance Driven"} ]
```