

Hybrid Force-Motion Control for One-Legged Robot in Operational Space

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Abstract—This paper presents a dynamic locomotion generation for a one-legged floating-base robot. Reference synthesis is performed by planning both swing motion of the foot and contact forces acting from the ground. A fifth-order polynomial is employed as the position reference to reduce the impact forces and ensure a steady transition between the swing and stance phases. Contact force references are designed utilizing the laws of linear and angular momentum conservation. A hybrid force-motion control framework is created in operational space for tracking generated references. Gait phase transition is proposed to assist the transition between the force and motion controller. A full-dynamics simulation environment is utilized to test the proposed control framework. Results supported the competence of the proposed control framework for the floating-base one-legged robot.

Index Terms—legged robot, force control, motion control, contact force planning, reference generation

I. INTRODUCTION

Legged robots have advantages over other robotic land platforms, such as wheeled robots on rough terrain. There are several methods to achieve legged robot locomotion. Since they are floating base robots that do not have any fixed attachment points on the world, the robot body's reference synthesis is fundamental to generate stable locomotion. In the literature, Zero Moment Point (ZMP) based stability criteria with Linear Inverted Pendulum Model (LIPM) is employed to generate stable walking motion [1]–[3]. Another technique is the bio-inspired Central Pattern Generator (CPG); in this method, stable periodic references are generated for robot locomotion [4], [5]. However, these reference generation techniques can be inadequate for dynamic locomotion. There are trajectory optimization methods to find optimal controls [6], [7]. Additionally, Kim et al. proposed a reference generation method with the aid of linear and angular momentum conservation laws for dynamic locomotion [8]. Spring-Loaded Inverted Pendulum (SLIP) [9], [10] techniques are more accurate for dynamic locomotion.

Reference generation is not sufficient to achieve stable robot locomotion single-handed. It is also necessary to track generated references exactly. Therefore, establishing an applicable control framework is a crucial task. In literature, several articles mentioned the legged robots' position control to achieve this task [3], [5]. Furthermore, force-controlled legged robots

have promising outcomes on dynamic locomotion [10], [11]. However, it is a challenging problem since there is no fixed point of the floating-base robots, their complex dynamics and high degrees of freedom (DOF).

Khatib proposed a motion and force control method of robot manipulators [12]. This article mentions the relation between configuration space robot dynamic equations and task space end-effector dynamic equations. It is called the operational space formulation. This formulation is a technique for controlling contact forces as it focuses on the end-effector dynamics. Sentis and Khatib extended the theory and control strategies in operational space to floating-base robots [13].

In this work, we employ a reference synthesis technique based on linear and angular momentum conservation laws. An optimization algorithm is also employed in order to produce stable dynamic locomotion references. A hybrid force-motion control is implemented in operational space to track generated force and position references.

This paper structured as follows: Section II-A describes modelling of a floating-base robots. There is brief explanation of the reference generation method and optimization algorithm in Section II-B. In Section III, the control framework is explained in details. Section IV presents simulation results. Discussion on the results are presented in Section V.

II. MODELLING AND REFERENCE GENERATION

A one-legged robot's motion equation is discussed in this section, and a reference generation method for a stable motion is proposed.

A. Floating-Base Motion Equation

However, the floating base system does not contain any fixed point in the world frame; it has contact with the ground. Thus, it is necessary to define a dynamic equation of the floating base system with respect to an inertial frame. The generalized coordinates of the robot are given as follows:

$$q = \begin{bmatrix} x_b \\ q_j \end{bmatrix}, \quad (1)$$

where $x_b \in SE(2)$ is the position and orientation of the body for the inertial frame. $q_j \in \mathbb{R}^2$ is the joint configuration of the one-legged robot with two revolute joints. The robot

is modelled with these generalized coordinates as it has non-actuated three virtual DOF (two translational and one rotational) located on the robot body. The motion equation of the floating base one-legged robot with contact can be written as:

$$M(q)\ddot{q} + C(q, \dot{q}) + G(q) + J_c(q)^T F_c = S^T \tau, \quad (2)$$

where $M(q) \in \mathbb{R}^{(3+2) \times (3+2)}$ is the inertia matrix, $C(q, \dot{q}) \in \mathbb{R}^{(3+2)}$ is the coriolis and centripetal effects, $G(q) \in \mathbb{R}^{(3+2)}$ is the gravitational force, $J_c(q) \in \mathbb{R}^{(2) \times (3+2)}$ is the contact jacobian of the robot with respect to the world frame, $F_c \in \mathbb{R}^2$ is the vector of two linearly independent forces are applied by the robot to the ground, $S = \begin{bmatrix} 0_{2 \times 3} & I_{2 \times 2} \end{bmatrix}$ is the selection matrix of the actuated joints, $\tau \in \mathbb{R}^2$ is the vector of joint torques. Since there is not actuation on the body, reference generation is essential for a stable motion. The kinematic model of the one-legged robot and attached body, foot, and joints frames are demonstrated with inertial frame in Figure 1.

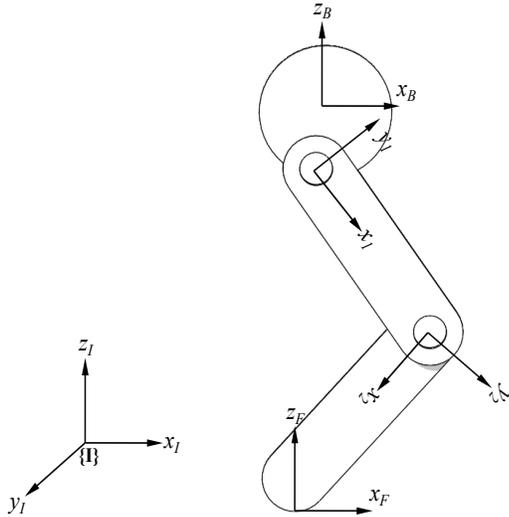


Fig. 1. The kinematic model and attached frames of the one-legged robot utilized in simulation environment.

B. Reference Generation

The floating base robot's motion is classified as a hybrid dynamic system [14]. It contains swing and stance phases that include different dynamics. During a swing, phase foot tracks a curvilinear trajectory until a contact. However, there are distinct dynamics in the swing phase including that the forces applied from the environment. These phases are connected with discrete events such as touch down and take-off. Due to the robot motion's hybrid dynamics, it is efficacious to produce references for swing and stance phases separately.

The stability of a floating base robot is mainly related with its body motion. The vertical contact force carry out the main duty to balance the robot body against the effects of gravity. Since there is not any contact force during swing

phase, the remaining impulse created by gravity in one step cycle, that is the summation of stance and swing times, must be compensated in stance phase by vertical contact forces. Therefore, planning vertical contact forces in dynamic motion is detrimental for stability of the motion. An optimization for planning suitable vertical contact forces through stance phase is utilized as follows:

$$\begin{aligned} \min_{\mathbf{F}_z} & \sum_{i=1}^{i_{st}} |\ddot{z}_{ref_i} - \ddot{z}_{b_i}(\mathbf{F}_z)|^2, \\ \text{s.t.} & \sum_{i=1}^{i_{st}} F_{z_i} = \frac{1}{\beta} \sum_{i=1}^{i_{st}} m_b g, \\ & |F_{z_{i+1}} - F_{z_i}| \leq \delta, \end{aligned} \quad (3)$$

where i_{st} is the ratio between stance time and sampling time. \ddot{z}_{ref} and \ddot{z}_b is the vertical acceleration reference and actual vertical acceleration of the body, respectively. F_z is the vertical contact force, β is duty ratio which is the ratio between stance time and step cycle time, m_b is the robot mass, g is the gravitational acceleration, and δ is a positive small number. The first constraint represents zero vertical momentum change in one cycle, and the second constraint prevents high fluctuations in planned contact forces and provides a continuity in time. The discretization is applied to quantities such as time for the optimization process. Further explanation can be found in [15].

High impact forces might cause issues in transition between phases in a hybrid motion. Stable transition among two different dynamics makes the hybrid system easier to control. In order to reduce the impact forces and carry out a stable transition, the swing foot trajectory is set to arrive to the floor with zero velocity and acceleration. With the addition of maximum vertical foot clearance, that is zero vertical velocity, and the stride length, there are five conditions that swing trajectory must satisfy:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix} = \begin{bmatrix} 0 \\ s_h \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (4)$$

and

$$\begin{aligned} a_1 &= [t_w, (t_w)^2, (t_w)^3, (t_w)^4, (t_w)^5] \\ a_2 &= [0.4t_w, (0.4t_w)^2, (0.4t_w)^3, (0.4t_w)^4, (0.4t_w)^5] \\ a_3 &= [0.4, 0.32t_w, 1.2(0.4t_w)^2, 1.6(0.4t_w)^3, 2(0.4t_w)^4] \\ a_4 &= [1, 2t_s, 3(t_w)^2, 4(t_w)^3, 5(t_w)^4] \\ a_5 &= [0, 2, 6t_w, 12(t_w)^2, 20(t_w)^3], \end{aligned} \quad (5)$$

where $a_{i=1\dots5} \in \mathbb{R}^5$ are row vectors contain time variable of the corresponding condition. t_w and s_h are the swing phase duration and maximum vertical foot clearance, respectively. The foot reaches its maximum vertical foot clearance within 40% of the swing time. The linear system's solution gives

coefficients ($c_{i=1\dots5}$) of the fifth-order polynomial. Here is the utilized fifth-order polynomial for swing phase reference:

$$x_{sw}(t) = -38104t^5 + 17147t^4 - 2572t^3 + 128.6t^2. \quad (6)$$

The results of the planned contact forces and swing phase references are presented in Figure 2.

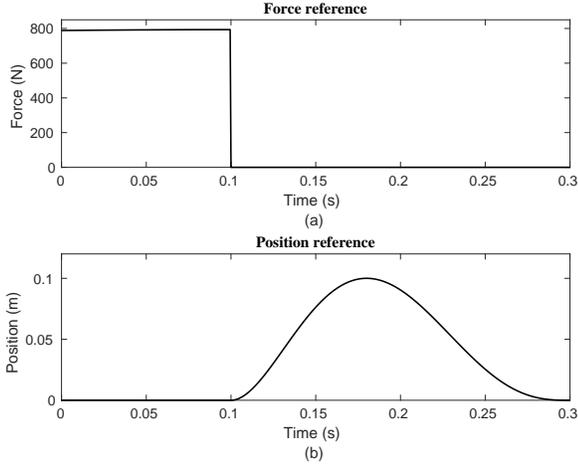


Fig. 2. (a) Planned contact force, (b) generated position reference in a step cycle duration.

III. CONTROL FRAMEWORK

Control of a hybrid dynamics system is a complex task. It is substantial to present robust and stable control law. A control framework of a system should be suitable for the desired task and work in harmony with the reference generation criterion. Therefore, the construction of a control framework is essential to achieve stable operation.

In this section, operational space dynamics of the floating-base robot is derived, motion and force control laws are introduced, the coordination of swing and stance phases are explained, and the final form of hybrid force-motion control framework is presented.

A. Operational Space Dynamics

It is essential to separate unactuated body dynamics from motion equation to represent floating base robot dynamics in operational space [13]. The generalized coordinates are updated with actuated joint selection:

$$\bar{q} = S_j \begin{bmatrix} x_b \\ q_j \end{bmatrix}, \quad (7)$$

where $S_j = [0_{2 \times 3} \quad I_{2 \times 2}]$ is the joint selection matrix. In order to mapping the generalized dynamics into joint space dynamics, generalized inverse of the joint selection matrix is utilized:

$$\hat{S}_j = M^{-1} S_j^T \bar{M} \quad (8)$$

where $\bar{M} = (S_j M^{-1} S_j^T)^{-1}$ is the joint space inertia matrix. The generalized inverse of joint selection matrix transpose (\hat{S}_j^T) is multiplied with (2) to transform system into the joint space dynamics:

$$\bar{M}(\bar{q})\ddot{\bar{q}} + \bar{C}(\bar{q}, \dot{\bar{q}}) + \bar{G}(\bar{q}) + \bar{J}_c(\bar{q})^T F_c = \tau. \quad (9)$$

Here $\bar{C}(\bar{q}, \dot{\bar{q}}) = \hat{S}_j^T C(q, \dot{q})$ is the coriolis and centrifugal effects, $\bar{G}(\bar{q}) = \hat{S}_j^T G(q)$ is the gravity term and $\bar{J}_c(\bar{q}) = J_c(q) \hat{S}_j$ is the jacobian of the actuated joint space.

Since the dynamic equations are mapped to the actuated joint space, operational space dynamics or end-effector dynamics can be obtained for floating-base robot with multiplying (9) with the generalized inverse of jacobian transpose ($\hat{J}_c(\bar{q})^T$):

$$\hat{J}_c^T \left(\bar{M} \ddot{\bar{q}} + \bar{C} + \bar{G} + \bar{J}_c^T F_c = \tau \right), \quad (10)$$

which equals to,

$$\Lambda(\bar{q})\ddot{x}_e + \mu(\bar{q}, \dot{\bar{q}}) + p(\bar{q}) + F_c = F_e, \quad (11)$$

where $\hat{J}_c = \bar{M} \bar{J}_c^T \Lambda$ is the dynamically consistent generalized inverse of actuated joint space jacobian that minimizes the instantaneous kinetic energy of robot [12]. Since kinematic relation between joint space and operation space is defined with $\dot{x}_e = \bar{J}_c \dot{\bar{q}}$ and $\ddot{x}_e = \bar{J}_c \ddot{\bar{q}} + \dot{\bar{J}}_c \dot{\bar{q}}$, the relation between the terms at actuated joint space and operational space is derived:

$$\begin{aligned} \Lambda(\bar{q}) &= (\bar{J}_c \bar{M}^{-1} \bar{J}_c^T)^{-1} \\ \mu(\bar{q}, \dot{\bar{q}}) &= \hat{J}_c^T \bar{C} - \Lambda \dot{\bar{J}}_c \dot{\bar{q}}, \\ p(\bar{q}) &= \hat{J}_c^T \bar{G}. \end{aligned} \quad (12)$$

Here $\Lambda(\bar{q})$, $\mu(\bar{q}, \dot{\bar{q}})$ and $p(\bar{q})$ are the operational space inertia, coriolis and centrifugal effects and gravity term, respectively. $F_e = \hat{J}_c^T \tau$ is the end effector force, $\dot{\bar{J}}_c(\bar{q}) = \dot{J}_c(q) \hat{S}_j$ is the time derivative of the actuated joint space jacobian and x_e is the position of end-effector in the operation space.

B. Control Algorithm

Resolved-acceleration control is utilized for decoupling motion control and force control for the hybrid dynamic system. During the swing phase, the acceleration based inverse dynamics control law is employed for motion control in the operation space. The control law is depicted in (13) and (14).

$$F_{e_m} = \Lambda \ddot{x}_{e_d} + \mu + p, \quad (13)$$

and

$$\ddot{x}_{e_d} = \left(K_{P_m} e_m + K_{I_m} \int_{t_0}^t e_m(\tau) d\tau + K_{D_m} \dot{e}_m \right). \quad (14)$$

Here x_{e_d} is the desired swing trajectory of the foot which is end-effector of the robot and K_{P_m} , K_{D_m} , and K_{I_m} are positive-definite motion control matrix gains. $e_m = x_{e_d} - x_e$ is the error between the desired and the actual position of the foot in the operational space. F_{e_m} is the motion control

force in the operational space. During the stance phase, the objective is to track planned contact force references. A direct force control law for tracking planned trajectories is presented in (15) and (16).

$$F_{e_f} = \Lambda \ddot{x}_e + \mu + p + F_c + F_d, \quad (15)$$

and

$$F_d = (K_{P_f} e_f + K_{D_f} \dot{e}_f). \quad (16)$$

Here K_{P_f} , and K_{D_f} are positive-definite proportional and derivative matrix gains, respectively. $e_f = F_r - F_c$ is the error between planned contact forces and actual contact forces. F_d is the additional force to track desired contact forces and F_{e_f} is force control force in the operational space.

C. Gait Phase Transition

As mentioned in Section II-B, the motion of the floating-base robot is a hybrid dynamic system. The motion includes swing and stance phases. Uniting these two different dynamics is significant to achieve a stable motion. Furthermore, the force and motion control algorithms for stance and swing phases is proposed in the previous section. Hence, it is essential to define the gait phase transition. The gait phase transition from swing phase to stance phase happens when the foot touches the ground. After the contact detected, the force control is triggered and the stance phase starts. When the time reaches the stance phase duration, the transition from the stance phase to the swing phase occurs. The motion control is activated after the transition into swing phase. The coordination between the force and motion control is ensured by the gait phase transition. A transition parameter is defined for phase transition as in (17).

$$t_p = \begin{cases} I_{(2 \times 2)}, & \text{stance phase} \\ 0_{(2 \times 2)}, & \text{swing phase} \end{cases} \quad (17)$$

The illustration of gait phase transition is in Figure 3.

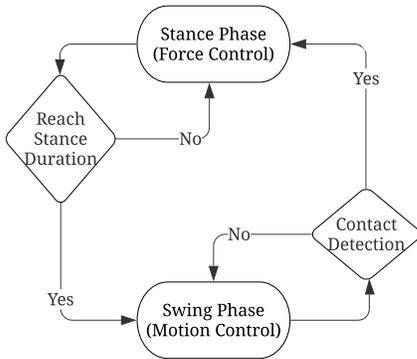


Fig. 3. Working scheme of the gait phase transition.

D. Hybrid Force-Motion Control

The force and motion control algorithms are presented separately in Section III-B. After defining gait phase transition, both control algorithms are concatenated into a single control law.

$$F_e = \Lambda (t_p \ddot{x}_e + (I_{(2 \times 2)} - t_p) \ddot{x}_{e_d}) + \mu + p + F_c + t_p F_d. \quad (18)$$

Here F_e is the hybrid force-motion control force in the operational space. The hybrid force-motion control torque is computed by

$$\tau = \bar{J}_c^T F_e. \quad (19)$$

Overall scheme of the control framework is presented as in Figure 4.

Instead of utilizing force sensors, the contact forces can be estimated with the help of kinematic constraints [16]. Let c is a contact point of the floating-base robot. Since point c is not able to move further, kinematic constraints of the robot is written as follows:

$$\begin{aligned} x_c &= const, \\ \dot{x}_c &= J_c \dot{q} = 0, \\ \ddot{x}_c &= J_c \ddot{q} + \dot{J}_c \dot{q} = 0, \end{aligned} \quad (20)$$

where x_c is the position of the contact point c with respect to initial frame, from the motion equation (2) and kinematics constraints (20), the contact force is estimated as:

$$F_c = (J_c M^{-1} J_c^T)^{-1} (J_c M^{-1} (S^T \tau - C - G) + \dot{J}_c \dot{q}). \quad (21)$$

This equation yields estimated contact forces based on the multi-body system dynamics without employing any force sensors [16].

IV. SIMULATION RESULTS

Simulations are employed for the verification of the hybrid force-motion control framework described in Section III. The simulation environment is built in MATLAB & Simulink. Simulation parameters are given in Table I. Generated foot motion trajectories and planned contact forces, which are explained in Section II-B, are utilized as references in simulation environment. These references are tracked with proposed control algorithm. Penalty based spring-damper system is used to evaluate ground contacts in the simulation environment. Detailed explanations about contact model can be found in [17].

Simulation results of hybrid force-motion controller in one step cycle are presented in Figure 5. Penalty based spring-damper system calculates contact force with penetration depth. In order to imitate a contact force in the simulation environment, the robot foot penetrates into the ground to a limited amount. Hence, there is some offset between position reference and simulation results of the foot position. Furthermore, there is contact force during the swing phase until foot completely

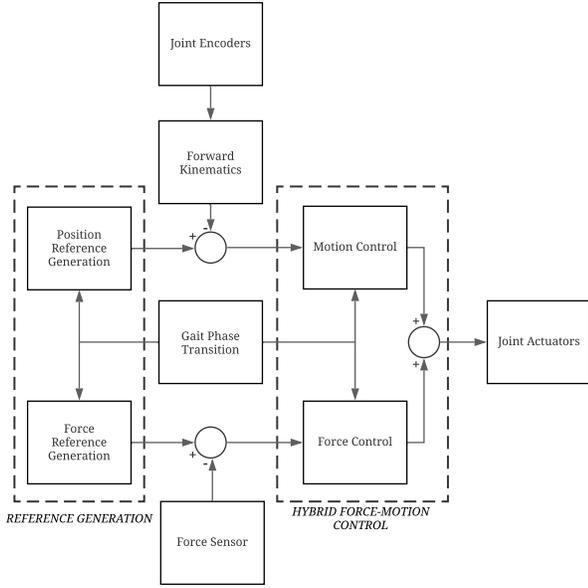


Fig. 4. Control framework of the hybrid force-motion controller.

TABLE I
SIMULATION AND LOCOMOTION PARAMETERS

Simulation Parameters		
Definition	Symbol	Value
Thigh length and width	$l_t - w_t$	0.4 – 0.05 m
Shank length and width	$l_s - w_s$	0.4 – 0.05 m
Body radius	r_b	0.15 m
Body and leg mass	$m_b - m_l$	35 – 5 kg
Body and leg inertia	$I_b - I_l$	0.315 – 0.267 kgm ²
Gravitational acceleration	g	9.81 kgm/s ²
Sampling time	dt	0.5 ms
Locomotion Parameters		
Definition	Symbol	Value
step cycle time	s_c	0.3 s
Swing time	t_w	0.2 s
Stance time	t_s	0.1 s
duty ratio	β	0.333
Step height	s_h	0.1 m
Step length	s_l	0.1 m

leaves the ground. By virtue of arriving to the floor with zero velocity and acceleration, there is no high impact forces.

The vertical body position during the ten-step simulation is shown in Figure 6. Since, there is no contact force input during swing phase, it is not straightforward to track the body acceleration reference completely. Therefore, an error occurs between the reference and simulation result. Nevertheless, this error is not significant and do not disturb the periodicity of the motion.

V. CONCLUSIONS

The full motion planning of a one-legged robot is presented in this paper. Due to the hybrid dynamics of the motion, force and position trajectories are generated separately for stance and swing phases. The control framework is proposed for tracking generated references. Operational Space Dynamics

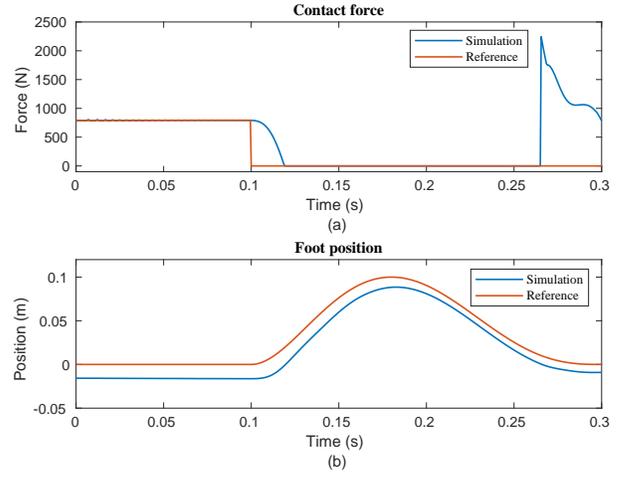


Fig. 5. Simulation results of the hybrid force-motion controller in one step cycle

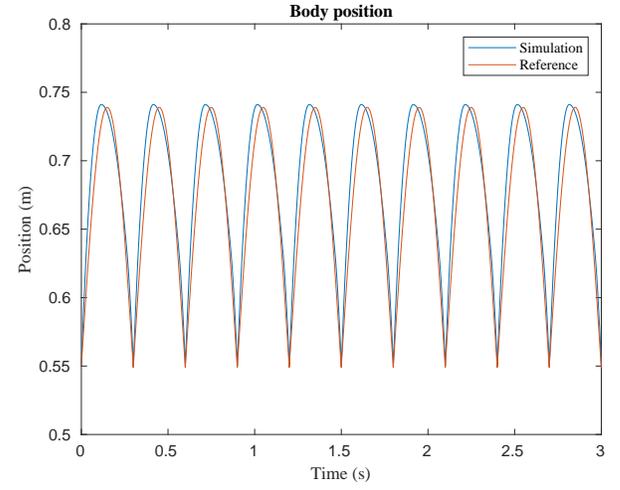


Fig. 6. Vertical body position in ten-step simulation

of a floating base robot is derived for the force and motion control. The transition parameter (t_p) is defined for smooth transition between controllers and gait phases. Hybrid force-motion control law is carried out in operational space.

We have carried out stable locomotion in the simulation environment. The robot can maneuver with a periodic behaviour without falling in the simulation environment. Simulation results verify the validity of proposed motion planning for one-legged robot. Furthermore, results are encouraging for extending the application of the presented method into biped and quadruped robots. It is foreseen that the proposed hybrid force and motion control method is convenient for dynamic locomotion.

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