PAIR IMPLEMENTATION

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ABSTRACT

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Keywords: Implementation, Nash implementation, network structures, pair structure

This thesis analyzes the implementation problem under complete information in a societal environment where all agents are grouped in pairs. We introduce a new equilibrium notion called pair-Nash equilibrium and define a related pair-monotonicity notion on social choice rules and show that it is necessary for implementation in pair-Nash equilibrium, i.e., pair-implementation. Moreover, we introduce the pair-implementability condition and prove that it suffices for pair-implementation. Our results extend implementation results of Maskin (1999) and Moore and Repullo (1990) to the pair-structure.

ÖZET

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Bu tezde, tam bilgi altında uygulamalar, toplumdaki her bireyin bir diğeriyle ikili eşleştiği durumlar (ikili-yapılar) için incelenmektedir. Bu analiz için, öncelikle yeni bir denge nosyonu olarak ikili-Nash dengesini ve bununla ilgili ikili-tekdüzelik kavramını tanımlayıp, bu ikili-tekdüzelik koşulunun sosyal seçim kurallarının ikili-Nash dengesi altında uygulanması (ikili-uygulama) için gerekli koşul olduğunu gösteriyoruz. Devamında, ikili-uygulanabilirlik koşulunu tanımlayarak bu koşulun sosyal seçim kurallarının ikili-uygulamaları için yeterli olduğunu kanıtlıyoruz. Elde ettiğimiz sonuçlar, Maskin (1999) and Moore and Repullo (1990) tarafından kurulan Nash uygulamaları sonuçlarının toplumda ikili-yapılara uyarlanmasını sağlamaktadır.

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1. INTRODUCTION

Implementation theory is concerned with the problem of a social planner, who would like to specify a social choice rule that selects a socially optimal outcome regarding the characteristics of agents in a society. As the planner lacks the information on characteristics of agents, the main objective of the planner is to extract this information via the design of an institution or a mechanism to deal with the following problem: While agents may have an incentive to reveal false information about their characteristics (e.g., types) to increase their individual welfare, the planner wishes to structure the mechanism that yields reliable information to be used in the selection of socially optimal outcomes foreseeing the outcome of the resulting strategic interaction among the agents.

With Hurwicz (1972) onwards, this problem is formalized and analyzed through a game form and a relevant game-theoretic equilibrium notion. So the main question became whether there can be a game form for the agents to play such that the equilibrium outcome(s) of the game coincide(s) with the outcomes specified by the social choice rule. Following literature started to structure the game forms as analoguous to real institutions by including the complexities of information structures and rationality assumptions. While Maskin (1999), Moore and Repullo (1990), Saijo (1988), Williams (1986) and Williams (2003) focused on implementation problems under perfect information, Palfrey and Srivastava (1987), Jackson (1991), Postlewaite and Schmeidler (1986) analyzed implementation problems under imperfect information. With the introduction of deficiencies of rationality such as status quo bias, attraction, temptation and framing effects by Thaler and Sunstein (2008) and Kahneman and Tversky (2013)[Chapter 6.], alternative models of implementation in the environments with boundedly-rational agents were studied (e.g. Eliaz (2002), Korpela (2012), de Clippel (2014)).

In recent studies of implementation theory, structures of informational networks gained importance, as the incentives of the agents in strategic settings may depend on the group incentives and cooperation of agents. Koray and Yildiz (2018) introduces implementation with rights structures, referred as Γ-implementation, which accounts for the coalitional incentives in such settings. Thus, they present the notions of image monotonicity and individual rights structure and prove them to be necessary and sufficient for Γ-implementability. Since their work relies on agents' consideration of how other agents can affect the outcomes by unilaterally changing their strategies and their use of rights structure focuses on how altering a state from one to another generates a change in the outcomes, our work elicits a different approach to group incentives in implementation for that we take groups of two agents and construct a new equilibrium concept according to this interaction structure of agents.

In this thesis, we analyze the set of agents grouped as pairs, because of the fact that the pair structure in societies and institutional settings is a commonly observed phenomenon. In contemporary societies, marriage systems, joint ventures in business environments, and bilateral diplomatic agreements present examples of cases that incentives of individuals in decision making are affected from their pairwise interference. When we consider the presence of pair structure in institutional level, trade agreements between two countries or companies, international agreements for national defense strategies can be regarded as crucial. For instance, long-lasted Brexit negotiations can be assessed as two agents', European Union's and United Kingdom's search for a profitable pairwise deviation. Also, in a business environment where two business partners discussing about a solution for increasing their recently dropped sales numbers, one may suggest lowering prices to boost sales while the other one cares about the perception of their company via selling high-end, luxurious and expensive products. In this case, these two partners take action in accordance with each other, thus they have to find a balanced solution that serves the concerns of all two parts of discussion.

At individual level, marriages, friendships and flat-sharing students represent exemples of pair structure in order to sustain these relationships. In many societies, marriage laws are constructed in the way permitting married couples live together and take their life decisions in line with both side's benefits. For instance, if one of the partners in a marriage is obliged to move to another location, codes of conduct in business environments are arranged in order not to separate the married couple and to sustain their mutual living. Thus, in a diversity of cases, laws can be binding a pair of agents (individually or institutionally) to act in harmony with each side's interests or without legal ties, agents may respect one another's advantages. In either cases, decision making processes as pairs need to be analyzed in a new framework that takes into account their incentives in accordance.

In order to deal with the implementation problem in pair structures, the need for a new concept of equilibrium and implementation emerges. Thus, we present pair-Nash equilibrium and pair-implementation concepts that build upon previous literature of implementation under perfect information.

Since the notions of pair-implementation and pair-Nash equilibrium builds upon the literature of implementation under complete information, we first present previous literature on Nash implementation which is related to our analysis. Maskin (1999) presents Nash equilibrium and Nash implementation where every agent knows about the others' preferences and reveals this information under a game form. In his pioneering study, Maskin introduces necessary and sufficient conditions for implementability of social choice rules and proves that monotonicity condition, which demands that when an alternative that is selected in a preference state and stays higher ranked in another state continues to be selected, is necessary for implementability of social choice rule. Together with the monotonicity condition, no-veto-power condition, which requires that an alternative which is highest ranked for every agent except one should be selected by the social choice rule so that none of the agents can veto an alternative that is desirable for other agents, is sufficient for Nash implementability, when the society consists of at least three agents.

Building upon Maskin's Nash implementation framework, Saijo (1988), Williams (1986) and Williams (2003) study Nash implementation with reduced strategy spaces, where the agents reveal the information about the preference characteristics of a smaller number of agents. When one thinks of a society consisting of large amount of individuals, the difficulty of gathering information from all agents about the characteristics of all the other agents and the related costs of storing and analyzing such a giant amount of data, the necessity of this simplification of game forms becomes crucial and inevitable. Despite this simplification, their work reaches a similar result that is monotonicity and no veto power are sufficient for Nash implementability of social choice rules, in an environment where the information structure of agents is limited to a cyclical observation of each other's characteristics under complete information.

Generalization of implementation problems to a larger set of social choice rules constitutes another important aspect of implementation literature. This intention brings the aspiration to less strict conditions of necessity and sufficiency for implementability of social choice rules. Moore and Repullo (1990) approaches this purpose by providing a weaker condition than Maskin's no-veto-power condition, so-called μ-condition and prove it to be both necessary and sufficient for Nash implementability.

In this thesis, we first formally describe a pair structure in a society, where every agent is paired with one other agent. This pair structure does not affect the individual utilities of agents, which depend on their own characteristics, referred as their state profiles, but their incentive for a profitable deviation depends on their partner's benefit as well as their own utility level. So, in this framework, a strategic action is profitable when it is profitable for both of the agents in a pair, increasing the utilities for both of them, or increasing the utility of one agent while not decreasing the other's.

Our framework of pair-implementability provides a new equilibrium concept, pair-Nash equilibrium, which is a collection of message profiles of a pair and responding strategies of other players to this pair's strategy, where none of the players can gather higher utilities by deviating as pairs, i.e., a collection of strategies from where no profitable pair-deviation is possible. So the main problem becomes implementation in pair-Nash equilibrium and we present necessary and sufficient conditions under characterization of individuals in pairs.

We introduce an analogous description of Maskin's monotonicity in pair structure as pair-monotonicity, meaning that if an alternative is chosen by a choice rule and ranked higher for every pair in another state, it should continue being socially optimal at the new state. We first prove that this condition is necessary for pair-implementability, i.e., if a social choice rule is pair-implementable, then it is pair-monotone.

For the sufficiency side of implementation, we present the pairwise analogous of Maskin's no-veto-power condition as pair-no-veto-power, that demands an alternative top ranked by all pairs in a society should be selected by the social choice rule. We also show that a weaker condition, pair-implementability condition which follows from condition-μ of Moore and Repullo (1990) with pair-monotonicity is sufficient for pair-implementability, so that if a social choice rule satisfies the pair-implementability condition, then it is pair-implementable. We also show that pair-no-veto-power condition implies our pair-implementability condition but not vice versa, as the former is a stronger condition while it leaves out a group of social choice rules that are pair-implementable. Hence, our study contributes to implementation literature by extending canonical framework of Moore and Repullo (1990) in pair structure.

The rest of this thesis is organized as follows: Chapter 2 presents the formalization of pair implementation framework with Section 2.1 introducing preliminaries of pair structure, pair-Nash equilibrium and pair-implementability, Section 2.2 following with necessity results. Section 2.3 presents our mechanism constructed for our sufficiency theorem and Chapter 3 concludes.

2. PAIR-IMPLEMENTATION

In this chapter, we establish the necessity and sufficiency conditions for a social choice correspondence to be implementable in pair-Nash equilibrium.

Many economic environments feature situations in which decision makers are paired with one another and have to address the concerns of their partner while making decisions. Households consisting of two individuals, law enforcement consisting of the police force and attorneys, bilateral agreements between institutions/corporations and inter-country agreements between two parts in diplomatic environments serve as an example of such situations. Notice that, in such situations, an agent must attribute importance to the benefit of his/her counterpart regardless of whether or not the agreements between two partners are legally binding.

In such a setting where the individuals are grouped as pairs, the strategic interaction between individuals depends on not only the individual motivation of decision makers but also concerns about the pairwise accordance of individuals. Here, the equilibrium outcome depends on the consensus between the partners, generating a *pair*-Nash equilibrium, different than the standard Nash equilibrium notion. Pair-Nash equilibrium of a game of n-agents is a collection of strategy profiles that provide the most preferred outcome(s) for each individual within the set of alternatives through pairwise deviations, while other individuals keep playing their part of equilibrium. So the outcome generated from a strategy profile of a player depends on his/her and his/her partner's strategy, together with the responses of other players to these strategies. Thus, to obtain a better outcome, the paired individuals should act in a coordinated manner, when they consider a potential deviation.

To understand the intuition underlying this equilibrium notion, consider the following scenario. Suppose that a tribe consisting of n individuals (n being an even number and hence allowing to match individuals as pairs) will be settled in a location, which will be determined according to a mechanism that is constructed by a social planner. Everybody has their own preference ordering for the locations about which the social planner has no information, yet every individual decides the message to send to the social planner in accordance with their companion, i.e., the individual that they are paired with, as every agent wants the optimal location for themselves and their companions. Consider that everybody plays their strategy and the resulting outcome is a pair-Nash equilibrium. In order for an individual to play another strategy, this deviation should be profitable not only for himself but also for his/her companion.

The purpose of the following sections is to introduce and formally define a setting to employ pair Nash-equilibrium concept and present its implications and some of its applications.

2.1 Notation and definitions

The society (the set of players) consists of agents indexed by $i = 1, 2, \ldots, n$. We let the set of players be given by $N = \{1, 2, \ldots, n\}$. For reasons of convenience, we restrict attention to cases where the cardinality of N is an even number.

As discussed above, the agents are paired with one another. Indeed, the matching between the agents are described by a bijection $\nu : N \to N$ such that for every agent $i \in N$, $\nu(i) \in N \setminus \{i\}$ denotes the agent matched with i. Thus, we have that $\nu(\nu(i)) = i$ for all $i \in N$.

A pair of players consisting of i and $j = \nu(i) \neq i$ is denoted by $(ij) \in N^2$. It is appropriate to emphasize that in our setting an agent can be paired with only one distinct individual, and vice versa. Thus, the *profile of pairs* in this economy is given by an $N \times N$ matrix $\mathcal{P}$ where the rows and columns are given by the agents N and $p_{ij} = 1$ if (ij) is a pair while $p_{ij} = 0$ otherwise. Clearly, as $i \neq j$ in any pair (ij), $p_{ii} = 0$. Thus, $\mathcal{P}$ is such that $\sum_j p_{ij} = 1$ and $\sum_i p_{ij} = 1$. We let the set of all possible pairs be denoted by $\mathscr{P}$. Abusing notation, $(ij) \in \mathcal{P}$ denotes the situation when players i and j are a pair in $\mathcal{P}$.

The *set of alternatives* of the economy is given by X. We denote the set of all non-empty subsets of X by $\mathcal{X}$. The *set of states* of the economy is given by Θ. An element of Θ will be denoted by θ and agents' preferences (continuous and complete preorders) on the set of alternatives depend on the state. Given a state $\theta \in \Theta$, the preferences of agent $i \in N$ on X are represented by a state dependent utility function $u_i(\cdot|\theta) : X \to \mathbb{R}$. In this environment, define *the lower contour set of $x \in X$ for an* *agent* $i \in N$ *at* $\theta \in \Theta$ as

$$LCS_i(x \mid \theta) = \{y \in X : u_i(x \mid \theta) \geq u_i(y \mid \theta)\}.$$

Then, *the lower contour set of* $x \in X$ *for a pair* $(ij) \in \mathcal{P}$ *(i.e.,* $i \in N$ *and* $j = \nu(i)$*) at* $\theta \in \Theta$ is defined as

$$L_{ij}(x \mid \theta) = LCS_i(x \mid \theta) \cap LCS_j(x \mid \theta).$$

A *social choice rule* $f : \Theta \to \mathcal{X}$ is a correspondence, which selects socially optimal alternatives in X for the state profile θ. A standard example of a social choice rule is efficiency: Given a state of the economy, the socially optimal allocations are those that are efficient (Pareto optimal) in that particular state.

In order to implement a given social choice rule f, the social planner constructs a strategic n-person game and specifies the rules of the game formalized as a *game form*, which is a tuple $\mu = (M, g)$, where $M = M_1 \times \ldots \times M_n$ is the set of a collection of individual message spaces with $m_i \in M_i$ being an individual message profile for individual i. For notational purposes, for any given $i \in N$ and $(ij) \in \mathcal{P}$, we let $m_{-i} \in M_{-i} := \times_{k \neq i} M_k$ and $m_{-ij} \in M_{-ij} := \times_{k \neq i,j} M_k$. Meanwhile, $g : M \to X$ is a mapping from the message space to the set of alternatives that associates every message profile with an outcome, being referred to as the *outcome function*. Thus, $\mu = (M, g)$ is equivalently called a *mechanism* and the social planner is also referred to as the mechanism designer.

This work concentrates on implementation under complete information. That is, it is assumed that each agent knows the state of the economy while the social planner does not. Consequently, the resulting question is whether or not the social planner can extract this information from the agents and implement a desired social choice rule (correspondence) using the appropriate concept of equilibrium suited to this setting.

The study of implementation in an environment where the individuals are grouped as pairs necessitates a new notion of equilibrium and a new notion of implementation. As in most equilibrium notions for strategic game forms, a player is required to optimally respond to the equilibrium strategies of the other players.

On the other hand, here a player's response is evaluated with not only his/her preferences but also with those of the player that he/she is paired with.

These are formalized in the following notion of equilibrium:

Definition 1. *Given a mechanism* $\mu = (M, g)$*, a message profile* $m^* \in M$ *is a Pair-Nash Equilibrium of* $\Gamma = \{N, \mathcal{P}, \langle u_i, M_i \rangle_{i \in N}\}$ *at* $\theta \in \Theta$ *if for all* $(ij) \in \mathcal{P}$*,*

$$u_i(g(m^*)) \geq u_i(g(m_i, m_j, m^*_{-ij})), \text{ and } u_j(g(m^*)) \geq u_j(g(m_i, m_j, m^*_{-ij})),$$

for all $(m_i, m_j) \in M_i \times M_j$*.*

In words, pair-Nash equilibrium of a mechanism describes a message profile $m^* = (m_i^*, m_j^*, m^*_{-ij})$ consisting of an optimal strategy for player i, m_i^*, an optimal strategy for player j, m_j^*, and optimal responses of other pairs, m^*_{-ij}, when i and j construct a pair (ij). The resulting alternatives from these optimal strategies provide higher utilities for *both* of the players i and j in pair (ij) at the given state profile θ, when compared with the utilities resulting from other message profiles, (m_i, m_j), at the same state profile θ.

In this equilibrium notion, individuals could generate a profitable deviation only when they deviate as a pair, i.e. by varying their own strategy while others play their part of the equilibrium. Thus, individual profitable deviation is only possible when deviation from a strategy is profitable for the other part of the pair as well. By the same example, if the pair (ij) change their strategy from $m^* = (m_i^*, m_j^*, m^*_{-ij})$ to $\tilde{m} = (\tilde{m}_i, \tilde{m}_j, m^*_{-ij})$, then resulting alternative $g(\tilde{m})$ should render higher utilities for both i and j. Notice that each player chooses his/her own strategy and each player's concern is his/her individual utility as long as it does not hurt his/her partner's utility. For each individual, the equilibrium outcome is among the chosen options within the set of outcomes that he or she can generate through pairwise deviations.

Example 1. *Suppose an environment with the set of agents* $N = \{1, 2, 3, 4\}$ *and the profile of pairs* $(12), (34) \in \mathcal{P}$ *and the set of actions* $M_i = \{a_1, a_2\}$ *for all* $i \in N$*. At a given state* θ*, the utilities of agents,* $u_i(\cdot, \cdot, \cdot, \cdot \mid \theta)$*, are given as follows:*

$$\begin{aligned}
&u_i(a_1, a_1, a_1, a_1 \mid \theta) = 1, \text{ for all } i = 1, \ldots, 4,\\
&u_i(a_2, a_2, a_2, a_2 \mid \theta) = 1, \text{ for all } i = 1, \ldots, 4,\\
&u_1(a_2, a_1, a_1, a_1 \mid \theta) = 2,\\
&u_2(a_2, a_1, a_1, a_1 \mid \theta) = u_3(a_2, a_1, a_1, a_1 \mid \theta) = u_4(a_2, a_1, a_1, a_1 \mid \theta) = 1/2,\\
&u_1(a_1, a_2, a_2, a_2 \mid \theta) = 1,\\
&u_2(a_1, a_2, a_2, a_2 \mid \theta) = u_3(a_1, a_2, a_2, a_2 \mid \theta) = u_4(a_1, a_2, a_2, a_2 \mid \theta) = 1/2,\\
&u_i(a \mid \theta) = 0 \text{ for all } a \in M \text{ not listed above.}
\end{aligned}$$

Taking pure strategies into consideration, notice that, in this example, (a_1, a_1, a_1, a_1)

is among pair-Nash equilibrium strategies while they are not among Nash equilibria. On the other hand, (a_1,a_2,a_2,a_2) *is a Nash equilibrium but not a pair-Nash equilibrium.*

In order to see that, let us start with (a_1,a_1,a_1,a_1)*. It is not a Nash equilibrium as* $u_1(a_1,a_1,a_1,a_1 \mid \theta) = 1 < 2 = u_1(a_2,a_1,a_1,a_1 \mid \theta)$*. On the other hand, for all* $(a,a') \in \{a_1,a_2\}^2 \setminus (a_1,a_1)$*,* $u_2(a,a',a_1,a_1 \mid \theta) < 1$*; thus,* (a_1,a_1,a_1,a_1) *is a pair-Nash equilibrium.*

On the other hand, (a_1,a_2,a_2,a_2) *is a Nash equilibrium because* $u_1(a_1,a_2,a_2,a_2 \mid \theta) = 1 = u_1(a_2,a_2,a_2,a_2 \mid \theta)$ *while for any* $a,a',a'' \in \{a_1,a_2\}^3$ *with* $(a,a',a'') \neq (a_2,a_2,a_2)$ *and* $(a,a',a'') \neq (a_1,a_1,a_1)$*, we have* $u_j(a_1,a,a',a'' \mid \theta) = 0 < 1/2 = u_j(a_1,a_2,a_2,a_2 \mid \theta)$ *for all* $j \neq 1$*. But,* (a_1,a_2,a_2,a_2) *is not a pair-Nash equilibrium: Consider* (a_2,a_2,a_2,a_2) *and notice that for* $(12) \in \mathcal{P}$ *we have that* (a_2,a_2) *is a profitable deviation because* $u_1(a_1,a_2,a_2,a_2 \mid \theta) = 1 = u_1(a_2,a_2,a_2,a_2 \mid \theta)$ *while* $u_2(a_1,a_2,a_2,a_2 \mid \theta) = 1/2 < 1 = u_2(a_2,a_2,a_2,a_2 \mid \theta)$*.*

These give rise to the following remark which is justified above:

Remark 1. There maybe Nash equilibria which are not pair-Nash while the reverse also holds.

Following the notion of pair-Nash equilibrium, the opportunity set of player i, the set of options that an individual can generate through unilateral deviations is defined as follows: Given a mechanism $\mu = (M,g)$, for any $(ij) \in \mathcal{P}$ and $\bar{m}_{-i} \in M_{-i} := \times_{j \neq i} M_j$,

$$G_i(\bar{m}_{-i}) = \{y \in X : g(m_i, \bar{m}_{-i}) = y \text{ with } m_i \in M_i\}.$$

Also, the opportunity set of pair $(ij) \in \mathcal{P}$, the set of options that this pair can generate through pairwise deviations, is defined as follows: Given $\bar{m}_{-ij} \in M_{-ij} := \times_{k \neq i,j} M_k$,

$$G_{ij}(\bar{m}_{-ij}) = \{y \in X : g(m_i, m_j, \bar{m}_{-ij}) = y \text{ with } (m_i, m_j) \in M_i \times M_j\}.$$

Equivalently, $G_{ij}(\bar{m}_{-ij}) = g(M_i, M_j, \bar{m}_{-ij})$.

Under this notation, notice that a strategy m^* constitutes a pair-Nash equilibrium of the game induced by mechanism $\mu = (M,g)$ at θ if

$$u_i(g(m^*) \mid \theta) \geq u_i(y \mid \theta), \text{ and } u_j(g(m^*) \mid \theta) \geq u_j(y \mid \theta), \text{ for all } y \in G_{ij}(m^*{}_{-ij})$$

for all $(ij) \in \mathcal{P}$.

Now, suppose that a social planner wishes to implement a social choice rule f in pair-Nash equilibrium. Let $f(\theta)$ be the resulting alternative relevant for the state profile θ, which is called f-optimal outcome. As the planner lacks the knowledge of preference profiles of players, he wishes to confirm that independent of what the current profile of preferences is and regardless of the specific equilibrium that is selected, f-optimal outcome will be realized. The following definition formalizes the conditions for that a social choice rule to be implemented in pair-Nash equilibrium.

Definition 2. *$f : \Theta \to \mathcal{X}$ is pair-implementable by $\mu = (M, g)$ if*

(1) *for all $\theta \in \Theta$ and for all $x \in f(\theta)$, there exists $m^* \in M$ such that for all $(ij) \in \mathcal{P}$*

$$u_i(g(m^*) \mid \theta) \geq u_i(g(m_i, m_j, m^*_{-ij}) \mid \theta) \text{ and}$$
$$u_j(g(m^*) \mid \theta) \geq u_j(g(m_i, m_j, m^*_{-ij}) \mid \theta),$$

for all $(m_i, m_j) \in M_i \times M_j$ and $g(m^*) = x$; and

(2) *for all $\theta \in \Theta$ and for all $m^* \in M$ with for all $(ij) \in \mathcal{P}$*

$$u_i(g(m^*) \mid \theta) \geq u_i(g(m_i, m_j, m^*_{-ij}) \mid \theta) \text{ and}$$
$$u_j(g(m^*) \mid \theta) \geq u_j(g(m_i, m_j, m^*_{-ij}) \mid \theta),$$

for all $(m_i, m_j) \in M_i \times M_j$, we have that $g(m^*) \in f(\theta)$.

That is, mechanism $\mu = (M, g)$ pair-implements a choice rule if it satisfies two requirements: First, for any state θ and for any f-optimal alternative x at θ, there exists a pair-Nash equilibrium message profile m^* that provides x via the outcome function g. Second, for any state θ, m^* being a pair-Nash equilibrium at θ implies that the resulting outcome $g(m^*)$ is f-optimal at θ. Thus, all pair-Nash equilibria obtained with mechanism should coincide with f-optimal alternatives, implying full-implementation.

2.2 Necessity

We now introduce the necessary condition and show that it must hold whenever the social choice correspondence is pair-implementable when attention is restricted to perfect information.

Definition 3. *$f : \Theta \to \mathcal{X}$ is pair-monotone if $x \in f(\theta)$ and*

$$L_{ij}(x \mid \theta) \subseteq L_{ij}(x \mid \theta'), \text{ for some } \theta' \in \Theta,$$

implies $x \in f(\theta')$.

In words, pair-monotonicity means that if an outcome chosen by a social choice rule at state θ and it becomes higher ranked for every pair at another state θ', then it should be f-optimal at θ'. Hence, pair-monotonicity property implies that whenever the social choice rule excludes a previously chosen outcome, then at least one pair has shifted this outcome down in their ranking. That is, if we denote the formerly chosen outcome by x, then at least one pair, who has previously preferred x to y now reverses this relation.

The following is our necessity theorem:

Theorem 1. *If a social choice rule $f : \Theta \to \mathcal{X}$ is pair-implementable, then it is pair-monotone.*

Proof. Let $\theta \in \Theta$ and $x \in f(\theta)$. Then, by (1) of pair-implementability, there exists $m^* \in M$ such that for all $(ij) \in \mathcal{P}$

$$\begin{aligned} &u_i(g(m^*) \mid \theta) \geq u_i(g(m_i, m_j, m^*_{-ij}) \mid \theta), \text{ and} \\ &u_j(g(m^*) \mid \theta) \geq u_j(g(m_i, m_j, m^*_{-ij}) \mid \theta) \end{aligned} \tag{2.1}$$

for all $(m_i, m_j) \in M_i \times M_j$ and $g(m^*) = x$.

Due to the hypothesis, let $\theta' \in \Theta$ be such that for all $(ij) \in \mathcal{P}$, $L_{ij}(x \mid \theta) \subseteq L_{ij}(x \mid \theta')$. Then, we need to show that $x \in f(\theta')$.

By (2.1), $g(m^*) = x$; and for any $y \in G_{ij}(m^*{}_{-ij})$, we have that $y \in L_{ij}(x \mid \theta)$. Hence, $G_{ij}(m^*{}_{-ij}) \subseteq L_{ij}(x \mid \theta) \subseteq L_{ij}(x \mid \theta')$ for all $(ij) \in \mathcal{P}$. As a result, $G_{ij}(m^*{}_{-ij}) \subseteq L_{ij}(x \mid \theta')$ implies for all $(ij) \in \mathcal{P}$,

$$u_i(g(m^*) \mid \theta') \geq u_i(g(m_i, m_j, m^*{}_{-ij}) \mid \theta')$$

and

$$u_j(g(m^*) \mid \theta') \geq u_j(g(m_i, m_j, m^*{}_{-ij}) \mid \theta')$$

for all $(m_i, m_j) \in M_i \times M_j$. Therefore, m^* is a pair-Nash equilibrium at θ' and by (2) of pair-implementability, $x \in f(\theta')$. □

A natural question concerns the relation between pair-monotonicity and the notion of monotonicity of Maskin (1999) which is defined below:

Definition 4. *A social choice rule* $f : \Theta \to \mathcal{X}$ *is Maskin monotone if* $x \in f(\theta)$ *and* $L_i(x \mid \theta) \subseteq L_i(x \mid \theta')$ *for all* $i \in N$ *and for some* $\theta' \in \Theta$ *implies* $x \in f(\theta')$*.*

In words, Maskin monotonicity requires the following: If an alternative x is f-optimal at some state θ and it does not get lower ranked for any agent at state θ', then x must be f-optimal at θ'.

Below, we establish that, every pair-monotone social choice rule must be Maskin monotone. That is, Maskin monotone social choice rules contain the set of pair-monotone social choice rules. On the other hand, showing whether or not the reverse of this relation holds is an open question.

Theorem 2. *If a social choice rule* $f : \Theta \to \mathcal{X}$ *is pair-monotone, then it is monotone.*

Proof. Suppose $x \in f(\theta)$ and $L_i(x \mid \theta) \subseteq L_i(x \mid \theta')$ for all $i \in N$. Therefore, for all $(ij) \in \mathcal{P}$, we have that $L_{ij}(x \mid \theta) = L_i(x \mid \theta) \cap L_j(x \mid \theta) \subseteq L_i(x \mid \theta') \cap L_j(x \mid \theta') = L_{ij}(x \mid \theta')$. As f is pair-monotone, we conclude that $x \in f(\theta')$ as desired. □

2.3 Sufficiency

In this section, we provide our sufficiency result and show that pair-monotonicity coupled with two conditions is sufficient to guarantee the pair-implementability of a social choice rule. These two conditions are weaker than the corresponding no-veto power property, i.e. they are implied by the pair version of the no-veto power property.

Before going into the details for this endeavor, we wish to present the mechanism to be employed in our sufficiency theorem.

2.3.1 Mechanism

Given a set of states Θ, a set of alternatives X, and a set of integers $K = \{1, \ldots, n\}$, the mechanism $\mu = (M, g)$ we are going to employ is one where for each agent $i \in N$ the message space is $M_i = \Theta \times X \times K$, and the outcome function is defined as follows:

Rule 1: If $m_i = (\theta, x, k)$, for all $i \in N$ and $x \in f(\theta)$, then

$$g(m) = x. \tag{2.2}$$

That is, if all agents propose the same strategy and the alternative x that they propose is f-optimal at their proposed profile θ, then the outcome of the game is x.

Rule 2: If $m_i = (\theta, x, k)$, for all $i \in N \setminus \{\tilde{i}\}$ and $(\tilde{i}\tilde{j}) \in \mathcal{P}$ and $x \in f(\theta)$ while $m_{\tilde{i}} = (\theta', x', k')$ with $m_{\tilde{i}} \neq m_i$, then

$$g(m) = \begin{cases} x' & \text{if } x' \in L_{\tilde{i}\tilde{j}}(x \mid \theta) \\ x & \text{if } x' \notin L_{\tilde{i}\tilde{j}}(x \mid \theta). \end{cases} \tag{2.3}$$

In words, *Rule 2* requires that, if all agents but one, $\tilde{i}$, play the same strategy and their proposed alternative x is f-optimal at their proposed state θ, the deviator gets his/her proposed alternative x' if and only if x' is in the pairwise lower contour set of x at θ of the pair the deviator belongs to. Otherwise, the deviator is ignored.

Rule 3: If $m_i = (\theta, x, k)$ for all $i \in N \setminus \{\tilde{i}, \tilde{j}\}$ and $(\tilde{i}\tilde{j}) \in \mathcal{P}$ and $x \in f(\theta)$ and $m_{\tilde{i}} \neq m_i$ and $m_{\tilde{j}} \neq m_i$ and $m_{\tilde{i}} = (\theta', \tilde{x}, k')$, while $m_{\tilde{j}} = (\theta'', \tilde{x}, k'')$ then

$$g(m) = \begin{cases} \tilde{x} & \text{if } \tilde{x} \in L_{\tilde{i}\tilde{j}}(x \mid \theta) \\ x & \text{if } \tilde{x} \notin L_{\tilde{i}\tilde{j}}(x \mid \theta). \end{cases} \tag{2.4}$$

That is, suppose all agents but one pair, $(\tilde{i}\tilde{j})$, play the same strategy and their proposed alternative x is f-optimal at their proposed state θ, while the deviating pair proposes the same alternative $\tilde{x}$ possibly different than the alternative proposed by the rest of the players. Then the deviating pair gets their proposed alternative $\tilde{x}$ if and only if $\tilde{x}$ is in their pairwise lower contour set of x at θ, i.e., in $\tilde{x} \in L_{\tilde{i}\tilde{j}}(x \mid \theta)$. Otherwise, the deviating pair is ignored.

Rule 4: If $m_i = (\theta, x, k)$ for all $i \in N \setminus \{\tilde{i}, \tilde{j}\}$ and $(\tilde{i}\tilde{j}) \in \mathcal{P}$ and $x \in f(\theta)$ and $m_{\tilde{i}} \neq m_i$

and $m_{\tilde{j}} \neq m_i$ and $m_{\tilde{i}} = (\theta', x', k')$ while $m_{\tilde{j}} = (\theta'', \tilde{x}, k'')$ with $x' \neq \tilde{x}$, then
(2.5)

$$g(m) = \begin{cases} x & \text{if either } \tilde{x}, x' \notin L_{\tilde{i}\tilde{j}}(x \mid \theta) \\ & \quad \text{or } \tilde{x}, x' \in L_{\tilde{i}\tilde{j}}(x \mid \theta) \text{ and } k', k'' \neq ((n-2)k + k' + k'')(\text{mod } n) + 1 \\ & \quad \text{or } \tilde{x}, x' \in L_{\tilde{i}\tilde{j}}(x \mid \theta) \text{ and } k', k'' = ((n-2)k + k' + k'')(\text{mod } n) + 1 \\ x' & \text{if either } x' \in L_{\tilde{i}\tilde{j}}(x \mid \theta) \text{ and } \tilde{x} \notin L_{\tilde{i}\tilde{j}}(x \mid \theta) \\ & \quad \text{or } x', \tilde{x} \in L_{\tilde{i}\tilde{j}}(x \mid \theta) \text{ and } k' = ((n-2)k + k' + k'')(\text{mod } n) + 1 \\ & \quad \text{and } k'' \neq ((n-2)k + k' + k'')(\text{mod } n) + 1 \\ \tilde{x} & \text{if either } \tilde{x} \in L_{\tilde{i}\tilde{j}}(x \mid \theta) \text{ and } x' \notin L_{\tilde{i}\tilde{j}}(x \mid \theta) \\ & \quad \text{or } \tilde{x}, x' \in L_{\tilde{i}\tilde{j}}(x \mid \theta) \text{ and } k'' = ((n-2)k + k' + k'')(\text{mod } n) + 1 \\ & \quad \text{and } k' \neq ((n-2)k + k' + k'')(\text{mod } n) + 1. \end{cases}$$

In words, suppose all agents but one pair, $(\tilde{i}\tilde{j}) \in \mathcal{P}$, play the same strategy and their proposed alternative x is f-optimal at their proposed state θ, while $\tilde{i}$ of the the deviating pair $(\tilde{i}\tilde{j})$ proposes x' and his/her partner $\tilde{j}$ proposes $\tilde{x}$ with $x' \neq \tilde{x}$, then the following holds:

(1) The deviating pair $(\tilde{i}\tilde{j})$ gets ignored whenever either x' and $\tilde{x}$ are both not in their pairwise lower contour set of x at θ (i.e., in $x', \tilde{x} \notin L_{\tilde{i}\tilde{j}}(x \mid \theta)$), or x' and $\tilde{x}$ are both in their pairwise lower contour set of x at θ but either none of or both of their integer choices equals $((n-2)k + k' + k'')(\text{mod } n) + 1$.

(2) The alternative x' proposed by $\tilde{i}$ of the deviating pair $(\tilde{i}\tilde{j})$ gets selected by the outcome function whenever either x' is in the pairwise lower contour set of $(\tilde{i}\tilde{j})$ of x at θ while $\tilde{x}$ is not (i.e., in $x' \in L_{\tilde{i}\tilde{j}}(x \mid \theta)$ and $\tilde{x} \notin L_{\tilde{i}\tilde{j}}(x \mid \theta)$), or both x' and $\tilde{x}$ are in their pairwise lower contour set of x at θ and $\tilde{i}$'s integer choice equals $((n-2)k + k' + k'')(\text{mod } n) + 1$ while $\tilde{j}$'s integer choice does not equal $((n-2)k + k' + k'')(\text{mod } n) + 1$.

(3) The alternative $\tilde{x}$ proposed by $\tilde{j}$ of the deviating pair $(\tilde{i}\tilde{j})$ gets selected by the outcome function whenever either $\tilde{x}$ is in the pairwise lower contour set of $(\tilde{i}\tilde{j})$ of x at θ while x' is not (i.e., in $\tilde{x} \in L_{\tilde{i}\tilde{j}}(x \mid \theta)$ and $x' \notin L_{\tilde{i}\tilde{j}}(x \mid \theta)$), or both x' and $\tilde{x}$ are in their pairwise lower contour set of x at θ and $\tilde{j}$'s integer choice equals $((n-2)k + k' + k'')(\text{mod } n) + 1$ while $\tilde{i}$'s integer choice does not equal $((n-2)k + k' + k'')(\text{mod } n) + 1$.

Rule 5: If m does not satisfy the hypothesis of any one of *Rules 1–4* and $m_i = (\theta^i, x^i, k^i)$ with $\theta^i \in \Theta$ and $x^i \in X$ and $k^i \in \{1, ..., n\}$ for all $i \in N$, then

(2.6) $$g(m) = x^{i^*} \text{ with } i^* \text{ such that } k^{i^*} = (\sum_i k^i)(\text{mod } n) + 1.$$

In words, *Rule 5* demands that in the cases that are not captured by *Rules 1–4*, the result is determined according to the conditions of modulo game that is the alternative proposed by the agent who claims the integer that beats the other integers claimed by other players.

2.3.2 Sufficiency Theorem for Pair Implementability

Next, we present our sufficiency condition, pair-implementability condition, to be employed in our sufficiency theorem. This condition turns out to be related with condition-μ of Moore and Repullo (1990).

Definition 5. *A social choice rule $f : \Theta \to \mathcal{X}$ satisfies the pair-implementability condition if all of the following hold:*

1. *For all $\theta \in \Theta$ and for all $x \in f(\theta)$, we have that $L_{ij}(x \mid \theta) \subseteq L_{ij}(x \mid \theta')$ for all $(ij) \in \mathcal{P}$ and for some $\theta' \in \Theta$ implies $x \in f(\theta')$.*
2. *For any $y \in X$, if $\theta \in \Theta$ is such that $L_{ij}(y \mid \theta) = X$ for all $(ij) \in \mathcal{P} \setminus \{(\tilde{i}\tilde{j})\}$ with $(\tilde{i}\tilde{j}) \in \mathcal{P}$, and $x \in L_{\tilde{i}\tilde{j}}(y \mid \theta)$ for some $\theta' \in \Theta$ and some $x \in f(\theta')$, then $y \in f(\theta)$.*
3. *If $y \in X$ is such that $L_{ij}(y \mid \theta) = X$ for all $(ij) \in \mathcal{P}$, then $y \in f(\theta)$.*

The first part of the pair-implementability condition requires the social choice rule f to satisfy pair-monotonicity. The second demands the following: if an alternative y is ranked at par with or above all the alternatives in X at state θ for all but one pair $(\tilde{i}\tilde{j})$ and there exists an alternative x which is f-optimal at some state θ' and is ranked below the alternative y for the pair $(\tilde{i}\tilde{j})$ at θ, then y must be f-optimal at θ. Finally, the last part of this condition demands *pair-unanimity*: if an alternative y is ranked at par with or above all the alternatives in X for all pairs at θ, then y has to be f-optimal at θ.

While the first and last items in our pair-implementability condition are standard, the second coupled with the third has a significant relation with the pair version of the no-veto-power property.

Definition 6. *A social choice rule $f : \Theta \to \mathcal{X}$ satisfies the pair-no-veto-power property if $\theta \in \Theta$ and $x \in X$ is such that $L_{ij}(x \mid \theta) = X$ for all $(ij) \in \mathcal{P} \setminus \{(\tilde{i}\tilde{j})\}$ with $(\tilde{i}\tilde{j}) \in \mathcal{P}$, then $x \in f(\theta)$.*

Pair-no-veto-power suggests that if an alternative x is at the top of $\frac{n}{2} - 1$ pairs' preferences, then a single pair cannot prevent the alternative x from being f-optimal.

One can simply notice that the pair-implementability condition is a weaker version of the combination of pair-monotonicity and pair-no-veto power. This observation is formally characterized and shown in the following theorem.

Theorem 3. *If a social choice rule $f : \Theta \to \mathcal{X}$ satisfies pair-monotonicity and pair-no-veto-power, then f satisfies the pair-implementability condition. Moreover, the reverse of this relation does not hold.*

Proof. Notice that the first step of the pair-implementability condition is exactly the same as the pair-monotonicity condition.

Next, suppose that a social choice rule f satisfies the pair-no-veto-power condition and assume that $y \in X$ and $\theta \in \Theta$ is such that $L_{ij}(y \mid \theta) = X$ for all $(ij) \in \mathcal{P} \setminus \{(\tilde{i}\tilde{j})\}$ with $(\tilde{i}\tilde{j}) \in \mathcal{P}$ and $x \in L_{\tilde{i}\tilde{j}}(y \mid \theta)$ for some $\theta' \in \Theta$ and some $x \in f(\theta')$. Then, clearly $L_{ij}(x \mid \theta) = X$ for all $(ij) \in \mathcal{P} \setminus \{(\tilde{i}\tilde{j})\}$ with $(\tilde{i}\tilde{j}) \in \mathcal{P}$. Thus, by the pair-no-veto-power property, $x \in f(\theta)$.

Finally, assume that $y \in X$ and $\theta \in \Theta$ is such that $L_{ij}(y \mid \theta) = X$ for all $(ij) \in \mathcal{P}$. Thus, trivially $y \in f(\theta)$ as $L_{ij}(x \mid \theta) = X$ for all $(ij) \in \mathcal{P} \setminus \{(\tilde{i}\tilde{j})\}$ with $(\tilde{i}\tilde{j}) \in \mathcal{P}$ is satisfied.

□

The following example illustrates a case where the reverse relation does not hold:

Example 2. *Let the set of alternatives be $X = \{x, y, z, t\}$. Suppose there are three pair of agents $(ab), (\tilde{a}\tilde{b}), (\hat{a}\hat{b}) \in \mathcal{P}$ having preference ordering on the alternatives in X at a state θ as follows:*

$u_i(x \mid \theta) \geq u_i(y \mid \theta) \geq u_i(z \mid \theta) \geq u_i(t \mid \theta)$ for all $i = a, b, \tilde{a}, \tilde{b}$ and $u_i(y \mid \theta) \geq u_i(z \mid \theta) \geq u_i(t \mid \theta) \geq u_i(x \mid \theta)$ for $i = \hat{a}, \hat{b}$.

Here, if the social choice rule f satisfies the pair-no-veto-power condition, then we have $f(\theta) = x$.

When we enrich this example by insisting that $x \notin f(\theta')$ for all $\theta' \in \Theta$, then we could obtain $f(\theta) = y$ under a suitable definition of efficiency. This maybe motivated by declaring z to be the acceptable alternative (alternatives providing higher utilities than z for all θ') for pair $(\hat{a}\hat{b})$ and requiring that $u_i(z \mid \theta') > u_i(x \mid \theta')$ for $\theta' \in \Theta$ and $i = \hat{a}, \hat{b}$. Thus, if we have a social choice rule f which selects only acceptable alternatives for each pair and existence of such alternatives is not an issue (say y is such an option), then we have $f(\theta) = y$ for this particular θ.

Then, the outcome of the social choice rule that satisfies the pair-no-veto-power condition and the outcome from the one that satisfies the pair-implementability condition does not coincide, we see in the latter case that the outcome is the alternative which is top-ranked by only one pair.

The following Theorem establishes that our pair-implementability condition is sufficient:

Theorem 4. *Suppose that the social choice rule $f : \Theta \to \mathcal{X}$ satisfies the pair-implementability condition. Then, f is pair-implementable.*

Proof. The proof is constructive by using the mechanism that we describe in Section 2.3.1 and employing two claims.

Claim 1. *For all $\theta \in \Theta$ and for all $x \in f(\theta)$, there is $m^x \in M$ of $\mu = (M, g)$ such that $u_i(g(m^x) \mid \theta) \geq u_i(g(m_i, m_j, m^x{}_{-ij}) \mid \theta)$ and $u_j(g(m^x) \mid \theta) \geq u_j(g(m_i, m_j, m^x{}_{-ij}) \mid \theta)$ for all $(ij) \in \mathcal{P}$ and for all $(m_i, m_j) \in M_i \times M_j$ and $g(m^x) = x$.*

Proof. Let $\theta \in \Theta$ and $x \in f(\theta)$ and define $m^x \in M$ by $m_i^x = (\theta, x, 1)$ for all $i \in N$.

Then, by *Rule 1*, $g(m^x) = x$.

Consider $(\tilde{i}\tilde{j}) \in \mathcal{P}$ (so $\tilde{i} \neq \tilde{j}$ as they are partners) and consider $m_{\tilde{i}} \neq m_i^x$ and $m_{\tilde{i}} \neq m_i^x$. That is, let $m_{\tilde{i}} = (\theta', x', k') \neq (\theta, x, 1) = m_i^x$ and $m_{\tilde{j}} = (\theta'', x'', k'') \neq (\theta, x, 1) = m_i^x$.

By *Rules 1–4*, $G_{\tilde{i}\tilde{j}}(m^x{}_{-\tilde{i}\tilde{j}}) = L_{\tilde{i}\tilde{j}}(x \mid \theta)$.

Therefore, for any $(m_{\tilde{i}}, m_{\tilde{j}}) \in M_{\tilde{i}} \times M_{\tilde{j}}$, it must be that the resulting alternative $y = g(m_{\tilde{i}}, m_{\tilde{i}}, m^x{}_{-\tilde{i}\tilde{j}})$ is in $G_{\tilde{i}\tilde{j}}(m^x{}_{-\tilde{i}\tilde{j}})$ and hence in $L_{\tilde{i}\tilde{j}}(x \mid \theta)$. Ergo, $u_{\tilde{i}}(g(m_{\tilde{i}}, m_{\tilde{j}}, m^x{}_{-\tilde{i}\tilde{j}}) \mid \theta) \leq u_{\tilde{i}}(g(m^x) \mid \theta)$ and $u_{\tilde{j}}(g(m_{\tilde{i}}, m_{\tilde{j}}, m^x{}_{-\tilde{i}\tilde{j}}) \mid \theta) \leq u_{\tilde{j}}(g(m^x) \mid \theta)$. Thus, m^x is a pair-Nash equilibrium at θ as $(\tilde{i}\tilde{j}) \in \mathcal{P}$ is arbitrary. □

The following is our second claim:

Claim 2. *Suppose that $m^* \in M$ is a pair-Nash equilibrium at $\theta \in \Theta$, i.e., for all $(ij) \in \mathcal{P}$, $u_i(g(m^*) \mid \theta) \geq u_i(g(m_i, m_j, m^*{}_{-ij}) \mid \theta)$ and $u_j(g(m^*) \mid \theta) \geq u_j(g(m_i, m_j, m^*{}_{-ij}) \mid \theta)$ for all $(m_i, m_j) \in M_i \times M_j$. Then, $g(m^*) \in f(\theta)$.*

Proof. In m^*, either one of *Rules 1–5* applies.

Case 1. Suppose *Rule 1* applies.

Thus, m^* is such that $m_i^* = (\theta', x, k)$ with $x \in f(\theta')$ for all $i \in N$. We need to show that $x \in f(\theta)$ because $g(m^*) = x$ under *Rule 1*.

As $m^* \in M$ is a pair-Nash equilibrium at θ (by the hypothesis of the claim at hand), for any $(\tilde{i}\tilde{j}) \in \mathcal{P}$, it must be that for any $m_{\tilde{i}}, m_{\tilde{j}} \in M_{\tilde{i}} \times M_{\tilde{j}}$ the resulting alternative $y = g(m_{\tilde{i}}, m_{\tilde{j}}, m^*_{-\tilde{i}\tilde{j}})$ is in $L_{\tilde{i}\tilde{j}}(x \mid \theta)$. Moreover, by *Rules 1–4*, $G_{\tilde{i}\tilde{j}}(m^*_{-\tilde{i}\tilde{j}}) = L_{\tilde{i}\tilde{j}}(x \mid \theta')$. Therefore, we have $L_{\tilde{i}\tilde{j}}(x \mid \theta') \subseteq L_{\tilde{i}\tilde{j}}(x \mid \theta)$. As $(\tilde{i}\tilde{j}) \in \mathcal{P}$ is arbitrary, $L_{\tilde{i}\tilde{j}}(x \mid \theta') \subseteq L_{\tilde{i}\tilde{j}}(x \mid \theta)$ for all $(\tilde{i}\tilde{j}) \in \mathcal{P}$. This coupled with $x \in f(\theta')$ implies (by pair-monotonicity – (1) of our pair implementability condition presented in Definition 5) $x \in f(\theta)$. □

Case 2. Suppose either one of *Rules 2–4* applies.

Then, m^* is such that $m_i{}^* = (\theta', x, k)$ for all $i \in N \setminus \{\tilde{i}, \tilde{j}\}$ for some $(\tilde{i}\tilde{j}) \in \mathcal{P}$ with $x \in f(\theta')$, and $m_{\tilde{i}} = (\theta^{\tilde{i}}, x', k^{\tilde{i}})$ and $m_{\tilde{j}} = (\theta^{\tilde{j}}, x'', k^{\tilde{j}})$.

Due to *Rules 1–4* we have that $G_{\tilde{i}\tilde{j}}(m^*{}_{-\tilde{i}\tilde{j}}) = L_{\tilde{i}\tilde{j}}(x \mid \theta')$ with $x \in f(\theta')$ and $g(m^*) \in G_{\tilde{i}\tilde{j}}(m^*{}_{-\tilde{i}\tilde{j}})$ as $(m_{\tilde{i}}, m_{\tilde{j}}) = (m^*_{\tilde{i}}, m^*_{\tilde{j}})$ is a possibility. That is, x and $g(m^*)$ are both in $G_{\tilde{i}\tilde{j}}(m^*{}_{-\tilde{i}\tilde{j}})$. As m^* is a pair-Nash equilibrium at θ, we have that $x \in L_{\tilde{i}\tilde{j}}(g(m^*) \mid \theta)$ and $x \in f(\theta')$.

Next, any agent $i \neq \tilde{i}$ may deviate from m_i^* and can generate an alternative x^i by choosing an appropriate integer due to *Rule 5*. Thus, $L_{ij}(g(m^*) \mid \theta) = X$ for all i paired with j and $i, j \neq \tilde{i}$ and $i, j \neq \tilde{j}$ since m^* is a pair-Nash equilibrium of μ at θ.

Hence, $\theta \in \Theta$ is such that $x \in L_{\tilde{i}\tilde{j}}(g(m^*) \mid \theta)$ for some $(\tilde{i}\tilde{j}) \in \mathcal{P}$ and $x \in f(\theta')$ while for all i paired with j and $i, j \neq \tilde{i}$ and $i, j \neq \tilde{j}$ it must be that $L_{ij}(g(m^*) \mid \theta) = X$. Ergo, by (2) of our pair-implementability condition (see Definition 5), we conclude that $g(m^*) \in f(\theta)$.

Case 3. Suppose *Rule 5* applies.

By deviating and making an appropriate integer choice, any one of players i with $(ij) \in \mathcal{P}$ can obtain any alternative $y \in X$. Therefore, m^* being a pair-Nash equilibrium at θ implies $L_{ij}(g(m^*) \mid \theta) = X$ for all $(ij) \in \mathcal{P}$. Hence, by (3) of our pair implementability condition presented in Definition 5, we obtain that $g(m^*) \in f(\theta)$.

Claims 1 and 2 conclude the proof of the Theorem. □

3. CONCLUDING REMARKS

This thesis explores the question of implementation in the environments in which agents are grouped as pairs and consider profitable deviations for the benefit of both agents consisting a pair. In the standard Nash equilbrium concept, a profitable deviation is a change in the strategy of an agent that will increase his/her own utility, while the rest of the agents keep their strategies unchanged. In societies, agents cooperate or interact when they are decision makers and often times they are grouped as pairs with legal agreements as in business partnerships or with implicit agreements as in friendships. In such settings, Nash equlibrium concept leaves this pair accordance out. So we introduce pair-Nash equilibrium to include this group structure into the analysis of the problem of implementation.

Formal analysis of pair-implementation starts with introducing the pair structure in a society, where every agent is grouped with one and only one other agent. Although every agent is only concerned about his/her own utility, in the suitable equilibrium notion, profitable deviations are defined as the shifts in strategies that are beneficial for both of the agents in a pair. The utility functions of agents depend on the characteristics of the agents, their state profiles that affect their preference ordering on the set of alternatives.

The formalization of pair-implementation follows from the literature on implementation under complete information. The pioneering analysis of Maskin (1999) shows that a condition called monotonicity is necessary for Nash implementation and together with monotonicity, no-veto-power condition is sufficient for Nash implementation when the society consists of more than three agents.

In this thesis, we introduce a new equilibrium notion, pair-Nash equilibrium that extends the Nash equilibrium concept in the pair structure. So, we describe pair-Nash equilibrium as a collection of strategies where any pair of agents can profitably deviate as a pair to increase their utility. Hence, our new implementation concept, pair-implementation concerns about social choice functions that are implementable in pair-Nash equilibrium. A social choice rule f is pair-implementable when f-optimal outcomes exactly coincide with pair-Nash equilibria, providing full implementation in pair structure.

The analysis of pair-implementation includes necessity and sufficiency theorems for pair-implementation. First, we introduce pair-monotonicity condition and prove as necessity condition that if a social choice rule is pair-implementable, then it is pair-monotone. In what follows, we describe our sufficiency condition, pair-implementability condition that is similar to the condition-μ of Moore and Repullo (1990) and prove that pair-implementability condition is sufficient for pair-implementation, i.e., if a social choice rule satisfies pair-implementability condition, then it is pair-implementable. Then, we give the pair structure analoguous of the standard no-veto-power condition, pair-no-veto-power and we show that pair-implementability condition is a weaker condition than pair-no-veto-power.

Our analysis relates with the previous work done by Maskin (1999) and Moore and Repullo (1990) by extending their approach to the pair-structure. Although informational networks are included in the analysis of implementation problem, network effect in incentives is a rather new concept in this field and our description of pairs as a graph structure opens a way to analyze larger groups and networks within the society.

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