Implementation with a Sympathizer[*]

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Abstract

This paper studies Nash implementation under complete information with the distinctive feature that the planner does not know individuals' state-contingent preferences and is completely ignorant of how individuals' payoff-relevant characteristics correspond to the states of the economy, on which the social goal depends. Our main question is whether or not the planner can learn individuals' underlying preferences and simultaneously implement the given social goal. In economic environments with at least three individuals, we show that the planner may Nash implement a social goal while extracting the desired information about individuals' state-contingent preferences from the society whenever this goal has standard monotonicity properties and one of the individuals, whose identity is not necessarily known to the planner and the other individuals, is a sympathizer. Vaguely put, such an agent is inclined toward the truthful revelation of how states of the economy are associated with individuals' preferences, while he is not inclined to reveal the realized "true" state of the economy. Then, in every Nash equilibrium of the mechanism we design, all individuals truthfully reveal the same information about individuals' choices.

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1 Introduction

In the implementation problem, a planner (she) is responsible for the decentralization of a social goal that depends on information that she seeks to elicit from the society via a mechanism. Her foresight of individuals' behavior is crucial for the design of such mechanisms. The standard approach assumes that the planner is informed of the one-to-one correspondence between the set of *states of the economy*, on which the social goal depends, and the payoff-relevant characteristics (*states of the world*). That is, the planner *knows* the association between individuals' preferences and the states of the economy. In this context, the seminal works of Maskin (1999, circulated in 1977), Moore and Repullo (1990), and Dutta and Sen (1991) provide characterizations of social goals that admit mechanisms the equilibria of which coincide with a given goal under complete information.[1] de Clippel (2014) extends this analysis to cases in which individuals' behavior does not necessarily satisfy the weak axiom of revealed preferences (WARP), which is generally regarded as rationality.

The critical difference of our setup is that the planner does not have any information about the connection between individuals' state-contingent preferences and states of the economy but still aims to implement a given social goal. Thus, our setting can be viewed as a situation with an extreme form of missing data concerning individuals' preferences.[2]

In a nutshell, we analyze full implementation under rationality and complete information with the distinctive feature that the planner does not know individuals' state-contingent preferences. Our main question concerns whether or not the planner can learn individuals' underlying preferences and simultaneously implement the given social goal.[3]

First, we establish that if the planner knows that a social choice correspondence (SCC) is implementable by a mechanism in Nash equilibrium, then she infers that there is a profile of sets *rational-consistent* with this SCC without necessarily knowing the full specification of sets that appear in this profile. Therefore, the knowledge of the existence of a profile rational-consistent with the SCC constitutes the minimal information pertinent to the association between individuals' preferences and the states of the economy in conjunction with the Nash implementability of that SCC. Moreover, the existence of a profile rational-consistent with a given SCC is equivalent to the well-

[1] Complete information involves situations when payoff-relevant characteristics are commonly known within the society but not to the planner. For more, see Maskin and Sjöström (2002), Palfrey (2002), and Serrano (2004).

[2] The planner could be an *implementation consulting agency* (e.g., McKinsey Implementation (McKinsey, 2018)) responsible for eliciting information about the financial and operational state of a client firm and implementing a given policy rule contingent on this information. Alternatively, the planner could be a court-appointed trustee authorized to run a company during its bankruptcy proceedings. In both cases, strategic interactions among subdivisions whose preferences the planner does not know emerge.

[3] For expositional purposes, we present the extensions of our setting and results to behavioral environments—by allowing, but not insisting on, violations of WARP—in the Appendix.

known Maskin monotonicity of this SCC.[4] On the other hand, our *second* and the main result is that with at least three individuals, if the planner knows that (*i*) the *environment* is *economic* and one of the individuals (whose identity is not necessarily known to the planner and the other individuals) is a *sympathizer* (*ii*) and that the given SCC possesses a rational-consistent profile of sets while the planner does not necessarily know the full specification of sets that appear in such a profile, then she infers the following: The given SCC is Nash implementable by a mechanism that elicits the desired information concerning rational-consistency from the society unanimously. Thus, the planner no longer needs to know the association between the payoff-relevant characteristics and the states of the economy to identify a profile of sets rational-consistent with the given SCC. She can simply ask the individuals, knowing that all announce the same profile of sets rational-consistent with the SCC.

We attain the notion of *sympathy* by modifying *partial honesty* of Dutta and Sen (2012) so that it involves only announcements of profiles of sets. To that regard, we restrict attention to mechanisms that involve each agent announcing a profile of sets. A *sympathizer* of the SCC, then, is an individual who strictly prefers an action that consists of the announcement of a profile rational-consistent with this SCC coupled with some messages to another action that involves announcing an inconsistent profile and the same messages, whenever both actions deliver this individual's most preferred alternatives among those he can sustain via unilateral deviations given others' actions. Thus, a sympathizer is not a snitch or an informer in the sense that he does not feel any obligation and/or inclination to reveal the state of the economy. Instead, he serves the planner as a guide.[5]

The *economic environment* assumption requires that agents' choices are not perfectly aligned: for any alternative and any state, there exist two individuals who do not choose that alternative in that state from the set of all alternatives. Therefore, it demands that there is some weak form of disagreement in the society at every state.[6]

[4]The behavioral version of rational-consistency, namely, consistency, is at the heart of de Clippel's characterization of Nash implementability in the behavioral domain. Given individuals' choices, a profile of sets indexed for an individual, a state, and a socially optimal alternative at that state, is said to be *consistent* with a social goal if (*i*) for all individuals, all states, and all socially optimal alternatives in that state, this alternative is chosen at that state by that individual from the corresponding set, and (*ii*) an alternative being socially optimal in the first state, but not in the second, implies that there exists an individual who does not choose that alternative at the second state from the set indexed for that individual and that alternative and the first state. Then, the *necessity* result establishes that the *opportunity sets* sustained by the mechanism that implements a given social goal, sets of alternatives that an individual can obtain by changing his messages while others' remain the same, form a profile of sets consistent with this social goal. Moreover, the existence of a consistent profile of sets can be used to modify the canonical mechanism—by utilizing this profile as opportunity sets—to deliver a *sufficiency* result. (Maskin, 1999; de Clippel, 2014)

[5]According to Cambridge Dictionary, a sympathizer is "a person who supports a political organization or believes in a set of ideas." Thus, a sympathizer can be thought of as a proponent of the policy the planner aims to implement.

[6]This assumption is also used in Bergemann and Morris (2008), Kartik and Tercieux (2012), Barlo and Dalkıran (2020), and Barlo and Dalkıran (2021).

The existence of a rational-consistent profile is at the core of the Nash implementability of a given SCC. Yet, the planner, completely ignorant of how states of the economy and payoff-relevant states are related, cannot identify/verify this central condition on her own. To extend our sufficiency result to a setting where the planner draws the inference of rational-consistency by herself, we provide the following result: The planner deduces the existence of a profile rational-consistent with the given SCC whenever she knows that this SCC possesses a Maskin monotonic extension to the set of all payoff-relevant states even if she does not know the full specification of this extension.

We extend our analysis and results to the behavioral domain (by allowing but not insisting on violations of WARP) in the Appendix.[7] We also consider extensions of our sufficiency result to noneconomic environments using the behavioral version of the no-veto property and continuing to work with three or more individuals. As a result, we attain another sufficiency result when the planner knows that the environment features *societal non-satiation* and contains at least two *strong sympathizers* the identities of whom are privately known to themselves, but not to the planner.[8]

From a technical point of view, the mechanism that we employ in our sufficiency results differs from the canonical mechanism in a particular manner: It asks every individual a profile of sets, the realized state of the economy, an alternative, and an integer. The distinctive feature is that the *opportunity sets*—alternatives that an individual attains by unilateral deviations given others' messages—associated with the situation in which all agents announce the same state and an alternative socially optimal at that state are determined according to the announced profiles of sets as long as profile announcements of all but one agree. Thus, the planner does not need to know individuals' state-contingent lower contour sets. The presence of a sympathizer ensures that in equilibrium, all agents announce the same profile of sets, which has to be rational-consistent.

Our paper is closely related to the literature on implementation with partial honesty, pioneered by Dutta and Sen (2012).[9] Their construction assumes that at least one of the individuals has a

[7] An incomplete list of papers on behavioral implementation contains Hurwicz (1986), Eliaz (2002), Barlo and Dalkiran (2009), Saran (2011), Korpela (2012), de Clippel (2014), Saran (2016), Barlo and Dalkıran (2020), and Hayashi et al. (2020).

[8] *Societal non-satiation* demands that for every alternative and every state, there exists an individual who does not choose that alternative at that state from the set of all alternatives. This restriction is weaker than the economic environment assumption and allows for more Nash equilibria in the mechanism we employ. But, with more Nash equilibria to handle comes the need for more power: instead of a single sympathizer, now we need at least two strong sympathizers. A *strong sympathizer* of the SCC is an individual who strictly prefers an action that consists of the announcement of a profile rational-consistent with this SCC coupled with some messages to another action that involves announcing an inconsistent profile and some other messages, whenever both actions deliver this individual's most preferred alternatives among those he can sustain via unilateral deviations given others' actions. So, a strong sympathizer is a sympathizer.

[9] An incomplete list in this literature consists of Matsushima (2008a), Matsushima (2008b), Kartik and Tercieux (2012), Kartik et al. (2014), Korpela (2014), Saporiti (2014), Ortner (2015), Doğan (2017), Kimya (2017), Lombardi and Yoshihara (2017), Mukherjee et al. (2017), Lombardi and Yoshihara (2018), Savva (2018), Hagiwara (2019), and Lombardi and Yoshihara (2020). See also Dutta (2019) for a survey of recent results in this literature. Another strand

preference for honesty. To formulate this, individuals' preferences on alternatives are extended to messages when dealing with mechanisms that involve the announcement of a state. A partially honest individual is assumed to strictly prefer a message involving the announcement of the 'true' state of the world when none of his deviations make him strictly better off. Then, that study shows that all SCCs satisfying the no-veto property can be implemented in Nash equilibrium whenever the society contains at least three individuals, one of whom, whose identity is privately known only by himself, is partially honest. This sufficiency result does not need Maskin monotonicity.

Sympathy involves an inclination toward the revelation of rational-consistent profiles of sets and not truthful announcements of the realized states of the economy. That is why, unlike many papers on implementation with partial honesty, we need a Maskin monotonicity type of requirement to extract information about the states of the economy. In Section A of the Appendix, we analyze the relation between sympathy and honesty in detail.

Another closely related paper is Barlo and Dalkıran (2021) which studies "suitable notions of implementation for environments in which planners do not observe all the data on individuals' choices and are partially informed about the association of individuals' preferences with states of the economy." That article differs from the current paper in three folds. In that paper, (*i*) the planner has missing data on individuals' choices and hence is not completely ignorant; (*ii*) there are no sympathizers and/or partially honest individuals in the society to help the planner; (*iii*) the equilibrium notion, while related to Nash equilibrium, is different.

The rest of the paper is organized as follows. We present the notations, definitions, and some preliminary results in Section 2. Our main result is in Section 3. Section 4 contains a result on the inference of the existence of a rational-consistent profile, while Section 5 concludes. Section A elaborates on the relation between sympathy and honesty. A behavioral formulation is presented in Section B, and our analysis of noneconomic environments in Section C of the Appendix. The proofs are in Section D of the Appendix.

2 Preliminaries

Let X be a set of *alternatives*, 2^X the set of all subsets of X, and $\mathcal{X} := 2^X \setminus \{\emptyset\}$. For all $x \in X$, let $\mathcal{X}_x$ be the set of all non-empty subsets of X containing x. $N = \{1, ..., n\}$ denotes a *society* with a finite set of individuals where $n \geq 2$.

Below, we introduce our setting under the *rational domain*. On the other hand, our construction

of related papers analyzes the characterization of jurors' preferences on rankings of contestants when jurors are not necessarily impartial and have incentives to misreport the true ranking of contestants. See Amorós (2009) and Amorós (2013). Yadav (2016) considers the effects of partial honesty in the model of Amorós (2013).

and results extend to the behavioral domain as well, and these are presented in the Appendix.

Ω denotes the set of all *feasible states* of the world and is in one-to-one correspondence with all the admissible payoff-relevant characteristics of the environment. The *preferences* of individual $i \in N$ at state $\omega \in \Omega$ is captured by a complete and transitive binary relation, a ranking, $R_i^\omega \subseteq X \times X$.[10] The *ranking profile of the society* is given by $\mathbf{R} = (R_i^\omega)_{i\in N,\ \omega\in\Omega}$ and it is in one-to-one correspondence with Ω. Given $i \in N$, $\omega \in \Omega$, and $x \in X$, $L_i^\omega(x) := \{y \in X \mid xR_i^\omega y\} \in \mathcal{X}_x$ denotes the *lower contour set of individual i at state ω of alternative* x, and we let $\mathbb{L}_i^\omega(x) := \{S \in \mathcal{X}_x \mid S \subset L_i^\omega(x)\}$ identify the collection of sets that contain x and are subsets of $L_i^\omega(x)$.

We let Θ be the set of *states of the economy*. A *social choice correspondence* (SCC) defined on the states of the economy is $f : \Theta \to \mathcal{X}$, a non-empty valued correspondence mapping Θ into $\mathcal{X}$. Given $\theta \in \Theta$, $f(\theta)$ denotes the set of alternatives that the planner desires to sustain at θ and is referred to as *f-optimal* alternatives at θ.

The *identification function* $\pi^* : \Theta \to \Omega$ captures the association of states of the economy with the underlying payoff-relevant characteristics (states), where $\pi^*(\theta) \in \Omega$ is in one-to-one correspondence with the ranking profile associated with $\theta \in \Theta$. To model a situation in which the planner does not know how to associate the states of the economy with the underlying payoff-relevant characteristics, we assume that the planner does not know $\pi^* : \Theta \to \Omega$.

We restrict attention to *complete information*. The information and knowledge requirements of our model are as follows:

(i) the planner knows N, X, Ω, Θ, and $f : \Theta \to \mathcal{X}$; and

(ii) N, X, Ω, Θ, $\pi^* : \Theta \to \Omega$, $f : \Theta \to \mathcal{X}$, and the realized state of the economy $\theta \in \Theta$ are common knowledge among the individuals; and

(iii) items (i) and (ii) are common knowledge among the individuals and the planner.

The essence of the asymmetry of information between the planner and the individuals involves the identification function π^* and the realized state of the economy θ.

A mechanism $\mu = (A, g)$ assigns each individual $i \in N$ a non-empty *message space* A_i and specifies an *outcome function* $g : A \to X$ where $A = \times_{j\in N} A_j$. $\mathcal{M}$ denotes the set of all mechanisms. Given a mechanism $\mu \in \mathcal{M}$ and $a_{-i} \in A_{-i} := \times_{j\neq i} A_j$, the *opportunity set* of individual i pertaining to others' message profile a_{-i} in mechanism μ is $O_i^\mu(a_{-i}) := g(A_i, a_{-i})$ where $g(A_i, a_{-i}) = \{g(a_i, a_{-i}) : a_i \in A_i\}$. Consequently, $a^* \in A$ is a ***Nash equilibrium*** of μ at $\omega \in \Omega$ if for all $i \in N$, $g(a^*)\, R_i^\omega\, g(a_i, \bar{a}^*_{-i})$

[10] A binary relation $R \subseteq X \times X$ is *complete* if for all $x, y \in X$ either xRy or yRx or both; *transitive* if for all $x, y, z \in X$ with xRy and yRz implies xRz.

for all $a_i \in A_i$ (equivalently, $g(a^*)\, R_i^{\omega}\, x$ for all $x \in O_i^{\mu}(a_{-i}^*)$). Given mechanism μ, the correspondence $NE^{\mu} : \Theta \twoheadrightarrow 2^X$ identifies Nash equilibrium outomes of μ at $\theta \in \Theta$ and is defined by $NE^{\mu}(\theta) := \{x \in X \mid \exists a^* \in A \text{ s.t. } a^* \text{ is a Nash equilibrium of } \mu \text{ at } \pi^*(\theta) \text{ and } g(a^*) = x\}$. Then, the notion of Nash implementation, which can be verified by an all-seeing party, is: An SCC $f : \Theta \to \mathcal{X}$ is ***implementable by a mechanism*** $\mu \in \mathcal{M}$ ***in Nash equilibrium***, if for all $\theta \in \Theta$, $f(\theta) = NE^{\mu}(\theta)$.

Below, we show that a variant of monotonicity of Maskin (1999), the rational version of consistency of de Clippel (2014), is related to Nash implementation:[11]

Definition 1. *A profile of sets* $\mathbf{S} := (S_i(x,\theta))_{i\in N,\ \theta\in\Theta,\ x\in f(\theta)}$ *is* **rational-consistent** *with the given SCC* $f : \Theta \to \mathcal{X}$ *if*

(i) *for all* $i \in N$*, all* $\theta \in \Theta$*, and all* $x \in f(\theta)$*,* $S_i(x,\theta) \in \mathbb{L}_i^{\pi^*(\theta)}(x)$*; and*

(ii) $x \in f(\theta)$ *and* $x \notin f(\tilde{\theta})$ *with* $\theta, \tilde{\theta} \in \Theta$ *implies there is* $j \in N$ *with* $S_j(x,\theta) \notin \mathbb{L}_j^{\pi^*(\tilde{\theta})}(x)$.

Let $\mathcal{S}(f)$ *denote the set of all profiles of sets that are rational-consistent with* f.

In words, a profile of sets $\mathbf{S}$ is rational-consistent with a given SCC f, if (i) for every individual i and state of the economy θ and alternative x in $f(\theta)$, x is one of the best alternatives according to $R_i^{\pi^*(\theta)}$ in the set $S_i(x,\theta)$; and (ii) if x is f-optimal at θ but not at $\tilde{\theta}$, then there exists $j \in N$ such that x is not among the best alternatives according to $R_j^{\pi^*(\tilde{\theta})}$ in $S_j(x,\theta)$.

When the planner knows that a mechanism $\mu^* = (A^*, g^*)$ implements SCC $f : \Theta \to \mathcal{X}$ in Nash equilibrium, then she infers the following: for all $\theta \in \Theta$ and all $x \in f(\theta)$, there is some $a^x \in A^*$ such that $g^*(a^x) = x$ and for all $i \in N$, $g^*(a^x) R_i^{\pi^*(\theta)} x'$ for all $x' \in O_i^{\mu^*}(a_{-i}^x)$, even though the planner does not know exactly what $\pi^*(\theta)$ is and precisely which message profile a^x corresponds to—unless there is a unique $a^x \in A$ delivering x. Therefore, for all $i \in N$, all $\theta \in \Theta$, and all $x \in f(\theta)$, the planner infers that there is a set $S_i(x,\theta) := O_i^{\mu^*}(a_{-i}^x)$ (the full specification of which she may not know) from which one of the top ranked alternatives of i at the true payoff-relevant ranking, $R_i^{\pi^*(\theta)}$, includes x. In other words, the planner infers that (i) of rational-consistency holds. For (ii) of rational-consistency, suppose that the planner knows that $x \in f(\theta)$ and $x \notin f(\tilde{\theta})$ for some $\theta, \tilde{\theta} \in \Theta$. Then, the planner (knowing that μ^* implements f in Nash equilibrium) infers that a^x cannot be a Nash equilibrium at the payoff-relevant state $\pi^*(\tilde{\theta})$ even though she does not know what the profile a^x and state $\pi^*(\tilde{\theta})$ are. This is because otherwise she figures out that a^x being a Nash equilibrium at $\pi^*(\tilde{\theta})$ implies, by (ii) of Nash implementation, $g^*(a^x) = x$ is in $f(\tilde{\theta})$. So, there is an individual $j \in N$ who does not rank x as the first alternative in $S_j(x,\theta) = O_j^{\mu^*}(a_{-j}^x)$ using ranking $R_j^{\pi^*(\tilde{\theta})}$. In other words, in this situation, the

[11] There are many variants of Maskin monotonicity in the literature. See for example, Eliaz (2002), Barlo and Dalkiran (2009), Sanver (2017), Koray and Yildiz (2018), and Lombardi and Yoshihara (2018).

planner infers that there is an individual j such that the underlying payoff-relevant state, $\pi^*(\tilde{\theta})$ that she does not know, is so that $S_j(x,\theta) \notin \mathbb{L}_j^{\pi^*(\tilde{\theta})}(x)$; enabling us to conclude that the planner deduces that (*ii*) of rational-consistency holds. These deliver the following necessity theorem proved above:

Theorem 1. *If the planner knows that the SCC $f : \Theta \to \mathcal{X}$ is Nash implementable, then the planner infers that $\mathcal{S}(f) \neq \emptyset$ without necessarily knowing the full specification of sets that appear in $\mathcal{S}(f)$.*

In our setup, the planner is completely ignorant and does not have any information that helps her associate the states of the economy with the underlying payoff-relevant characteristics. But, she needs this information to design desired mechanisms. We argue that the planner obtaining this information from knowing the full specification of a rational-consistent profile of sets beats the purpose. Indeed, it is natural to consider mechanisms in which the planner *asks* individuals' help. Nevertheless, this endeavor is fruitful only when there is some hope for Nash implementation, i.e., when the planner infers that $\mathcal{S}(f) \neq \emptyset$; or else, by Theorem 1, she deduces that f is not Nash implementable. In what follows, we establish that if the planner knows the existence (but not the full specification) of a rational-consistent profile with a given SCC, then she can extract the rest of the information about this profile from the society while implementing this SCC, whenever there exists a partially honest guide among the individuals.

3 The Planner Asking for Guidance

The planner aims to elicit the information about the full specification of a rational-consistent profile of sets from the society. To that end, the planner employs a *sympathizer* of the social goal, a partially honest guide who is inclined toward the truthful revelation of a rational-consistent profile but not the realized state of the economy.

To formalize these, for any SCC $f : \Theta \to \mathcal{X}$, we restrict attention to mechanisms in which one of the components of each individual's messages involves the announcement of a profile of sets indexed for $i \in N$, $\theta \in \Theta$, and $x \in f(\theta)$. We refer to such game forms as *guidance mechanisms* and denote them by $\mathcal{M}^S \subset \mathcal{M}$. To that regard, we let $\mathcal{S}$ denote the set of all profile of sets of alternatives $\mathbf{S} = (S_i(x,\theta))_{i\in N,\ \theta\in\Theta,\ x\in f(\theta)}$ with the property that $x \in S_i(x,\theta)$ for all $i \in N$, $\theta \in \Theta$, and $x \in f(\theta)$. The guidance mechanism $\mu \in \mathcal{M}^S$ is such that $A_i := \mathcal{S} \times M_i$ for each $i \in N$ for some non-empty M_i and $M := \times_{i\in N} M_i$ and a generic message (action) $a_i \in A_i$ is $a_i = (\mathbf{S}^{(i)}, m_i)$. We note that for any SCC f, the set of rational-consistent profiles, $\mathcal{S}(f)$, is contained in $\mathcal{S}$.

In what follows, we provide an extension of individuals' preferences over alternatives to choices on messages in guidance mechanisms.

For any $f : \Theta \to \mathcal{X}$, any $\mu \in \mathcal{M}^S$, and any $\omega \in \Omega$, the correspondence $BR_i^{\omega} : A_{-i} \twoheadrightarrow A_i$ identifies individual i's ***best responses*** at ω to others' messages. In particular, if individual i is a standard economic agent, and *not a sympathizer* of f at $\omega \in \Omega$, then for all $a_{-i} \in A_{-i}$,

$$a_i \in BR_i^{\omega}(a_{-i}) \text{ if and only if } g(a_i, a_{-i}) \, R_i^{\omega} \, g(a_i', a_{-i}) \text{ for all } a_i' \in A_i.$$

For sympathizers, the following holds:

Definition 2. *Given an SCC $f : \Theta \to \mathcal{X}$ and a guidance mechanism $\mu \in \mathcal{M}^S$, we say that individual $i \in N$ is a* **sympathizer** of f at $\omega \in \Omega$ *if for all $a_{-i} \in A_{-i}$,*

(*i*) $\mathbf{S} \in \mathcal{S}(f)$, $\tilde{\mathbf{S}} \notin \mathcal{S}(f)$, *and* $m_i \in M_i$ *implies* $(\mathbf{S}, m_i) \in BR_i^{\omega}(a_{-i})$ *and* $(\tilde{\mathbf{S}}, m_i) \notin BR_i^{\omega}(a_{-i})$ *if*

$$g((\mathbf{S}, m_i), a_{-i}) \, R_i^{\omega} \, g(a_i', a_{-i}) \textit{ for all } a_i' \in A_i, \textit{ and}$$
$$g((\tilde{\mathbf{S}}, m_i), a_{-i}) \, R_i^{\omega} \, g(a_i'', a_{-i}) \textit{ for all } a_i'' \in A_i; \textit{ and}$$

(*ii*) *in all other cases,* $a_i \in BR_i^{\omega}(a_{-i})$ *if and only if* $g(a_i, a_{-i}) \, R_i^{\omega} \, g(a_i', a_{-i})$ *for all* $a_i' \in A_i$.

We say that the environment satisfies the **sympathizer property** *with respect to SCC f if, for every state $\omega \in \Omega$, there exists at least one sympathizer of f at ω, while the identity of each sympathizer of f at ω is privately known only by himself.*

In words, a sympathizer i of f at ω strictly prefers a rational-consistent profile $\mathbf{S}$ coupled with a message profile m_i to a non-rational-consistent profile $\tilde{\mathbf{S}}$ coupled with the same message profile m_i whenever both action profiles, $(\mathbf{S}, m_i)$ and $(\tilde{\mathbf{S}}, m_i)$, lead to alternatives among the best according to R_i^{ω}. Therefore, individuals' best responses in a guidance mechanism μ at ω are obtained using the usual preference maximization along with an additional lexicographic tie-breaking rule favoring the announcement of rational-consistent profiles of sets. In fact, i's best responses are *standard* if $\mu \notin \mathcal{M}^S$ and/or the announcement of a rational-consistent profile coupled with some messages does not deliver the top-ranked alternative in i's opportunity set and/or i is not a sympathizer of f at ω.

On the other hand, if the guidance mechanism μ associated with f is such that i is a sympathizer of f at ω and can obtain her top-ranked alternative in her opportunity set via the announcement of a rational-consistent profile, then her best responses are not in one-to-one correspondence with her preferences R_i^{ω}. To reflect the novel nature of Nash equilibrium obtained from such best responses, we introduce the concept of Nash* equilibrium: Given a mechanism $\mu \in \mathcal{M}$, $a^* \in A$ is a ***Nash* equilibrium*** of μ at $\omega \in \Omega$ if, for all $i \in N$, $a_i^* \in BR_i^{\pi^*(\theta)}(a_{-i}^*)$. Nash and Nash* equilibrium coincide when $\mu \notin \mathcal{M}^S$ and/or there are no sympathizers of f at ω and/or i is a sympathizer of f at ω but there is no $(\mathbf{S}, m_i)$ with $\mathbf{S} \in \mathcal{S}(f)$ and $g((\mathbf{S}, m_i), a_{-i}^*) \, R_i^{\omega} \, g(a_i', a_{-i}^*)$ for all $a_i' \in A_i$ while a^* is a

Nash equilibrium at ω. In general, the set of Nash* equilibrium at ω is subset of the set of Nash equilibrium at ω of the same mechanism. The notion of Nash* implementation is the following:

Definition 3. *We say that an SCC* $f : \Theta \to \mathcal{X}$ *is* **implementable by a mechanism** $\mu \in \mathcal{M}$ **in Nash* equilibrium***, if*

(i) *for any* $\theta \in \Theta$ *and* $x \in f(\theta)$*, there exists* $a^x \in A$ *such that* $g(a^x) = x$ *and* $a^x_i \in BR_i^{\pi^*(\theta)}(a^x_{-i})$ *for all* $i \in N$*; and*

(ii) *for any* $\theta \in \Theta$*,* $a^* \in A$ *with* $a^*_i \in BR_i^{\pi^*(\theta)}(a^*_{-i})$ *for all* $i \in N$ *implies* $g(a^*) \in f(\theta)$*.*

When the mechanism in this definition is not in $\mathcal{M}^S$, Nash* implementation coincides with Nash implementation. Furthermore, the necessary condition we attain employing Nash* implementation is not independent of the mechanism. Hence, it is not helpful in constructing mechanisms that can be employed in the sufficiency direction.

Our main result uses the following assumption:

Definition 4. *We say that the* **economic environment** *assumption holds whenever for all* $\omega \in \Omega$ *and all* $x \in X$*, there are two individuals* $i, j \in N$ *with* $i \neq j$ *and there are two alternatives* $y^i, y^j \in X$ *such that* $y^i P_i^\omega x$ *and* $y^j P_j^\omega x$*.*

The economic environment assumption demands that for every state and alternative, there are two individuals not choosing that alternative from the set of all alternatives at that given state. This assumption, therefore, needs a weak form of disagreement in the society.

The following is our main result:

Theorem 2. *Suppose* $n \geq 3$*. Suppose that the planner knows that*

(i) *the environment is economic, and it satisfies the sympathizer property, and*

(ii) *the SCC* $f : \Theta \to \mathcal{X}$ *has a rational-consistent profile of sets, i.e.,* $\mathcal{S}(f) \neq \emptyset$*, while she does not necessarily know the full specification of the sets that appear in* $\mathcal{S}(f)$*.*

Then, the planner infers that f *is Nash* implementable by a guidance mechanism* $\mu \in \mathcal{M}^S$*, and for any state of the economy* $\theta \in \Theta$ *and any Nash* equilibrium* $\bar{a} = (\bar{\mathbf{S}}^{(i)}, \bar{m}_i)_{i \in N}$ *of mechanism* μ *at state* $\pi^*(\theta)$*,* $\bar{\mathbf{S}}^{(i)} = \mathbf{S}$ *for some rational-consistent profile* $\mathbf{S} \in \mathcal{S}(f)$ *for all* $i \in N$*.*

Theorem 2 establishes sufficiency for three or more individuals by utilizing a guidance mechanism that extracts the information about rational-consistency from the society unanimously and implements the desired goal if the following hold: The planner knows that the environment is economic and satisfies the sympathizer property while there is a rational-consistent profile of sets.

4 Inference of Rational-Consistency

Below, we present a way of ensuring the planner's inference of the existence of a rational-consistent profile of sets. It involves Maskin monotonicity: We say that a correspondence mapping Ω, payoff-relevant states, into 2^X, (possibly empty) subsets of alternatives, is an *extension of an SCC* $f : \Theta \to \mathcal{X}$ *to* Ω, denoted by $f_\Omega : \Omega \to 2^X$, if $f(\theta) = f_\Omega(\pi^*(\theta))$ for all $\theta \in \Theta$. Thus, f_Ω is non-empty-valued for all $\omega \in \pi^*(\Theta)$. Moreover, if the SCC $f : \Theta \to \mathcal{X}$ possesses an extension to Ω, then $f(\theta) \neq f(\theta')$ implies $\pi^*(\theta) \neq \pi^*(\theta')$, where $\theta, \theta' \in \Theta$. That is why there is no loss of generality to restrict attention to injective identification functions π^*.[12] The notion of Maskin monotonicity formulated for correspondences defined on Ω is as follows:

Definition 5. *A correspondence* $\phi : \Omega \to 2^X$ *is* **Maskin monotonic** *if* $x \in \phi(\omega)$ *and* $L_i^\omega(x) \subseteq L_i^{\tilde{\omega}}(x)$ *for all* $i \in N$ *implies* $x \in \phi(\tilde{\omega})$, *where* $\omega, \tilde{\omega} \in \Omega$.[13]

The following result provides a sufficient condition for the planner's inference of the existence of a profile rational-consistent with a given SCC:

Proposition 1. *If the planner knows that SCC* $f : \Theta \to \mathcal{X}$ *has a Maskin monotonic extension even if she does not know the full specification of this extension, she infers that* $\mathcal{S}(f)$ *is non-empty without necessarily knowing the specification of sets that appear in* $\mathcal{S}(f)$.

Proposition 1 establishes the following: Suppose that the planner knows that $f : \Theta \to \mathcal{X}$ has a Maskin monotonic extension to Ω, $f_\Omega : \Omega \to 2^X$, while she does not know its full specification. She knows only $f_\Omega(\pi^*(\theta))$ which equals $f(\theta)$ while she does not know $\pi^*(\theta)$. Thus, she is completely ignorant of the shape of f_Ω on $\Omega \setminus \pi^*(\Theta)$. Still, the planner figures out that $\mathbf{L}^{\pi^*(\Theta)} := (L_i^\omega(x))_{i \in N,\ \omega \in \pi^*(\Theta),\ x \in f_\Omega(\omega)}$ is a rational-consistent profile with $f_\Omega|_{\pi^*(\Theta)} = f$, without knowing the full specifications of (*i*) the identification function $\pi^* : \Theta \to \Omega$, (*ii*) the lower contour sets that appear in $\mathbf{L}^{\pi^*(\Theta)}$, and (*iii*) the Maskin monotonic extension f_Ω.[14]

[12] A function $\psi : X \to Y$ is *injective* if it maps distinct elements of its domain, X, to distinct elements in its range, Y; it is *surjective* if for every element in its range, $y \in Y$, there is an element in its domain, $x \in X$, with $\psi(x) = y$. A function $\psi : X \to Y$ is a *bijection* if it is injective and surjective.

[13] For SCC defined on Ω, $\phi : \Omega \to \mathcal{X}$, the existence of a rational-consistent profile of sets with ϕ on Ω is equivalent to Maskin monotonicity of ϕ: For *sufficiency*, suppose that there is a profile of sets $\mathbf{S} = (S_i(x, \omega))_{i \in N,\ \omega \in \Omega,\ x \in \phi(\omega)}$ that is rational-consistent with ϕ and $x \in \phi(\omega)$ but $x \notin \phi(\tilde{\omega})$. Then, by (*ii*) of rational-consistency, there is $j \in N$ such that $S_j(x, \omega) \notin \mathbb{L}_j^{\tilde{\omega}}(x)$, i.e., $S_j(x, \omega) \in \mathcal{X}_x$ is not a subset of $L_j^{\tilde{\omega}}(x)$. But, by (*i*) of rational-consistency we observe that $S_j(x, \omega) \in \mathbb{L}_j^\omega(x)$ and hence $S_j(x, \omega)$ is a subset of $L_j^\omega(x)$ that contains x. Thus, we conclude that $j \in N$ is such that $L_j^\omega \nsubseteq L_j^{\tilde{\omega}}$, which establishes that f is Maskin monotonic. For *necessity*, suppose that ϕ is Maskin monotonic and let $\mathbf{S}$ be given by $S_i(x, \omega) = L_i^\omega(x)$ for all $i \in N$, all $\omega \in \Omega$, and all $x \in \phi(\omega)$. Then, (*i*) of rational-consistency is trivially satisfied. For (*ii*) of rational-consistency, suppose that $x \in \phi(\omega)$ and $x \notin \phi(\tilde{\omega})$ for some $\omega, \tilde{\omega} \in \Omega$. By Maskin monotonicity, there is $j \in N$ such that $L_j^\omega(x) \nsubseteq L_j^{\tilde{\omega}}(x)$. So, $L_j^\omega(x) = S_j(x, \omega)$ implies $S_j(x, \omega)$ not in $\mathbb{L}_j^{\tilde{\omega}}(x)$.

[14] Proposition 2 offers an extension of this result to the behavioral domain using consistency of de Clippel (2014).

As a result, the information the planner infers from knowing that SCC $f : \Theta \to \mathcal{X}$ has a Maskin monotonic extension, the specification of which she does not know, does not suffice to construct the standard canonical mechanisms employed in Maskin (1999), Moore and Repullo (1990), Dutta and Sen (1991), and de Clippel (2014). That is because the planner does not necessarily know individuals' lower contour sets, which, in these mechanisms, are equal to their opportunity sets for cases when all individuals announce the same state and alternative.[15]

5 Concluding Remarks

We consider full implementation under complete information with the additional feature that the planner is completely ignorant of individuals' underlying state-contingent choices. Our main result is that if there are at least three individuals and the planner knows that the environment is economic, satisfies the sympathizer property, and there is a rational-consistent profile of sets with the given SCC, then she infers the following: This SCC is implementable by a guidance mechanism under Nash* equilibrium by eliciting the information concerning rational-consistency from the society. Moreover, in every Nash* equilibrium, all individuals announce the same profile that is rational-consistent with the given SCC.

[15]If the planner were to know that the environment is economic, the full specification of a correspondence $f_\Omega : \Omega \to 2^X$, and that f_Ω is a Maskin monotonic extension of $f : \Theta \to \mathcal{X}$ to Ω, then she can construct a variant of the canonical mechanism using her knowledge about $(L_i^\omega(x))_{i\in N,\ \omega\in\Omega,\ x\in f_\Omega(\omega)}$. Then, she infers that this mechanism, $\mu^* = (A^*, g^*)$, Nash implements f_Ω and hence $f = f_\Omega \mid_{\pi^*(\Theta)}$ (since she knows $\pi^*(\Theta) \subset \Omega$ even though she does not know the exact form of π^*): $A_i^* := X \times X \times \Theta \times \Omega \times \mathbb{N}$ where each $a_i = (x^{(i)}, y^{(i)}, \theta^{(i)}, \omega^{(i)}, k^{(i)}) \in A_i^*$ obeys the requirement that $x^{(i)} \in f(\theta^{(i)}) \cap f_\Omega(\omega^{(i)})$, $y^{(i)} \in X$, $\theta^{(i)} \in \Theta$, $\omega^{(i)} \in \Omega$, and $k^{(i)} \in \mathbb{N}$. The outcome function $g^* : A^* \to X$ is as follows: Rule 1: $g^*(a) = x$ if $a_i = (x, y, \theta, \omega, \cdot)$ for all $i \in N$; Rule 2: $g^*(a)$ equals y' if $y' \in L_j^\omega(x)$ and x otherwise, whenever $a_i = (x, y, \theta, \omega, \cdot)$ for all $i \in N \setminus \{j\}$ and $a_j = (x', y', \theta', \omega', \cdot) \neq (x, y, \theta, \omega, \cdot)$; Rule 3: $g^*(a) = x^{(i^*)}$ where $i^* = \min\{j \in N : k^{(j)} \geq \max_{i' \in N} k^{(i')}\}$.

Appendix

A Sympathy versus Honesty

In this section, we analyze whether or not standard implementation results with partial honesty can be applied to our framework. To that regard, we adopt the convention that a state under complete information is to encompass all the information that is common knowledge among the individuals. Consequently, we define a *grand state* as the combination of a state of the economy, a payoff-relevant state, and a mapping $\pi : \Theta \to \Omega$. Let $\Sigma := \{(\theta, \omega, \pi) \in \Theta \times \Omega \times \Pi \mid \pi(\theta) = \omega\}$ be the set of grand states where a generic member $\sigma \in \Sigma$ is $\sigma = (\theta, \omega, \pi)$ with $\pi(\theta) = \omega$ and $\Pi := \{\pi' \mid \pi' : \Theta \to \Omega\}$. The SCC $f : \Theta \to \mathcal{X}$ is defined on Θ; thus, we consider its natural extension onto Σ: $f(\sigma) = f(\theta)$ for all $\sigma = (\theta, \omega, \pi) \in \Sigma$.

To formalize partial honesty, we consider mechanisms that involve the announcement of a grand state, $\mathcal{M}^{\Sigma}$, which consists of mechanisms of the form $\mu = (A, g)$ with $A_i = (\sigma^{(i)}, m_i) \in \Sigma \times M_i$ for some message space M_i, for all $i \in N$. Given a mechanism $\mu \in \mathcal{M}^{\Sigma}$, we say that individual $i \in N$ is ***partially honest at the realized state*** $\sigma = (\theta, \omega, \pi) \in \Sigma$ if for all $a_{-i} \in A_{-i}$, (*i*) $\tilde{\sigma} \in \Sigma \setminus \{\sigma\}$ and $m_i, \tilde{m}_i \in M_i$ implies $(\sigma, m_i) \in BR_i^{\omega}(a_{-i})$ and $(\tilde{\sigma}, \tilde{m}_i) \notin BR_i^{\omega}(a_{-i})$ if $g((\sigma, m_i), a_{-i}) R_i^{\omega} g(a_i', a_{-i})$ for all $a_i' \in A_i$, and $g((\tilde{\sigma}, \tilde{m}_i), a_{-i}) R_i^{\omega} g(a_i'', a_{-i})$ for all $a_i'' \in A_i$; and (*ii*) in all other cases, $a_i \in BR_i^{\omega}(a_{-i})$ if and only if $g(a_i, a_{-i}) R_i^{\omega} g(a_i', a_{-i})$ for all $a_i' \in A_i$. On the other hand, if i is not partially honest at $\sigma = (\theta, \omega, \pi) \in \Sigma$, then $a_i \in BR_i^{\omega}(a_{-i})$ if and only if $g(a_i, a_{-i}) R_i^{\omega} g(a_i', a_{-i})$ for all $a_i' \in A_i$.

Then, by Dutta and Sen (2012, Theorem 1), if the planner knows that for every state $\sigma \in \Sigma$, there is a partially honest individual at σ (even if she does not know the identity of this agent) and that the SCC $f : \Theta \to \mathcal{X}$ satisfies the no-veto property, then she infers that f is Nash* implementable and in every such equilibrium all but one announce a grand set aligned with the realized grand set.[16]

To compare sympathy with honesty, we introduce a weaker notion of partial honesty in $\mathcal{M}^{\Sigma}$ where at a grand state $\sigma = (\theta, \omega, \pi)$, the individual at hand is partially honest with respect to the announcement of π but not (θ, ω): we say that individual $i \in N$ is ***weakly partially honest at the realized state*** $\sigma = (\theta, \omega, \pi) \in \Sigma$ if for all $a_{-i} \in A_{-i}$, (*i*) $\tilde{\sigma} \in \Sigma$ with $\tilde{\sigma} = (\tilde{\theta}, \tilde{\omega}, \pi)$ and $\hat{\sigma} = (\hat{\theta}, \hat{\omega}, \hat{\pi})$ with $\hat{\pi} \neq \pi$ and $\tilde{m}_i, \hat{m}_i \in M_i$ implies $(\tilde{\sigma}, \tilde{m}_i) \in BR_i^{\omega}(a_{-i})$ but $(\hat{\sigma}, \hat{m}_i) \notin BR_i^{\omega}(a_{-i})$ if $g((\tilde{\sigma}, \tilde{m}_i), a_{-i}) R_i^{\omega} g(a_i', a_{-i})$ for all $a_i' \in A_i$ and $g((\hat{\sigma}, \hat{m}_i), a_{-i}) R_i^{\omega} g(a_i'', a_{-i})$ for all $a_i'' \in A_i$, and (*ii*) otherwise, $a_i \in BR_i^{\omega}(a_{-i})$ if and only if $g(a_i, a_{-i}) R_i^{\omega} g(a_i', a_{-i})$ for all $a_i' \in A_i$. But, if i is not weakly partially honest at $\sigma = (\theta, \omega, \pi) \in \Sigma$, then $a_i \in BR_i^{\omega}(a_{-i})$ if and only if $g(a_i, a_{-i}) R_i^{\omega} g(a_i', a_{-i})$ for all $a_i' \in A_i$.

[16] An SCC $f : \Theta \to \mathcal{X}$ satisfies the no-veto property under rationality if for any $\sigma = (\theta, \omega, \pi) \in \Sigma$, $x R_i^{\omega} y$ for all $y \in X$, all $i \in N \setminus \{j\}$ for some $j \in N$ implies $x \in f(\theta)$.

Now, consider agent $i \in N$ who is weakly partially honest at the realized state $\sigma = (\theta, \omega, \pi^*) \in \Sigma$ so that the realized association between the states of the economy Θ and payoff-relevant states Ω is π^* as in our setup. Let $\tilde{\sigma} = (\tilde{\theta}, \tilde{\omega}, \pi^*) \in \Sigma$ and $\hat{\sigma} = (\hat{\theta}, \hat{\omega}, \hat{\pi}) \in \Sigma$ with $\hat{\pi} \neq \pi^*$ and $\tilde{m}_i, \hat{m}_i \in M_i$. So, if $g((\tilde{\sigma}, \tilde{m}_i), a_{-i}) R_i^{\omega} g(a'_i, a_{-i})$ for all $a'_i \in A_i$, and $g((\hat{\sigma}, \hat{m}_i), a_{-i}) R_i^{\omega} g(a''_i, a_{-i})$ for all $a''_i \in A_i$, then, as i is weakly partially honest, we conclude that $((\tilde{\theta}, \tilde{\omega}, \pi^*), \tilde{m}_i) \in BR_i^{\omega}(a_{-i})$ while $((\hat{\theta}, \hat{\omega}, \hat{\pi}), \hat{m}_i) \notin BR_i^{\omega}(a_{-i})$, equivalently, $(\pi^*, (\tilde{\theta}, \tilde{\omega}, \tilde{m}_i)) \in BR_i^{\omega}(a_{-i})$ and $(\hat{\pi}, (\hat{\theta}, \hat{\omega}, \hat{m}_i)) \notin BR_i^{\omega}(a_{-i})$.

We wish to emphasize that a sympathizer is defined for guidance mechanisms $\mathcal{M}^S$ consisting of $\mu^S = (A^S, g^S)$ where the individuals are to announce profiles of sets $\mathbf{S} \in \mathcal{S}$ and choose some messages, i.e., $A_i^S := \mathcal{S} \times M_i^S$ for some message set M_i^S. We, now, observe that the definition of a weakly partially honest individual resembles our definition of a strong sympathizer:[17] Given an SCC $f : \Theta \to \mathcal{X}$, if we replace π^* with $\mathbf{S} \in \mathcal{S}(f)$ and $\hat{\pi}$ with $\hat{\mathbf{S}} \notin \mathcal{S}(f)$ in the specifications elaborated in the previous paragraph at the realized state $\sigma = (\theta, \omega, \pi^*) \in \Sigma$ with $\tilde{m}_i^S = (\tilde{\theta}, \tilde{\omega}, \tilde{m}_i)$ and $\hat{m}_i^S = (\hat{\theta}, \hat{\omega}, \hat{m}_i)$, we attain a definition akin to the one of a strong sympathizer of f at $\pi^*(\theta) = \omega$. Indeed, if the planner is informed of π^*, then she can construct the set of rational-consistent profiles $\mathcal{S}(f)$. But, she cannot necessarily identify π^* uniquely if she is informed of an element $\mathbf{S}$ in $\mathcal{S}(f)$.

Thus, in our construct, to implement a given SCC, the extent of information the planner seeks to elicit with the help of a sympathizer is "less" than the extent of information the planner obtains thanks to a weakly partially honest individual. Moreover, if the planner aims to implement an SCC by extracting the information about the relation between Θ and Ω from the society with the help of a weakly partially honest individual (who at the realized grand state $(\theta, \omega, \pi^*) \in \Sigma$, is inclined toward the truthful announcement of π^* but not (θ, ω)), then rational-consistency/monotonicity type of requirements concerning (θ, ω) emerge.[18]

B A Behavioral Formulation

To facilitate extended exposition, we present a behavioral formulation of our setting that allows (but does not insist on) violations of WARP. We restate and prove our results with this formulation that encompasses the rational domain.

The *(individual) choice* of agent $i \in N$ at a feasible state $\omega \in \Omega$ is captured by the choice correspondence $C_i^{\omega} : \mathcal{X} \to \mathcal{X}$ with the requirement that for any $S \in \mathcal{X}$, $C_i^{\omega}(S) \subset S$. Given

[17] Under rationality, strong sympathy is defined as follows: Given an SCC $f : \Theta \to \mathcal{X}$ and $\mu^S \in \mathcal{M}^S$, $i \in N$ is a *strong sympathizer of f at $\omega \in \Omega$* if for all $a_{-i} \in A_{-i}^S$, (*i*) $\mathbf{S} \in \mathcal{S}(f)$, $\hat{\mathbf{S}} \notin \mathcal{S}(f)$, and $m_i, \hat{m}_i \in M_i^S$ implies $(\mathbf{S}, m_i) \in BR_i^{\omega}(a_{-i})$ and $(\hat{\mathbf{S}}, \hat{m}_i) \notin BR_i^{\omega}(a_{-i})$ if $g^S((\mathbf{S}, m_i), a_{-i}) R_i^{\omega} g^S(a'_i, a_{-i})$ for all $a'_i \in A_i$, and $g^S((\hat{\mathbf{S}}, \hat{m}_i), a_{-i}) R_i^{\omega} g^S(a''_i, a_{-i})$ for all $a''_i \in A_i$; and (*ii*) in all other cases, $a_i \in BR_i^{\omega}(a_{-i})$ if and only if $g^S(a_i, a_{-i}) R_i^{\omega} g^S(a'_i, a_{-i})$ for all $a'_i \in A_i$. But, if $i \in N$ is not a strong sympathizer at $\sigma = (\theta, \omega, \pi) \in \Sigma$, then $a_i \in BR_i^{\omega}(a_{-i})$ if and only if $g^S(a_i, a_{-i}) R_i^{\omega} g^S(a'_i, a_{-i})$ for all $a'_i \in A_i$.

[18] See Lombardi and Yoshihara (2018) that obtains a necessary condition for partially honestly Nash implementability, namely, partial-honesty monotonicity.

alternative $x \in X$, individual $i \in N$, and state $\omega \in \Omega$, we refer to a set $S \in \mathcal{X}$ with $x \in C_i^{\omega}(S)$ as a *choice set* of individual i at state ω for alternative x. The societal choice topography on Ω is given by the profile of individual choice correspondences $C(\Omega) := (C_i^{\omega}(S))_{i \in N,\ \omega \in \Omega,\ S \in \mathcal{X}}$.[19]

Given a mechanism $\mu \in \mathcal{M}$, $a^* \in A$ constitutes a ***behavioral Nash equilibrium*** of μ at a state $\omega \in \Omega$ if $g(a^*) \in \cap_{i \in N} C_i^{\omega}(O_i^{\mu}(a_{-i}^*))$. Then, ***behavioral Nash implementability*** is: an SCC $f : \Theta \to \mathcal{X}$ is implementable by a mechanism $\mu \in \mathcal{M}$ in behavioral Nash equilibrium if (i) for any $\theta \in \Theta$ and $x \in f(\theta)$, there is $a^x \in A$ such that $g(a^x) = x$ and $x \in \cap_{i \in N} C_i^{\pi^*(\theta)}(O_i^{\mu}(a_{-i}^x))$; and (ii) for any $\theta \in \Theta$, $a^* \in A$ with $g(a^*) \in \cap_{i \in N} C_i^{\pi^*(\theta)}(O_i^{\mu}(a_{-i}^*))$ implies $g(a^*) \in f(\theta)$.

If an SCC $f : \Theta \to \mathcal{X}$ is implementable by a mechanism $\mu \in \mathcal{M}$ in behavioral Nash equilibrium, we define the profile of sets sustained by μ as follows: $\mathbf{S}^{\mu} := (S_i^{\mu}(x, \theta))_{i \in N,\ \theta \in \Theta,\ x \in f(\theta)}$ with $S_i^{\mu}(x, \theta) := O_i^{\mu}(a_{-i}^x)$ for any $i \in N$, $\theta \in \Theta$, and $x \in f(\theta)$ while $a^x \in A$ is such that $g(a^x) = x$ and $g(a^x) \in C_i^{\pi^*(\theta)}(O_i^{\mu}(a_{-i}^x))$ for all $i \in N$. Then, the necessity result of de Clippel (2014) tells us that if f is behavioral Nash implementable by a mechanism $\mu \in \mathcal{M}$, then $\mathbf{S}^{\mu}$ is a profile consistent with f:

Definition 6. *Given SCC $f : \Theta \to \mathcal{X}$, a profile $\mathbf{S} := (S_i(x, \theta))_{i \in N,\ \theta \in \Theta,\ x \in f(\theta)}$ is* **consistent** *with $f : \Theta \to \mathcal{X}$ if*

(i) *for all $\theta \in \Theta$ and all $x \in f(\theta)$, $x \in \cap_{i \in N} C_i^{\pi^*(\theta)}(S_i(x, \theta))$; and*

(ii) *$x \in f(\theta)$ and $x \notin f(\theta')$ for some $\theta, \theta' \in \Theta$ implies $x \notin \cap_{i \in N} C_i^{\pi^*(\theta')}(S_i(x, \theta))$.*

$\mathcal{S}(f)$ denotes the set of all profiles of sets that are consistent with f.

Under WARP, rational-consistency and consistency are equivalent.[20] Moreover, using de Clippel's necessity result and following similar arguments leading to Theorem 1, enable us to conclude the following: If the planner knows that f is behavioral Nash implementable, then she infers that $\mathcal{S}(f) \neq \emptyset$ without necessarily knowing the full specification of sets that appear in $\mathcal{S}(f)$.

Now, we extend the notion of sympathy to the behavioral domain: For any $f : \Theta \to \mathcal{X}$, any $\mu \in \mathcal{M}^S$, and any $\omega \in \Omega$, the correspondence $BR_i^{\omega} : A_{-i} \twoheadrightarrow A_i$ constitutes i's behavioral best responses at ω given others' messages. If i is a standard economic agent, *not a sympathizer* of f at $\omega \in \Omega$, then for all $a_{-i} \in A_{-i}$, $a_i \in BR_i^{\omega}(a_{-i})$ if and only if $g(a_i, a_{-i}) \in C_i^{\omega}(O_i^{\mu}(a_{-i}))$. For sympathizers, the following holds:

[19]This setting encompasses the rational domain: Under rationality, every individual's choice correspondence satisfies WARP at every feasible state. So, for any given $i \in N$ and $\omega \in \Omega$, there exists a complete and transitive binary preference relation $R_i^{\omega} \subseteq X \times X$ such that for any $x, y \in X$, $x R_i^{\omega} y$ if and only if $x \in C_i^{\omega}(\{x, y\})$. Therefore, for any given $i \in N$ and $\omega \in \Omega$ and $S \in \mathcal{X}$, $C_i^{\omega}(S) = \{x^* \in S \mid x^* \ R_i^{\omega}\ y, \text{ for all } y \in S\}$.

[20]A profile of sets $\mathbf{S}$ is consistent with a given SCC $f : \Theta \to \mathcal{X}$, if (i) the set $S_i(x, \theta)$ is a choice set of alternative x by individual i at state $\pi^*(\theta)$ for all $i \in N$, all $\theta \in \Theta$, and all $x \in f(\theta)$; and (ii) if x is f-optimal at θ but not at θ' for some $\theta, \theta' \in \Theta$, then there exists $j \in N$ such that x is not chosen from $S_j(x, \theta)$ by j at $\pi^*(\theta')$.

Definition 7. *Given an SCC $f : \Theta \to \mathcal{X}$ and a guidance mechanism $\mu \in \mathcal{M}^S$, individual $i \in N$ is a*

1. **behavioral sympathizer** *of f at $\omega \in \Omega$ if for all $a_{-i} \in A_{-i}$,*
   - (i) *$g((\mathbf{S}^{(i)}, m_i), a_{-i}), g((\tilde{\mathbf{S}}^{(i)}, m_i), a_{-i}) \in C_i^{\omega}(O_i^{\mu}(a_{-i}))$ with $\mathbf{S}^{(i)} \in \mathcal{S}(f)$, $\tilde{\mathbf{S}}^{(i)} \in \mathcal{S} \setminus \mathcal{S}(f)$, and $m_i \in M_i$ implies $(\mathbf{S}^{(i)}, m_i) \in BR_i^{\omega}(a_{-i})$ and $(\tilde{\mathbf{S}}^{(i)}, m_i) \notin BR_i^{\omega}(a_{-i})$; and*
   - (ii) *in all other cases, $a_i \in BR_i^{\omega}(a_{-i})$ if and only if $g(a_i, a_{-i}) \in C_i^{\omega}(O_i^{\mu}(a_{-i}))$.*
2. **strong behavioral sympathizer** *of f at $\omega \in \Omega$ if for all $a_{-i} \in A_{-i}$,*
   - (i) *$g((\mathbf{S}^{(i)}, m_i), a_{-i}), g((\tilde{\mathbf{S}}^{(i)}, \tilde{m}_i), a_{-i}) \in C_i^{\omega}(O_i^{\mu}(a_{-i}))$ with $\mathbf{S}^{(i)} \in \mathcal{S}(f)$, $\tilde{\mathbf{S}}^{(i)} \in \mathcal{S} \setminus \mathcal{S}(f)$, and $m_i, \tilde{m}_i \in M_i$ implies $(\mathbf{S}^{(i)}, m_i) \in BR_i^{\omega}(a_{-i})$ and $(\tilde{\mathbf{S}}^{(i)}, \tilde{m}_i) \notin BR_i^{\omega}(a_{-i})$; and*
   - (ii) *in all other cases, $a_i \in BR_i^{\omega}(a_{-i})$ if and only if $g(a_i, a_{-i}) \in C_i^{\omega}(O_i^{\mu}(a_{-i}))$.*

The environment satisfies the behavioral sympathizer property *(*strong behavioral sympathizer property*) with respect to SCC f if for all $\omega \in \Omega$, there is at least* one *behavioral sympathizer (at least* two *strong behavioral sympathizers, resp.) of f at ω, while the identity of each behavioral sympathizer (strong behavioral sympathizer, resp.) of f at ω is privately known only by himself.*

An immediate consequence of this definition is that given an SCC f and guidance mechanism μ, every strong behavioral sympathizer of f at ω is a behavioral sympathizer of f at ω.[21]

The notion of behavioral Nash* implementation is obtained by modifying Definition 3 using the behavioral best response correspondences specified in Definition 7.

Some of our results adopt the following assumptions:

Definition 8. *We say that*

(i) *the environment features* **societal non-satiation** *if for any state $\omega \in \Omega$ and any alternative $x \in X$, there is an individual $i \in N$ such that $x \notin C_i^{\omega}(X)$.*

(ii) *the* **behavioral economic environment** *assumption holds if for any state $\omega \in \Omega$ and any alternative $x \in X$, there are two agents $i, j \in N$ with $i \neq j$ such that $x \notin C_i^{\omega}(X) \cup C_j^{\omega}(X)$.*

(iii) *an SCC $f : \Theta \to \mathcal{X}$ satisfies the* **behavioral no-veto property** *if for any state of the economy $\theta \in \Theta$, $x \in \cap_{i \in N \setminus \{j\}} C_i^{\pi^*(\theta)}(X)$ for some $j \in N$ implies $x \in f(\theta)$.*

[21] The first part of Definition 7 says the following: Given an SCC f, guidance mechanism μ, any one of others' actions a_{-i}, and any state $\omega \in \Omega$, a behavioral sympathizer i of f at ω chooses to announce a consistent profile of sets $\mathbf{S}^{(i)}$ as well as a message profile m_i; he does not choose to announce an inconsistent profile $\tilde{\mathbf{S}}^{(i)}$ and to select the same message profile m_i whenever both action profiles, $(\mathbf{S}^{(i)}, m_i)$ and $(\tilde{\mathbf{S}}^{(i)}, m_i)$, lead to alternatives which are among those chosen by individual i at state ω from his opportunity set corresponding to others' behavior a_{-i} (namely, $O_i^{\mu}(a_{-i})$). On the other hand, the second part of Definition 7 demands the following: A strong behavioral sympathizer i of f at ω chooses to announce a consistent profile of sets $\mathbf{S}^{(i)}$ while selecting a message profile m_i; he does not choose to announce an inconsistent profile $\tilde{\mathbf{S}}^{(i)}$ coupled with selecting some other message profile $\tilde{m}_i$ whenever both action profiles, $(\mathbf{S}^{(i)}, m_i)$ and $(\tilde{\mathbf{S}}^{(i)}, \tilde{m}_i)$, result in alternatives that are among the chosen by i at ω from $O_i^{\mu}(a_{-i})$.

We note that the behavioral economic environment assumption implies societal non-satiation. Moreover, the behavioral no-veto property vacuously holds in behavioral economic environments.[22]

Before going into our results, we wish to emphasize that under WARP, behavioral Nash equilibrium is equivalent to Nash equilibrium, behavioral Nash* equilibrium to Nash* equilibrium, and the corresponding implementation notions are equivalent. Also, consistency is equivalent to rational-consistency, a behavioral sympathizer to a sympathizer (while we refer to a strong behavioral sympathizer as a strong sympathizer), the behavioral no-veto property, and behavioral economic environment assumption to their rational versions, respectively. When the meaning is clear, we refer to these behavioral notions without spelling out the 'behavioral' label.

C Noneconomic Environments

To extend our analysis to noneconomic environments, we need to discuss the construction of the mechanism employed in the proof of Theorem 2. Our mechanism asks each individual i to announce a feasible profile of choice sets $\mathbf{S}^{(i)} \in \mathcal{S}$; a state of the economy $\theta^{(i)} \in \Theta$; an alternative $x^{(i)} \in X$; a natural number $k^{(i)}$. Rule 1 decrees that if all but one individual announce the same profile, $\mathbf{S}$, while all agents' announcements involve θ and x with $x \in f(\theta)$, then the outcome equals x. Rule 2 demands that the outcome is x whenever all but one individual i' announce the same profile, $\mathbf{S}$, and the messages of all but one individual j involve θ and x with $x \in f(\theta)$ while j sends message x' and θ' provided that x' is not in $S_j(x, \theta)$ listed in $\mathbf{S}$. If x' were to be in $S_j(x, \theta)$ listed in $\mathbf{S}$ in the contingency discussed in the previous sentence, then Rule 2 decrees that the outcome is x'. Rule 3 encompasses all the other situations and involves the *integer game*: the outcome equals the alternative chosen by the agent with the lowest index among those who choose the highest integer.

The economic environment assumption dispenses with the Nash* equilibria that may arise under Rules 2 and 3 as well as some that may emerge under Rule 1. Equilibria that arise under Rule 3 are not desirable because, in such equilibria, all individuals apart from the sympathizers do not need to announce a consistent profile of sets. As a result, the relevant information about the societal choice topography cannot be extracted in equilibrium from these individuals. Fortunately, societal non-satiation is sufficiently strong to rule out such equilibria.

If we adopt societal non-satiation along with the no-veto property, then we allow for some addi-

[22] Societal non-satiation requires that for any given state, all individuals do not choose the same alternative from the set of all alternatives at that state. The behavioral economic environment assumption demands that for every state and alternative, there are two agents not choosing that alternative from the set of all alternatives at that state. The behavioral no-veto property demands that if an alternative is chosen from the set of all alternatives at a state by every individual but one, then that alternative has to be f-optimal at the corresponding state of the economy. This notion ignores the welfare of the agent who does not agree with the rest of the society. Benoit and Ok (2006) and Barlo and Dalkiran (2009) obtain implementation results with *limited-veto-power*, a weaker condition than the no-veto property.

tional equilibria under Rules 1 and 2. Then, for any state, we need *at least two strong sympathizers*. This is because our mechanism is such that when we deal with an equilibrium at a state under Rules 1 or 2 in which all but one individual announce the same profile of sets while the odd man out is announcing a different profile, by changing his announcement concerning the profile, each agent different from the odd man out can trigger Rule 3, and hence, obtain any alternative he desires by also changing his integer choice. Because we need the equilibrium announcement of the profile of sets by all but the odd man out to be consistent with the social goal, we have to make sure that there is a strong sympathizer among those announcing the same profile; sympathy does not suffice as this agent also needs to change his integer choice.[23]

Another interesting consequence of the additional equilibria that emerge under Rules 1 and 2 is that, now, all but one agent announce the same consistent profile.

Our second sufficiency result also provides a robustness check for Theorem 2:

Theorem 3. *Let $n \geq 3$ and the SCC $f : \Theta \to \mathcal{X}$ be given. Suppose that*

(i) *the planner knows that the environment features societal non-satiation and satisfies the strong sympathizer property, and*

(ii) *without necessarily knowing the full specification of sets that appear in $\mathcal{S}(f)$, the planner knows that $\mathcal{S}(f) \neq \emptyset$ and that f satisfies the no-veto property.*

Then, the planner infers that f is Nash* implementable by a guidance mechanism $\mu \in \mathcal{M}^S$, and for any state of the economy $\theta \in \Theta$ and any Nash* equilibrium $\bar{a} = (\bar{\mathbf{S}}^{(i)}, \bar{m}_i)_{i \in N}$ of mechanism μ at state $\pi^*(\theta)$, $\bar{\mathbf{S}}^{(i)} = \mathbf{S}$ for some rational-consistent profile $\mathbf{S} \in \mathcal{S}(f)$ for all $i \in N \setminus \{j\}$ for some $j \in N$.

Theorem 3 justifies that noneconomic environments impose more knowledge requirements on the planner seeking to elicit information about consistency from the society. Indeed, the knowledge of the existence (but not necessarily the full specification) of a consistent profile no longer suffices even with the help of two strong sympathizers. The hypothesis of Theorem 3 includes the assumption that the planner knows that the SCC satisfies the no-veto property, a piece of information that the planner cannot verify herself since she does not know $\pi^* : \Theta \to \Omega$ and hence individuals' state-contingent choices.

[23]The need to have an additional partially honest agent does not appear in Dutta and Sen (2012). They work in the rational domain with an informed planner (knowing the identification function $\pi^* : \Theta \to \Omega$) and assume that a *partially honest* agent strictly prefers to reveal the state truthfully when he is indifferent. To see why they do not need an additional partially honest individual, consider the canonical mechanism without the announcement of a profile of choice sets and a Nash equilibrium in which the rule that implies the opportunity sets of all but one individual, i^*, equals X. Then, they do not need to guarantee that one of those individuals $i \neq i^*$ (different from the odd man out i^*) is partially honest as the no-veto property delivers the desired conclusion. However, no-veto does not help in our case, and we need to ensure that one of those individuals $i \neq i^*$ is a sympathizer and hence announces a consistent profile.

Using arguments leading to Proposition 1, the following result, presented without proof, establishes that the planner infers (*ii*) of Theorem 3 if she knows the following: f has an extension to Ω, f_Ω, the full specification of which the planner does not know, that satisfies the no-veto property and possesses a consistent profile.

Proposition 2. *Suppose that the planner knows that SCC $f : \Theta \to \mathcal{X}$ has an extension $f_\Omega : \Omega \to 2^X$ that possesses a consistent profile of sets and satisfies the no-veto property, while she does not know the full specification of f_Ω. Then, she infers that f satisfies the no-veto property and $\mathcal{S}(f)$ is non-empty without necessarily knowing the specification of sets that appear in $\mathcal{S}(f)$.*

D Proofs

D.1 Proof of Theorem 2

For extended applicability, we prove Theorem 2 in the behavioral domain.

The construction featured in the proof utilizes the guidance mechanism $\mu \in \mathcal{M}^S$ with $\mu = (A, g)$ defined as follows: $A_i := \mathcal{S} \times \Theta \times X \times \mathbb{N}$ where a generic member $a_i = (\mathbf{S}^{(i)}, \theta^{(i)}, x^{(i)}, k^{(i)}) \in A_i$ with $\mathbf{S}^{(i)} \in \mathcal{S}$, $\theta^{(i)} \in \Theta$, $x^{(i)} \in X$, and $k^{(i)} \in \mathbb{N}$ with the convention that $m_i = (\theta^{(i)}, x^{(i)}, k^{(i)})$ and $M_i := \Theta \times X \times \mathbb{N}$. The outcome function is defined via the rules specified in Table 1. We note that planner's knowledge enables her to construct this mechanism without knowing $\pi^* : \Theta \to \Omega$.

Rule 1 :	$g(a) = x$	if $\mathbf{S}^{(i)} = \mathbf{S}$ for all $i \in N \setminus \{i'\}$ for some $i' \in N$, and $m_j = (\theta, x, \cdot)$ for all $j \in N$ with $x \in f(\theta)$,
Rule 2 :	$g(a) = \begin{cases} x' & \text{if } x' \in S_j(x,\theta) \text{ where } S_j(x,\theta) = \mathbf{S}\vert_{j,\theta,x\in f(\theta)}, \\ x & \text{otherwise.} \end{cases}$	if $\mathbf{S}^{(i)} = \mathbf{S}$ for all $i \in N \setminus \{i'\}$ for some $i' \in N$, and $m_i = (\theta, x, \cdot)$ for all $i \in N \setminus \{j\}$ with $x \in f(\theta)$, and $m_j = (\theta', x', \cdot) \neq (\theta, x, \cdot)$,
Rule 3 :	$g(a) = x^{(i^*)}$ where $i^* = \min\{j \in N \mid k^{(j)} = \max_{i' \in N} k^{(i')}\}$	otherwise.

Table 1: The outcome function of the mechanism with three or more individuals.

The proof is presented via two claims. The first establishes that the planner infers (*i*) of Nash* implementation holds, while the second delivers her inference of (*ii*) of Nash* implementation.

Claim 1. *Even if the planner does not know $\mathcal{S}(f)$ and the realized state $\pi^*(\theta)$, she makes the following deduction for all $\theta \in \Theta$ and for all $x \in f(\theta)$: If $a^x \in A$ were $a_i^x = (\mathbf{S}, \theta, x, 1)$ for some $\mathbf{S} \in \mathcal{S}(f)$,* *for all $i \in N$, then a^x would be a Nash* equilibrium of μ at $\pi^*(\theta)$ (i.e., $a_i^x \in BR_i^{\pi^*(\theta)}(a_{-i}^x)$ for all $i \in N$) and $g(a^x) = x$.*

Proof. The planner does not know $\mathcal{S}(f)$, the realized state θ, and the association $\pi^* : \Theta \to \Omega$. But still, she deduces that if the individuals were to use this action profile, then Rule 1 would apply and $g(a^x) = x$. As she contemplates on agents choosing such that $\mathbf{S}^{(i)} = \mathbf{S} \in \mathcal{S}(f)$ for all $i \in N$, she infers that individual deviations can only result in Rules 1 and 2. Hence, she deduces that $O_i^\mu(a_{-i}^x) = S_i(x,\theta)$ where $S_i(x,\theta) = \mathbf{S}|_{i,\theta,x\in f(\theta)}$ due to the definition of the mechanism as she is informed of $\mathbf{S}$ by the society on account of observing a^x. Thus, she infers that if i were not a sympathizer of f at $\pi^*(\theta)$, then, by (i) of consistency, $x \in C_i^{\pi^*(\theta)}(S_i(x,\theta))$, which is equivalent to $a_i^x \in BR_i^{\pi^*(\theta)}(a_{-i}^x)$. This is a deduction she makes without knowing $\pi^*(\theta)$. She also deduces that if i were a sympathizer of f at $\pi^*(\theta)$, then $\mathbf{S} \in \mathcal{S}(f)$ and $x \in C_i^{\pi^*(\theta)}(S_i(x,\theta))$ (which she infers due to (i) of consistency without knowing $\pi^*(\theta)$) would imply $a_i^x \in BR_i^{\pi^*(\theta)}(a_{-i}^x)$. ■

Claim 2. *Even if the planner does not know $\mathcal{S}(f)$ and the realized state $\pi^*(\theta)$, she makes the following deduction for all $\theta \in \Theta$: If $a^* \in A$ were a Nash* equilibrium of $\mu \in \mathcal{M}^S$ at $\pi^*(\theta)$ for some $\theta \in \Theta$, then $g(a^*)$ would be in $f(\theta)$.*

Proof. The planner knows that contemplating a Nash* equilibrium a^* at $\pi^*(\theta)$ for some θ under Rule 1 such that $a_i^* = (\mathbf{S}', \theta', x', k')$ with $x' \in f(\theta')$, and $a_i^* \in BR_i^{\pi^*(\theta)}(a_{-i}^*)$ for all $i \in N$ implies that, as Rule 1 holds, $g(a^*) = x'$ and $O_i^\mu(a_{-i}^*) = S_i(x',\theta') = \mathbf{S}'|_{i,\theta',x'\in f(\theta')}$ for all $i \in N$ due to Rules 1 and 2.

Then, the planner deduces that $\mathbf{S}' \in \mathcal{S}(f)$, or else individual i, the sympathizer of f at $\pi^*(\theta)$ who she knows exists, has a profitable deviation: i could deviate to $a_i' = (\mathbf{S}'', m_i^*, a_{-i}^*)$ with $\mathbf{S}'' \in \mathcal{S}(f)$ and $m_i^* = (\theta', x', k')$ implies $g(\mathbf{S}'', m_i^*, a_{-i}^*) = g(a^*) = x' \in C_i^{\pi^*(\theta)}(S_i(x',\theta'))$ (due to $a_i^* \in BR_i^{\pi^*(\theta)}(a_{-i}^*)$ implying $x' = g(a^*) \in C_i^{\pi^*(\theta)}(O_i^\mu(a_{-i}^*))$ and $O_i^\mu(a_{-i}^*) = S_i(x',\theta')$—inferences the planner makes without knowing $\pi^*(\theta)$ but by contemplating such a Nash* equilibrium at $\pi^*(\theta)$). So, she deduces that $(\mathbf{S}'', m_i^*) \in BR_i^{\pi^*(\theta)}(a_{-i}^*)$ and $(\mathbf{S}', m_i^*) \notin BR_i^{\pi^*(\theta)}(a_{-i}^*)$, which constitutes a contradiction to a^* being a Nash* equilibrium at $\pi^*(\theta)$.

Next, the planner infers that $x' \notin f(\theta)$ leads to an impasse: She deduces that if $x' \in f(\theta')$, $x' \notin f(\theta)$, and $\mathbf{S}' \in \mathcal{S}(f)$, then there is $j \in N$ (whose identity the planner does not know) such that $x' \notin C_j^{\pi^*(\theta)}(S_j(x',\theta'))$. Recall that she knows $O_j^\mu(a_{-j}^*) = S_j(x',\theta')$. So, she infers $x' \notin C_j^{\pi^*(\theta)}(S_j(x',\theta'))$ implies $a_j^* \notin BR_j^{\pi^*(\theta)}(a_{-j}^*)$ and hence a^* cannot be a Nash* at $\pi^*(\theta)$, which delivers a contradiction.

Another type of Nash* equilibrium a^* at $\pi^*(\theta)$ under Rule 1 the planner needs to consider is one where there exists an individual i' such that $a_{i'}^* = (\mathbf{S}'', \theta', x', k')$ whereas $a_i^* = (\mathbf{S}', \theta', x', k')$ for all $i \in N \setminus \{i'\}$ with $\mathbf{S}' \neq \mathbf{S}''$. Then she figures out that, by Rules 1 and 3, $O_i^\mu(a_{-i}^*) = X$ for all $i \in N \setminus \{i'\}$

as any one of $i \neq i'$ could deviate to $a_i = (\mathbf{S}, \theta', y, k)$ with $\mathbf{S} \neq \mathbf{S}'$, $y \in X$ and $k > k'$. Since a^* is a Nash* equilibrium at $\pi^*(\theta)$, she deduces that $g(a^*) \in C_i^{\pi^*(\theta)}(X)$ for all $i \neq i'$ which she knows is a contradiction to the environment being economic.

The planner also makes the deduction that there cannot be a Nash* equilibrium under Rule 2 or 3: If there were a such Nash* equilibrium $\bar{a} \in A$, then, thanks to the definition of the mechanism, she infers that $O_i^{\mu}(\bar{a}_{-i}) = X$ for all $i \in N \setminus \{j\}$ for some $j \in N$. By her hypothesis that $\bar{a}$ is Nash* under either Rule 2 or 3, she figures out that $\bar{a}_{i'} \in BR_{i'}^{\pi^*(\theta)}(\bar{a}_{-i'})$ for all $i' \in N$ implies $g(\bar{a}) \in \cap_{i \in N \setminus \{j\}} C_i^{\pi^*(\theta)}(X)$. She knows that this is not possible due to the economic environment assumption. ■

D.2 Proof of Proposition 1

Suppose that the planner knows that an SCC $f : \Theta \to \mathcal{X}$ possesses a Maskin monotonic extension $f_\Omega : \Omega \to 2^X$, but she does not know its full specification. Still, she infers that $\mathbf{S}$ given by $S_i(x, \theta) = L_i^{\pi^*(\theta)}(x)$ for all $i \in N$, all $\theta \in \Theta$, and all $x \in f(\theta)$ must be so that (*i*) of rational-consistency holds trivially (even though she knows neither $\pi^*(\theta)$ nor $L_i^{\pi^*(\theta)}(x)$ while she infers that $f_\Omega(\pi^*(\theta))$ equals $f(\theta)$). For her inference of (*ii*) of rational-consistency, suppose that she knows $x \in f(\theta)$ and $x \notin f(\tilde{\theta})$ for some $\theta, \tilde{\theta} \in \Theta$. As she knows that $f(\theta) = f_\Omega(\pi^*(\theta))$ and $f(\tilde{\theta}) = f_\Omega(\pi^*(\tilde{\theta}))$ and f_Ω is Maskin monotonic, she infers that (even though she does not know $\pi^*(\theta)$ and $\pi^*(\tilde{\theta})$) it must be that there exists $j \in N$ such that $L_j^{\pi^*(\theta)}(x) \nsubseteq L_j^{\pi^*(\tilde{\theta})}(x)$, and hence $L_j^{\pi^*(\theta)}(x) = S_j(x, \theta)$ delivers the desired conclusion of her inference of $S_j(x, \theta) \notin \mathbb{L}_j^{\pi^*(\tilde{\theta})}(x)$.

D.3 Proof of Theorem 3

Instead of using the no-veto property, we prove our second sufficiency theorem with a weaker condition, (*ii'*) stated below. Combining it with societal non-satiation and consistency delivers a condition akin to condition μ of Moore and Repullo (1990), condition λ of Korpela (2012)), and strong consistency of de Clippel (2014).

(*ii'*) without necessarily knowing the full specification of sets that appear in $\mathcal{S}(f)$, the planner knows that $\mathcal{S}(f) \neq \emptyset$ and the following hold:

For any $\theta \in \Theta$, for any $\mathbf{S} \in \mathcal{S}(f)$, $x \in C_j^{\pi^*(\theta)}(S_j(x', \theta'))$ where $j \in N$, $\theta' \in \Theta$, $x' \in f(\theta')$, $S_j(x', \theta') = \mathbf{S}|_{j, \theta', x' \in f(\theta')}$, and $x \in C_i^{\pi^*(\theta)}(X)$ for all $i \in N \setminus \{j\}$ implies $x \in f(\theta)$.

We note that (*ii*) of Theorem 3 implies (*ii'*) above.

The proof employs mechanism μ used in the proof of Theorem 2 (involving rules specified in Table 1). Moreover, every strong sympathizer of f at $\pi^*(\theta)$ is a sympathizer of f at $\pi^*(\theta)$. Thus, the proof of Claim 1 can be used without any modifications to establish that for all $\theta \in \Theta$ and for all $x \in f(\theta)$, the planner infers the following: if every individual were to play $(\mathbf{S}, \theta, x, 1)$ for some

$\mathbf{S} \in \mathcal{S}(f)$ (even if the planner does not know $\mathcal{S}(f)$ and the function π^*), then this action profile would be Nash* at $\pi^*(\theta)$ and $g(a^x) = x$. Therefore, what remains to be shown is:

Claim 3. *Even if the planner does not know $\mathcal{S}(f)$ and the realized state $\pi^*(\theta)$, she makes the following deduction for all $\theta \in \Theta$: If $a^* \in A$ were a Nash* equilibrium of $\mu \in \mathcal{M}^S$ at $\pi^*(\theta)$ for some $\theta \in \Theta$, then $g(a^*)$ would be in $f(\theta)$.*

Proof. The proof of the claim involves the analysis of three cases.

Case 1. *The planner contemplates the situation where $a^* \in A$ be a Nash* equilibrium at $\pi^*(\theta)$ for some $\theta \in \Theta$ such that Rule 1 holds: $a_i^* = (\mathbf{S}^{(i)}, \theta', x', k')$ for all $i \in N$ with $\mathbf{S}^{(i')} = \mathbf{S}$ for all $i' \neq j$ for some $j \in N$ and $x' \in f(\theta')$. Then, she infers that $g(a^*) = x'$.*

Proof of Claim 3 under Case 1. First, we prove that the planner deduces that $\mathbf{S} \in \mathcal{S}(f)$. Therefore, in all Nash* equilibria under Case 1, she infers that all but one player announce the same profile of sets that must be among the consistent profiles of sets with the SCC f.

If the planner considers $\mathbf{S}^{(j)} = \mathbf{S}$, then letting the first player be one of the strong sympathizers of f at $\pi^*(\theta)$ without a loss of generality, the planner infers the following: If $\mathbf{S} \notin \mathcal{S}(f)$, then deviating to $\bar{a}_1 = (\bar{\mathbf{S}}, \theta', x', k')$ with $\bar{\mathbf{S}} \in \mathcal{S}(f)$ results in $g(\bar{\mathbf{S}}, m_1^*, a_{-1}^*) = g(\mathbf{S}, m_1^*, a_{-1}^*) = x'$ (due to Rule 1) and $x' \in C_i^{\pi^*(\theta)}(O_i^{\mu}(a_{-i}^*))$ for all $i \in N$ (since a^* is a Nash* equilibrium at $\pi^*(\theta)$) where $m_1^* = (\theta', x', k')$; thus, $a_1^* \notin BR_1^{\pi^*(\theta)}(a_{-1}^*)$, a contradiction to a^* being Nash* at $\pi^*(\theta)$.

If the planner contemplates on $\mathbf{S}^{(j)} \neq \mathbf{S}$, $\mathbf{S} \notin \mathcal{S}(f)$, j not being a strong sympathizer of f at $\pi^*(\theta)$ while one of the strong sympathizers of f at $\pi^*(\theta)$ being the first player, then she infers the following: Agent 1 deviating to $\bar{a}_1 = (\bar{\mathbf{S}}, \theta', x', \bar{k})$ with $\bar{\mathbf{S}} \in \mathcal{S}(f)$ and $\bar{k} > k'$ results in $g(\bar{\mathbf{S}}, \bar{m}_1, a_{-1}^*) = g(\mathbf{S}, m_1^*, a_{-1}^*) = x'$ (due to Rules 1 and 3) and $x' \in C_i^{\pi^*(\theta)}(O_i^{\mu}(a_{-i}^*))$ for all $i \in N$ (since a^* is a Nash* equilibrium at $\pi^*(\theta)$) where $m_1^* = (\theta', x', k')$ and $\bar{m}_1 = (\theta', x', \bar{k})$; ergo, $a_1^* \notin BR_1^{\pi^*(\theta)}(a_{-1}^*)$, a contradiction to a^* being a Nash* equilibrium at $\pi^*(\theta)$.[24]

The same reasoning detailed in the previous paragraph delivers the planner's inference of a contradiction if $\mathbf{S}^{(j)} \neq \mathbf{S}$, $\mathbf{S} \notin \mathcal{S}(f)$, and j is a strong sympathizer of f at $\pi^*(\theta)$. Because then she makes the deduction that there would be another strong sympathizer of f at $\pi^*(\theta)$ and he would have a profitable deviation opportunity.[25]

[24] This is why we have to strengthen sympathy to strong sympathy, as the deviating individual has to change his integer choice as well.

[25] The need for an additional strong sympathizer arises due to this case. To see this, suppose that there is only one strong sympathizer of f at $\pi^*(\theta)$ and consider the situation when $\mathbf{S}^{(j)} \neq \mathbf{S}$ and j is the only strong sympathizer of f at $\pi^*(\theta)$. Then, $\mathbf{S} \notin \mathcal{S}(f)$ does not necessarily result in a contradiction as there is no other strong sympathizer of f at $\pi^*(\theta)$ among those who are announcing an inconsistent profile of sets $\mathbf{S}$. Hence, one of the agents whose opportunity set equals X must be a strong sympathizer of f at $\pi^*(\theta)$.

Suppose that the planner considers the situation where $g(a^*) = x' \notin f(\theta)$ because otherwise she would be done with Case 1. Then, she knowing that $x' \in f(\theta')$ and $x \notin f(\theta)$ and $\mathbf{S} \in \mathcal{S}(f)$ with $\theta, \theta' \in \Theta$ implies (due to (ii) of consistency) there exists $i^* \in N$ such that $x' \notin C_{i^*}^{\pi^*(\theta)}(S_{i^*}(x', \theta'))$ where $S_{i^*}(x', \theta') = \mathbf{S}|_{i^*, \theta', x' \in f(\theta')}$. There appears two subcases she needs to check. The first is one where a^* is such that $\mathbf{S}^{(i)} = \mathbf{S}$ for all $i \in N$. Then, $O_i^{\mu}(a_{-i}^*) = S_i(x', \theta')$ (by Rules 1 and 2) and, as a^* is Nash* at $\pi^*(\theta)$, it must be that $x' \in C_i^{\pi^*(\theta)}(O_i^{\mu}(a_{-i}^*))$ for all $i \in N$. So, she obtains a contradiction as $x' \in C_{i^*}^{\pi^*(\theta)}(S_{i^*}(x', \theta'))$. The second subcase that the planner needs to consider is one where a^* is such that $\mathbf{S}^{(i)} = \mathbf{S}$ for all $i \in N \setminus \{j\}$ for some $j \in N$ and $\mathbf{S}^{(j)} \neq \mathbf{S}$. Then, by Rules 1 and 2 and 3, $O_i^{\mu}(a_{-i}^*) = X$ for all $i \neq j$ and $O_j^{\mu}(a_{-j}^*) = S_j(x', \theta')$ where $S_j(x', \theta') = \mathbf{S}|_{j, \theta', x' \in f(\theta')}$ and $\mathbf{S} \in \mathcal{S}(f)$ as was shown above. Because that a^* is a Nash* equilibrium at $\pi^*(\theta)$, the planner infers that $x' \in C_i^{\pi^*(\theta)}(X)$ for all $i \neq j$ while $x' \in C_j^{\pi^*(\theta)}(S_j(x', \theta'))$ and $x' \in f(\theta')$. As the planner knows (ii') holds, she concludes $x \in f(\theta)$, which in contradiction to $x \notin f(\theta)$. ■

Case 2. *Suppose that the planner considers a Nash* equilibrium a^* at $\pi^*(\theta)$ in which Rule 2 applies: $a_i^* = (\mathbf{S}^{(i)}, m_i^*)$ with $\mathbf{S}^{(i)} = \mathbf{S}$ for all $i \in N \setminus \{i'\}$ for some $i' \in N$ and $m_j^* = (\theta', x', k')$ for all $j \in N \setminus \{\ell\}$ for some $\ell \in N$ with $\theta' \in \Theta$ and $x' \in f(\theta')$ while $m_\ell^* = (\theta'', x'', k'') \neq (\theta', x', k')$.*

Proof of Claim 3 under Case 2. The first step is to prove the planner's inference of $\mathbf{S} \in \mathcal{S}(f)$. We point out that this establishes the observation that the planner deduces that in all Nash* equilibria in which Rule 2 applies, all but one individual announce the same profile of sets, which has to be one of the profiles consistent with the SCC f.

Suppose that the planner contemplates on $\mathbf{S} \notin \mathcal{S}(f)$. Then, she knows that there is a strong sympathizer of f at $\pi^*(\theta)$, individual $i^* \neq i'$, with $\mathbf{S}^{(i^*)} = \mathbf{S}$ as she knows that there are at least two strong sympathizers of f at $\pi^*(\theta)$. She imagines (without a loss of generality) $i^* = 1$. If $\mathbf{S}^{(i')} \neq \mathbf{S}$, player 1 deviating to $\bar{a}_1 = (\bar{\mathbf{S}}, \bar{m}_1)$ where $\bar{\mathbf{S}} \in \mathcal{S}(f)$ and $\bar{m}_1 = (\tilde{\theta}, g(a^*), \bar{k})$ with $\tilde{\theta} \in \Theta$ and $\bar{k} > k', k''$ implies that Rule 3 applies and as a result $g(\bar{\mathbf{S}}, \bar{m}_1, a_{-1}^*) = g(\mathbf{S}, m_1^*, a_{-1}^*) = g(a^*)$ which is in $C_1^{\pi^*(\theta)}(O_1^{\mu}(a_{-1}^*))$ due to a^* being a Nash* equilibrium at $\pi^*(\theta)$. But then, the planner infers that as player 1 is a strong sympathizer of f at $\pi^*(\theta)$, $a_1^* \notin BR_1^{\pi^*(\theta)}(a_{-1}^*)$, a contradiction to a^* being Nash* at $\pi^*(\theta)$. If $\mathbf{S}^{(i')} = \mathbf{S}$, then the planner deduces that all players are announcing $\mathbf{S}$; and hence, player 1 deviating to $\bar{a}_1 = (\bar{\mathbf{S}}, m_1^*)$ where $\bar{\mathbf{S}} \in \mathcal{S}(f)$ implies no deviations apart from individual 1's announcing $\bar{\mathbf{S}}$ instead of $\mathbf{S}$ and as a result $g(\bar{\mathbf{S}}, m_1^*, a_{-1}^*) = g(\mathbf{S}, m_1^*, a_{-1}^*) = g(a^*) \in C_1^{\pi^*(\theta)}(O_1^{\mu}(a_{-1}^*))$ as a^* is Nash* at $\pi^*(\theta)$. However, player 1 being a strong sympathizer of f at $\pi^*(\theta)$ implies $a_1^* \notin BR_1^{\pi^*(\theta)}(a_{-1}^*)$, contradicting to a^* being a Nash* equilibrium at $\pi^*(\theta)$.

Having established the planners inference of $\mathbf{S} \in \mathcal{S}(f)$, we note the following: If she considers $\mathbf{S}^{(i')} \neq \mathbf{S}$, then she knows that $O_i^{\mu}(a_{-i}^*) = X$ for all $i \neq i'$ (by any one of such $i \neq i'$ deviating to

$\mathbf{S}^{(i)} \neq \mathbf{S}$ and choosing the highest integer and any alternative) while $O^{\mu}_{i'}(a^*_{-i'}) = S_{i'}(x', \theta')$ if $i' = \ell$ and $O^{\mu}_{i'}(a^*_{-i'}) = X$ if $i \neq \ell$ (by i' deviating to $m'_{i'} \neq (\theta', x', k')$ and making Rule 3 apply). Thus, if $i' = \ell$, $\mathbf{S} \in \mathcal{S}(f)$ and a^* being Nash* at $\pi^*(\theta)$ implying $g(a^*) \in C_i^{\pi^*(\theta)}(X)$ for all $i \neq i'$ and $g(a^*) \in C_{i'}^{\pi^*(\theta)}(S_{i'}(x', \theta'))$ with $x' \in f(\theta')$ enables the planner to employ condition (ii') and conclude that $g(a^*) \in f(\theta)$. But if she contemplates on $i' \neq \ell$, then she figures out that $\mathbf{S} \in \mathcal{S}(f)$ and a^* being Nash* at $\pi^*(\theta)$ imply $g(a^*) \in C_i^{\pi^*(\theta)}(X)$ for all $i \in N$, which is in contradiction to societal non-satiation. So, she deduces that in all Nash* equilibria in which Rule 2 applies, all but one individual announce the same profile of sets which has to be one of profiles consistent with f.

If the planner considers $\mathbf{S}^{(i')} = \mathbf{S}$, then she infers that $O^{\mu}_{j}(a^*_{-j}) = X$ for all $j \neq \ell$ (by any one of such $j \neq \ell$ deviating to $m'_j \neq m^*_j$) while $O^{\mu}_{\ell}(a^*_{-\ell}) = S_{\ell}(x', \theta')$ (by Rule 2). Hence, she deduces that $\mathbf{S} \in \mathcal{S}(f)$, a^* being Nash* at $\pi^*(\theta)$ implying $g(a^*) \in C_j^{\pi^*(\theta)}(X)$ for all $j \neq \ell$ and $g(a^*) \in C_{\ell}^{\pi^*(\theta)}(S_{\ell}(x', \theta'))$ with $x' \in f(\theta')$, and condition (ii') conduce to $g(a^*) \in f(\theta)$. ■

Case 3. *The planner infers that under Rule 3, there cannot be a Nash* equilibrium at* $\pi^*(\theta)$ *for any* $\theta \in \Theta$.

Proof of Claim 3 under Case 3. If the planner contemplates on such a Nash* equilibrium, a^*, she infers that $O^{\mu}_{i}(a^*_{-i}) = X$ for all $i \in N$ and $g(a^*) \in C_i^{\pi^*(\theta)}(X)$ for all $i \in N$ (on account of a^* being Nash* at $\pi^*(\theta)$). This, she concludes, results in a contradiction with societal non-satiation. ■

These conclude the proof of Claim 3, and hence, the proof of Theorem 3. ■

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