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QPO frequency derivative–frequency correlation indicates non-Keplerian boundary layer with a maximum in rotation rate

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ABSTRACT

The correlation between the frequency and the absolute value of the frequency derivative of the kilohertz quasi-periodic oscillations (QPOs) observed for the first time from 4U 1636-53 is a simple consequence and indicator of the existence of a non-Keplerian rotation rate in the accretion disc boundary layer. This Letter interprets the observed correlation, showing that the observations provide strong evidence in support of the fundamental assumption of disc accretion models around slow rotators, that the boundary layer matches the Keplerian disc to the neutron star magnetosphere.

Key words: accretion, accretion discs – stars: neutron – X-rays: binaries.

1 INTRODUCTION

Quasi-periodic oscillations (QPO) from neutron stars in low-mass X-ray binaries (LMXB; van der Klis et al. 1985; van der Klis 2010) are likely to contain information on the neutron star in interaction with surrounding material. The beat-frequency model (Alpar & Shaham 1985), interpreted the first discovered QPOs (later called 'horizontal branch oscillations') from the LMXB GX 5-1 (van der Klis et al. 1985) as the beat frequency between the rotation rate of the neutron star and the Keplerian frequency at the inner edge of the accretion disc. Disc modes in association with horizontal and normal branch QPOs were further explored by Alpar et al. (1992) and Alpar & Yılmaz (1997) and many other researchers (see van der Klis 2010 and references therein). With the discovery of QPOs at kilohertz frequencies (Strohmayer et al. 1996; van der Klis et al. 1996; van der Klis 2000), the interaction between the neutron star magnetosphere and the accretion disc boundary layer was posed as the likely source of these high-frequency modulations of the X-ray luminosity (Campana 2000; Cui 2000; van der Klis 2010). Modes in the boundary layer of a thin gaseous accretion disc with inner radius quite close to the neutron star could be modulating the accretion flow on to the neutron star at such high frequencies (Wagoner 1999; Kato 2001; Erkut & Alpar 2004; Alpar & Psaltis 2008; Erkut, Psaltis & Alpar 2008; Kato 2009, 2012). The rotation rates $\Omega(r)$ in the disc boundary layer are likely to be non-Keplerian, as the boundary layer matches the Keplerian flow in the disc to the rotation frequency Ω_* of the neutron star and its corotating magnetosphere. LMXBs contain 'slow rotators', with the neutron star rotation rate Ω_* less than $\Omega_K(r_{in})$, the Keplerian rotation rate in the inner disc beyond the boundary layer. The rotation rate $\Omega(r)$ within the boundary layer must clearly go through a maximum value Ω_{max}, as it deviates from the Keplerian flow in the disc to match the stellar rotation rate Ω_* at the magnetosphere. Alpar & Psaltis (2008) noted that this deviation from Keplerian flow in the boundary layer can significantly modify constraints on the neutron star mass and radius obtained by associating kilohertz QPO frequencies with Keplerian frequencies at the disc-magnetosphere boundary. The mode frequencies to be associated with upper or lower kilohertz QPOs are of the form $\kappa(r) \pm m\Omega(r)$, where m is an integer and the epicyclic frequency $\kappa(r)$ is defined through $\kappa^2 \equiv 2\Omega(2\Omega + r\mathrm{d}\Omega/\mathrm{d}r)$. The rotation rate $\Omega(r)$ and the epicyclic frequency $\kappa(r)$ depend on the radial position r within the boundary layer (Erkut et al. 2008). The frequencies and widths of the QPOs are likely to be affected by the regions of the boundary layer that are dominant in the modulation of the accretion flow to the neutron star. The observation of a correlation between the lower kilohertz QPO frequency and the rate of change of this frequency with time (Sanna et al. 2012) provides direct evidence for the non-Keplerian nature of the rotation rate $\Omega(r)$ in the boundary layer with the existence of a maximum $\Omega(r)$. It is nice and rather rare that such a simple analytical property of a model is verified by an astrophysical observation.

2 NON-KEPLERIAN FLOW IN THE BOUNDARY LAYER IS INDICATED BY OBSERVATIONS

Sanna et al. (2012) have measured, for the first time, short time-scale time derivatives of the lower kilohertz QPO frequency from the neutron-star LMXB 4U 1636-53. They measure positive and negative time derivatives as the QPO frequency wanders. The data exhibit a remarkable, simple correlation, that the absolute value of the time derivative of the lower kilohertz QPO frequency decreases as that QPO frequency increases:

$$\frac{\mathrm{d}|\dot{\nu}|}{\mathrm{d}\nu} < 0. \quad (1)$$

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Following Sanna et al. (2012), I will employ frequencies ν in Hz. If the observed kilohertz QPO frequencies are mode frequencies $\nu(r)$ at some radial positions r in the boundary layer, applying the chain rule, and using $\mathrm{d}r/\mathrm{d}t = V_\mathrm{r}$, the radial velocity of matter in the boundary layer, equation (1) leads to

$$\begin{aligned} \frac{\mathrm{d}|\dot{\nu}|}{\mathrm{d}\nu} &= \frac{\mathrm{d}}{\mathrm{d}\nu}\left|\frac{\mathrm{d}\nu}{\mathrm{d}r}\right||V_\mathrm{r}| \\ &= \frac{\mathrm{d}r}{\mathrm{d}\nu}\frac{\mathrm{d}}{\mathrm{d}r}\left|\frac{\mathrm{d}\nu}{\mathrm{d}r}\right||V_\mathrm{r}| \\ &= |V_\mathrm{r}|\left|\frac{\mathrm{d}r}{\mathrm{d}\nu}\right|\frac{\mathrm{d}^2\nu}{\mathrm{d}r^2} < 0. \end{aligned} \tag{2}$$

The radial velocity V_r is assumed to be uniform throughout the boundary layer, and constant on the short time-scales of variation of the kilohertz QPO frequencies, so that additional terms in equation (2) representing radial and temporal variations of V_r are neglected. In the Shakura & Sunyaev (1973) model, V_r depends on the mass-inflow rate only to the power 1/3; $V_\mathrm{r} \propto \dot{M}_\mathrm{inflow}^{1/3}$, as noted by Sanna et al. (2012). Fluctuations in the mass-inflow rate $\dot{M}_\mathrm{inflow}$ from the companion are likely to be smoothed by the viscous transport through the disc. Equation (2) shows that the run of rotation rate $\nu(r)$ in the boundary layer has a negative second derivative,

$$\frac{\mathrm{d}^2\nu}{\mathrm{d}r^2} < 0. \tag{3}$$

This means that $\nu(r)$ has a continuous derivative going through zero at a smooth maximum at some finite radius r_0 in the boundary layer. All of the choices for ν_QPO as a mode frequency in the boundary layer, like $(1/(2\pi)$ times) $\kappa(r)$, $\Omega(r)$, $\kappa(r) \pm \Omega(r)$ indeed have such a maximum in the boundary layer. In fact, this is the characteristic property of boundary layer modes as pointed out by Alpar & Psaltis (2008). QPO models employing general relativistic frequencies (Stella & Vietri 1998) and resonances (e.g. Kluzniak et al. (2004)) also have frequency maxima at radii around r_ISCO, the innermost stable circular orbit. General relativistic effects will fold in and play a secondary role to viscous and hydrodynamic effects leading to the non-Keplerian flow in the boundary layer. If the lower kilohertz QPO frequency were a Kepler frequency, or were related to the upper kilohertz QPO frequency with a linear relation as discussed by Sanna et al. (2012) and the upper kilohertz QPO were Keplerian, then we would have

$$|\dot{\nu}| \propto \left|\frac{\mathrm{d}\nu_\mathrm{K}}{\mathrm{d}r}\right|V_\mathrm{r} = \frac{3}{2}\frac{\nu_\mathrm{K}}{r}V_\mathrm{r}, \tag{4}$$

which increases as the QPO frequency increases, contrary to the correlation observed by Sanna et al. (2012). Substituting measured values of $|\dot{\nu}|$ and assuming $\nu_\mathrm{QPO} \cong \nu_\mathrm{K}$ gives a very low value of the radial flow rate V_r in the boundary layer, requiring a very low value of the Shakura–Sunyaev thin disc α parameter, $\alpha \sim 10^{-6}$ (Sanna et al. 2012). Associating the kilohertz QPO frequencies with mode frequencies in the boundary layer solves both of these problems and explains the results of Sanna et al. (2012) naturally. If the lower kHz QPO frequency is either κ or $\kappa - \Omega$,

$$|\dot{\nu}| = \frac{1}{2\pi}\left[\frac{\partial\kappa}{\partial\Omega},\ or\ \left(\frac{\partial\kappa}{\partial\Omega} - 1\right)\right]\left|\frac{\partial\Omega}{\partial r}\right|V_\mathrm{r}. \tag{5}$$

The frequency band is determined by the location near the maximum of Ω and κ of those parts of the boundary region that pre-dominate in the formation of the QPO signal. Here $|\frac{\partial\Omega}{\partial r}|$ and therefore $|\dot{\nu}|$ do decrease as the frequency increases towards the maximum. As $|\frac{\partial\Omega}{\partial r}|$ is small and $[\frac{\partial\kappa}{\partial\Omega},$ or $(\frac{\partial\kappa}{\partial\Omega} - 1)]$ are of O(1), V_r and α are not unduly small.

3 DISCUSSION AND CONCLUSIONS

In models involving the disc boundary, the QPO signals in the accretion luminosity are due to variations in the flow through the boundary layer that may be intrinsic or excited through resonant interactions with the neutron star. These variations are wave-packets of the boundary layer normal modes visualized as 'blobs'. The frequencies and growth or decay time-scales of the boundary layer normal modes are calculated by modelling the local dynamics at each radial position r. The growth and decay times are likely to be much longer than the ~100 s time-scales of the observations analysed by Sanna et al. (2012) if the sound speed $c_\mathrm{s} < 0.5\Omega r$ for $\alpha < 0.01$, or $c_\mathrm{s} < 0.2\Omega r$ if $\alpha = 0.1$ (Erkut et al. 2008). Wave-packets that effect the accretion with discernibly high-quality QPOs are likely to arise in the parts of the boundary layer where the frequency spread $\delta\nu = \partial\nu/\partial r\delta r$ is relatively small, i.e. near the mode frequency maximum. The boundary layer is in interaction with the neutron star magnetosphere so that its excitations are imprinted on the accretion flow and show up as QPOs in the accretion luminosity. With this reasonable assumption, as the wave-packet moves through the boundary layer with speed V_r, the QPO frequency will have a derivative as employed in the derivation leading to equation (2). The QPO frequency derivative is positive or negative, with similar absolute values as observed by Sanna et al. (2012), depending on the sign of $\partial\nu/\partial r$, positive when the wave-packet is closer to the star than the location of the maximum, and negative when the wave-packet is beyond that location. Fig. 2 of Sanna et al. (2012) shows that the frequency spread of the lower kilohertz QPO decreases to about 2.5 Hz at a QPO frequency of about 920 Hz. Fig. 3 shows that the absolute value of the QPO frequency derivative decreases to about one-fifth of its value at 650 Hz. The few frequency derivative measurements just above 920 Hz have larger error bars, and the frequency spread and its errors are also larger. These trends are suggestive that the maximum mode frequency associated with the lower kilohertz QPOs may be about 920 Hz. The presence of frequency maxima may hold clues to other QPO phenomenology. For example, a 3:2 ratio of frequencies corresponds to the $\kappa + \Omega$ and κ modes near their maxima, where $\kappa = 2\Omega$ (see Erkut 2011 for an application to black hole QPO). Parallel QPO frequency-count rate tracks (Mendez et al. 1999) may correspond to runs of mode frequencies in the boundary layer at epochs of different $< \dot{M}_\mathrm{inflow} >$ (van der Klis 2001) with different inner disc radii and maximum mode frequencies. The observation that the absolute value of the time derivative of the lower kilohertz QPO frequency decreases as that QPO frequency increases, provides direct evidence, within models that associate the kilohertz QPOs with modes of the accretion disc boundary layer, of the existence of a maximum in the non-Keplerian rotation rate $\Omega(r)$ in the boundary layer. The boundary layer rotation curve is expected to have a maximum as it evolves from the Keplerian rotation curve in the disc to match the ('slow rotator') neutron star's rotation rate Ω_* at the magnetospheric boundary. The astrophysical observation of Sanna et al. (2012) verifies this simple analytical property of disc boundary layer models.

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