Implementation with a Sympathizer*

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Abstract

This paper considers the Nash implementation problem in which the planner does not know individuals' state-contingent choices that may involve violations of rationality. In economic environments with at least three individuals, we show that the planner may Nash implement a social choice correspondence while extracting information about individuals' state-contingent choices from the society whenever one of the individuals, whose identity is not necessarily known to the planner and the other individuals, is a weak sympathizer. Such an agent is weakly inclined toward truthful revelation of individuals' state-contingent choices but not the "true" state. Then, in every Nash equilibrium of the mechanism we design, all individuals except one truthfully reveal the same information about individuals' choices.

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1 Introduction

The analysis of the design of mechanisms the equilibria of which coincide with a given social goal is essential to economic theory. The seminal works of Maskin (1999, circulated in 1977), Moore and Repullo (1990), and Dutta and Sen (1991) deal with this question under *complete information*: the situation when payoff-relevant characteristics is commonly known within the society but not to the planner.[1] de Clippel (2014) extends this analysis to cases in which individuals' behavior does not necessarily satisfy the weak axiom of revealed preferences (WARP), generally regarded as representing rationality.

The condition at the heart of the desired characterization turns out to be *consistency*, a notion that implies the well-known Maskin monotonicity when individuals' behavior satisfies the WARP.[2] The *opportunity sets* sustained by the mechanism that implements a given social goal, subsets of alternatives that an individual can obtain by changing his messages while others' remain the same, form a profile of sets consistent with this social goal. Moreover, the existence of a consistent profile of choice sets allows us to modify the canonical mechanism (by using this profile as opportunity sets) to obtain a *sufficiency* result.[3] Checking whether or not a given profile of sets is consistent with a social choice rule requires that the planner know the choices of each individual at each state from the corresponding member of that profile. The planner is then able to elicit the information concerning the state from the society via the canonical mechanism.

What if the planner does not know individuals' state-contingent choices? Can he

[1]For more on implementation under complete information, see Maskin and Sjöström (2002), Palfrey (2002), and Serrano (2004). On the other hand, Postlewaite and Schmeidler (1986), Palfrey and Srivastava (1987), and Jackson (1991) analyze the case of incomplete information.

[2]Given individuals' choice behavior, a profile of subsets of alternatives where each subset is indexed for an individual and a state and a socially optimal alternative at that state, is said to be *consistent* with a social choice correspondence if (i) for every individual and every state and every socially optimal alternative in that state, this alternative is chosen at that state by that individual from the corresponding choice set, and (ii) an alternative being socially optimal in the first state, but not in the second, implies that there exists an individual who does not choose that alternative at the second state from the choice set indexed for that individual and that alternative and the first state.

[3]See Maskin (1999) and de Clippel (2014).

learn individuals' choices as he attempts to elicit information about the state?

The current paper considers full implementation under complete information allowing, but not insisting on, violations of rationality with the additional feature that the planner does not know individuals' state-contingent choices. Our main result is that in *economic environments* with at least three individuals, the planner may Nash implement a social choice correspondence (SCC) possessing a consistent profile of sets by also eliciting the information concerning consistency from the society whenever one of the individuals (whose identity is not necessarily known to the planner and the other individuals) is a *weak sympathizer*. In every Nash equilibrium, all individuals save one announce the same consistent profile of sets. Thus, the planner no longer needs to know the *societal choice topography*, the state-contingent choice behavior of individuals from every possible subset of alternatives, to identify a profile of sets consistent with the given SCC. He can simply ask the individuals, knowing that all but one will truthfully announce the same profile of sets consistent with the social goal. We provide a tangible display of these by presenting a motivating example in which the hypothesis of our main result holds.

We attain the notion of *sympathy* by modifying *partial honesty* of Dutta and Sen (2012) so that it involves only announcements of profiles of sets and not the states of the world. Restricting our attention to mechanisms that involve the announcement of a profile of sets enables us to introduce a *weak sympathizer*: an individual who strictly prefers the truthful revelation of a consistent profile of sets coupled with some messages whenever none of his lies (announcements of inconsistent profiles of sets) makes him strictly better off while he keeps on using the same remaining parts of his messages. Indeed, a weak sympathizer is not a snitch or an informer in the sense that he does not feel any obligation and/or inclination to reveal the state of the economy; instead, he serves the planner as a guide without asking for compensation of any sort.

The *economic environment* assumption requires that individuals' choices are not perfectly aligned: for any alternative and for any state, there exist two individuals who do not choose that alternative in that state from the set of all alternatives. Therefore, it demands that there is some weak form of disagreement in the society at every state.[4]

We provide comprehensive robustness checks for our main result and attain two additional sufficiency results.

The first involves replacing the economic environment assumption with other conditions such as the no-veto property while continuing to work with three or more individuals. What we obtain is a choice version of condition μ of Moore and Repullo (1990), in line with condition λ of Korpela (2012), and the strong consistency of de Clippel (2014). This enables us to provide another sufficiency result similar to those in these papers with the additional feature that the information about consistency is extracted from the society when the environment features *societal non-satiation* and contains at least two *sympathizers* the identities of whom are privately known to themselves, but not the planner.

Societal non-satiation demands that for every alternative and every state, there exists an individual who does not choose that alternative at that state from the set of all alternatives. This restriction is weaker than the economic environment assumption and allows for more Nash equilibria in the mechanism we employ. But, with more Nash equilibria to handle comes the need for more power: instead of a single weak sympathizer, now we need at least two sympathizers. A *sympathizer* is an individual who strictly prefers a message profile consisting of the announcement of a consistent profile of sets and the rest of his messages whenever none of his deviations, consisting of lies (announcements of inconsistent profiles) coupled with some other messages, can make him strictly better off. Therefore, every sympathizer is a weak sympathizer.

The second robustness check involves the case of two individuals. Because "the two-agent model is the leading case for applications to contracting or bargaining" (Moore & Repullo, 1990), this particular robustness check also has an intrinsic stand-alone value. Our two-individual sufficiency result requires that both of the individuals are sympathizers, and a slightly stronger version of the two-individual necessity condition (in line with condition $\mu 2$ of Moore and Repullo (1990) and condition β of Dutta and Sen

[4]Our economic environment assumption is in line with the one in Kartik and Tercieux (2012), and weaker than that in Jackson (1991) and Bergemann and Morris (2008).

(1991)) holds. Vaguely put, the novel requirement in our sufficiency condition demands that the requirements that appear in the two-individual necessity condition hold across any pair of profiles of sets each satisfying the two-individual necessity condition. To exhibit the practicality of our two-individual sufficiency result, we present an example concerning a social goal that seeks compromise; we show that the hypothesis of this result is satisfied.

In the standard approach to the implementation problem, there is a one-to-one correspondence between the set of states and the payoff-relevant characteristics of the environment. Thus, the planner knows individuals' choices contingent on any given state. But he does not know the realized state. In our setup, the critical difference is that the planner does not know individuals' state-contingent choices. Hence, as far as the planner's knowledge is concerned, the set of states is not necessarily in one-to-one correspondence with the payoff-relevant characteristics.

To exhibit our contributions within the framework of the standard approach, we offer the following interpretation: As in the standard setting, the set of *feasible states* is in one-to-one correspondence with the possible payoff-relevant characteristics. Indeed, one can easily construct the set of all possible choice/preference profiles using the set of alternatives. On the other hand, not all feasible states can emerge in our environment. The states that can emerge form the set of *prevailing states*. In other words, the support of the distribution determining whether or not a state can emerge equals the set of prevailing states. Meanwhile, the SCC maps the *states of the economy* into non-empty subsets of alternatives. The set of feasible states and how it corresponds to payoff-relevant characteristics, the set of states of the economy, and the SCC are common knowledge among the individuals and the planner. But, how the states of the economy correspond to payoff-relevant characteristics is not known to the planner, yet it is common knowledge among the agents.

As a result, the planner seeks to implement a social goal contingent on the states of the economy even though he does not know individuals' preferences/choices corresponding to those states. That is why, the planner can be viewed as an outsider who is delegated the responsibility of implementing the social goal at hand.[5]

The states of the economy may also arise due to *categorization*: The feasible states are clustered into classes based on some criteria, and the social goal and the planner adopt these classes. For example, in a society with n individuals where $n \geq 3$ and n is odd, each agent's state is either 0 or 1 (where 0 stands for the bad state and 1 for the good state) resulting in the set of feasible states $\{0, 1\}^n$. The social goal and the planner adopt the categorization saying that the aggregate state of the economy is 'good' if more than half of the individuals' states are 1 and otherwise it is 'bad'. The planner seeks to implement a social goal contingent on the aggregate state of the economy, 'good' or 'bad', while he does not know individuals' preferences/choices corresponding to these states.

This example also shows that our construction cannot be interpreted as a *domain restriction*. To see this, consider the states of the economy specified above. Then, the union of the feasible states corresponding to the 'good' aggregate state of the economy and those corresponding to the 'bad' equals the set of all feasible states.

Identification of a mechanism that implements a social goal on the set of all feasible states ensures that the revelation of the realized state is equivalent to the revelation of individuals' preferences/choices at that state. Moreover, the same mechanism implements any restriction of that social goal on a smaller set of states, and hence, this equivalence continues to hold on this set of states as well. Therefore, our results are most useful when the social goal is not implementable on the set of all feasible states. When the social goal is implementable on the set of all feasible states by a 'grand' mechanism, our results display that we can replace this mechanism (possibly on grounds of complexity) and use ours defined solely on a smaller set of states of the economy with the novel feature of asking the society the information concerning their preferences/choices.

Our paper is closely related to the literature on implementation with partial honesty,

[5]For example, the planner could be an *implementation consulting agency* (e.g., McKinsey Implementation (McKinsey, 2018)) responsible to elicit information about the financial and operational state of a client firm to implement a given policy contingent on this information that results in a strategic interaction among subdivisions within the firm. Or the planner could be a court-appointed trustee authorized to run a company during its bankruptcy proceedings.

pioneered by Dutta and Sen (2012).[6, 7] Their construction assumes that at least one of the individuals has a preference for honesty. To formulate this, individuals' preferences on alternatives are extended to messages when dealing with mechanisms that involve the announcement of a state. A partially honest individual is assumed to strictly prefer a message involving the announcement of the 'true' state of the world when none of his deviations make him strictly better off. Then, that study shows that all SCCs satisfying the no-veto property can be implemented in Nash equilibrium whenever the society contains at least three individuals one of whom, whose identity is privately known only by himself, is partially honest. This sufficiency result does not need Maskin monotonicity.

Sympathy involves an inclination toward the revelation of consistent profiles of choice sets and not truthful announcements of the states. That is why, unlike the majority of papers on implementation with partial honesty, we need a Maskin monotonicity type of requirement to extract information about the states of the world.

Our analysis allows for individuals' choices to violate the WARP and hence is related to behavioral implementation literature. The two papers in line with ours are Korpela (2012) and de Clippel (2014).[8] These papers provide necessary as well as sufficient conditions for (behavioral) Nash implementation when individuals display systematic deviations from rationality.

The rest of the paper is organized as follows. We present a motivating example in Section 2, the notations and definitions and some preliminary results in Section 3. Our main result and its first robustness check are in Section 4. Section C of the Appendix

[6]An incomplete list of papers in this literature consists of Matsushima (2008a), Matsushima (2008b), Kartik and Tercieux (2012), Kartik, Tercieux, and Holden (2014), Korpela (2014), Saporiti (2014), Ortner (2015), Doğan (2017), Kimya (2017), Lombardi and Yoshihara (2017), Mukherjee, Muto, and Ramaekers (2017), Lombardi and Yoshihara (2018), Savva (2018), and Lombardi and Yoshihara (2019). See also Dutta (2019) for a survey of recent results in this literature.

[7]Another strand of related papers in the rational domain analyzes the characterization of jurors' preferences on rankings of contestants when jurors are not necessarily impartial and have incentives to misreport the true ranking of contestants. See Amorós (2009) and Amorós (2013). Yadav (2016) considers the effects of partial honesty in the model of Amorós (2013).

[8]Hurwicz (1986), Eliaz (2002), Barlo and Dalkiran (2009), Saran (2011), Saran (2016), Koray and Yildiz (2018), and Barlo and Dalkiran (2019) are among the papers presenting an analysis when the society is not necessarily composed of individuals whose choice behavior satisfies the WARP.

deals with the case of two individuals. All the proofs are in the Appendix.

2 A motivating example

Suppose that there are three individuals, denoted by $N = \{1, 2, 3\}$. The set of alternatives pertinent to implementation is $X = \{a, b, c\}$.

We assume it is common knowledge among the individuals and the planner that individuals' choices can be represented by strict preferences (asymmetric and negatively transitive binary relations).[9] Therefore, we face 6^3 contingencies, each corresponding to a possible preference profile. If all of them were to be seen as possible, the set of *feasible states* would be $\Omega = \{(P_1, P_2, P_3) : P_i \in \{abc, acb, bac, bca, cab, cba\}, i = 1, 2, 3\}$, with xyz denoting the strict preference order where x is strictly preferred to y and y to z with $x, y, z \in \{a, b, c\}$ and $x \neq y$ and $x \neq z$ and $y \neq z$.

If it were to be common knowledge about this environment that, in addition to the above, the best alternative of each individual is distinct and there is strong conflict of interest between individuals 1 and 3 (meaning that individual 1's top-ranked alternative must be individual 3's bottom-ranked and vice versa) then the set of *admissible states* would be

$$\begin{aligned}\Omega^* = \{&(abc, bac, cba), (abc, bca, cba), (acb, cab, bca), (acb, cba, bca), (bac, abc, cab),\\ &(bac, acb, cab), (bca, cba, acb), (bca, cab, acb), (cab, acb, bac), (cab, abc, bac),\\ &(cba, bca, abc), (cba, bac, abc)\} \subset \Omega. \end{aligned} \tag{1}$$

In our example, we suppose that the planner knows that only two of the admissible states may prevail. But, he does not know which two. We model this by letting the *states of the economy* be denoted by $\Theta = \{\theta, \theta'\}$. Then, the individuals (but not the planner) know that θ corresponds to (abc, bac, cba) and θ' to (bac, acb, cab); we refer to $\{(abc, bac, cba), (bac, acb, cab)\}$ as the *set of prevailing states*. This can be sustained by requiring that before the realization of the true state of the world, it becomes common knowledge among the individuals that individual 1's best alternative is not c, and

[9] A binary relation $P \subseteq X \times X$ is *asymmetric* if xPy implies *not* yPx; and, P is *negatively transitive* if xPy implies either zPx or yPz for all $z \neq x$ and $z \neq y$.

individual 2's preferences are either *acb* or *bac*; this piece of information is not known to the planner.

We wish to note that the set of feasible states is in one-to-one correspondence with all the possible preference profiles as in most of the implementation papers. However, in many applications, as in this example, the set of states of the economy is not in one-to-one correspondence with all the possible preference profiles.

The social choice goal to be implemented, $f : \Theta \to X$, is given exogenously to the planner and is as follows: $f(\theta) = b$ and $f(\theta') = c$. We wish to note that the planner does not know the preference profile associated with the states of the economy, but still has to implement this social choice function.

The following summarizes the knowledge and information requirements needed in our example: It is common knowledge

(i) among the individuals and the planner that the set of individuals is $\{1, 2, 3\}$; the set of feasible states is $\Omega = \{(P_1, P_2, P_3) : P_i \in \{abc, acb, bac, bca, cab, cba\}, i = 1, 2, 3\}$; the set of admissible states is Ω^* as defined in equation (1); the set of states of the economy is $\Theta = \{\theta, \theta'\}$; the social choice function $f : \Theta \to X$ such that $f(\theta) = b$ and $f(\theta') = c$; and that each individual observes the realized state of the economy $\tilde{\theta} \in \Theta$; and

(ii) among the individuals that the set of individuals is $\{1, 2, 3\}$; the set of feasible states is $\Omega = \{(P_1, P_2, P_3) : P_i \in \{abc, acb, bac, bca, cab, cba\}, i = 1, 2, 3\}$; the set of admissible states is Ω^* as defined in equation (1); the set of states of the economy is $\Theta = \{\theta, \theta'\}$; θ and θ' correspond to the prevailing states (abc, bac, cba) and (bac, acb, cab), respectively; the social choice function $f : \Theta \to X$ such that $f(\theta) = b$ and $f(\theta') = c$; and that each individual observes the realized state of the economy $\tilde{\theta} \in \Theta$; and

(iii) the planner knows that 'the prevailing states corresponding to the states of the economy θ and θ' are common knowledge among the individuals.'

de Clippel (2014) establishes a necessary condition for Nash implementation when individuals are not necessarily rational. This condition, *consistency*, is a notion that extends the well-known Maskin monotonicity to the domain when the individuals' choices do not satisfy the WARP. In the current setting, consistency requires that there are choice sets for every individual $i = 1, 2, 3$, at states θ and θ', for alternative b, which is f-optimal at θ, and for alternative c, f-optimal at θ', such that there exists an agent $j \in \{1, 2, 3\}$ who does not choose b at θ' from his choice set corresponding to θ and b, and there is an individual $k \in \{1, 2, 3\}$ not choosing c at θ from his choice set associated with θ' and c. That is, there is a profile of sets $\mathbf{S} = (S_i(b, \theta), S_i(c, \theta'))_{i=1,2,3}$ such that $b \in C_i^{\theta}(S_i(b, \theta))$ and $c \in C_i^{\theta'}(S_i(c, \theta'))$ for all $i = 1, 2, 3$, while $b \notin C_j^{\theta'}(S_j(b, \theta))$ and $c \notin C_k^{\theta}(S_k(c, \theta'))$ for some $j, k = 1, 2, 3$.

Let us start the identification of the consistent profile of choice sets of our example:
$\underline{S_1(b, \theta)}$: *abc* implies two possibilities for $S_1(b, \theta)$: $\{b, c\}$ and $\{b\}$.
$\underline{S_2(b, \theta)}$: *bac* implies four possibilities for $S_2(b, \theta)$: $\{a, b, c\}$, $\{a, b\}$, $\{b, c\}$, and $\{b\}$.
$\underline{S_3(b, \theta)}$: *cba* results in two possibilities for $S_3(b, \theta)$: $\{a, b\}$ and $\{b\}$.
$\underline{S_1(c, \theta')}$: *bac* implies the requirement that $S_1(c, \theta')$ equals $\{c\}$.
$\underline{S_2(c, \theta')}$: *acb* implies two possibilities for $S_2(c, \theta')$: $\{b, c\}$ and $\{c\}$.
$\underline{S_3(c, \theta')}$: *cab* results in four possibilities for $S_3(c, \theta')$: $\{a, b, c\}$, $\{a, c\}$, $\{b, c\}$, and $\{c\}$.
In addition to the above, $b \notin C_2^{\theta'}(\{b, c\})$ and $c \notin C_2^{\theta}(\{b, c\})$. Therefore, the profile of sets $\mathbf{S} = (S_i(b, \theta), S_i(c, \theta'))_{i=1,2,3}$ specified as follows is one of the profiles of sets consistent with the social choice function f:

$$\begin{array}{ll} S_1(b, \theta) = \{b, c\} & S_1(c, \theta') = \{c\} \\ S_2(b, \theta) = \{b, c\} & S_2(c, \theta') = \{b, c\} \\ S_3(b, \theta) = \{a, b\} & S_3(c, \theta') = \{a, b, c\}. \end{array}$$

The planner does not know individuals' preferences in states θ and θ', and hence, cannot identify a profile of sets consistent with f in order to construct a mechanism that implements f in Nash equilibrium.

What if the planner were to ask the individuals to reveal a profile of sets consistent with f as a part of the mechanism?

In that regard, we consider mechanisms that require the announcement of a profile of alleged choice sets along with some other actions. Indeed, the truthful revelation of a profile of choice sets consistent with the given SCC can be achieved when there is a *weak sympathizer* in the society: an individual who strictly prefers to announce a consistent profile and choose some messages whenever none of his lies (inconsistent profiles of sets) makes him strictly better off while he keeps on using the same remaining parts of his messages. A weak sympathizer is not a snitch or an informer, and does not feel any obligation and/or inclination to reveal the state of the economy truthfully.

When it is commonly known among the individuals and the planner that the society contains a weak sympathizer (the identity of whom is privately known only by himself) our main result, Theorem 1, shows the following: in *economic environments* with at least three individuals, the implementation of a social goal can be achieved while almost unanimously eliciting information about a consistent profile of choice sets from the society provided that there is such a profile.

The economic environment condition requires that, at any state, any one of the alternatives should not be chosen from the set of all alternatives by at least two individuals. In our example, this condition holds because individuals' choices from $\{a, b, c\}$ are such that at θ, alternative a is not chosen by individuals 2 and 3, b by 1 and 3, and c by 1 and 2; while at θ', alternative a is not chosen by individuals 1 and 3, b by 2 and 3, and c by 1 and 2.

Therefore, Theorem 1 applies, and it implies that the social choice function of our example can be implemented while eliciting the relevant information pertaining to individuals' state-contingent choices.

3 Notations, definitions, and preliminaries

Let X be a set of *alternatives*, and $\mathcal{X}$ the set of all nonempty subsets of X. $N = \{1, ..., n\}$ denotes a *society* with a finite set of individuals where $n \geq 2$.

Ω denotes the finite set of all *feasible states* of the world and it is assumed to be in one-to-one correspondence with all the payoff-relevant characteristics of the environment. Indeed, one may derive this set of states using the set of alternatives X. The distribution on Ω determining the realized state is denoted by ϕ where for any $\omega \in \Omega$, $\phi(\omega) \in [0, 1]$ identifies the probability that ω occurs. In many economic applications of interest, one may consider an *interim* set of states, a set of *admissible states* of the world which we denote by Ω^*. Naturally, $\Omega^* \subseteq \Omega$. The set of *prevailing states* is given by $\Omega^P := \text{supp}(\phi) = \{\omega \in \Omega : \phi(\omega) \neq 0\}$. We assume that $\Omega^P \subset \Omega^*$.[10]

To model the information/knowledge formalities, we adopt the following: Let Θ be the set of *states of the economy* defined with the requirement that $\#\Theta = \#\Omega^P$. The bijection $\pi : \Theta \to \Omega^P$ is called the *identification function* where $\pi(\theta) \in \Omega^*$ identifies the particulars of the payoff-relevant characteristics associated with the state of the economy $\theta \in \Theta$.[11] As $\pi : \Theta \to \Omega^P$ is a bijection, we refer to a state of the economy θ also as a prevailing state and to the set of the states of the economy Θ as the set of prevailing states. When the meaning is clear, we refer to a prevailing state as a state.

An alternative interpretation involves the *categorization/aggregation* of admissible states: Using a surjective *categorization/aggregation function* $\kappa : \Omega^* \to \Theta$ where for any admissible state $\omega \in \Omega^*$, $\kappa(\omega)$ is the category/aggregate state of ω in the set categories/aggregate states, Θ (previously specified as the set of states of the economy). The *identification function with categories/aggregation* is an injective mapping $\upsilon : \Theta \to \Omega^*$ associating a given category/aggregate state $\theta \in \Theta$ with the 'true' payoff-relevant characteristics $\upsilon(\theta) \in \Omega^*$ with the requirement that for any $\omega \in \Omega^*$, $\upsilon(\kappa(\omega)) = \omega$. In what follows, we work with a given set of states of the economy Θ and injective identification function $\pi : \Theta \to \Omega^*$. Notwithstanding, by replacing the identification function π with the identification function with categories/aggregation υ (while selecting an appropriate categorization/aggregation function κ), one can interpret our findings using the categorization/aggregation approach.

The *(individual) choice* of agent $i \in N$ at a feasible state $\omega \in \Omega$ is captured by the choice correspondence $C_i^\omega : \mathcal{X} \to \mathcal{X}$ with the feasibility requirement that for any $S \in \mathcal{X}$, $C_i^\omega(S) \subset S$. Given alternative $x \in X$, individual $i \in N$, and feasible state $\omega \in \Omega$, we refer to a set $S \in \mathcal{X}$ with $x \in C_i^\omega(S)$ as a *choice set* of individual i at

[10]Situations in which X and Ω are not finite and ϕ is a measurable distribution can be handled by constructing a σ-algebra on Ω and insisting on some standard measurability requirements. To refrain from technicalities, we restrict our attention to finite sets in this paper.

[11]A function $\psi : X \to Y$ is *injective* if it maps distinct elements of its domain, X, to distinct elements in its range, Y; it is *surjective* if for every element in its range, $y \in Y$, there is an element in its domain, $x \in X$, with $\psi(x) = y$. A function $\psi : X \to Y$ is a *bijection* if it is injective and surjective.

state ω for alternative x. The *societal choice topography on* Ω is given by the profile of individual choice correspondences $\mathcal{C}(\Omega) := (C_i^{\omega}(S))_{i\in N,\ \omega\in\Omega,\ S\in\mathcal{X}}$.

Under rationality, every individual's choice correspondence satisfies the WARP at every state of the world. So, for any given $i \in N$ and $\omega \in \Omega$, there exists a reflexive, complete, and transitive binary preference relation $R_i^{\omega} \subseteq X \times X$ such that for any $x, y \in X$, $x\ R_i^{\omega}\ y$ if and only if $x \in C_i^{\omega}(\{x, y\})$.[12] Therefore, for any given $i \in N$ and $\omega \in \Omega$ and $S \in \mathcal{X}$, we have that $C_i^{\omega}(S) = \{x^* \in S : x^*\ R_i^{\omega}\ y, \forall y \in S\}$. See Sen (1971) for the formal treatment. In what follows, we allow for violations of WARP.

A *social choice correspondence* (SCC) defined on the states of the economy is $f : \Theta \to \mathcal{X}$, a non-empty valued correspondence mapping Θ into X (i.e., $f(\theta) \in \mathcal{X}$ for every $\theta \in \Theta$). Given $\theta \in \Theta$, the set of alternatives $f(\theta)$ denotes the alternatives that the planner desires to sustain at θ and are referred to as *f-optimal* alternatives at θ.

We restrict our attention to *complete information*. The information and knowledge requirements of our model are as follows:

(*i*) N, X, Ω, Ω^*, $\mathcal{C}(\Omega)$, Θ, $f : \Theta \to \mathcal{X}$, and that 'every individual observes the realized state of the economy' are common knowledge among the individuals and the planner; and

(*ii*) N, X, Ω, Ω^*, $\mathcal{C}(\Omega)$, Θ, the injection $\pi : \Theta \to \Omega^*$, $f : \Theta \to \mathcal{X}$, and the realized state of the economy $\theta \in \Theta$ are common knowledge among the individuals; and

(*iii*) the planner knows that 'the injective identification function $\pi : \Theta \to \Omega^*$ is common knowledge among the individuals'.

The essence of the asymmetry of information/knowledge between the planner and the individuals involves the identification function π and the realized state of the economy θ. Recall that $\pi : \Theta \to \Omega^P$ is a bijection and $\Omega^P \subset \Omega^*$. Thus, $\pi : \Theta \to \Omega^*$ is an injection.

The following is the notion of consistency (de Clippel, 2014):

[12] A binary relation $R \subseteq X \times X$ is *reflexive* if for all $x \in X$, xRx; *complete* if for all $x, y \in X$ either xRy or yRx or both; and *transitive* if for all $x, y, z \in X$ with xRy and yRz implies xRz.

Definition 1. *Given an SCC* $f : \Theta \to \mathcal{X}$*, a profile of sets* $\mathbf{S} := (S_i(x,\theta))_{i \in N,\ \theta \in \Theta,\ x \in f(\theta)}$ *is* consistent *with* $f : \Theta \to \mathcal{X}$ *if*

(*i*) $x \in C_i^{\pi(\theta)}(S_i(x,\theta))$*, for all* $i \in N$ *and* $\theta \in \Theta$ *and* $x \in f(\theta)$*; and*

(*ii*) $x \in f(\theta)$ *and* $x \notin f(\theta')$ *for some* $\theta, \theta' \in \Theta$ *implies there is* $j \in N$ *such that* $x \notin C_j^{\pi(\theta')}(S_j(x,\theta))$.

$\mathcal{S}(f)$ *denotes the set of all profiles of sets that are consistent with* f.

In words, a profile of sets $\mathbf{S}$ is consistent with a given SCC $f : \Theta \to \mathcal{X}$, if (*i*) the set $S_i(x,\theta)$ is a choice set of alternative x by individual i at state $\pi(\theta)$, for every $i \in N$ and $\theta \in \Theta$ and $x \in f(\theta)$; and (*ii*) if alternative x is f-optimal at state θ but not at state θ' for some $\theta, \theta' \in \Theta$, then there exists an individual $j \in N$ such that x is not chosen from $S_j(x,\theta)$ by j at $\pi(\theta')$.

A mechanism $\mu = (A, g)$ assigns each individual $i \in N$ a non-empty *message space* A_i and specifies an *outcome function* $g : A \to X$ where $A = \times_{j \in N} A_j$. $\mathcal{M}$ denotes the set of all mechanisms.

Given a mechanism $\mu \in \mathcal{M}$ and $a_{-i} \in A_{-i} := \times_{j \neq i} A_j$, the *opportunity set* of individual i pertaining to others' message profile a_{-i} in mechanism μ is $O_i^{\mu}(a_{-i}) := g(A_i, a_{-i})$ where $g(A_i, a_{-i}) = \{g(a_i, a_{-i}) : a_i \in A_i\}$.

In our setup, a *sympathizer* of the social goal is assumed to be inclined toward the truthful revelation of the societal choice topography to the planner. As mentioned before, a sympathizer is not a snitch or an informer in the sense that he does not feel any obligation and/or inclination to reveal the state of the economy, but serves the planner as a guide (without asking for compensation of any sort). Our results show that the revelation of the whole choice topography is not needed, and announcements of profiles of sets consistent with the SCC f are sufficient for implementation. In other words, we do not need all the information pertaining to individuals' state-contingent choice behavior in order to implement the SCC f. That is why, in what follows, we restrict our attention to the truthful revelation of consistency.

As a result, one can assume the sympathizer is *inclined toward truthful revelation of consistency.* To define this, we consider mechanisms in which one of the components of each individual's message space involves the announcement of a profile of choice sets indexed for $i \in N$, $\theta \in \Theta$, and $x \in f(\theta)$. We refer to such game forms as *mechanisms involving the announcement of a profile of choice sets* and denote the set of such mechanisms by $\mathcal{M}^* \subset \mathcal{M}$. To that regard, we let $\mathcal{S}$ denote the set of all profile of sets of alternatives $\mathbf{S} = (S_i(x, \theta))_{i \in N,\ \theta \in \Theta,\ x \in f(\theta)}$ with the property that $x \in S_i(x, \theta)$ for all $i \in N$, $\theta \in \Theta$, $x \in f(\theta)$. Naturally, $\mathcal{S}(f) \subset \mathcal{S}$. As a result, the mechanism $\mu \in \mathcal{M}^*$ is such that $A_i := \mathcal{S} \times M_i$ for each $i \in N$ for some non-empty M_i and $M := \times_{i \in N} M_i$ and a generic message (alternatively, action) $a_i \in A_i$ is denoted by $a_i = (\mathbf{S}^{(i)}, m_i)$.

In what follows, we extend individuals' choices on alternatives to choices on messages concerning mechanisms $\mu \in \mathcal{M}^*$.

For any given $\mu \in \mathcal{M}^*$ and state $\omega \in \Omega$, the correspondence $BR_i^\omega : A_{-i} \twoheadrightarrow A_i$ identifies the individual choices of agent i on his message space A_i (i.e., $BR_i^\omega(a_{-i}) \subset A_i$ identifies i's chosen messages at ω when others' message profile is given by a_{-i}). In particular, if individual i is a standard economic agent who is *not weakly inclined toward consistency* (i.e., not a weak sympathizer) at $\omega \in \Omega$, then for all $a_{-i} \in A_{-i}$,

$$a_i \in BR_i^\omega(a_{-i}) \text{ if and only if } g(a_i, a_{-i}) \in C_i^\omega(O_i^\mu(a_{-i})).$$

Otherwise, the following must hold:

Definition 2. *Given a mechanism $\mu \in \mathcal{M}^*$, we say that individual $i \in N$ is*

1. weakly inclined toward consistency *at a state $\omega \in \Omega$ if for all $a_{-i} \in A_{-i}$,*

   (i) $g((\mathbf{S}^{(i)}, m_i), a_{-i}), g((\tilde{\mathbf{S}}^{(i)}, m_i), a_{-i}) \in C_i^\omega(O_i^\mu(a_{-i}))$ *with* $\mathbf{S}^{(i)} \in \mathcal{S}(f)$, $\tilde{\mathbf{S}}^{(i)} \in \mathcal{S} \setminus \mathcal{S}(f)$, *and* $m_i \in M_i$ *implies* $(\mathbf{S}^{(i)}, m_i) \in BR_i^\omega(a_{-i})$ *and* $(\tilde{\mathbf{S}}^{(i)}, m_i) \notin BR_i^\omega(a_{-i})$*; and*

   (ii) in all other cases, $a_i \in BR_i^\omega(a_{-i})$ *if and only if* $g(a_i, a_{-i}) \in C_i^\omega(O_i^\mu(a_{-i}))$.

2. inclined toward consistency *at a state $\omega \in \Omega$ if for all $a_{-i} \in A_{-i}$,*

   (i) $g((\mathbf{S}^{(i)}, m_i), a_{-i}), g((\tilde{\mathbf{S}}^{(i)}, \tilde{m}_i), a_{-i}) \in C_i^\omega(O_i^\mu(a_{-i}))$ *with* $\mathbf{S}^{(i)} \in \mathcal{S}(f)$, $\tilde{\mathbf{S}}^{(i)} \in \mathcal{S} \setminus \mathcal{S}(f)$, *and* $m_i, \tilde{m}_i \in M_i$ *implies* $(\mathbf{S}^{(i)}, m_i) \in BR_i^\omega(a_{-i})$ *and* $(\tilde{\mathbf{S}}^{(i)}, \tilde{m}_i) \notin BR_i^\omega(a_{-i})$*; and*

(ii) *in all other cases,* $a_i \in BR_i^{\omega}(a_{-i})$ *if and only if* $g(a_i, a_{-i}) \in C_i^{\omega}(O_i^{\mu}(a_{-i}))$.

We call an individual weakly inclined toward consistency at a state $\omega \in \Omega$ *a* weak sympathizer *at* ω, *and one who is inclined toward consistency at* $\omega \in \Omega$ *a* sympathizer *at* ω.

The first part of Definition 2 says the following: Given mechanism μ, any one of others' actions a_{-i}, and any state $\omega \in \Omega$, a weak sympathizer i at ω chooses to announce a consistent profile of sets $\mathbf{S}^{(i)}$ as well as a message profile m_i; he does not choose to announce an inconsistent profile $\tilde{\mathbf{S}}^{(i)}$ and to select the same message profile m_i whenever both action profiles, $(\mathbf{S}^{(i)}, m_i)$ and $(\tilde{\mathbf{S}}^{(i)}, m_i)$, lead to alternatives which are among those chosen by individual i at state ω from his opportunity set corresponding to others' behavior a_{-i} (namely, $O_i^{\mu}(a_{-i})$). On the other hand, the second part of Definition 2 demands the following: Given mechanism μ, any of others' actions a_{-i}, and state $\omega \in \Omega$, a sympathizer i at ω chooses to announce a consistent profile of sets $\mathbf{S}^{(i)}$ while selecting a message profile m_i; he does not choose to announce an inconsistent profile $\tilde{\mathbf{S}}^{(i)}$ coupled with selecting some other message profile $\tilde{m}_i$ whenever both action profiles, $(\mathbf{S}^{(i)}, m_i)$ and $(\tilde{\mathbf{S}}^{(i)}, \tilde{m}_i)$, result in alternatives that are among the chosen by i at ω from $O_i^{\mu}(a_{-i})$.

As a consequence, every individual inclined toward consistency at a state ω is also weakly inclined toward consistency at ω. That is, every sympathizer is a weak sympathizer. We employ the following sympathy properties in our results:

Definition 3. *We say that the environment satisfies the* weak-sympathizer property *if, for every admissible state* $\omega \in \Omega^*$, *there exits at least one weak sympathizer at* ω. *Moreover, the* sympathizer property *holds if, for every admissible state* $\omega \in \Omega^*$, *there are at least two sympathizers at* ω. *The identity of each (weak) sympathizer associated with a given admissible state is privately known only by himself.*

Given a mechanism $\mu \in \mathcal{M}$, $a^* \in A$ constitutes a *Nash equilibrium* of μ at a state $\theta \in \Theta$ if $a_i^* \in C_i^{\pi(\theta)}(O_i^{\mu}(a_{-i}^*))$ for all $i \in N$. When attention is restricted to $\mu \in \mathcal{M}^*$, we say that $a^* \in A$ is a *Nash** *equilibrium* of μ at a state $\theta \in \Theta$ if $a_i^* \in BR_i^{\pi(\theta)}(a_{-i}^*)$

for all $i \in N$. We note that when there are no sympathizers, the two notions coincide. Otherwise, the set of Nash* equilibrium of a mechanism might be a proper subset of the set of Nash equilibrium of the same mechanism.

Nash implementability is defined as follows: an SCC $f : \Theta \to \mathcal{X}$ is *(fully) implementable by a mechanism* $\mu \in \mathcal{M}$ *in Nash equilibrium* if (i) for any $\theta \in \Theta$ and $x \in f(\theta)$, there exists $a^x \in A$ such that $g(a^x) = x$ and $x \in C_i^{\pi(\theta)}(O_i^{\mu}(a_{-i}^x))$ for all $i \in N$; and (ii) for any $\theta \in \Theta$, $a^* \in A$ with $g(a^*) \in C_i^{\pi(\theta)}(O_i^{\mu}(a_{-i}^*))$ for all $i \in N$ implies $g(a^*) \in f(\theta)$.

If an SCC $f : \Theta \to \mathcal{X}$ is implementable by a mechanism $\mu \in \mathcal{M}$ in Nash equilibrium, we define the *profile of sets sustained by* μ as follows: $\mathbf{S}^{\mu} := (S_i^{\mu}(x,\theta))_{i \in N,\ \theta \in \Theta,\ x \in f(\theta)}$ with $S_i^{\mu}(x,\theta) := O_i^{\mu}(a_{-i}^x)$ for any $i \in N$, $\theta \in \Theta$, and $x \in f(\theta)$ while $a^x \in A$ is such that $g(a^x) = x$ and $g(a^x) \in C_i^{\pi(\theta)}(O_i^{\mu}(a_{-i}^x))$ for all $i \in N$.

The necessity result of de Clippel (2014) tells us that if an SCC f is Nash implementable by a mechanism $\mu \in \mathcal{M}$, then $\mathbf{S}^{\mu} \in \mathcal{S}(f)$. That is, the profile of sets sustained by μ is consistent with f.

On the other hand, the notion of Nash* implementation is the following:

Definition 4. *We say that an SCC* $f : \Theta \to \mathcal{X}$ *is* (fully) implementable by a mechanism $\mu \in \mathcal{M}$ in Nash* equilibrium*, if*

(i) *for any* $\theta \in \Theta$ *and* $x \in f(\theta)$*, there exists* $a^x \in A$ *such that* $g(a^x) = x$ *and* $a_i^x \in BR_i^{\pi(\theta)}(a_{-i}^x)$ *for all* $i \in N$*; and*

(ii) *for any* $\theta \in \Theta$*,* $a^* \in A$ *with* $a_i^* \in BR_i^{\pi(\theta)}(a_{-i}^*)$ *for all* $i \in N$ *implies* $g(a^*) \in f(\theta)$*.*

When the mechanism μ in this definition is in $\mathcal{M} \setminus \mathcal{M}^*$, Nash* implementation coincides with Nash implementation; being a (weak) sympathizer does not put any additional restrictions on individuals' choices.

In Appendix A, we present a necessity result with Nash* implementation.[13] The necessary condition we attain is not independent of the mechanism μ, and hence, it is not helpful in constructing mechanisms that will be employed in the sufficiency direction.

[13]This result shows that the necessary condition we obtain differs from consistency whenever the mechanism Nash* implementing a given SCC, μ, is in $\mathcal{M}^*$ and a (weak) sympathizer announces an inconsistent profile of sets in a Nash* equilibrium.

Some of our results adopt the following assumptions:

Definition 5. *We say that the environment features* societal non-satiation *whenever, for any admissible state* $\omega \in \Omega^*$ *and any alternative* $x \in X$*, there is an individual* $i \in N$ *such that* $x \notin C_i^{\omega}(X)$*. Moreover, we say that the* economic environment *assumption holds whenever, for any admissible state* $\omega \in \Omega^*$ *and any alternative* $x \in X$*, there are two individuals* $i, j \in N$ *with* $i \neq j$ *such that* $x \notin C_i^{\omega}(X) \cup C_j^{\omega}(X)$*.*

The economic environment assumption implies societal non-satiation. The latter requires that for any given admissible state, all individuals do not choose the same alternative from the set of all alternatives at that state. The former demands that for every admissible state and alternative, there are two individuals not choosing that alternative from the set of all alternatives at that given state. The economic environment assumption involves a weak form of disagreement among the society when selecting an alternative from the set of all alternatives.

Next, we define the no-veto property:

Definition 6. *We say that an SCC* $f : \Theta \to \mathcal{X}$ *satisfies the no-veto property if for any* $\theta \in \Theta$*,* $x \in C_i^{\pi(\theta)}(X)$ *for all* $i \in N \setminus \{j\}$ *for some* $j \in N$ *implies* $x \in f(\theta)$*.*

This property demands that if an alternative is chosen from the set of all alternatives at a state by every individual but one, then that alternative has to be f-optimal at that state. Evidently, the welfare of the individual who does not agree with the rest of the society is ignored in this notion.[14] Moreover, we wish to note that the no-veto property never applies in economic environments.

4 Three or more individuals

The following is the first of our sufficiency results for three or more individuals. It utilizes a mechanism involving the announcement of a profile of choice sets and extracting the desired information about consistency from the society.

[14] Benoit and Ok (2006) and Barlo and Dalkiran (2009) obtain full implementation results employing a property called *limited-veto-power*, a weaker condition than the no-veto property.

Theorem 1. *Suppose $n \geq 3$. Consider an economic environment satisfying the weak-sympathizer property, and assume that the SCC $f : \Theta \to \mathcal{X}$ is such that $\mathcal{S}(f) \neq \emptyset$. Then, f is Nash* implementable by a mechanism $\mu \in \mathcal{M}^*$. Furthermore, for any $\theta \in \Theta$, in every Nash* equilibrium $\bar{a} = (\bar{\mathbf{S}}^{(i)}, \bar{m}_i)_{i \in N} \in A$ of mechanism μ at θ, $\bar{\mathbf{S}}^{(i)} = \mathbf{S}$ for some $\mathbf{S} \in \mathcal{S}(f)$ for all $i \in N \setminus \{j\}$ for some $j \in N$.*

Theorem 1 provides a clear and useful message in economic environments satisfying the weak-sympathizer property: The relevant information about societal choice topography can be elicited from individuals almost unanimously whenever there exists a profile of sets consistent with that social goal. The identity of the weak sympathizer associated with a given prevailing state is not known to the planner and the other individuals.

The following is an implication of this theorem: If the set of prevailing states is not in one-to-one correspondence with all the admissible payoff-relevant characteristics of this environment (i.e., Ω^P is a strict subset of Ω^*), the planner needs to 'know' individuals' state-contingent choices to identify a consistent profile of choice sets. What Theorem 1 implies is that the designer can learn the realized state, as well as the relevant information about the societal choice topography from the individuals, using the same mechanism whenever there is a profile of choice sets consistent with that social goal. Thus, the economic environment assumption and the weak-sympathizer property empower us to dispose of the planner's need to know individuals' state-contingent choices whenever the social goal has a consistent profile of sets.

The existence of a profile of choice sets consistent with the given SCC f is akin to a feasibility requirement: the necessity result tells us that $\mathcal{S}(f) = \emptyset$ implies that f is not Nash implementable. That is, Nash implementability of the SCC f is *unattainable*. Thus, insisting on the existence of a consistent profile of sets in our theorem is natural. On the other hand, if one was to restrict attention to the sufficiency direction, the following concern may arise: How would the planner know if there is a profile of choice sets consistent with the SCC he wishes to implement? This is a relevant question, as the designer does not know individuals' state-contingent choices. An auxiliary result helps us deal with this issue: the planner infers that there is a profile of sets consistent with the SCC f whenever he knows that there is a larger set of states and an extension of f onto that set such that this extension has a consistent profile of choice sets.

We can obtain a tangible display of this by considering the efficiency notion introduced and analyzed by de Clippel (2014): The *de Clippel efficient SCC* is $f^{eff} : \Omega \to \mathcal{X}$ and it is defined for any feasible state $\omega \in \Omega$ by $f^{eff}(\omega) := \{x \in X \mid \exists \mathbf{Y} := (Y_i)_{i \in N}$ with $Y_i \in \mathcal{X}$ and $x \in C_i^{\omega}(Y_i), \forall i \in N$, and $\cup_{i \in N} Y_i = X\}$. It turns out that f^{eff} is Nash implementable on all domains $\Omega' \subset \Omega$ (de Clippel, 2014, Proposition 3). Therefore, for any set of states of the economy Θ and injective identification function $\pi : \Theta \to \Omega^*$, the planner, without knowing π, infers that $\mathcal{S}(f^{eff} \mid_{\pi(\Theta)}) \neq \emptyset$ where $\mathcal{S}(f^{eff} \mid_{\pi(\Theta)}) = \{\mathbf{S} \in \mathcal{S} : \mathbf{S} = (S_i(x, \omega))_{i \in N,\ \omega \in \Omega,\ x \in f^{eff}(\omega)}$ such that $\pi^{-1}(\omega) \in \Theta\}$. That is, the designer deduces that, for any Θ and injective $\pi : \Theta \to \Omega^*$, the restriction of f^{eff} on $\pi(\Theta)$ has a consistent profile of choice sets.

In fact, our auxiliary result establishes that existence of a profile of sets consistent with an SCC f follows whenever there is a profile of sets consistent with an extension of the SCC f onto the set of admissible states.

We need the following for our auxiliary result: given an SCC $f : \Theta \to \mathcal{X}$ and an identification function $\pi : \Theta \to \Omega^*$, an *extension of f onto Ω^** is an SCC $f_{\Omega^*} : \Omega^* \to \mathcal{X}$ such that $f_{\Omega^*}(\pi(\theta)) = f(\theta)$ for all $\theta \in \Theta$.

Lemma 1. *Suppose there exists an extension of the SCC $f : \Theta \to \mathcal{X}$ onto the set of admissible states Ω^*, $f_{\Omega^*} : \Omega^* \to \mathcal{X}$, such that f_{Ω^*} possesses a consistent profile of sets. Then, $\mathcal{S}(f) \neq \emptyset$.*

Combining the necessity of Nash implementation and this lemma implies the following: $\mathcal{S}(f) \neq \emptyset$ whenever there exists an extension of the SCC $f : \Theta \to \mathcal{X}$ onto Ω^*, $f_{\Omega^*} : \Omega^* \to \mathcal{X}$, such that f_{Ω^*} is Nash implementable on the domain Ω^*. It is useful to point out that the hypothesis of Lemma 1 does not involve the Nash implementability of an extension of the given SCC on the domain of admissible states, but rather the existence of a profile of sets consistent with an extension of the given SCC onto the domain of admissible states. Indeed, the extension of f onto Ω^*, f_{Ω^*}, that may be used in the lemma, may violate some needed properties (such as no-veto property and/or unanimity) at states $\omega \in \Omega^* \setminus \Omega^P$; consequently, the sufficiency result of de Clippel (2014) may not apply to f_{Ω^*}.

As a result, using Lemma 1 we can restate our main result in the following corollary presented without a proof:

Corollary 1. *Suppose $n \geq 3$. Consider an economic environment satisfying the weak-sympathizer property, and assume that there exists an extension of the SCC $f : \Theta \to \mathcal{X}$ onto Ω^*, $f_{\Omega^*} : \Omega^* \to \mathcal{X}$, which possesses a consistent profile of choice sets. Then, f is Nash* implementable by a mechanism $\mu \in \mathcal{M}^*$. Moreover, for any $\theta \in \Theta$, in every Nash* equilibrium $\bar{a} = (\bar{\mathbf{S}}^{(i)}, \bar{m}_i)_{i \in N} \in A$ of mechanism μ at θ, $\bar{\mathbf{S}}^{(i)} = \mathbf{S}$ for some $\mathbf{S} \in \mathcal{S}(f)$ for all $i \in N \setminus \{j\}$ for some $j \in N$.*

Next, we analyze how to weaken the economic environment assumption specified in Definition 5. To that regard, we need to discuss the construction of the mechanism employed in the proof of Theorem 1. Our mechanism asks each individual i to choose a feasible profile of choice sets $\mathbf{S}^{(i)} \in \mathcal{S}$; a state of the economy $\theta^{(i)} \in \Theta$; an alternative $x^{(i)} \in X$; a natural number $k^{(i)}$. Rule 1 decrees that if all but one individual announce the same feasible profile of choice sets, $\mathbf{S}$, while the remaining choices of all individuals involve θ and x with $x \in f(\theta)$, then the outcome function equals x. On the other hand, Rule 2 demands that the outcome equals x whenever all but one individual i' announce the same feasible profile of choice sets, $\mathbf{S}$, and the choices of all but one individual j involve θ and x with $x \in f(\theta)$ while j chooses x' and θ' provided that x' is not in the choice set $S_j(x, \theta)$ listed in the profile $\mathbf{S}$. If x' were to be in the choice set $S_j(x, \theta)$ listed in the profile $\mathbf{S}$ in the contingency discussed in the previous sentence, then Rule 2 decrees that the outcome is x'. Rule 3 encompasses all the other situations and involves the *integer game*: the outcome equals the alternative chosen by the agent with the lowest index among those who choose the highest natural number.

The economic environment assumption dispenses with the Nash* equilibria that may arise under Rules 2 and 3 as well as that under Rule 1. Equilibria that arise under Rule 3 are not desirable because of the following: in an equilibrium under Rule 3, all individuals apart from the sympathizers do not need to choose a consistent profile of sets. As a result, the relevant information about the societal choice topography cannot be extracted in equilibrium from these individuals. Fortunately, societal non-satiation is sufficiently strong to rule out such equilibria.

If we adopt societal non-satiation along with the no-veto property, then we allow for some additional equilibria under Rules 1 and 2. Then, for any prevailing state, we need *at least two sympathizers*. This is because our mechanism is such that when we deal with an equilibrium at a prevailing state under Rules 1 or 2 in which all but one individual announce the same profile of sets while the odd man out is announcing a different profile, by changing his announcement concerning the profile, each agent different from the odd man out can trigger Rule 3, and hence, obtain any alternative he desires by also changing his integer choice. Because we need the equilibrium announcement of the profile of sets by all but the odd man out to be consistent with the social goal, we have to make sure that there is a sympathizer (at this state) among those announcing the same profile; weak sympathy does not suffice as this agent also needs to change his integer choice.[15]

In what follows, we provide the formal presentation and execution of the above-discussed result by replacing the no-veto property with a weaker condition that is inspired by condition μ of Moore and Repullo (1990) (alternatively, condition λ of Korpela (2012)) and delivers a similar requirement as in de Clippel (2014). It says that for any profile of choice sets consistent with the social goal the following must hold: (i) an alternative x must be f-optimal at θ whenever it is chosen by all but one individual at $\pi(\theta)$ from the set of all alternatives while the odd man out chooses x at some other state $\pi(\theta')$ from the choice set (associated with the consistent profile)

[15] The need to have an additional sympathizer is a *novel* point that does not appear in Dutta and Sen (2012) and it is one of the reasons why they can conduct their analysis with only one *partially honest* individual. They work in the rational domain where states are in one-to-one correspondence with the payoff-relevant aspects and assume that a partially honest individual strictly prefers to reveal the state truthfully when he is indifferent. To see why they do not need an additional partially honest individual, consider the canonical mechanism without the announcement of a profile of choice sets and a Nash equilibrium in which the rule that implies the opportunity sets of all but one individual, i^*, equals X. Then, they do not need to guarantee that one of those individuals $i \neq i^*$ (different from the odd man out i^*) is partially honest as the no-veto property delivers the desired conclusion. However, in our case, we need to worry about "sympathy" to ensure that one of those individuals $i \neq i^*$ is inclined toward the truthful revelation of a consistent profile of sets.

corresponding to θ' and some f-optimal alternative x' in θ'; and (ii) the social goal has to *respect unanimity*, meaning that if an alternative x is chosen by every individual at a state $\pi(\theta)$ from the set of all alternatives, then x has to be f-optimal at θ. The no-veto property implies this condition. On the other hand, in an environment featuring societal non-satiation, unanimity holds vacuously; thus, it can be dispensed with.

Now, we state our second sufficiency result. Indeed, this theorem can be regarded as a robustness check for Theorem 1.

Theorem 2. *Suppose $n \geq 3$. Consider an environment satisfying the sympathizer property, and assume that the SCC $f : \Theta \to \mathcal{X}$ is such that $\mathcal{S}(f) \neq \emptyset$ and the following hold: For any $\theta \in \Theta$,*

(i) *for any $\mathbf{S} \in \mathcal{S}(f)$, $x \in C_j^{\pi(\theta)}(S_j(x', \theta'))$ where $j \in N$, $\theta' \in \Theta$, $x' \in f(\theta')$, $S_j(x', \theta') = \mathbf{S}|_{j,x',\theta'}$, and $x \in C_i^{\pi(\theta)}(X)$ for all $i \in N \setminus \{j\}$ implies $x \in f(\theta)$; and*

(ii) *$x \in C_i^{\pi(\theta)}(X)$ for all $i \in N$ implies $x \in f(\theta)$.*

Then, f is Nash* implementable by a mechanism $\mu \in \mathcal{M}^*$. Moreover, if the environment features societal non-satiation, then (ii) above can be dispensed with and for any $\theta \in \Theta$, in every Nash* equilibrium $\hat{a} = (\hat{\mathbf{S}}^{(i)}, \hat{m}_i)_{i \in N} \in A$ of μ at θ, $\hat{\mathbf{S}}^{(i)} = \mathbf{S}$ for some $\mathbf{S} \in \mathcal{S}(f)$ for all $i \in N \setminus \{j\}$ for some $j \in N$.

It is straightforward to see that the no-veto property implies (i) and (ii) of the hypothesis of Theorem 2. Therefore, we obtain the following immediate corollary:

Corollary 2. *Suppose $n \geq 3$. Consider an environment featuring societal non-satiation and the sympathizer property, and assume that the SCC $f : \Theta \to \mathcal{X}$ is such that $\mathcal{S}(f) \neq \emptyset$ and satisfies the no-veto property. Then, f is Nash* implementable by a mechanism $\mu \in \mathcal{M}^*$ such that in every Nash* equilibrium $\hat{a} = (\hat{\mathbf{S}}^{(i)}, \hat{m}_i)_{i \in N} \in A$ of μ at θ, $\hat{\mathbf{S}}^{(i)} = \mathbf{S}$ for some $\mathbf{S} \in \mathcal{S}(f)$ for all $i \in N \setminus \{j\}$ for some $j \in N$.*

Theorem 2 demands that there exists a profile of sets consistent with the SCC f with some requirements that cannot be verified by the planner, who does not know individuals' state-contingent choices.[16] However, using the arguments leading to Lemma 1 we can modify Corollary 2 (which uses the no-veto property instead of the hypothesis of Theorem 2) to attain the conclusion that the planner infers $\mathcal{S}(f) \neq \emptyset$ whenever he knows that f has an extension that possesses a consistent profile of sets:[17]

Corollary 3. *Suppose $n \geq 3$. Consider an environment featuring societal non-satiation and the sympathizer property, and assume that the SCC $f : \Theta \to \mathcal{X}$ satisfies the no-veto property and possesses an extension onto Ω^*, $f_{\Omega^*} : \Omega^* \to \mathcal{X}$, which has a consistent profile of sets. Then, f is Nash* implementable by a mechanism $\mu \in \mathcal{M}^*$ such that in every Nash* equilibrium $\hat{a} = (\hat{\mathbf{S}}^{(i)}, \hat{m}_i)_{i \in N} \in A$ of μ at θ, $\hat{\mathbf{S}}^{(i)} = \mathbf{S}$ for some $\mathbf{S} \in \mathcal{S}(f)$ for all $i \in N \setminus \{j\}$ for some $j \in N$.*

[16]Taking steps similar to those discussed following Theorem 1 does not suffice to ease the extent of the information requirement of our hypothesis. Even with societal non-satiation, we cannot modify Theorem 2 with a condition that demands that the planner knows that there exists an extension of f onto the set of admissible states Ω^*, $f_{\Omega^*} : \Omega^* \to \mathcal{X}$, such that for any profile of sets consistent with f_{Ω^*}, denoted by $\mathbf{S}_{\Omega^*} = (S_i^*(x, \omega))_{i \in N,\ \omega \in \Omega^*,\ x \in f_{\Omega^*}(\omega)}$, it must be that $x \in C_j^{\omega}(S_j^*(x', \omega'))$ with $j \in N$, $\omega' \in \Omega^*$, $x' \in f_{\Omega^*}(\omega')$, and $x \in C_i^{\omega}(X)$ for all $i \in N$ with $i \neq j$ implies $x \in f_{\Omega^*}(\omega)$. However, this is not sufficient as it does not empower us to 'extend' a profile of sets consistent with f, $\mathbf{S}_\Theta := (S_i(x, \theta))_{i \in N,\ \theta \in \Theta,\ x \in f(\theta)}$, to a profile of sets consistent with f_{Ω^*}, $\bar{\mathbf{S}}_{\Omega^*} = (\bar{S}_i^*(x, \omega))_{i \in N,\ \omega \in \Omega^*,\ x \in f_{\Omega^*}(\omega)}$, such that $S_i(x, \theta) = \bar{S}_i^*(x, \pi(\theta))$ for all $\theta \in \Theta$ and $x \in f(\theta)$. As a result, what the planner needs to know in addition to the above is: for all profiles of sets consistent with f, $\mathbf{S}_\Theta := (S_i(x, \theta))_{i \in N,\ \theta \in \Theta,\ x \in f(\theta)}$, there exists a profile of sets consistent with f_{Ω^*}, $\bar{\mathbf{S}}_{\Omega^*} = (\bar{S}_i^*(x, \omega))_{i \in N,\ \omega \in \Omega^*,\ x \in f_{\Omega^*}(\omega)}$, such that $S_i(x, \theta) = \bar{S}_i^*(x, \pi(\theta))$ for all $\theta \in \Theta$ and $x \in f(\theta)$. So, the addition of this condition does not help ease the need for the designer to know individuals' state-contingent choices.

[17]If the extension of f onto the set of admissible states, Ω^*, satisfies the no-veto property, then the planner infers that f satisfies the no-veto property on the set of states of the economy, Θ, as well.

Appendix

A A necessity result with Nash* implementation

Using Definition 2 delivers the following *necessity result for Nash* implementation* under weak sympathy. This result also applies to the case under sympathy because every sympathizer is a weak sympathizer. However, the necessity condition we obtain is not independent of the mechanism that Nash* implements the SCC at hand.

Proposition 1. *Suppose an SCC $f : \Theta \to X$ is Nash* implementable by a mechanism $\mu \in \mathcal{M}$. If $\mu \in \mathcal{M} \setminus \mathcal{M}^*$, then there exists a consistent profile of sets with f. On the other hand, if $\mu \in \mathcal{M}^*$, then there exists a profile of sets $\mathbf{S}^\mu := (S_i^\mu(x,\theta))_{i\in N,\ \theta\in\Theta,\ x\in f(\theta)}$ such that*

(*i*) *$x \in C_i^{\pi(\theta)}(S_i^\mu(x,\theta))$ for all $i \in N$, $\theta \in \Theta$, and $x \in f(\theta)$; and*

(*ii*) *$x \in f(\theta)$ and $x \notin f(\theta')$ for some $\theta, \theta' \in \Theta$ implies there is $j \in N$ such that*

- (*ii*.1) *either j is not a weak sympathizer at state $\pi(\theta')$ and $x \notin C_j^{\pi(\theta')}(S_j^\mu(x,\theta))$;*
- (*ii*.2) *or j is a weak sympathizer at state $\pi(\theta')$ with $x \in C_j^{\pi(\theta')}(S_j^\mu(x,\theta))$ and*
  - (*ii*.2.*a*) *there exists $\bar{\mathbf{S}} \in \mathcal{S}(f)$ such that $g((\bar{\mathbf{S}}, m_j^x), a_{-j}^x) \in C_j^{\pi(\theta')}(S_j^\mu(x,\theta))$ and*
  - (*ii*.2.*b*) *for all $\tilde{\mathbf{S}} \in \mathcal{S}(f)$, we have that $g((\tilde{\mathbf{S}}, m_j^x), a_{-j}^x) \notin C_j^{\pi(\theta)}(S_j^\mu(x,\theta))$ whenever j is a weak sympathizer at $\pi(\theta)$*

where $a^x = (\mathbf{S}^{(i),x}, m_i^x)_{i\in N} \in A$ is such that $g(a^x) = x$ and $a_i^x \in BR_i^{\pi(\theta)}(a_{-i}^x)$.

Proof. The existence of a consistent profile of sets follows from de Clippel (2014) when $\mu \in \mathcal{M} \setminus \mathcal{M}^*$. Now, suppose $\mu \in \mathcal{M}^*$. Let $\mathbf{S}^\mu := (S_i^\mu(x,\theta))_{i\in N,\ \theta\in\Theta,\ x\in f(\theta)}$ with $S_i^\mu(x,\theta) := O_i^\mu(a_{-i}^x)$ for any $i \in N$, $\theta \in \Theta$, and $x \in f(\theta)$, where $a^x = (\mathbf{S}^{(i),x}, m_i^x)_{i\in N} \in A$ is such that $g(a^x) = x$ and $a_i^x \in BR_i^{\pi(\theta)}(a_{-i}^x)$, i.e., a^x is a Nash* equilibrium associated with alternative $x \in f(\theta)$.

Then, (*i*) of the proposition follows from (*i*) of Nash* implementation.

For (*ii*), suppose that $x \in f(\theta)$ and $x \notin f(\theta')$ for some $\theta, \theta' \in \Theta$. Then, we need to have $j \in N$ such that $a_j^x \notin BR_j^{\pi(\theta')}(a_{-j}^x)$ because otherwise a^x would be Nash* at θ', and hence, $x \in f(\theta')$ by (*ii*) of Nash* implementation; a contradiction.

If j is not a weak sympathizer at $\pi(\theta')$, then $(ii.1)$ of the proposition follows.

If j is a weak sympathizer at $\pi(\theta')$ with $x \in C_j^{\pi(\theta')}(S_j^{\mu}(x,\theta))$, then this coupled with $a_j^x \notin BR_j^{\pi(\theta')}(a_{-j}^x)$ implies that $\mathbf{S}^{(j),x} \notin \mathcal{S}(f)$ and there exists $\bar{\mathbf{S}} \in \mathcal{S}(f)$ such that $g((\bar{\mathbf{S}}, m_j^x), a_{-j}^x) \in C_j^{\pi(\theta')}(S_j^{\mu}(x,\theta))$, because otherwise we would obtain a contradiction to j being a weak sympathizer at $\pi(\theta')$.[18] Also, if j were to be a weak sympathizer at $\pi(\theta)$, $x \in f(\theta)$ and a^x being Nash* at θ requires that $g((\tilde{\mathbf{S}}, m_j^x), a_{-j}^x) \notin C_j^{\pi(\theta)}(S_j^{\mu}(x,\theta))$ for all $\tilde{\mathbf{S}} \in \mathcal{S}(f)$ since $\mathbf{S}^{(j),x} \notin \mathcal{S}(f)$ and $x \in C_j^{\pi(\theta)}(S_j^{\mu}(x,\theta))$. ■

B Proofs

B.1 The proof of Theorem 1

The construction featured in the proof utilizes the following mechanism $\mu \in \mathcal{M}^*$ with $\mu = (A, g)$ defined as follows: $A_i := \mathcal{S} \times \Theta \times X \times \mathbb{N}$ where a generic member $a_i = (\mathbf{S}^{(i)}, \theta^{(i)}, x^{(i)}, k^{(i)}) \in A_i$ with $\mathbf{S}^{(i)} \in \mathcal{S}$, $\theta^{(i)} \in \Theta$, $x^{(i)} \in X$, and $k^{(i)} \in \mathbb{N}$ with the convention that $m_i = (\theta^{(i)}, x^{(i)}, k^{(i)})$ and $M_i := \Theta \times X \times \mathbb{N}$. The outcome function is defined via the Rules specified in Table 1.

The proof is presented via two claims. The first establishes (i) of Nash* implementation holds, while the second delivers (ii) of Nash* implementation.

Claim 1. *For all $\theta \in \Theta$ and for all $x \in f(\theta)$, define $a^x \in A$ by $a_i^x = (\mathbf{S}, \theta, x, 1)$ with $\mathbf{S} \in \mathcal{S}(f)$. Then, $a_i^x \in BR_i^{\pi(\theta)}(a_{-i}^x)$ for all $i \in N$ and $g(a^x) = x$.*

Proof. Rule 1 applies and $g(a^x) = x$. As $\mathbf{S}^{(i)} = \mathbf{S}$ for all $i \in N$, the individual deviations can only result in Rules 1 and 2. Hence, $O_i^{\mu}(a_{-i}^x) = S_i(x,\theta)$. Thus, if i is not a weak sympathizer at state $\pi(\theta)$, then (i) of consistency saying that $x \in C_i^{\pi(\theta)}(S_i(x,\theta))$ is equivalent to $a_i^x \in BR_i^{\pi(\theta)}(a_{-i}^x)$. If i is a weak sympathizer at $\pi(\theta)$, then $\mathbf{S} \in \mathcal{S}(f)$ and $x \in C_i^{\pi(\theta)}(S_i(x,\theta))$ (due to (i) of consistency) imply $a_i^x \in BR_i^{\pi(\theta)}(a_{-i}^x)$. ■

[18] Here, individual j announces an inconsistent profile of sets, $\mathbf{S}^{(j),x}$, in the Nash* equilibrium sustaining alternative x at state θ in mechanism $\mu \in \mathcal{M}^*$ even though he is a weak sympathizer at state $\pi(\theta)$. This displays that information extraction concerning the choice topography and the state demands the construction of the right mechanism because a weak sympathizer is not obliged to reveal the realized state and/or refrain from manipulating the outcome via inconsistent announcements.

Rule 1:	$g(a) = x$	if $\mathbf{S}^{(i)} = \mathbf{S}$ for all $i \in N \setminus \{i'\}$ for some $i' \in N,$ and $m_j = (\theta, x, \cdot)$ for all $j \in N$ with $x \in f(\theta),$
Rule 2:	$g(a) = \begin{cases} x' & \text{if } x' \in S_j(x,\theta) \text{ and } S_j(x,\theta) = \mathbf{S}\mid_{j,x,\theta}, \\ x & \text{otherwise.} \end{cases}$	if $\mathbf{S}^{(i)} = \mathbf{S}$ for all $i \in N \setminus \{i'\}$ for some $i' \in N,$ and $m_i = (\theta, x, \cdot)$ for all $i \in N \setminus \{j\}$ with $x \in f(\theta),$ and $m_j = (\theta', x', \cdot) \neq (\theta, x, \cdot),$
Rule 3:	$g(a) = x^{(i^*)}$ where $i^* = \min\{j \in N : k^{(j)} \geq \max_{i' \in N} k^{(i')}\}$	otherwise.

Table 1: The outcome function of the mechanism with three or more individuals.

Claim 2. *If $a^* \in A$ is a Nash* equilibrium of $\mu \in \mathcal{M}^*$ at some $\theta \in \Theta$, then $g(a^*) \in f(\theta)$. That is, $a^* \in A$ such that $a_i^* \in BR_i^{\pi(\theta)}(a_{-i}^*)$ for all $i \in N$ for some $\theta \in \Theta$ implies $g(a^*) \in f(\theta)$.*

Proof. Consider a Nash* equilibrium a^* at θ under Rule 1 such that $a_i^* = (\mathbf{S}', \theta', x', k')$ with $x' \in f(\theta')$, and $a_i^* \in BR_i^{\pi(\theta)}(a_{-i}^*)$ for all $i \in N$. Then, as Rule 1 holds $g(a^*) = x'$ and $O_i^\mu(a_{-i}^*) = S_i(x', \theta')$ for all $i \in N$ due to Rule 1 and Rule 2.

Notice that $\mathbf{S}' \in \mathcal{S}(f)$. Because, otherwise, $\mathbf{S}' \notin \mathcal{S}(f)$ and letting i be the weak sympathizer associated with the state $\pi(\theta)$ and i deviating to $a_i' = (\mathbf{S}'', m_i^*, a_{-i}^*)$ with $\mathbf{S}'' \in \mathcal{S}(f)$ and $m_i^* = (\theta', x', k')$ implies $g(\mathbf{S}'', m_i^*, a_{-i}^*) = g(a^*) = x' \in C_i^{\pi(\theta)}(S_i(x', \theta'))$ (due to $a_i^* \in BR_i^{\pi(\theta)}(a_{-i}^*)$ implying $x' = g(a^*) \in C_i^{\pi(\theta)}(O_i^\mu(a_{-i}^*))$ and $O_i^\mu(a_{-i}^*) = S_i(x', \theta')$) while $(\mathbf{S}'', m_i^*) \in BR_i^{\pi(\theta)}(a_{-i}^*)$ and $(\mathbf{S}', m_i^*) \notin BR_i^{\pi(\theta)}(a_{-i}^*)$, a contradiction to a^* being a Nash* equilibrium at θ.

Assume that $x' \notin f(\theta)$ or else the proof concludes. Because that $x' \in f(\theta')$ and $x' \notin f(\theta)$ and $\mathbf{S}' \in \mathcal{S}(f)$, by *(ii)* of consistency, there exists $j \in N$ such that $x' \notin C_j^{\pi(\theta)}(S_j(x', \theta'))$. Recall that $O_j^\mu(a_{-j}^*) = S_j(x', \theta')$. The desired contradiction is achieved because then $x' \notin C_j^{\pi(\theta)}(S_j(x', \theta'))$ implies $a_j^* \notin BR_j^{\pi(\theta)}(a_{-j}^*)$.

Another type of Nash* equilibrium a^* at θ under Rule 1 is one where there exists an individual i' such that $a^*_{i'} = (\mathbf{S}'', \theta', x', k')$ whereas $a^*_i = (\mathbf{S}', \theta', x', k')$ for all $i \in N \setminus \{i'\}$ with $\mathbf{S}' \neq \mathbf{S}''$. Then, by Rule 1 and Rule 3, $O^\mu_i(a^*_{-i}) = X$ for all $i \in N \setminus \{i'\}$ as any one of $i \neq i'$ could deviate to $a_i = (\mathbf{S}, \theta', y, k)$ with $\mathbf{S} \neq \mathbf{S}'$, $y \in X$ and $k > k'$. Since a^* is a Nash* equilibrium at θ, we observe that $g(a^*) \in C^{\pi(\theta)}_i(X)$ for all $i \neq i'$, a contradiction to the economic environment assumption.

Next, we establish that there cannot be a Nash* equilibrium under Rule 2 or 3. Consider any Nash* equilibrium $\bar{a} \in A$ under either Rule 2 or 3 in order to obtain a contradiction. In both of the cases, $O^\mu_j(\bar{a}_{-j}) = X$ for at least $n-1$ individuals j and $\bar{a}_i \in BR^{\pi(\theta)}_i(\bar{a}_{-i})$ for all $i \in N$ implies for at least $n-1$ individuals j we have $g(\bar{a}) \in C^{\pi(\theta)}_j(X)$. This is not possible due to the economic environment assumption. ■

B.2 The proof of Lemma 1

By hypothesis, we have that $\pi(\Theta) \subset \Omega^*$; $f_{\Omega^*} : \Omega^* \to \mathcal{X}$ is such that $f_{\Omega^*}(\pi(\theta)) = f(\theta)$ for all $\theta \in \Theta$. Let $\mathbf{S}_{\Omega^*} := (S^*_i(x, \omega))_{i \in N,\ \omega \in \Omega^*,\ x \in f_{\Omega^*}(\omega)}$ be a profile of sets consistent with f_{Ω^*}: (i) it must be that $x \in C^\omega_i(S^*_i(x, \omega))$ for all $i \in N$ and $\omega \in \Omega^*$ and $x \in f_{\Omega^*}(\omega)$; and (ii) having $x \in f_{\Omega^*}(\omega)$ and $x \notin f_{\Omega^*}(\omega')$ with $\omega, \omega' \in \Omega^*$ implies there is $j \in N$ such that $x \notin C^{\omega'}_j(S^*_j(x, \omega))$. Define $\mathbf{S}_\Theta := (S_i(x, \theta))_{i \in N,\ \theta \in \Theta,\ x \in f(\theta)}$ with $S_i(x, \theta) := S^*_i(x, \pi(\theta))$ for all $i \in N$ and $\theta \in \Theta$ and $x \in f(\theta)$.

Then, as $f(\theta) = f_{\Omega^*}(\pi(\theta))$ and $\pi(\Theta) \subset \Omega^*$, $x \notin C^{\pi(\theta')}_j(S_j(x', \theta'))$ for some $j \in N$ and $\theta' \in \Theta$ and $x' \in f(\theta')$ implies $x \notin C^{\pi(\theta')}_j(S^*_j(x', \pi(\theta')))$ for $j \in N$ and $\pi(\theta') \in \Omega^*$ and $x' \in f_{\Omega^*}(\pi(\theta'))$. Thus, $\mathbf{S}_\Theta$ satisfies (i) of consistency.

For (ii) of consistency suppose that $x \in f(\theta)$ and $x \notin f(\theta')$ for some $\theta, \theta' \in \Theta$. Then, as $\pi : \Theta \to \Omega^*$ we have that $\pi(\theta), \pi(\theta') \in \Omega^*$ and due to $f_{\Omega^*}(\pi(\tilde{\theta})) = f(\tilde{\theta})$ for all $\tilde{\theta} \in \Theta$ we have that $x \in f_{\Omega^*}(\pi(\theta))$ and $x \notin f_{\Omega^*}(\pi(\theta'))$. As $\mathbf{S}_{\Omega^*}$ is consistent with f_{Ω^*}, by (ii) of consistency, there is $j \in N$ such that $x \notin C^{\pi(\theta')}_j(S^*_j(x, \pi(\theta)))$. By construction, $S_j(x, \theta) = S^*_j(x, \pi(\theta))$. Thus, $x \notin C^{\pi(\theta')}_j(S_j(x, \theta))$. Hence, $\mathbf{S}_\Theta$ satisfies (ii) of consistency.

B.3 The proof of Theorem 2

The proof employs mechanism μ used in the proof of Theorem 1 (involving rules specified in Table 1). Moreover, every sympathizer at $\pi(\theta)$ is a weak sympathizer at

$\pi(\theta)$. Thus, the proof of Claim 1 can be used without any modifications to prove that for all $\theta \in \Theta$ and for all $x \in f(\theta)$, $a^x \in A$ defined by $a_i^x = (\mathbf{S}, \theta, x, 1)$ with $\mathbf{S} \in \mathcal{S}(f)$ is such that $a_i^x \in BR_i^{\pi(\theta)}(a_{-i}^x)$ for all $i \in N$ and $g(a^x) = x$. What remains to be shown is:

Claim 3. *If $a^* \in A$ is a Nash* equilibrium of $\mu \in \mathcal{M}^*$ at some $\theta \in \Theta$, then $g(a^*) \in f(\theta)$. I.e., $a^* \in A$ such that $a_i^* \in BR_i^{\pi(\theta)}(a_{-i}^*)$ for all $i \in N$ for some $\theta \in \Theta$ implies $g(a^*) \in f(\theta)$.*

Proof. The proof of the claim involves the analysis of three cases.

Case 1. *Let $a^* \in A$ be a Nash* equilibrium at $\theta \in \Theta$ such that Rule 1 holds. That is, $a_i^* = (\mathbf{S}^{(i)}, \theta', x', k')$ for all $i \in N$ with $\mathbf{S}^{(i')} = \mathbf{S}$ for all $i' \neq j$ for some $j \in N$ and $x' \in f(\theta')$. Then, $g(a^*) = x' \in f(\theta)$.*

Proof of Case 1. First, we prove that $\mathbf{S} \in \mathcal{S}(f)$. Therefore, in such Nash* equilibria, all but one player announce the same profile of sets that must be among the consistent profiles of sets with the SCC f.

If $\mathbf{S}^{(j)} = \mathbf{S}$, then without loss of generality letting the first player be one of the sympathizers at $\pi(\theta)$, we observe the following: If $\mathbf{S} \notin \mathcal{S}(f)$, then deviating to $\bar{a}_1 = (\bar{\mathbf{S}}, \theta', x', k')$ with $\bar{\mathbf{S}} \in \mathcal{S}(f)$ results in $g(\bar{\mathbf{S}}, m_1^*, a_{-1}^*) = g(\mathbf{S}, m_1^*, a_{-1}^*) = x'$ (due to Rule 1) and $x' \in C_i^{\pi(\theta)}(O_i^{\mu}(a_{-i}^*))$ for all $i \in N$ (since a^* is a Nash* equilibrium) where $m_1^* = (\theta', x', k')$. Thus, $a_1^* \notin BR_1^{\pi(\theta)}(a_{-1}^*)$, a contradiction to a^* being Nash* at θ.

However, if $\mathbf{S}^{(j)} \neq \mathbf{S}$ and j is not a sympathizer at $\pi(\theta)$, then we let without loss of generality one of the sympathizers at $\pi(\theta)$ be the first player and assume that $\mathbf{S} \notin \mathcal{S}(f)$. Then, agent 1 playing $\bar{a}_1 = (\bar{\mathbf{S}}, \theta', x', \bar{k})$ with $\bar{\mathbf{S}} \in \mathcal{S}(f)$ and $\bar{k} > k'$ results in $g(\bar{\mathbf{S}}, \bar{m}_1, a_{-1}^*) = g(\mathbf{S}, m_1^*, a_{-1}^*) = x'$ (due to Rules 1 and 3) and $x' \in C_i^{\pi(\theta)}(O_i^{\mu}(a_{-i}^*))$ for all $i \in N$ (since a^* is a Nash* equilibrium) where $m_1^* = (\theta', x', k')$ and $\bar{m}_1 = (\theta', x', \bar{k})$. Ergo, $a_1^* \notin BR_1^{\pi(\theta)}(a_{-1}^*)$, a contradiction to a^* being a Nash* equilibrium at θ.[19]

If $\mathbf{S}^{(j)} \neq \mathbf{S}$ but j is a sympathizer at $\pi(\theta)$, then we need another sympathizer $i^* \neq j$ at $\pi(\theta)$: Without loss of generality, let that individual be player 1 and assume $\mathbf{S} \notin \mathcal{S}(f)$.

[19]This is why we have to strengthen weak sympathy to sympathy, as the deviating individual has to change his integer choice as well.

Then, the same arguments of the previous paragraph delivers a contradiction.[20]

Suppose that $g(a^*) = x' \notin f(\theta)$ because otherwise the proof of Case 1 concludes. Then, $x' \in f(\theta')$ and $x \notin f(\theta)$ and $\mathbf{S} \in \mathcal{S}(f)$ with $\theta, \theta' \in \Theta$ implies (due to (ii) of consistency) there exists $i^* \in N$ such that $x' \notin C_{i^*}^{\pi(\theta)}(S_{i^*}(x', \theta'))$. There appears two subcases we need to check. The first is one where a^* is such that $\mathbf{S}^{(i)} = \mathbf{S}$ for all $i \in N$. Then, $O_i^{\mu}(a_{-i}^*) = S_i(x', \theta')$ (by Rules 1 and 2) and as a^* is Nash* it must be that $x' \in C_i^{\pi(\theta)}(O_i^{\mu}(a_{-i}^*))$ for all $i \in N$. Ergo, the desired contradiction is obtained as $x' \in C_{i^*}^{\pi(\theta)}(S_{i^*}(x', \theta'))$. The second subcase that we need to consider is one where a^* is such that $\mathbf{S}^{(i)} = \mathbf{S}$ for all $i \in N \setminus \{j\}$ for some $j \in N$ and $\mathbf{S}^{(j)} \neq \mathbf{S}$. Then, by Rules 1 and 2 and 3, $O_i^{\mu}(a_{-i}^*) = X$ for all $i \neq j$ and $O_j^{\mu}(a_{-j}^*) = S_j(x', \theta')$ where $S_j(x', \theta') \in \mathbf{S}$ and $\mathbf{S} \in \mathcal{S}(f)$ as was shown above. Because that a^* is a Nash* equilibrium at θ, we observe that $x' \in C_i^{\pi(\theta)}(X)$ for all $i \neq j$ while $x' \in C_j^{\pi(\theta)}(S_j(x', \theta'))$ and $x' \in f(\theta')$. But then (i) of Theorem 2 implies $x \in f(\theta)$, a contradiction with $x \notin f(\theta)$. ■

Case 2. *Consider a Nash* equilibrium a^* at θ in which Rule 2 applies. That is, let $a_i^* = (\mathbf{S}^{(i)}, m_i^*)$ with $\mathbf{S}^{(i)} = \mathbf{S}$ for all $i \in N \setminus \{i'\}$ for some $i' \in N$ and $m_j^* = (\theta', x', k')$ for all $j \in N \setminus \{\ell\}$ for some $\ell \in N$ with $\theta' \in \Theta$ and $x' \in f(\theta')$ while $m_\ell^* = (\theta'', x'', k'') \neq (\theta', x', k')$. Then, $g(a^*) \in f(\theta)$.*

Proof of Case 2. The first step is to prove that $\mathbf{S} \in \mathcal{S}(f)$. It should be pointed out that this establishes the observation that in all Nash* equilibria in which Rule 2 applies, all but one individual announce the same profile of sets which has to be one of the profiles of sets consistent with the SCC f.

Suppose that $\mathbf{S} \notin \mathcal{S}(f)$ and notice that there exists a sympathizer at $\pi(\theta)$ individual $i^* \neq i'$ with $\mathbf{S}^{(i^*)} = \mathbf{S}$ as there are at least two sympathizers at $\pi(\theta)$. Without loss of generality, let $i^* = 1$. If $\mathbf{S}^{(i')} \neq \mathbf{S}$, player 1 deviating to $\bar{a}_1 = (\bar{\mathbf{S}}, \bar{m}_1)$ where $\bar{\mathbf{S}} \in \mathcal{S}(f)$ and $\bar{m}_1 = (\tilde{\theta}, g(a^*), \bar{k})$ with $\tilde{\theta} \in \Theta$ and $\bar{k} > k', k''$ implies that Rule 3 applies and as a result $g(\bar{\mathbf{S}}, \bar{m}_1, a_{-1}^*) = g(\mathbf{S}, m_1^*, a_{-1}^*) = g(a^*)$ which is in $C_1^{\pi(\theta)}(O_1^{\mu}(a_{-1}^*))$ due to

[20] The need for an additional sympathizer arises due to this case. To see this, suppose that there is only one sympathizer at $\pi(\theta)$ and consider the situation when $\mathbf{S}^{(j)} \neq \mathbf{S}$ and j is the only sympathizer at $\pi(\theta)$. Then, $\mathbf{S} \notin \mathcal{S}(f)$ does not necessarily result in a contradiction as there is no other sympathizer at $\pi(\theta)$ among those who are announcing an inconsistent profile of sets $\mathbf{S}$. Hence, one of the agents whose opportunity set equals X must be inclined toward truthful revelation of consistency at $\pi(\theta)$.

a^* being a Nash* equilibrium at θ. But then, as player 1 is a sympathizer at $\pi(\theta)$, $a_1^* \notin BR_1^{\pi(\theta)}(a_{-1}^*)$, a contradiction to a^* being Nash*. If $\mathbf{S}^{(i')} = \mathbf{S}$, then all players are announcing $\mathbf{S}$; and hence, player 1 deviating to $\bar{a}_1 = (\bar{\mathbf{S}}, m_1^*)$ where $\bar{\mathbf{S}} \in \mathcal{S}(f)$ implies that Rule 1 applies and as a result $g(\bar{\mathbf{S}}, m_1^*, a_{-1}^*) = g(\mathbf{S}, m_1^*, a_{-1}^*) = g(a^*) \in C_1^{\pi(\theta)}(O_1^{\mu}(a_{-1}^*))$ as a^* is Nash* at θ. However, player 1 being a sympathizer at $\pi(\theta)$ implies $a_1^* \notin BR_1^{\pi(\theta)}(a_{-1}^*)$, contradicting to a^* being a Nash* equilibrium at θ.

Having established $\mathbf{S} \in \mathcal{S}(f)$, we note the following:

If $\mathbf{S}^{(i')} \neq \mathbf{S}$, then $O_i^{\mu}(a_{-i}^*) = X$ for all $i \neq i'$ (by any one of such $i \neq i'$ deviating to $\mathbf{S}^{(i)} \neq \mathbf{S}$ and choosing the highest integer and any alternative) while $O_{i'}^{\mu}(a_{-i'}^*) = S_{i'}(x', \theta')$ if $i' = \ell$ and $O_{i'}^{\mu}(a_{-i'}^*) = X$ if $i \neq \ell$ (by i' deviating to $m_{i'}' \neq (\theta', x', k')$ and making Rule 3 apply). Thus, if $i' = \ell$, $\mathbf{S} \in \mathcal{S}(f)$ and a^* being Nash* at θ implying $g(a^*) \in C_i^{\pi(\theta)}(X)$ for all $i \neq i'$ and $g(a^*) \in C_{i'}^{\pi(\theta)}(S_{i'}(x', \theta'))$ with $x' \in f(\theta')$ enable us to employ condition (i) of the hypothesis of Theorem 2 and conclude that $g(a^*) \in f(\theta)$. But if $i' \neq \ell$, then $\mathbf{S} \in \mathcal{S}(f)$ and a^* being Nash* at θ implying $g(a^*) \in C_i^{\pi(\theta)}(X)$ for all $i \in N$ results in $g(a^*) \in f(\theta)$ due to condition (ii) of the hypothesis of Theorem 2. Observe that when the environment features societal non-satiation and (ii) of the hypothesis of Theorem 2 is dispensed with, we cannot have a Nash* equilibrium a^* at θ in which Rule 2 applies and $\mathbf{S}^{(i')} \neq \mathbf{S}$ and $i' \neq \ell$. Because otherwise, $g(a^*) \in C_i^{\pi(\theta)}(X)$ for all $i \in N$, a contradiction to societal non-satiation.

If $\mathbf{S}^{(i')} = \mathbf{S}$, then $O_j^{\mu}(a_{-j}^*) = X$ for all $j \neq \ell$ (by any one of such $j \neq \ell$ deviating to $m_j' \neq m_j^*$) while $O_\ell^{\mu}(a_{-\ell}^*) = S_\ell(x', \theta')$ (by Rule 2). Hence, $\mathbf{S} \in \mathcal{S}(f)$ and a^* being Nash* at θ implying $g(a^*) \in C_j^{\pi(\theta)}(X)$ for all $j \neq \ell$ and $g(a^*) \in C_\ell^{\pi(\theta)}(S_\ell(x', \theta'))$ with $x' \in f(\theta')$ and condition (i) of Theorem 2 conduce to $g(a^*) \in f(\theta)$. ■

Case 3. *Let a^* be Nash* at θ under Rule 3. Then, $g(a^*) \in f(\theta)$.*

Proof of Case 3. Clearly, $O_i^{\mu}(a_{-i}^*) = X$ for all $i \in N$ and $g(a^*) \in C_i^{\pi(\theta)}(X)$ for all $i \in N$ (on account of a^* being Nash* at θ) leads to the conclusion that $g(a^*) \in f(\theta)$ thanks to condition (ii) of the hypothesis of the current theorem.

Insisting on societal non-satiation eliminates such Nash* equilibria: $O_i^{\mu}(a_{-i}^*) = X$ for all $i \in N$ and $g(a^*) \in C_i^{\pi(\theta)}(X)$ for all $i \in N$ result in a contradiction to societal non-satiation which requires that there is $j \in N$ such that $g(a^*) \notin C_j^{\pi(\theta)}(X)$. ■

These conclude the proof of the Claim 3, and hence, the proof of Theorem 2. ■

C Two individuals

For reasons of completeness, we present the notion of two-individual consistency, a necessary condition for Nash implementation with two individuals:

Definition 7. *Let $n = 2$. Given an SCC $f : \Theta \to \mathcal{X}$, a profile of choice sets $\mathbf{S} := (S_i(x, \theta))_{i\in\{1,2\},\ \theta\in\Theta,\ x\in f(\theta)}$ is* two-individual consistent *with $f : \Theta \to \mathcal{X}$ if*

(i) $x \in C_i^{\pi(\theta)}(S_i(x, \theta))$, for all $i \in \{1, 2\}$ and $\theta \in \Theta$ and $x \in f(\theta)$; and

(ii) $x \in f(\theta)$ and $x \notin f(\theta')$ for some $\theta, \theta' \in \Theta$ implies that either $x \notin C_i^{\pi(\theta')}(S_i(x, \theta))$ or $x \notin C_j^{\pi(\theta')}(S_j(x, \theta))$ for $i, j \in \{1, 2\}, i \neq j$; and

(iii) there exists a function $e : X \times \Theta \times X \times \Theta \to X$ such that for any $\theta, \theta' \in \Theta$, $x \in f(\theta)$, and $x' \in f(\theta')$

(iii.1) $e(x, \theta, x', \theta') \in S_1(x, \theta) \cap S_2(x', \theta')$; and

(iii.2) $e(x, \theta, x', \theta') \in f(\theta^*)$ if $e(x, \theta, x', \theta') \in C_1^{\pi(\theta^*)}(S_1(x, \theta)) \cap C_2^{\pi(\theta^*)}(S_2(x', \theta'))$.

$\mathcal{S}^{II}(f)$ denotes the set of profiles of sets $\mathbf{S}$ that are two-individual consistent with f.

In words, a profile $\mathbf{S} := (S_i(x, \theta))_{i=1,2,\ \theta\in\Theta,\ x\in f(\theta)}$ is two-individual consistent with an SCC $f : \Theta \to \mathcal{X}$ if (i) and (ii) of consistency (Definition 1) along with the following hold: (iii) for any pair of states and f-optimal alternatives for the corresponding states (x, θ) and (x', θ'), we have that $(iii.1)$ there exists a function e mapping this pair into the intersection of the corresponding choice sets, $S_1(x, \theta) \cap S_2(x', \theta')$, such that $(iii.2)$ the resulting alternative $e(x, \theta, x', \theta')$ is f-optimal at state θ^* whenever it is chosen at $\pi(\theta^*)$ by individuals 1 and 2 from $S_1(x, \theta)$ and $S_2(x', \theta')$), respectively. Item (iii) is novel to the case with two individuals.

Now, for reasons of completeness, we state and prove the corresponding necessity result discussed in de Clippel (2014).

Theorem 3. *Let $n = 2$. If an SCC $f : \Theta \to \mathcal{X}$ is Nash implementable by a mechanism $\mu \in \mathcal{M}$, then $\mathcal{S}^{II}(f) \neq \emptyset$. That is, the existence of a two-individual consistent profile is necessary for Nash Implementation.*

Proof of Theorem 3. Suppose that SCC $f : \Theta \to \mathcal{X}$ is Nash implementable by a mechanism $\mu = (g, A) \in \mathcal{M}$. Thus, for any $\theta \in \Theta$ and $x \in f(\theta)$, (by (i) of Nash implementation) there exists $a^x \in A$ such that $g(a^x) = x$ and $x \in C_i^{\pi(\theta)}(O_i^\mu(a_{-i}^x))$ for $i = 1, 2$. For any $i = 1, 2$ and $\theta \in \Theta$ and $x \in f(\theta)$, define $\mathbf{S}^\mu := (S_i^\mu(x, \theta))_{i=\{1,2\},\ \theta\in\Theta,\ x\in f(\theta)}$ by $S^\mu(x, \theta) := O_i^\mu(a_{-i}^x)$ where $a^x \in A$ is such that $g(a^x) = x$ and $x \in C_i^{\pi(\theta)}(O_i^\mu(a_{-i}^x))$.

First, we show that $\mathbf{S}^\mu$ satisfies (i) and (ii) of Definition 7: notice that for any $i \in \{1, 2\}$ and $\theta \in \Theta$ and $x \in f(\theta)$ it must be that $x \in C_i^{\pi(\theta)}(S_i^\mu(x, \theta))$ as $g(a^x) = x$ and $x \in C_i^{\pi(\theta)}(O_i^\mu(a_{-i}^x))$ and $S_i^\mu(x, \theta) = O_i^\mu(a_{-i}^x)$ for all $i \in \{1, 2\}$. Moreover, for any $\theta, \theta' \in \Theta$ with $x \in f(\theta)$ but $x \notin f(\theta')$, there exists $j \in \{1, 2\}$ such that $g(a^x) \notin C_j^{\pi(\theta')}(O_j^\mu(a_{-j}^x))$ (because otherwise a^x would be Nash at θ' and by (ii) of Nash implementation x must be in $f(\theta')$). Hence, $x \notin C_j^{\pi(\theta')}(S_j^\mu(x, \theta))$.

Let $e : X \times \Theta \times X \times \Theta \to X$ be such that $e(x, \theta, x', \theta') := g(a_1^x, a_2^{x'})$ where $a^x \in A$ is such that $g(a^x) = x$ and $x \in C_i^{\pi(\theta)}(O_i^\mu(a_{-i}^x))$ for $i = 1, 2$, and $a^{x'} \in A$ is such that $g(a^{x'}) = x'$ and $x' \in C_j^{\pi(\theta)}(O_j^\mu(a_{-j}^{x'}))$ for $j = 1, 2$. Then $g(a_1^x, a_2^{x'}) \in O_1^\mu(a_2^{x'}) \cap O_2^\mu(a_1^x) = S_1^\mu(x, \theta) \cap S_2^\mu(x', \theta')$, implying $(iii.1)$ of Definition 7. Moreover, if $e(x, \theta, x', \theta') \in C_1^{\pi(\theta^*)}(S_1^\mu(x, \theta)) \cap C_2^{\pi(\theta^*)}(S_2^\mu(x', \theta'))$, then $g(a_1^x, a_2^{x'}) \in C_1^{\pi(\theta^*)}(O_1^\mu(a_2^{x'})) \cap C_2^{\pi(\theta^*)}(O_2^\mu(a_1^x))$. This means that $(a_1^x, a_2^{x'})$ is a Nash equilibrium of μ at θ^*. Hence, by (ii) of Nash implementation, we must have $g(a_1^x, a_2^{x'}) \in f(\theta^*)$, implying $(iii.2)$ of Definition 7. ■

The following is a sufficiency result with two individuals using a mechanism that elicits the relevant information about the societal choice topography from the two individuals. It can be also regarded as a two-individual robustness check for Theorem 1. Unsurprisingly, with two individuals the sufficiency conditions become more stringent as in Moore and Repullo (1990), Dutta and Sen (1991), and Dutta and Sen (2012).

Theorem 4. *Suppose $n = 2$, both of the individuals are sympathizers at every $\pi(\theta)$ with $\theta \in \Theta$, the SCC $f : \Theta \to \mathcal{X}$ is such that $\mathcal{S}^{II}(f) \neq \emptyset$, and the following hold:*

(i) *for any $\mathbf{S} \in \mathcal{S}^{II}(f)$, $x \in C_i^{\pi(\theta)}(S_i(x', \theta'))$ with $\theta' \in \Theta$, $x' \in f(\theta')$, $S_i(x', \theta') =$*

$\mathbf{S}\,|_{i,x',\theta'}$, *and* $x \in C_j^{\pi(\theta)}(X)$ *with* $i,j = 1,2$ *and* $j \neq i$ *implies* $x \in f(\theta)$*; and*

(*ii*) $x \in C_1^{\pi(\theta)}(X) \cap C_2^{\pi(\theta)}(X)$ *implies* $x \in f(\theta)$*; and*

(*iii*) *for any* $\mathbf{S}, \tilde{\mathbf{S}} \in \mathcal{S}^{II}(f)$, $S_1(x,\theta) \cap \tilde{S}_2(x',\theta') \neq \emptyset$ *with* $\theta, \theta' \in \Theta$, $x \in f(\theta)$, $x' \in f(\theta')$, $S_1(x,\theta) = \mathbf{S}|_{1,x,\theta}$, *and* $\tilde{S}_2(x',\theta') = \tilde{\mathbf{S}}|_{2,x',\theta'}$*; and*

(*iv*) *for any* $\mathbf{S}, \tilde{\mathbf{S}} \in \mathcal{S}^{II}(f)$, $x^* \in C_1^{\pi(\theta^*)}(S_1(x,\theta)) \cap C_2^{\pi(\theta^*)}(\tilde{S}_2(x',\theta'))$ *with* $\theta, \theta' \in \Theta$, $x \in f(\theta)$, $x' \in f(\theta')$, $S_1(x,\theta) = \mathbf{S}|_{1,x,\theta}$, *and* $\tilde{S}_2(x',\theta') = \tilde{\mathbf{S}}|_{2,x',\theta'}$ *implies* $x^* \in f(\theta^*)$.

Then, f *is Nash** *implementable by a mechanism* $\mu \in \mathcal{M}^*$*, and for each* $\theta \in \Theta$*, in every Nash** *equilibrium of* μ *at* θ*, each agent announces a profile of sets two-individual consistent with* f*.*

We show that the hypothesis of this theorem is satisfied in the two-individual example we present in Section C.1.

Theorem 4 keeps the hypothesis of Theorem 2: the existence of a profile of sets two-individual consistent with f and (*i*) and (*ii*) along with the need of having at least two sympathizers at every prevailing state. Even then, the planner has to make sure that the mechanism elicits the information about the choice topography and the state; this is no easy task as being a sympathizer does not imply that the agent is compelled to reveal the realized state and/or refrain from manipulating the outcome (via announcements of inconsistent profiles) when the mechanism is not designed right.

The rest of the hypothesis of Theorem 4 is specific to the case of two individuals.

An interesting and useful observation concerning the case with two individuals is about the relation between an individual's actions and the other's opportunity sets. Given mechanism $\mu = (A, g)$, any $a_i \in A_i$ corresponds to $O_j^{\mu}(a_i)$, $i, j = 1,2$ and $i \neq j$. Therefore, individual i choosing action a_i can be thought of as offering agent j the opportunity set $O_j^{\mu}(a_i)$, $i, j = 1,2$ and $i \neq j$.

In the mechanism that we employ to prove Theorem 4, each individual announces a profile of choice sets. Sympathy ensures that, in equilibrium, each agent announces a two-individual consistent profile of sets, but not necessarily the same. Consequently, we obtain hypothesis (*iii*) of Theorem 4, ensuring that the two-individual mechanism is well-defined (under Rule 3 specified in Table 2): because a choice set listed in individual 1's announcement corresponds to an opportunity set of agent 2 and vice versa, each of the choice sets of individual 1 listed in any one of the two-individual consistent profile of sets for agent 1 must have a non-empty intersection with each of the choice sets of individual 2 listed in any one of the two-individual consistent profile of sets for agent 2. In other words, hypothesis (*iii*) of Theorem 4 compels the non-emptiness requirement of two-individual consistency, specified by (*iii*.1) of Definition 7, to hold across any pair of profiles of sets, both two-individual consistent with f.

Item (*iii*.1) of two-individual consistency implies that there is a selection (function) identifying the alternative corresponding to the intersection of the actions where the action of individual i corresponds to the choice set of j listed in i's announcement, $i, j = 1, 2$ and $i \neq j$. On the other hand, item (*iii*.2) ensures that if each of such alternatives is chosen by both individuals at a state $\pi(\theta^*)$ from the corresponding choice sets, then it must be f-optimal at θ^*. Hypothesis (*iv*) of Theorem 4 strengthens this requirement by demanding the following: it holds for any alternatives—and not just for the ones identified by the function specified in item (*iii*.1)—that are chosen at some state $\pi(\theta^*)$ by both individuals from the choice set of individual 1 listed in any one of the two-individual consistent profile of sets for agent 1 and from agent 2's choice set listed in any one of the two-individual consistent profile of sets for individual 2. Hypothesis (*iv*) implies that (*iii*.2) of two-individual consistency holds across any pair of profiles of sets, both of which are two-individual consistent with f. We may weaken this requirement a bit further by employing an identification function as is done in two-individual consistency. This is precisely what is done in Footnote 21. However, we think the current form of the hypothesis with items (*iii*) and (*iv*) of Theorem 4 is more user friendly.

Items (*iii*) and (*iv*) demand the associated requirements hold across any pair of two-individual consistent profiles of sets. Therefore, they involve slightly stronger requirements when compared with those implied by the necessary condition, the two-individual consistency. On the other hand, when there is a unique profile of choice sets two-individual consistent with the SCC, the alternative hypothesis in Footnote 21 is equivalent to two-individual consistency.

Proof of Theorem 4. We employ the following mechanism $\mu \in \mathcal{M}^*$ with $\mu = (A, g)$ defined as follows: A_i equals

$$\left\{(\mathbf{S}^{(i)}, \theta^{(i)}, c^{(i)}, x^{(i)}, y^{(i)}, k^{(i)}) \in \mathcal{S}^* \times \Theta \times \{F, NF\} \times X \times X \times \mathbb{N} : x^{(i)} \in f(\theta^{(i)})\right\} \quad (2)$$

where a generic member is $a_i = (\mathbf{S}^{(i)}, \theta^{(i)}, c^{(i)}, x^{(i)}, y^{(i)}, k^{(i)})$ with $\mathbf{S}^{(i)} \in \mathcal{S}^*$ which is to be defined in the following paragraph, $\theta^{(i)} \in \Theta$, $c^{(i)} \in \{F, NF\}$ and $x^{(i)} \in f(\theta^{(i)})$, $y^{(i)} \in X$, and $k^{(i)} \in \mathbb{N}$ with the convention that $m_i = (\theta^{(i)}, c^{(i)}, x^{(i)}, y^{(i)}, k^{(i)})$ and $M_i := \Theta \times \{F, NF\} \times X^2 \times \mathbb{N}$.

$\mathcal{S}^*$ consists of profiles of sets in $\mathcal{S}$ such that for any $\theta, \theta' \in \Theta$ and $x \in f(\theta)$ and $x' \in f(\theta')$, it must be that $S_1(x, \theta) \cap S_2(x', \theta') \neq \emptyset$ for any $\mathbf{S}, \mathbf{S}'$ such that $S_1(x, \theta) = \mathbf{S}|_{1,x,\theta}$ and $S_2(x', \theta') = \mathbf{S}'|_{2,x',\theta'}$. Observe that (iii) of the hypothesis of Theorem 4 entails the implication that $\mathcal{S}^{II}(f) \subset \mathcal{S}^*$. This coupled with $\mathcal{S}^{II}(f) \neq \emptyset$ implies that $\mathcal{S}^* \neq \emptyset$.

The outcome function $g : A \to X$ is specified via the rules presented in Table 2.[21]

The proof of Theorem 4 follows from the following: Claims 4 and 5 establish (i) and (ii) of Nash* implementation, respectively (see Definition 4).

Claim 4. *Let $\theta \in \Theta$ and $x \in f(\theta)$ and define $a^x \in A$ by $a_i^x = (\mathbf{S}, \theta, NF, x, x, 1)$ with $\mathbf{S} \in \mathcal{S}(f)$ and $i = 1, 2$. Then, a^x is a Nash* equilibrium of μ at θ, i.e., $a_i^x \in BR_i^{\pi(\theta)}(a_j^x)$ for $i, j = 1, 2$ and $i \neq j$, and $g(a^x) = x$.*

Proof. Rule 1 applies. Thus, $g(a^x) = x$. Due to Rules 1, 2.1, 2.2, and 3, $O_i^\mu(a_j^x) = S_i(x, \theta)$ with $S_i(x, \theta) = \mathbf{S}|_{i,x,\theta}$ and $\mathbf{S} \in \mathcal{S}^{II}(f)$ for $i, j = 1, 2$ and $i \neq j$. As i is a

[21] Hypothesis (iii) and (iv) of Theorem 4 can be replaced with the following slightly weaker condition, which coincides with (iii) of two-individual consistency whenever there is a unique two-individual consistent profile of sets, i.e., $\#\mathcal{S}^{II}(f) = 1$ (in that case, $\mathcal{S}^{II}(f) \in \mathcal{S}^*$ follows by definition of two-individual consistency):

(iii) Suppose that $\mathcal{S}^{II}(f) \subset \mathcal{S}^*$ and there exists a function $e : \mathcal{S}^* \times \mathcal{S}^* \times X \times \Theta \times X \times \Theta \to X$ such that for any $(\mathbf{S}, \mathbf{S}', x, \theta, x', \theta')$ with $\mathbf{S}, \mathbf{S}' \in \mathcal{S}^*$ and $\theta, \theta' \in \Theta$ and $x \in f(\theta)$ and $x' \in f(\theta')$

- $(iii.1)$ $e(\mathbf{S}, \mathbf{S}', x, \theta, x', \theta') \in S_1'(x', \theta') \cap S_2(x, \theta)$ where $S_1'(x', \theta') = \mathbf{S}'|_{1,x',\theta'}$ and $S_2(x, \theta) = \mathbf{S}|_{2,x,\theta}$; and
- $(iii.2)$ $e(\mathbf{S}, \mathbf{S}', x, \theta, x', \theta') \in C_1^{\pi(\theta^*)}(S_1'(x', \theta')) \cap C_2^{\pi(\theta^*)}(S_2(x, \theta))$ with $\theta^* \in \Theta$ and $S_1'(x', \theta') = \mathbf{S}'|_{1,x',\theta'}$ and $S_2(x, \theta) = \mathbf{S}|_{2,x,\theta}$ implies $e(\mathbf{S}, \mathbf{S}', x, \theta, x', \theta') \in f(\theta^*)$.

Then, modifying the outcome function by requiring $g(a) = e(\mathbf{S}^{(1)}, \mathbf{S}^{(2)}, x^{(1)}, \theta^{(1)}, x^{(2)}, \theta^{(2)})$ in Rule 3 ensures that the same proof goes through.

Rule 1 :	$g(a) = x'$	if $a_1 = a_2 = (\mathbf{S}', \theta', NF, x', y', k')$.
Rule 2.1 :	$g(a) = \begin{cases} y^{(1)} & \text{if } y^{(1)} \in S_1(x^{(2)}, \theta^{(2)}) \\ & \quad \text{and } S_1(x^{(2)}, \theta^{(2)}) = \mathbf{S}^{(2)}\|_{1,x^{(2)},\theta^{(2)}}, \\ x^{(2)} & \text{otherwise.} \end{cases}$	if $a_1 = (\mathbf{S}^{(1)}, \theta^{(1)}, F, x^{(1)}, y^{(1)}, k^{(1)})$ and $a_2 = (\mathbf{S}^{(2)}, \theta^{(2)}, NF, x^{(2)}, y^{(2)}, k^{(2)})$.
Rule 2.2 :	$g(a) = \begin{cases} y^{(2)} & \text{if } y^{(2)} \in S_2(x^{(1)}, \theta^{(1)}) \\ & \quad \text{and } S_2(x^{(1)}, \theta^{(1)}) = \mathbf{S}^{(1)}\|_{2,x^{(1)},\theta^{(1)}}, \\ x^{(1)} & \text{otherwise.} \end{cases}$	if $a_1 = (\mathbf{S}^{(1)}, \theta^{(1)}, NF, x^{(1)}, y^{(1)}, k^{(1)})$ and $a_2 = (\mathbf{S}^{(2)}, \theta^{(2)}, F, x^{(2)}, y^{(2)}, k^{(2)})$.
Rule 3 :	$g(a) \in S_1(x^{(2)}, \theta^{(2)}) \cap S_2(x^{(1)}, \theta^{(1)})$ with $S_1(x^{(2)}, \theta^{(2)}) = \mathbf{S}^{(2)}\|_{1,x^{(2)},\theta^{(2)}}$ and $S_2(x^{(1)}, \theta^{(1)}) = \mathbf{S}^{(1)}\|_{2,x^{(1)},\theta^{(1)}}$.	if $a_1 = (\mathbf{S}^{(1)}, \theta^{(1)}, NF, x^{(1)}, y^{(1)}, k^{(1)})$ and $a_2 = (\mathbf{S}^{(2)}, \theta^{(2)}, NF, x^{(2)}, y^{(2)}, k^{(2)})$. and $a_1 \neq a_2$.
Rule 4 :	$g(a) = \begin{cases} x^{(1)} & \text{if } k^{(1)} \geq k^{(2)}, \\ x^{(2)} & \text{otherwise.} \end{cases}$	otherwise.

Table 2: The outcome function of the mechanism with two individuals.

sympathizer and $\mathbf{S} \in \mathcal{S}^{II}(f)$, $x \in C_i^{\pi(\theta)}(S_i(x, \theta))$ (following from (i) of two-individual consistency) implies $a_i^x \in BR_i^{\pi(\theta)}(a_j^x)$, with $i, j = 1, 2$ and $i \neq j$. ■

Claim 5. *If $a^* \in A$ is a Nash* equilibrium of $\mu \in \mathcal{M}^*$ at some $\theta \in \Theta$, then $g(a^*) \in f(\theta)$. That is, $a^* \in A$ such that $a_i^* \in BR_i^{\pi(\theta)}(a_j^*)$ for all $i, j = 1, 2$ with $i \neq j$ for some $\theta \in \Theta$ implies $g(a^*) \in f(\theta)$.*

Proof. In what follows, we go over four possible cases:

Case 1. *Let $a^* \in A$ be a Nash* equilibrium at $\theta \in \Theta$ such that Rule 1 holds. That is, $a_1^* = a_2^* = (\mathbf{S}', \theta', NF, x', y', k')$. Then, $g(a^*) \in f(\theta)$.*

Proof of Case 1. We first establish that $\mathbf{S}' \in \mathcal{S}^{II}(f)$. Therefore, in every Nash* equilibrium under Rule 1, both of the individuals announce the same profile of sets that must be two-individual consistent with f.

Suppose $\mathbf{S}' \notin \mathcal{S}^{II}(f)$, and consider $a_1' = (\bar{\mathbf{S}}, m_1')$ where $\bar{\mathbf{S}} \in \mathcal{S}^{II}(f)$ and $m_1' = (\theta', F, x', x', 1)$. Because $x' \in f(\theta')$ by the defining property of A_1 given in (2), Rule

2.1 applies, and using x' as $y^{(1)}$ with the property that $x' \in S_1(x', \theta')$ with $S_1(x', \theta') = \mathbf{S}'|_{1,x',\theta'}$ (recall that $\mathbf{S}' \in \mathcal{S}^* \subset \mathcal{S}$ and the defining property of $\mathcal{S}$ requires $x' \in S_1(x', \theta')$ regardless whether or not $\mathbf{S}'$ is consistent) implies $g(\bar{\mathbf{S}}, m_1', a_2^*) = g(\mathbf{S}', m_1^*, a_2^*) = x' \in C_1^{\pi(\theta)}(O_1^{\mu}(a_2^*))$ (due to a^* being a Nash* equilibrium) where $m_1^* = (\theta', NF, x', y', k')$. This coupled with $\bar{\mathbf{S}} \in \mathcal{S}^{II}(f)$ and $\mathbf{S}' \notin \mathcal{S}^{II}(f)$ results in $a_1^* \notin BR_1^{\pi(\theta)}(a_2^*)$ (as individual 1 is a sympathizer), contradicting to a^* being Nash* at θ.

If $g(a^*) = x' \in f(\theta)$, the proof concludes. Hence, assume that $x' \notin f(\theta)$. Notice that $O_i^{\mu}(a_j^*) = S_i(x', \theta')$ due to Rules 1 and 2.i and 3 with $i, j = 1, 2$ and $i \neq j$. Then, $\mathbf{S}' \in \mathcal{S}^{II}(f)$ and (by (ii) of two-individual consistency) $x' \in f(\theta')$ and $x' \notin f(\theta)$ imply that there exists $i = 1, 2$ such that $x' \notin C_i^{\pi(\theta)}(S_i(x', \theta'))$; this is equivalent to there being $i = 1, 2$ such that $x' \notin C_i^{\pi(\theta)}(O_i^{\mu}(a_j^*))$ with $j \neq i$. But this is in contradiction with a^* being a Nash* equilibrium at θ. ■

Case 2. *Let $a^* \in A$ be Nash* at $\theta \in \Theta$ such that Rule 2.1 holds. That is, $a_1^* = (\mathbf{S}^{(1)}, \theta^{(1)}, F, x^{(1)}, y^{(1)}, k^{(1)})$ and $a_2^* = (\mathbf{S}^{(2)}, \theta^{(2)}, NF, x^{(2)}, y^{(2)}, k^{(2)})$. Then, $g(a^*) \in f(\theta)$.*

Proof of Case 2. Observe that by (2), $a_i^* \in A_i$ implies $x^{(i)} \in f(\theta^{(i)})$ for $i = 1, 2$. Our first step is to prove that $\mathbf{S}^{(2)} \in \mathcal{S}^{II}(f)$. Suppose not. Consider $a_2' = (\bar{\mathbf{S}}, \bar{m}_2)$ where $\bar{\mathbf{S}} \in \mathcal{S}^{II}(f)$ and $\bar{m}_2 = (\theta^{(2)}, F, g(a^*), y^{(2)}, \bar{k})$ with $\bar{k} > k^{(1)}$. Then, Rule 4 applies and $g(\bar{\mathbf{S}}, \bar{m}_2, a_1^*) = g(\mathbf{S}^{(2)}, m_2^*, a_1^*) = g(a^*)$, which must be in $C_2^{\pi(\theta)}(O_2^{\mu}(a_1^*))$ as a^* is a Nash* equilibrium where $m_2^* = (\theta^{(2)}, NF, x^{(2)}, y^{(2)}, k^{(2)})$. But as $\mathbf{S}^{(2)} \notin \mathcal{S}^{II}(f)$ and individual 2 is a sympathizer, $a_2^* \notin BR_2^{\pi(\theta)}(a_1^*)$; contradicting to a^* being Nash* at θ.

Before showing $g(a^*)$ must be in $f(\theta)$, we wish to prove that $\mathbf{S}^{(1)} \in \mathcal{S}^{II}(f)$ as well. This establishes that both individuals announce a two-individual consistent profile of sets, yet not necessarily the same, in every Nash* equilibrium under Rule 2.1. Suppose $\mathbf{S}^{(1)} \notin \mathcal{S}^{II}(f)$ and consider $a_1' = (\bar{\mathbf{S}}, m_1^*)$ where $\bar{\mathbf{S}} \in \mathcal{S}^{II}(f)$ and $m_1^* = (\theta^{(1)}, F, x^{(1)}, y^{(1)}, k^{(1)})$. So, Rule 2.1 still applies at (a_1', a_2^*), and we have $g(\bar{\mathbf{S}}, m_1^*, a_2^*) = g(\mathbf{S}^{(1)}, m_1^*, a_2^*) = g(a^*)$, which must be in $C_1^{\pi(\theta)}(O_1^{\mu}(a_2^*))$ as a^* is a Nash* equilibrium. Because $\mathbf{S}^{(1)} \notin \mathcal{S}^{II}(f)$ and agent 1 is a sympathizer, $a_1^* \notin BR_1^{\pi(\theta)}(a_2^*)$; a contradiction.

Notice that due to Rules 1, 2.1, and 3 we have that $O_1^{\mu}(a_2^*) = S_1(x', \theta')$ with $x' \in f(\theta')$ and $S_1(x', \theta') = \mathbf{S}^{(2)}|_{1,x',\theta'}$ while $\mathbf{S}^{(2)}$ is consistent with f, i.e., $\mathbf{S}^{(2)} \in \mathcal{S}^{II}(f)$. Note that $g(a^*) \in C_1^{\pi(\theta)}(S_1(x', \theta'))$ follows from $O_1^{\mu}(a_2^*) = S_1(x', \theta')$ and a^* being a

Nash* equilibrium at θ. Moreover, $O_2^\mu(a_1^*) = X$, which follows from Rules 2.1 and 4. Thus, $g(a^*) \in C_2^{\pi(\theta)}(X)$ is due to a^* being Nash* at θ. Therefore, (i) of the hypothesis of Theorem 4 applies, and we conclude $g(a^*) \in f(\theta)$. ■

We omit the proof of the case of Nash* equilibria under Rule 2.2 due to symmetry.

Case 3. *Let $a^* \in A$ be a Nash* equilibrium at $\theta \in \Theta$ such that Rule 3 holds. That is, $a_1^* = (\mathbf{S}^{(1)}, \theta^{(1)}, NF, x^{(1)}, y^{(1)}, k^{(1)})$ and $a_2^* = (\mathbf{S}^{(2)}, \theta^{(2)}, NF, x^{(2)}, y^{(2)}, k^{(2)})$ where $a_1^* \neq a_2^*$. Then, $g(a^*) \in f(\theta)$.*

Proof of Case 3. Notice that by (2), $a_i^* \in A_i$ implies $x^{(i)} \in f(\theta^{(i)})$ for $i = 1, 2$. Let $g(a^*)$ be an arbitrary member of $S_1(x^{(2)}, \theta^{(2)}) \cap S_2(x^{(1)}, \theta^{(1)}) \neq \emptyset$ with $S_1(x^{(2)}, \theta^{(2)}) = \mathbf{S}^{(2)}|_{1,x^{(2)},\theta^{(2)}}$ and $S_2(x^{(1)}, \theta^{(1)}) = \mathbf{S}^{(1)}|_{2,x^{(1)},\theta^{(1)}}$ where $\mathbf{S}^{(1)}, \mathbf{S}^{(2)} \in \mathcal{S}^*$. As we intend to make use of (iv) of the hypothesis of Theorem 4, we first have to prove that $\mathbf{S}^{(1)}, \mathbf{S}^{(2)} \in \mathcal{S}^{II}(f)$. Thus, both individuals announce a two-individual consistent profile of sets (but not necessarily the same) in every Nash* equilibrium under Rule 3.

Suppose that $\mathbf{S}^{(1)} \notin \mathcal{S}^{II}(f)$ and consider $a_1' = (\bar{\mathbf{S}}, \bar{m}_1)$ where $\bar{\mathbf{S}} \in \mathcal{S}^{II}(f)$ and $\bar{m}_1 = (\theta^{(1)}, F, x^{(1)}, g(a^*), k^{(1)})$. Note that then Rule 2.1 applies and as $g(a^*) \in S_1(x^{(2)}, \theta^{(2)})$ (due to $g(a^*) \in S_1(x^{(2)}, \theta^{(2)}) \cap S_2(x^{(1)}, \theta^{(1)})$) we have that $g(\bar{\mathbf{S}}, \bar{m}_1, a_2^*) = g(\mathbf{S}^{(1)}, m_1^*, a_2^*) = g(a^*)$, which is in $C_1^{\pi(\theta)}(O_1^\mu(a_2^*))$ as a^* is Nash* at θ with $m_1^* = (\theta^{(1)}, NF, x^{(1)}, y^{(1)}, k^{(1)})$. Since $\mathbf{S}^{(1)} \notin \mathcal{S}^{II}(f)$ and agent 1 is a sympathizer, $a_1^* \notin BR_1^{\pi(\theta)}(a_2^*)$; contradicting to a^* being Nash* at θ. Showing $\mathbf{S}^{(2)} \in \mathcal{S}^{II}(f)$ involves replicating the arguments presented in this paragraph for individual 2.

Rules 1, 2.1, and 3 imply $O_i^\mu(a_j^*) = S_i(x^{(j)}, \theta^{(j)})$ with $x^{(j)} \in f(\theta^{(j)})$ (due to (2)) and $S_i(x^{(j)}, \theta^{(j)}) = \mathbf{S}^{(j)}|_{i,x^{(j)},\theta^{(j)}}$ while $\mathbf{S}^{(j)} \in \mathcal{S}^{II}(f)$, $i, j = 1, 2$ and $i \neq j$. Note that $g(a^*) \in C_i^{\pi(\theta)}(S_i(x^{(j)}, \theta^{(j)}))$ follows from $O_i^\mu(a_j^*) = S_i(x^{(j)}, \theta^{(j)})$, and a^* being Nash* at θ, for both $i, j = 1, 2$ and $i \neq j$. Therefore, (iv) of the hypothesis of Theorem 4 applies, and we conclude $g(a^*) \in f(\theta)$. ■

Case 4. *Let a^* be Nash* at θ under Rule 4. Then, $g(a^*) \in f(\theta)$.*

Proof of Case 4. It is clear that $O_i^\mu(a_j^*) = X$ for all $i, j = 1, 2$ and $i \neq j$. $g(a^*) \in C_i^{\pi(\theta)}(X)$ for all $i = 1, 2$ thanks to a^* being Nash* at θ, which leads to the conclusion that $g(a^*) \in f(\theta)$ due to condition (ii) of the hypothesis of the theorem.

Next, we display that in every Nash* equilibrium a^* under Rule 4 at θ, it must be that $a_i^* = (\mathbf{S}^{(i)}, m_i^*)$ with $\mathbf{S}^{(i)} \in \mathcal{S}^{II}(f)$, $i = 1, 2$. Suppose $\mathbf{S}^{(i)} \notin \mathcal{S}^{II}(f)$. Then, $a_i' = (\bar{\mathbf{S}}, m_i')$ with $\bar{\mathbf{S}} \in \mathcal{S}^{II}(f)$ and m_i' involves individual i winning the integer game (i.e., $k' > k^{(j)}$ with $i, j = 1, 2$ and $i \neq j$) while the prize individual i goes for is $g(a^*)$. In particular, $m_i' = (\theta^{(i)}, c^{(i)}, g(a^*), y^{(i)}, k')$ with $k' > k^{(j)}$, and $i, j = 1, 2$ and $i \neq j$. Then, $g(\bar{\mathbf{S}}, m_i', a_j^*) = g(\mathbf{S}^{(i)}, m_i^*, a_j^*) = g(a^*)$ while $g(a^*) \in C_i^{\pi(\theta)}(X)$ for all $i, j = 1, 2$ with $i \neq j$ thanks to a^* being Nash* at θ. As individual i is a sympathizer, we obtain the desired contradiction because $a_i^* \notin BR_i^{\pi(\theta)}(a_j^*)$, $i, j = 1, 2$ and $i \neq j$. ■ ■ ■

C.1 An example with two individuals

In what follows, we present an example with two rational individuals and a social goal that seeks compromise; we show that the hypothesis of Theorem 4, as well as the alternative one presented in Footnote 21, hold. Also, this example exhibits a tangible setting in which our contributions in terms of information extraction are on display.

Let $X = \{a, b, c\}$ be the set of alternatives. The set of states of the economy is equal to $\Theta = \{\theta, \theta'\}$. The society is composed of Ann and Bob, who are rational; their state-contingent individual choices at state $\theta \in \Theta$ are captured by the choice correspondences, $C_A^{\pi(\theta)} : \mathcal{X} \to \mathcal{X}$ and $C_B^{\pi(\theta)} : \mathcal{X} \to \mathcal{X}$, which are specified in Table 3. Let the set of admissible states, Ω^*, be given by all possible strict ranking profiles

S	$C_A^{\pi(\theta)}$	$C_B^{\pi(\theta)}$	$C_A^{\pi(\theta')}$	$C_B^{\pi(\theta')}$
$\{a, b, c\}$	$\{b\}$	$\{c\}$	$\{a\}$	$\{c\}$
$\{a, b\}$	$\{b\}$	$\{a\}$	$\{a\}$	$\{b\}$
$\{a, c\}$	$\{a\}$	$\{c\}$	$\{a\}$	$\{c\}$
$\{b, c\}$	$\{b\}$	$\{c\}$	$\{b\}$	$\{c\}$

Table 3: Individual choices of Ann and Bob.

on $\{a, b, c\}$, and $\pi : \Theta \to \Omega^*$ be such that $bP_A^{\pi(\theta)}aP_A^{\pi(\theta)}c$ and $cP_B^{\pi(\theta)}aP_B^{\pi(\theta)}b$ at θ and $aP_A^{\pi(\theta')}bP_A^{\pi(\theta')}c$ and $cP_B^{\pi(\theta')}bP_B^{\pi(\theta')}a$ at θ'.

The SCC $f : \Theta \to \mathcal{X}$ the planner seeks to implement without knowing individuals' state-contingent preferences is defined by $f(\theta) = \{a\}$ and $f(\theta') = \{b\}$.[22]

[22] This social choice function corresponds to the Rawlsian welfare criterion: the desired alternative at a state of the economy is one that maximizes the welfare of the worst treated individual at that

Let us start with condition (i) of Definition 7, i.e., $x \in C_i^{\pi(\theta)}(S_i(x,\theta))$, for all $i \in N$ and $\theta \in \Theta$ and $x \in f(\theta)$:

$\underline{S_A(a,\theta)}$: $bP_A^{\pi(\theta)}aP_A^{\pi(\theta)}c$ and $a \in C_A^{\pi(\theta)}(S_A(a,\theta))$ imply two possibilities: $\{a,c\}$ and $\{a\}$.
$\underline{S_A(b,\theta')}$: $aP_A^{\pi(\theta')}bP_A^{\pi(\theta')}c$ and $b \in C_A^{\pi(\theta')}(S_A(b,\theta'))$ imply two candidates: $\{b,c\}$ and $\{b\}$.
$\underline{S_B(a,\theta)}$: $cP_B^{\pi(\theta)}aP_B^{\pi(\theta)}b$ and $a \in C_B^{\pi(\theta)}(S_B(a,\theta))$ result in two cases: $\{a,b\}$ and $\{a\}$.
$\underline{S_B(b,\theta')}$: $cP_B^{\pi(\theta')}bP_B^{\pi(\theta')}a$ and $b \in C_B^{\pi(\theta')}(S_B(b,\theta'))$ imply two candidates: $\{a,b\}$ and $\{b\}$.

Next, we check whether or not the above sets satisfy condition (ii) of Definition 7, i.e., $x \in f(\theta)$ and $x \notin f(\theta')$ for some $\theta, \theta' \in \Theta$ implies there is $j \in N$ such that $x \notin C_j^{\pi(\theta')}(S_j(x,\theta))$:

$\underline{a \in f(\theta) \text{ and } a \notin f(\theta')}$: Hence, either $a \notin C_A^{\pi(\theta')}(S_A(a,\theta))$ or $a \notin C_B^{\pi(\theta')}(S_B(a,\theta))$. This implies that we cannot have $S_A(a,\theta) = S_B(a,\theta) = \{a\}$ simultaneously. All other combinations of the listed candidate sets satisfy condition (ii) of Definition 7.

$\underline{b \in f(\theta') \text{ and } b \notin f(\theta)}$: Thus, either $b \notin C_A^{\pi(\theta)}(S_A(b,\theta'))$ or $b \notin C_B^{\pi(\theta)}(S_B(b,\theta'))$. So, it cannot be $S_A(b,\theta') = S_B(b,\theta') = \{b\}$ simultaneously. Similar to the above, all the other combinations of the candidates satisfy condition (ii) of Definition 7.

Now, we consider the following implication of condition (iii) of Definition 7: We must have $S_A(a,\theta) \cap S_B(b,\theta') \neq \emptyset$ and $S_A(b,\theta') \cap S_B(a,\theta) \neq \emptyset$ for a function $e : X \times \Theta \times X \times \Theta \to X$ to satisfy $(iii.1)$ of two-individual consistency:

$\underline{S_A(a,\theta) \text{ is either } \{a,c\} \text{ or } \{a\}}$: For $S_A(a,\theta) \cap S_B(b,\theta') \neq \emptyset$, $S_B(b,\theta')$ cannot be $\{b\}$. Hence, it must be $S_B(b,\theta') = \{a,b\}$.

$\underline{S_A(b,\theta') \text{ is either } \{b,c\} \text{ or } \{b\}}$: For $S_A(b,\theta') \cap S_B(a,\theta) \neq \emptyset$, $S_B(a,\theta)$ cannot be $\{a\}$. Therefore, we must have $S_A(b,\theta') = \{a,b\}$.

That is, we must have $S_B(b,\theta') = \{a,b\}$ whereas $S_A(a,\theta)$ is either $\{a,c\}$ or $\{a\}$. This implies $S_A(a,\theta) \cap S_B(b,\theta') = \{a\}$. On the other hand, $S_B(a,\theta) = \{a,b\}$ whereas $S_A(b,\theta')$ is either $\{b,c\}$ or $\{b\}$. Hence, $S_A(b,\theta') \cap S_B(a,\theta) = \{b\}$. This further implies that for any function $e : X \times \Theta \times X \times \Theta \to X$ which satisfies condition (iii) of Definition 7, we must have $e(a,\theta,b,\theta') = a$ and $e(b,\theta',a,\theta) = b$. It is straigthforward to show that such a function will always satisfy condition (iv) of Definition 7.

state. In our example, there is strong disagreement between the individuals: at any prevailing state, every alternative that is top-ranked by one of the individual is ranked at the bottom by the other.

Therefore, there are four profiles of sets, denoted by $\mathbf{S}$, $\mathbf{S}'$, $\tilde{\mathbf{S}}$, and $\hat{\mathbf{S}}$, that satisfy the two-individual consistency (Definition 7) with our SCC f, i.e., $\mathcal{S}^{II}(f) = \{\mathbf{S}, \mathbf{S}', \tilde{\mathbf{S}}, \hat{\mathbf{S}}\}$, and they are specified in Table 4.

$\mathbf{S}$:	$S_A(a,\theta) = \{a,c\}$	$S_A(b,\theta') = \{b,c\}$	$S_B(a,\theta) = \{a,b\}$	$S_B(b,\theta') = \{a,b\}$
$\mathbf{S}'$:	${S'}_A(a,\theta) = \{a\}$	${S'}_A(b,\theta') = \{b,c\}$	${S'}_B(a,\theta) = \{a,b\}$	${S'}_B(b,\theta') = \{a,b\}$
$\tilde{\mathbf{S}}$:	$\tilde{S}_A(a,\theta) = \{a,c\}$	$\tilde{S}_A(b,\theta') = \{b\}$	$\tilde{S}_B(a,\theta) = \{a,b\}$	$\tilde{S}_B(b,\theta') = \{a,b\}$
$\hat{\mathbf{S}}$:	$\hat{S}_A(a,\theta) = \{a\}$	$\hat{S}_A(b,\theta') = \{b\}$	$\hat{S}_B(a,\theta) = \{a,b\}$	$\hat{S}_B(b,\theta') = \{a,b\}$

Table 4: The two-individual consistent profiles of sets: $\mathcal{S}^{II}(f) = \{\mathbf{S}, \mathbf{S}', \tilde{\mathbf{S}}, \hat{\mathbf{S}}\}$.

It is easy to check that the hypothesis of Theorem 4, conditions (*i*), (*ii*), (*iii*), and (*iv*) (along with the alternative hypothesis of Theorem 4 defined in Footnote 21), are satisfied for f with $\mathcal{S}^{II}(f) = \{\mathbf{S}, \mathbf{S}', \tilde{\mathbf{S}}, \hat{\mathbf{S}}\}$. Thus, the mechanism described in Table 2 Nash* implements the SCC f when both of the individuals are sympathizers at $\pi(\theta)$ and $\pi(\theta')$.

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