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Modelling Integrated Multi-item Supplier Selection with Shipping Frequencies

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Abstract

There are many benefits for coordination of multiple suppliers when single supplier cannot satisfy buyer demands. In addition, buyer needs to purchase multiple items in a real supply chain. So, a model that satisfies these requests has many advantages. We extend the existing approaches in the literature that assume all suppliers need to be put on a common replenishment cycle and each supplier delivers exactly once in a cycle. More specifically, inspired by approaches that perform well for the Economic Lot Scheduling Problem, we assume an integer number of times a supplier can ship available items in an overall replenishment cycle. Because of complexity issue, a new approach based on genetic algorithm is employed to solve the presented model. Results depict that new model is more beneficial and practical.
Keywords: Integrated supply chain, Multi-item, Frequent shipping, Multi-supplier, Supplier selection.

1. Introduction

The design of the supply base is a core strategic area in *SCM*[1]. Following make-or-buy decisions, the determination of the size of the supply base and the selection of the suppliers are important decision problems (Benton, 2010). On the tactical level, the allocation of requirements to suppliers has to be determined and on the operational level, order quantities need to be determined and scheduled. An important trade-off when designing the supply base is the balance between Economies of Scale advocating few suppliers versus risk diversification favoring many suppliers.

Other than these arguments, especially for many industries where large buyers acquire and develop several small suppliers in developing countries, finite production rates where a single supplier is too small to satisfy the buyer's requirements drive larger supply bases.

The importance of integration in a supply chain was considered by Thomas and Griffin (1996). They argue that in order to achieve effective supply chain management, planning and coordination among all entities in a supply chain is needed.

Therefore, multiple supplier and inventory coordination problems have received considerable attention in the literature. The idea of joint optimization for buyer and vendor was initiated by Goyal (1976) and later supported by Banerjee (1986). Banerjee (1986) introduced *JELS*[2] model for a single vendor and single buyer to minimize joint total relevant cost. *JELS* was a single-source model that means all items should be purchased from selected supplier and allocation was ignored.

Kheljani et al. (2009) study the coordination problem between one buyer and multiple potential suppliers in the supplier selection process. In the objective function of their model, the total cost of the whole supply chain is minimized rather than only the buyer's cost. The total cost of the supply chain includes the buyer's cost and suppliers' costs. Finally, they solved their model by applying mixed integer nonlinear programming. The obtained model supports single-item to coordinate the supply chain.

Another problem that surfaces is integration of supply chain when multiple items should be ordered. Various interdependencies could exist among the different products and taking generated synergetic cooperation into account through multi-item models is profitable both for buyer and suppliers.

Aliabadi et al. (2013) develop Kheljani's model to coordinate an integrated supply chain when multiple item should be purchased from multiple supplier in integrated framework. They solve their model with a meta-heuristic approach which was based on hierarchical-structured

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1 Supply chain management

2 Joint economic lot size

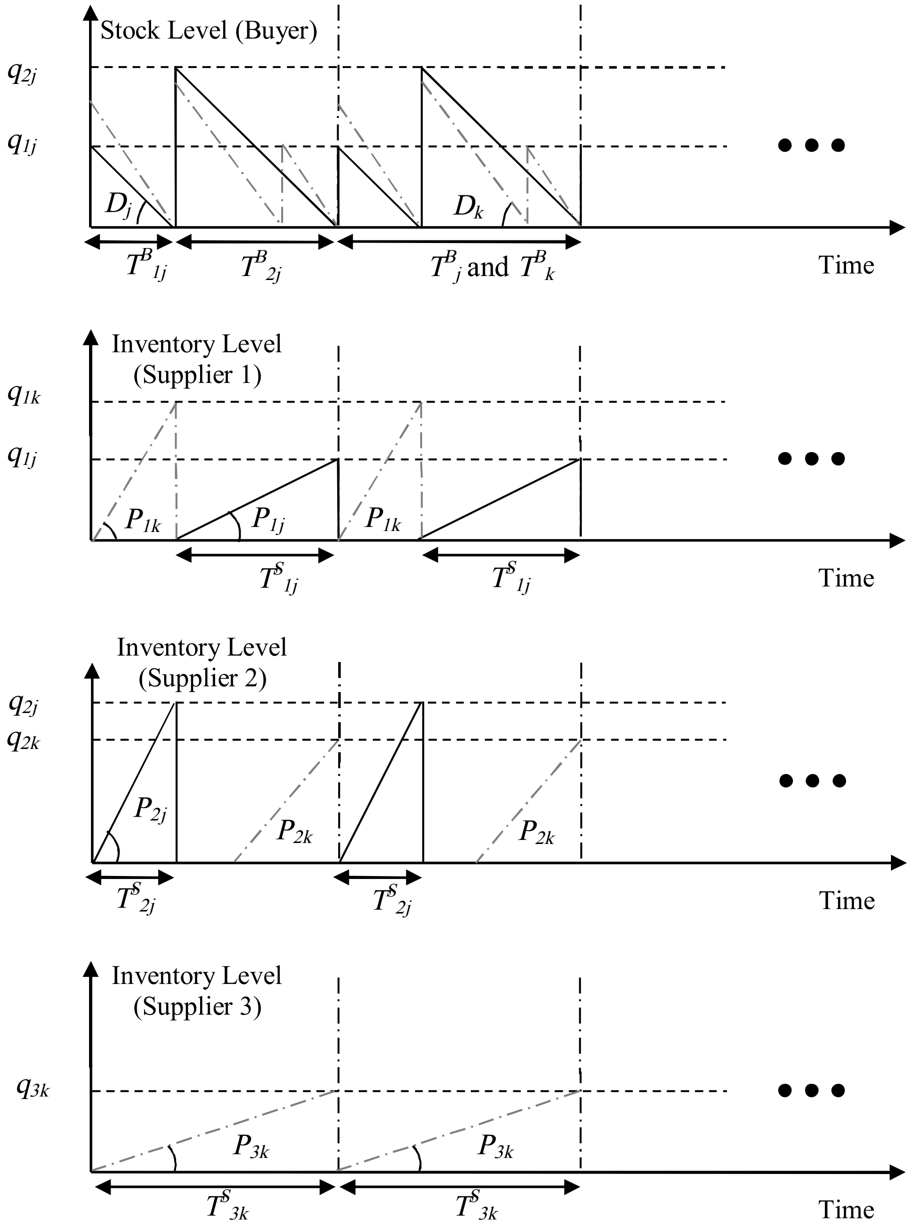

Fig. 1. Inventory levels of one buyer and three suppliers in multi-item model without shipping frequency

9) Suppliers' unsold opportunity cost is supposed intangible.
10) Finally, items quality is independent of their price. Hence, the only factor in the holding cost of buyer for items is related to types of items.

The buyer as a central decision maker has to take hierarchical interdependent decisions:

- As strategic decision, the choice of suppliers, modeled by a binary variable $Y_i \in \{0, 1\}$.
- As a tactical decision, the allocation of annual demand to i^{th} supplier for j^{th} item, modeled by sourcing fraction X_{ij}, ($0 \le X_{ij} \le 1$). Also, the number of shipping from i^{th} supplier for j^{th} item is modeled by an integer number n_{ij}, ($n_{ij} \ge 1$ and integer).

3. A Multi-item Model with Shipping Frequencies

After reviewing the previous versions of integrated supply chain models in the literature and our modeling assumptions, we propose a new multi-item model that has advantages of both Aliabadi et al. (2013) and Minner and Pourghannad (2010) models. We will prove that our new model can outperform in terms of total benefit of whole chain.

First of all, we introduce the notations, used in the mathematical modeling. The mathematical model will be developed according to the following notations:

$$\max TB: \sum_{j=1}^{n} D_j V_j - \sum_{j=1}^{n}\sum_{i=1}^{m}\left[\frac{n_{ij}D_j\left(A_{ij}+S_{ij}+b_{ij}\right)sgn\left(X_{ij}\right)}{Q_j} + \frac{X_{ij}^2 Q_j}{2n_{ij}P_{ij}}\left(h_j^B P_{ij}+h_{ij}D_j\right) + D_j X_{ij}\left(Z_{ij}+t_{ij}\right)\right] \quad (6)$$

Subject to :

$$\sum_{i=1}^{m} X_{ij} = 1 \qquad \forall j \qquad (7)$$

$$\sum_{j=1}^{n} \frac{D_j X_{ij}}{P_{ij}} \leq 1 \qquad \forall i \qquad (8)$$

$$\frac{K_{ij}Y_i}{D_j} \leq X_{ij} \leq \frac{P_{ij}Y_i}{D_j} \qquad \forall i,j \qquad (9)$$

$$int\; n_{ij} \geq sgn\left(X_{ij}\right) \qquad \forall i,j \qquad (10)$$

As Q_j* is optimum order quantity and it is possible to substitute R_{ij} with $(A_{ij}+S_{ij}+b_{ij}) \times sgn(X_{ij})$; Q_j* for fixed X_{ij} and n_{ij} can be obtained by taking first derivative of Eq. (3) :

$$\frac{\partial TB}{\partial Q_j} = 0 \Rightarrow Q_j^* = \sqrt{\frac{2D_j \sum_{i=1}^{m} n_{ij}{}^2 P_{ij} R_{ij}}{\sum_{i=1}^{m} X_{ij}^2\left(h_j^B P_{ij}+h_{ij}D_j\right)}} \qquad (11)$$

Zence, the objective function is reformed as follows:

$$\max TB: \sum_{j=1}^{n} D_j V_j - \sum_{j=1}^{n}\sum_{i=1}^{m}\left[\frac{n_{ij}D_j\left(A_{ij}+S_{ij}+b_{ij}\right)sgn\left(X_{ij}\right)}{\sqrt{\frac{2D_j \sum_{i=1}^{m} n_{ij}{}^2 P_{ij} R_{ij}}{\sum_{i=1}^{m} X_{ij}^2\left(h_j^B P_{ij}+h_{ij}D_j\right)}}} + \sqrt{\frac{2D_j \sum_{i=1}^{m} n_{ij}{}^2 P_{ij} R_{ij}}{\sum_{i=1}^{m} X_{ij}^2\left(h_j^B P_{ij}+h_{ij}D_j\right)}} \times \frac{X_{ij}^2\left(h_j^B P_{ij}+h_{ij}D_j\right)}{2n_{ij}P_{ij}} + D_j X_{ij}\left(Z_{ij}+t_{ij}\right)\right] \qquad (12)$$

As mentioned earlier, n is the number of suppliers and m is the number of items. X_{ij} is the fraction of j^{th} item's demand that is supplied by i^{th} supplier, Y_i is a binary variable that indicates decision of buyer to select i^{th} supplier and n_{ij} is an integer number that expresses number of shipment from i^{th} supplier for j^{th} item within a cycle.

The objective function (12) is used to maximize the total benefit of the aforementioned integrated supply chain. The total benefit is obtained from the difference between the incomes (the first term in the objective function) and the costs (the second term in the objective function) in the integrated supply chain model. The constraint set (7) ensures that the sum of orders from suppliers for j^{th} item is equal to the j^{th} item's demand. The constraint set (8) indicates that the i^{th} supplier is capable of producing all the items that the buyer orders. The constraint set (9) guarantees the minimum permissible order of j^{th} item from i^{th} supplier if the i^{th} supplier is selected by the buyer. Also, the constraint set (9) guarantees that the fraction of j^{th} item's demand that the buyer orders from the i^{th} supplier is not more than the i^{th} supplier's production capacity for j^{th} item. Also, this constraint set indicates that if a supplier is not selected, the fraction of j^{th} item's demand which is assigned to this supplier is zero. Finally, constraint set (10) states that the number of shipment should be considered whenever i^{th} supplier is selected to supply j^{th} item. Hence, from the constraint set (9) one can easily infer that X_{ij} is a bounded variable between $[K_{ij}Y_i/D_j,\; P_{ij}Y_i/D_j]$. It can limit our feasible space and accelerate our search. It is the lifeblood of our presented approach to solving the model effectively.

4. Structural Properties of Model

In this section, a research about structural properties of objective function is intended. For the first step, we should investigate convexity of objective function. If we prove that it is a convex function, we can solve the problem for given suppliers and fixed shipping numbers n_{ij} using standard convex analysis. On the contrary, we should develop a meta-heuristic algorithm to find a good solution in reasonable *CPU* time.

Our investigation shows that the objective function is not a convex function in general condition. (For more details see appendix A.) Also, we need to prove that our new model yields better results in terms of total benefits compared with Aliabadi et al. (2013).

Theorem 1. The proposed model always yields better or at least the same results in terms of objective function compared with the single shipment model.

Proof. Suppose S_1 be the feasible region of our problem when all n_{ij} be equal to 1 then S_1 will also be feasible for single shipment model. Therefore, the optimal solution of S_1 is the same in both models with the same value of *TB*.

$$\%Benefit = \left(\frac{C_{new\ 2LGA} - C_{2LGA}}{C_{2LGA}}\right) \times 100 \qquad (13)$$

Where $C_{new\ 2LGA}$ is the cost of proposed *2LGA* and C_{2LGA} is the objective function which is obtained from previous *2LGA* in Aliabadi et al. (2013) model.

The results show efficacy in comparison to the previous model. The average improvement is about 1.68%.

From mathematical point of view, this improvement in global optimum is proved by Theorem 1. But when there is no guarantee to find global optimum, it is possible to find worse value for objective function due to intricacy of new model. But our proposed solution algorithm expresses good quality in proposed samples. Even in some instances, the efficiency is over 9%. Despite of small improvement in some instances, in general it depends on the problem's nature and in some cases a major benefit is achieved which is reasonable regarding to the insignificant increases in calculation time.

Also for more complicated problem, *CPU* time for solving problem sets #3, and #7 by *LINGO* are 13800 and 28800, respectively. As a matter of fact, we try larger samples; however, *LINGO* failed to solve these problems in a reasonable time and memory usage. This point, even, highlights the value of our work, because our proposed procedure can deal with big problems relatively fast.

Table 1
The comparison between new 2LGA and previous 2LGA with all n_{ij}=1

Problem Number	*n*	*m*	New *2LGA*		*2LGA* (n_{ij}=1)		*LINGO* (n_{ij}=1)	*%Benefit*
			Objective Function	*CPU Time (s)*	*Objective Function*	*CPU Time (s)*	*Objective Function*	
1	2	2	37461.81	49.59	37362.253	30.40	37388.69	0.266464
2	3	5	391401.29	190.8	358519.79	116.27	361801.1	9.17146
3	4	4	160414.32	157.2	159486.92	88.98	159460.7	0.58149
4	2	3	2230810.57	75.02	2229756.4	70.04	2207729	0.047277
5	3	2	2096214.68	97.18	2093837.4	45.92	2094784	0.113537
6	3	3	64448.741	93.96	63551.482	70.97	64464.54	1.411862
7	5	5	194591.04	175.038	194292.56	101.34	193263	0.153624

7. Conclusion and Further Research

We have extended the integrated supplier selection, supply allocation and order scheduling approach by Aliabadi et al. (2013) to allow for multiple shipments within an order cycle. Rather than using a general purpose non-linear programming solver (such as *LINGO*), we employed a multi-layer genetic algorithm derived from the structural properties of developed model. By considering the results, one can easily infer that the new modeling exposed more efficiency in comparison to the previous model. The average efficacy in the proposed *2LGA* is about 1.68%. This survey can be used as a starting point for extending the model into other directions making it more realistic.

Although our proposed *2LGA* works well and outperforms the method used by Aliabadi et al. (2013), in this paper, our purpose was not to find the best method to solve the problem. Hence, investigation to find a possible exact method or other heuristic methods to solve the problem is a valuable future work. On the other hand, in this study, all parameters are assumed to be deterministic. Considering stochastic demands is another worthwhile direction for the future works. Besides, further attention is also required to include the routing problem along with supplier selection problem.

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