Optimal Subsidization of Business Start-ups*

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Abstract

This paper studies efficient allocation of resources in an economy in which agents are initially heterogeneous with regard to their wealth levels and whether they have productive ideas or not. An agent with an idea can start a business that generates random returns. Agents have private information about (1) their initial types, (2) how they allocate their resources between consumption and investment, and (3) the realized returns. I show that, under informational frictions, a society that targets productive efficiency should subsidize poor agents with ideas, and choose the amount and timing of subsidies carefully in order to ensure that other agents do not mimic poor agents with ideas and receive subsidies. Then, I provide an implementation of the start-up subsidies in a market framework that resembles the U.S. Small Business Administration's Business Loan Program.

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1 Introduction

Starting a business requires two main ingredients: a productive idea and resources to invest in that idea. Unfortunately, it is not necessarily the case that whoever has one of these ingredients also has the other one. Consequently, there is a potential mismatch among individuals in a society in terms of who holds productive resources and who can use them most efficiently. In a frictionless world, a solution to this mismatch is provided by private markets: those with ideas (potential start-ups) can borrow from those with resources, invest, and then pay back. This paper explores how a society should cope with this mismatch in an environment in which informational frictions limit market's ability to finance investment in productive ideas. I show that, under informational frictions, a society that targets productive efficiency has to (1) subsidize agents with ideas, (2) choose the amount and timing of transfers carefully in order to ensure that agents without ideas do not mimic those with ideas and receive subsidies. Then, I provide an implementation of the start-up subsidies in a market framework that resembles the U.S. Small Business Administration's Business Loan Program.

Individuals in the model economy live for two periods and are risk-neutral. In period one, agents are heterogeneous with respect to wealth levels and whether they have ideas or not. Agents with ideas can create businesses that generate risky returns in period two. The returns depend on the amount of capital invested and there is diminishing marginal returns to capital. In the absence of informational frictions, efficient resource allocation involves two separate steps: (1) productive efficiency requires transferring resources to poor and productive agents in period one to ensure that they all can invest at the socially efficient level; (2) distributive efficiency then requires making transfers between agents so as to achieve the desired consumption distribution, which depends on the welfare criterion of the society.

Unfortunately, it is hardly the case that all relevant information about business start-ups are known publicly.[1] The paper assumes that agents' ex ante types (wealth-idea), how they allocate their resources, and ex post returns to business start-ups are private information. Under the unobservability of the returns, productive efficiency implies poor agents with ideas should be subsidized so as to get them operate their businesses at the efficient scale. The assumption that agents' wealth-idea types are private information implies that the government might be limited in the amount of subsidies it can transfer to agents with ideas: large transfers might induce people without ideas to mimic agents with ideas and receive transfers. The constrained efficient level of start-up subsidies arise from this productive efficiency vs. incentives trade-off.

In order to understand the intuition for the subsidy result, one first needs to know what society cares about in this economy. I assume the social welfare function to be utilitarian with equal weights on every agent. This assumption, together with risk neutrality of agents, implies that society has a preference only for the amount of total consumption, not for how it is distributed across agents. The society is only concerned about agents making right amounts of investment. Therefore, the problem that the society is facing is maximizing production subject to incentive compatibility and feasibility.

The intuition for the subsidy result is simple. Since there are diminishing marginal returns to investment in start-ups, there is a socially efficient level of investment in each start-up.

[1] See Hubbard (1998) for a survey of the literature on informational problems in capital markets.

However, since returns to start-ups are unobservable, agents cannot write contracts with state-contingent repayment schedules. This market incompleteness then implies that agents can, at most, borrow an amount that they can pay back the next period in the lowest return state.[2] This borrowing constraint binds for poor agents with ideas when they want to invest at their efficient level. If the society can transfer some of its resources to these individuals, it would relax their budget constraints, enabling them to produce at a level closer to the social optimal, which is the social objective.

Consequently, this paper focuses solely on productive efficiency, leaving aside distributional concerns. The motivation for subsidy in this model is the need to finance the investment of poor agents with ideas. In fact, due to the choice of social welfare function and risk-neutrality assumption, this is the only reason why subsidy is socially desirable.

If the society knew who were the poor agents with ideas, then it would be very easy to implement the subsidy. However, when there are benefits at stake, such as a subsidy, people can pretend to be poor and to have productive ideas, get the subsidy, and consume it. As a result, the amount of subsidy going to poor agents with ideas is constrained by incentive compatibility: agents should not find it optimal to lie about their wealth and ideas, and use the subsidy for reasons other than investment.

Of course, it is possible that the society can try to understand whether people's ideas are productive, and monitor their wealth and how they use the subsidy. However, these activities are all costly. The assumption that it is impossible to pursue such monitoring activities corresponds to assuming that monitoring costs are prohibitively high. I accept that this is an extreme assumption; however, assuming that agents' wealth, ideas, and actions are perfectly, costlessly observable is also extreme. I focus on the less studied of the extremes. I conjecture that the subsidy result would still be true if I allowed for monitoring technologies as long as the cost of monitoring is not zero.

It is important to note that the subsidy result is not an artifact of risk neutrality; it survives even if agents have strictly concave utility functions. The fact that poor agents with ideas might be borrowing constrained due to their inability to sign state-contingent debt contracts and as a result might end up investing at suboptimal levels under laissez-faire is not coming from risk-neutrality assumption. Thus, the desirability of subsidies remain valid even if we assume that people are risk-averse. However, in that case, and with a conventional social welfare function, society would also have a taste for equality that would create a motive for redistribution from the rich to the poor. Furthermore, under risk aversion, society would like to smooth people's consumption across states and periods. The risk-

[2] Observe that I do not allow for default in the model. Following Diamond (1984), one can add default to this model by assuming that if a start-up continues to operate after period two, this brings a continuation value to the owner; if not, then at least some strictly positive fraction of this value gets destroyed. Then, agents can write state-contingent contracts by conditioning the continuation of a start-up business on the level of repayment. The fear of losing a fraction of the continuation value can make the agent make the payment associated with her true return level. In such a world, θ_l state can be interpreted as a default state. Even though in such a model poor agents with ideas would be able to borrow more than they can in the original model, one can show that this level would still be strictly less than the amount they need to finance socially efficient investment level. Therefore, the subsidy result would still be true under this alternative model. The reason why such an extension can be interesting is because it can make the details of the efficient social contract more realistic, giving rise to a more realistic implementation. A paper along these lines is currently work in progress.

neutrality assumption makes it possible to abstract away from these additional distributive forces and allows us to focus solely on what productive efficiency dictates for fiscal policy regarding business start-ups. I provide a further discussion of risk-neutrality assumption in Section 3.2.

A corollary that follows from the subsidy result is that how productive activity (distribution of investment in the current context) *should be* organized in the economy depends on the distribution of wealth. This result depends crucially on the existence of informational frictions. The result and the assumptions behind it are further explained in Section 3.

The paper provides a decentralization of the constrained efficient allocation in an incomplete markets setup where people trade risk-free bonds in a competitive market. Given that markets cannot provide subsidies on their own, an incomplete markets equilibrium under laissez-faire cannot attain constrained efficiency.[3] In order to implement the efficient allocation, the paper introduces two separate institutions to the market environment: a government and a government agency that deals with start-up firms. The government taxes all agents in a lump-sum manner and subsidizes its agency from its budget. The agency then subsidizes some individuals from a pool of applicants based only on their level of bond holdings. The tax-subsidy system is chosen such that only agents with ideas get subsidized. A comparison of the implementation with the U.S. Small Business Administration's (SBA) Business Loan Program is provided in Section 4.

Related Literature. This is not the first paper to put forth the idea that, under informational frictions, subsidizing a certain group of individuals in a society may increase productive efficiency. Aghion and Bolton (1997) show that when there are moral hazard problems with limited wealth constraints, as in Sappington (1983), then an economy with a more egalitarian wealth distribution dominates one with a less egalitarian distribution in terms of total long-run output. This result might be interpreted to support the notion that redistributing resources from the rich to the poor might be optimal for a society that cares about total output maximization. However, it is important to note that Aghion and Bolton (1997) does not explicitly analyze the problem of a government which aims to transfer resources from the rich to the poor. Therefore, they do not deal with the problem of designing subsidies that maximize total output subject to incentive compatibility constraints, which is exactly what the current paper does.[4]

Loury (1981), Banerjee and Newman (1991), and Galor and Zeira (1993) are also related to the current paper. These papers share a common result: in the presence of capital market imperfections, the distribution of wealth affects the distribution of investment, and hence aggregate output.[5] This is akin to the following result I derive in this paper: the distri-

[3] Note that I do not allow for markets to open ex ante, meaning before agents know whether they are rich or poor and whether they have ideas or not. If that is allowed, then the interpretation of the optimal contract would be completely different. Instead of calling the transfers in the optimal social contract subsidies, we would call them state-contingent payment schedules of the optimal financial contract written between agents behind the veil of ignorance. This is the implementation technique proposed in Prescott and Townsend (1984). Thus, constrained efficiency requires either markets to open ex ante or government to execute subsidies.

[4] Also, note that in Aghion and Bolton (1997), all the people in the economy have the same entrepreneurial ability (they all have ideas), so even if they were to analyze the optimal design of subsidies, their government would not have to worry about targeting subsidies to agents with ideas.

[5] Aghion et al. (1999), Section 2, proves a similar result and provides a discussion of related papers.

bution of wealth affects the distribution of productive activity in the constrained efficient allocation. However, there is an important distinction between the two results. All the papers mentioned above assume some form of *exogenous* market incompleteness and show that the wealth distribution affects the equilibrium distribution of investment under this assumption. The current paper, on the other hand - instead of making arbitrary assumptions on the space of contracts available to agents - takes as given informational frictions and shows that the distribution of wealth affects productive activity in an economy *even* in the constrained efficient allocation. Consequently, this paper directly establishes that it is due to informational frictions that the distribution of wealth affects the distribution of productive activity.[6] Quintin (2008) is similar to the current paper in this sense: it shows that under limited enforcement frictions, a la Kehoe and Levine (1993), the distribution of wealth affects the organization of production in the constrained efficient allocation.

Another strand of literature that is related to this paper is on optimal venture capital contracts since both this literature and the current paper consider the question of how to finance business start-ups.[7] In general, the venture capital literature focuses on characterizing the structure of optimal contracts in principal-agent relationships in which venture capitalists monitor everything but entrepreneurs' effort. Keuschnigg and Nielsen (2003) and Keuschnigg and Nielsen (2004) analyze optimal tax policy when venture capital financing is available to business start-ups. The current paper assumes less transparency between investors and start-ups by assuming that no agent in the economy can monitor whether others have business ideas that are worth investing, and once the business is set up how much they invest in their businesses. This rules out the existence of venture capital in the current model.[8] Consequently, this paper deals with the complementary problem of how a government should design its entrepreneurial policy in an environment in which, due to informational problems, venture capitalists do not exist.

The rest of the paper is organized as follows. Section 2 introduces the baseline model formally and analyzes the full information benchmark. Section 3 defines and solves for the constrained efficient allocation. Section 4 provides an implementation of constrained efficient allocation similar to the U.S. SBA's loan program. Finally, Section 5 concludes. The proofs of the propositions are presented in a separate online Appendix.

2 Model

2.1 Environment

The economy is populated by a continuum of unit measure of agents who live for two periods. Agents are risk-neutral with the instantaneous utility function $u : \Re \to \Re$ defined as $u(c) = c$, for $c \geq 0$ and $u(c) = -\infty$, for $c < 0$. Allowing for negative consumption but setting

[6]Banerjee and Newman (1993) is closer to the current paper in the sense that it explicitly models an informational friction that causes the market imperfection. However, it restricts the contract space available to the agents arbitrarily. Therefore, essentially, it also focuses on some exogenously specified equilibrium notion, not on constrained efficiency.

[7]See Admati and Pfleiderer (1994) and Gompers (1995) for important contributions to this literature.

[8]The paper does not claim that venture capital does not exist in real life or it is not important. However, given that it requires some resources that are limited in supply (like time of experts) and, hence, serves a relatively small portion of business start-ups, an alternative less transparent relationship is also present.

utility derived from it to negative infinity is a convenient way of securing non-negativity of consumption in the solution. They are expected utility maximizers with

$$E_1\{u(c_1) + \beta u(c_2)\},$$

where c_t is period t consumption and $\beta \in (0, 1)$ is the discount factor.

At the beginning of period one, some agents are born with ideas and some without. Let i denote whether an agent is bornwith an idea or not. Those who have ideas are called $i = 1$ types, and those who do not are called $i = 0$ types. Let $I = \{0, 1\}$. The fraction of agents born with (without) an idea is η_1 (η_0). Only agents with ideas can start businesses. Agents are also born with different levels of initial endowment of the only consumption good, $w \in W = \{p, r\}$, $p < r$. Fraction ζ_w are born with initial wealth level w. There is no endowment in period two. The distribution of wealth across agents is independent from the distribution of ideas. So, there are four types of agents initially, at the beginning of period one: $\{(p, 0), (p, 1), (r, 0), (r, 1)\}$.

When an agent of type (w, i) starts a business in period one by investing k units, she gets the following return in period two

$$y = i\left[\theta g(k) + (1 - \kappa)k\right],$$

where κ is the depreciation rate, θ is the random return on capital, and y is the random output produced in period two. The function $g(\cdot)$ is a diminishing returns to scale production function with usual properties: $g(0) = 0$ and $g', -g'' > 0$. θ is drawn from the set $\Theta = \{\theta_l, \theta_h\}$, where $\theta_l < \theta_h$, according to the probability distribution μ, independently across agents. The probability of drawing θ_l is μ_l and θ_h is μ_h. The assumption that the cardinality of the set of returns is two is immaterial for any result. All the results go through if Θ has any finite number of elements. An agent gets to learn the realization of return after the investment is made. Hence, agents face idiosyncratic investment risk. The term i is in the production function to denote that only agents with ideas can start businesses. To ease notation, define

$$f(k, \theta) = \left[\theta g(k) + (1 - \kappa)k\right].$$

What is important is that the value of the start-up is higher than zero in the lowest return state, i.e., $f(k, \theta_l) > 0$, for $k > 0$. This is true if either $\kappa < 1$, meaning there is no full depreciation, or $\theta_l > 0$. As we will see in Section 3 where we analyze the constrained efficient allocation, under this assumption, agents without ideas find it costly to pretend to have ideas, which allows the planner to subsidize poor agents with ideas in an incentive compatible way.

There is also a risk-free, linear storage technology that allows people to transfer resources from period one to two at a gross return A. This technology is available to all agents and it is private, meaning the amount of consumption good each agent stores is unobservable to other agents. The storage technology can be interpretted literally as putting part of the consumption good in a private storage unit. In that case, the return A is one if the good is perfectly storable or is strictly less than one if part of the good perishes during storage. In this paper, I allow for A to be greater than one, which allows for a broader interpretation of the storage technology: it can also be thought as a production unit that people operate privately in their backyards. Importantly, I assume that $A < 1/\beta$. This means that storage is an inferior technology since immediate consumption of a unit of good gives higher utility than storing and consuming it in period two. Below I formally state this assumption.

Assumption 1. $A < 1/\beta$.

The information structure and timing of events are as follows: An agent's initial type, actions, and period two realized returns are private information. The rest of the data of the economy is public information. Given her initial type, an agent chooses how much to consume, invest, and store in period one. Then, in period two, θ is realized and hence output is produced, and the agent consumes. Whether θ is realized in period one or two is immaterial for the results; the important thing is that it is realized after the investment decision is made so that agents face investment risk.

One way to think about resource allocation is to consider a benevolent social planner who chooses allocations for agents. Since consumption-investment choice is unobservable, the planner cannot choose allocations directly. Instead, each period the planner makes transfers between agents based on their reports of their private histories. This way the planner manipulates agents' actions. In addition, there is no outside party, which means the planner cannot save or borrow resources through time. All results would go through if, instead, the planner could borrow and save at a risk-free rate of $1/\beta$.

An allocation in this economy is a vector $(c, k, s, \delta) \equiv (c_1, c_2, k_1, s_1, \delta_1, \delta_2)$, where

$$c_1, c_2 : W \times I \to \Re; \quad k_1, s_1 : W \times I \to \Re_+; \quad \delta_1 : W \times I \to \Re,\ \delta_2 : W \times I \times \Theta \to \Re.$$

Here, $c_1(w, i)$, $k_1(w, i)$, and $s_1(w, i)$ refer to period one levels of consumption, investment, and storage of the agent who has initial wealth w and idea i. Similarly, $c_2(w, i, \theta)$ is the consumption level of the agent of type (w, i) who has a realized return θ in period two. Since an agent with no idea cannot start a business, her period two consumption is independent of θ, meaning $c_2(w, 0, \theta_l) = c_2(w, 0, \theta_h)$. $\delta_1(w, i)$ and $\delta_2(w, i, \theta)$ are the levels of transfers received by corresponding types.

Feasibility. An allocation (c, k, s, δ) is *feasible* if

$$\begin{aligned} \textstyle\sum_{w,i} \zeta_w \eta_i \delta_1(w, i) \leq 0, \\ \textstyle\sum_{w,i} \sum_{\theta} \zeta_w \eta_i \mu_\theta \delta_2(w, i, \theta) \leq 0, \end{aligned} \tag{1}$$

and for every $(w, i) \in W \times I$

$$\begin{aligned} c_1(w, i) + k_1(w, i) + s_1(w, i) \leq w + \delta_1(w, i), \\ c_2(w, i, \theta) \leq f(k_1(w, i), \theta) i + A s_1(w, i) + \delta_2(w, i, \theta), \end{aligned} \tag{2}$$

$$k_1(w, i), s_1(w, i) \geq 0. \tag{3}$$

Here, (1) is period by period *aggregate feasibility* condition regarding transfers, (2) is *individual feasibility* and stands for the fact that allocation assigned to each agent should be affordable by him, and (3) is just the non-negativity constraint on investment and storage.

Incentive compatibility. Using the terminology of mechanism design literature, there are two sources of private information in the model. First, there is hidden information: an agent's initial type and period two investment returns are observed privately by the agent. Second, agents are involved in hidden action: their consumption and investment levels are hidden. Hence, people can deviate from an allocation recommended by the planner both by lying about their private information and/or by choosing an investment level that is different from what the planner recommended. I invoke a powerful revelation principle introduced by Myerson (1982) and characterize the set of incentive compatible allocations as follows.

Let $(\tilde{w}, \tilde{\imath}) \in W \times I$ and $\tilde{\theta} : \Theta \to \Theta$ be agent's period one and period two reporting strategies, respectively. Also, define $(\tilde{k}_1, \tilde{s}_1) \in \Re^2_+$ as agent's investment strategy. Then, $\tilde{\gamma} \equiv (\tilde{w}, \tilde{\imath}, \tilde{\theta}, \tilde{k}_1, \tilde{s}_1)$ is a complete strategy of agent (w, i).[9] Let Γ be the set of all complete strategies. Given allocation (c, k, s, δ), for any (w, i), the value of following strategy $\tilde{\gamma}$ is:

$$V_{w,i}(\tilde{\gamma}; c, k, s, \delta) \equiv u[w + \delta_1(\tilde{w}, \tilde{\imath}) - \tilde{k}_1 - \tilde{s}_1] + \beta \sum_{\theta} \mu_\theta u[f(\tilde{k}_1, \theta)i + A\tilde{s}_1 + \delta_2(\tilde{w}, \tilde{\imath}, \tilde{\theta}(\theta))].$$

Define $\gamma \equiv (w, i, \theta^*, k_1, s_1)$ to be the strategy consisting of truthful reporting and obeying recommendations, where $\theta^*(\theta) = \theta$ denotes the truth-telling period two reporting strategy.

An allocation (c, k, s, δ) is *incentive compatible* if for each $(w, i) \in W \times I$,

$$V_{w,i}(\gamma; c, k, s, \delta) \geq V_{w,i}(\tilde{\gamma}; c, k, s, \delta), \text{for all } \tilde{\gamma} \in \Gamma. \tag{4}$$

2.2 Benchmark: Full Information Efficiency

The aim of this section is to analyze the allocation society can achieve when everything in the economy is publicly observable. We call such allocation full information efficient or simply *efficient* allocation. The efficient allocation turns out to be a useful benchmark for what the society can achieve under informational frictions. Under the utilitarian objective, the efficient allocation is the solution to the following problem:

$$\max_{c,k} \sum_{w,i} \zeta_w \eta_i \Big\{ u(c_1(w,i)) + \beta \sum_{\theta} \mu_\theta u(c_2(w,i,\theta)) \Big\}$$

s.t.

$$\sum_{w,i} \zeta_w \eta_i \Big\{ c_1(w,i) + k_1(w,i) + s_1(w,i) \Big\} \leq \sum_{w} \zeta_w w,$$

$$\sum_{w,i} \zeta_w \eta_i \sum_{\theta} \mu_\theta c_2(w,i,\theta) \leq \sum_{w,i} \zeta_w \eta_i \sum_{\theta} \mu_\theta \Big\{ f(k_1(w,i),\theta)i + As_1(w,i) \Big\},$$

$$k_1(w,i), s_1(w,i) \geq 0, \text{ for all } (w,i) \in W \times I.$$

Since s_1 is wasteful, in the full information efficient allocation $s_1(w, i) = 0$, for all (w, i).

Assuming that total initial wealth is large enough to finance the aggregate level of efficient investment, the first-order optimality condition for investment of agents with ideas reads:

$$1 = \beta \left[g'(k_1(w,1)) E\{\theta\} + (1 - \kappa) \right],$$

[9] Myerson (1982) calls this *participation strategy.* Also, note that consumption is not a part of the strategy since it is implied by the choice of other actions.

where $E\{\theta\} = \sum_\theta \mu_\theta \theta$. The left-hand side of the equation is the marginal social cost of investing an additional unit in terms of period one utility. The right-hand side is the marginal social benefit of investment in the same units. This condition defines

$$k_1(w,1) = g'^{-1}\Big\{\frac{1-\beta(1-\kappa)}{\beta E\{\theta\}}\Big\} \equiv \bar{k}$$

as the efficient level of investment in start-ups provided that the following assumption holds.[10]

Assumption 2. *Total resources in the economy in period one are sufficient to finance $\bar{k}$ investment for each $(w,1)$ agent, or $\eta_1 \bar{k} \leq \sum_w \zeta_w w$.*

Assumption 2 formally states that cumulative initial wealth is sufficiently large to finance the aggregate level of efficient investment. It is made solely for expositional purposes. If it does not hold, then the efficient level of investment will be a corner solution, $\frac{\sum_w \zeta_w w}{\eta_1}$, and all the results of the paper go unchanged.

Observe that the full information efficient investment level is independent of agents' wealth level. This makes sense as those with ideas operate identical entrepreneurial technologies independent of their wealth levels. Moreover, looking at the objective function of the full information problem, one can see that utilitarian welfare with equal weights and risk neutrality together imply that society has no preference for how total consumption should be distributed, as long as no one gets negative consumption. The society is only concerned about the right agents making the right amounts of investment.

The next section analyzes a problem with the same objective function, but with a different constraint set due to private information. Hence, that problem will be one of maximizing production subject to feasibility and incentive compatibility. Thanks to the benchmark analysis, it is clear that the challenge that awaits the society under private information is to make agents with ideas invest as close to the full information efficient level as possible.

3 Constrained Efficient Allocation

While analyzing the benchmark case of full information, I only made one assumption, and that compared the total level of efficient investment to total initial wealth. However, with private information, the comparison of individual wealth levels and efficient level of investment - p, r and $\bar{k}$ - becomes important. The first assumption about this comparison is the following:

Assumption 3. $p < \bar{k} < r$.

The first part of this assumption, $p < \bar{k}$, says that the initial wealth of the poor is not large enough to cover the full information level of investment. Thus, a poor agent with an idea cannot operate her idea at the most efficient level on her own. If, to the contrary, $p \geq \bar{k}$ were the case, the economy would reach full information without agents interacting at all. Obviously, this case is neither interesting nor realistic. The second part of the assumption, $\bar{k} < r$, says that a rich agent with an idea can invest at the efficient level even under autarky.

[10] $g'^{-1}(\cdot)$ is a well-defined function since $g'' < 0$.

The remainder of this section first defines and then characterizes the efficient allocation under informational problems. Throughout the paper, I refer to this allocation as the *constrained efficient* allocation to distinguish it from the full information efficient allocation.

Constrained Efficient Allocation. *An allocation* $(c^*, k^*, s^*, \delta^*)$ *is called constrained efficient if it solves the following social planner's problem:*

$$\max_{c,k,\delta} \sum_{w,i} \zeta_w \eta_i \Big\{ u(c_1(w,i)) + \beta \sum_{\theta} \mu_\theta u(c_2(w,i,\theta)) \Big\} \quad \textit{subject to } (1), (2), (3), \textit{ and } (4).$$

As in the benchmark case, the objective function clearly shows that society does not care about how consumption is going to be distributed among individuals. Consequently, the above problem is one of constrained productive efficiency. This implies there can be many constrained efficient allocations, all of which have the same investment allocation and hence the same total production and welfare, but different consumption allocation across agents. Even though we are not interested in who consumes how much, I would like to note that incentive compatibility arising from private information does put some discipline on the distribution of consumption across agents compared to the full information benchmark.

3.1 Characterizing the Constrained Efficient Allocation

First, observe that any transfer mechanism in which a period two transfer depends on a period two shock, θ, cannot be incentive compatible since if period two transfers depend on θ, then all the agents would report to be of the θ type that brings the highest level of period two transfers. Consequently, without loss of generality, the rest of the paper restricts attention to allocations in which transfers in both periods are functions of period one announcements only, $\delta_1, \delta_2 : W \times I \to \Re$. Now I make the second assumption comparing p and $\bar{k}$.

Assumption 4. $\frac{\bar{k}-p}{\beta} > f(\bar{k}, \theta_l)$.

To understand why this assumption is important, suppose it does not hold. Observe that in order to invest at the full information efficient level, the poor agent with an idea needs at least $\bar{k} - p$ additional resources in period one. Moreover, the most this agent can pay back in period two in low-return state is $f(\bar{k}, \theta_l)$. When Assumption 4 does not hold, poor agents with ideas can sign debt contracts that promise to pay an interest rate of $1/\beta$ with other agents in the economy, borrow $\bar{k} - p$, invest in their businesses, and pay back $\frac{\bar{k}-p}{\beta}$ with certainty next period. The society can implement the full information outcome by just making sure that such simple, not state-contingent debt contracts are perfectly enforced. However, that even in the worst case an entrepreneur can pay back her debt is highly unrealistic, especially for businesses that are newly forming.[11]

Proposition 1 below formally shows that when Assumption 4 does not hold, the full information efficient allocation is trivially reached by a mechanism in which net present value (NPV) of transfers going to each agent is zero. Before getting to the proposition,

[11] That the lowest return is sufficiently dire is a standard assumption in financial contracting literature. Among others, see Diamond (1984), Gale and Hellwig (1985), Bolton and Scharfstein (1990), and DeMarzo and Fishman (2007).

define $\Delta(w,i) = \delta_1(w,i) + \beta\delta_2(w,i)$ as the NPV of transfers an agent gets under a given allocation. An agent (w,i) is said to be *subsidized* under an allocation if $\Delta(w,i) > 0$.

Proposition 1. *Suppose that $\frac{\bar{k}-p}{\beta} \leq f(\bar{k}, \theta_l)$. Then, in the constrained efficient allocation:*[12]

1. *For all $w \in W$, $k_1^*(w,1) = \bar{k}$ and $k_1^*(w,0) = 0$.*
2. *For all $w \in W$, $\delta_1^*(w,1) = \bar{k} - p$, $\delta_2^*(w,1) = \frac{\bar{k}-p}{\beta}$, and $(\delta_1^*(w,0), \delta_2^*(w,0))$ is chosen to satisfy $\Delta^*(w,0) = 0$ and (1)-(2), with non-negative consumption for all agents.*

Proof. Relegated to the Appendix. □

From now on, the paper analyzes the more interesting case in which Assumption 4 holds: the lowest return to an idea under efficient investment, $f(\bar{k}, \theta_l)$, is sufficiently low. The main result of this section is Proposition 4 which provides a thorough characterization of the constrained efficient allocation under Assumption 4. I first show two intermediate results, Lemma 2 and Proposition 3, which pave the way to Proposition 4.

Remember that, under the ex ante welfare criterion, the only reason why the planner intervenes in this economy ($\delta \neq 0$) is because, under autarky, poor agents with ideas invest at a very low level, p, relative to the efficient level. In order to make her invest at the full information level, the planner has to set $\delta_1(p,1) \geq \bar{k} - p$. Since returns to business start-ups, θ, are private information, period two transfers cannot depend on the returns. Therefore, an agent who is poor and has an idea can pay back to the society an amount that is at most equal to the output she produces in the low-return state, $\delta_2(p,1) \geq -f(\bar{k}, \theta_l)$. This implies that to attain full information efficiency, the NPV of transfers going to poor agents with ideas should at least be $\bar{\Delta} \equiv \bar{k} - p - \beta f(\bar{k}, \theta_l)$, which is strictly positive by Assumption 4.

In what follows, without loss of generality, I restrict attention to constrained efficient allocations in which $\Delta^*(p,1) \leq \bar{\Delta}$. This is without loss of generality for the following reason. The discussion in the above paragraph shows that if $\Delta^*(p,1) = \bar{\Delta}$, then $k_1^*(p,1) = \bar{k}$, meaning full information efficiency is attained. Thus, in any allocation in which NPV of transfers going to $(p,1)$ is higher than $\bar{\Delta}$, the value of the objective function in the social planner's problem under informational frictions is equal to the full information level. Thus, increasing the NPV of transfers going to $(p,1)$ above $\bar{\Delta}$ does not change social objective but only changes the distribution of consumption across agents (increases the welfare of poor agents with ideas at the expense of others, thereby also tightening incentive constraints). Since we are only interested in productive efficiency, any allocation that can be achieved by a transfer system where $\Delta^*(p,1) = \bar{\Delta}$ is as good as another where $\Delta^*(p,1) > \bar{\Delta}$.

Lemma 2. *In the constrained efficient allocation, $\delta_1^*(p,1) = k_1^*(p,1) - p$ and $\delta_2^*(p,1) = -f(k_1^*(p,1), \theta_l)$.*

Proof. Relegated to the Appendix. □

[12] In fact, there is a set of constrained efficient allocations that are unique up to the distribution of consumption. Since the current paper is not concerned with consumption distribution, I refer to this whole set as "the constrained efficient allocation."

In words, Lemma 2 states that, in the constrained efficient allocation, poor agents with ideas invest all of their first period resources in their start-ups, and do not consume at all in the first period and in the low state of the second period.

Proposition 3. *In the constrained efficient allocation,* $\Delta^*(p,1) \geq 0$.

Proof. Relegated to the Appendix. □

Proposition 3 shows that any allocation that has the potential for qualifying to be constrained efficient has to satisfy $\Delta(p,1) \geq 0$. Therefore, in our search for the constrained efficient allocation, we can disregard all the allocations in which $\Delta(p,1) < 0$.

Now, we show that society has to increase the NPV of transfers going to poor agents with ideas, $(p,1)$, so as to bring these agents' investment levels closer to the full information level. To see this, consider the allocation that we can achieve by a transfer mechanism in which $\Delta(p,1) = 0$. By Lemma 2, we know that in this allocation, $(p,1)$ agent's investment level is given by

$$k - p - \beta f(k, \theta_l) = 0.$$

Now, we show that in order to increase $k_1(p,1)$ above k and towards the efficient level, $\bar{k}$, one needs to keep increasing $\Delta(p,1)$ towards $\bar{\Delta}$. To see this, observe that

$$\begin{aligned}
\Delta(p,1) &= k_1(p,1) - p - \beta f(k_1(p,1), \theta_l) \\
&= k_1(p,1) - p - \beta \left[\theta_l g(k_1(p,1)) + (1-\kappa) k_1(p,1)\right].
\end{aligned}$$

Taking the derivative of both sides with respect to $k_1(p,1)$ gives

$$\frac{d\Delta(p,1)}{dk_1(p,1)} = 1 - \beta \left[\theta_l g'(k_1(p,1)) + (1-\kappa)\right].$$

Now observe that for $\Delta(p,1) \geq 0$,

$$\begin{aligned}
k_1(p,1) - p - \beta \left[\theta_l g(k_1(p,1)) + (1-\kappa) k_1(p,1)\right] &\geq 0 \\
k_1(p,1) - \beta \left[\theta_l g(k_1(p,1)) + (1-\kappa) k_1(p,1)\right] &\geq p \\
1 - \beta \left[\theta_l \frac{g(k_1(p,1))}{k_1(p,1)} + (1-\kappa)\right] &\geq \frac{p}{k_1(p,1)} > 0.
\end{aligned}$$

Since $g(\cdot)$ is a strictly concave function, we have $g'(k) < \frac{g(k)}{k}$, which implies

$$\frac{d\Delta(p,1)}{dk_1(p,1)} = 1 - \beta \left[\theta_l g'(k_1(p,1)) + (1-\kappa)\right] > 1 - \beta \left[\theta_l \frac{g(k_1(p,1))}{k_1(p,1)} + (1-\kappa)\right] > 0.$$

Therefore, $\Delta(p,1)$ is strictly increasing in $k_1(p,1)$, for $\Delta(p,1) \geq 0$. This implies $\Delta(p,1)$ is a one-to-one function of $k_1(p,1)$, as long as $\Delta(p,1) \geq 0$. As a result, the converse is also true: in order to increase $k_1(p,1)$, the planner needs to increase $\Delta(p,1)$. We established that society has to increase the NPV of transfers going to $(p,1)$ so as to bring these agents' investment levels closer to the full information level and thus to bring social welfare closer to the full information level.

Proposition 4, the main result of this section, provides an exact calculation of the constrained efficient allocation. It should be clear from the argument of the previous paragraph that, increasing the NPV of transfers that poor agents with ideas receive, $\Delta(p, 1)$, improves productive efficiency. On the other hand, increasing $\Delta(p, 1)$ also makes other agents more likely to lie to be of type $(p, 1)$, meaning there is an incentive cost to increasing such transfers. Proposition 4 shows that the constrained efficient allocation arises from this trade-off. It is immediate from the characterization in Proposition 4 that agents with ideas - potential start-ups - are subsidized in NPV terms in the constrained efficient allocation. The other result of this section, Corollary 5, formalizes this result.

Assumption 5 below is a technical assumption. There are two parts to this assumption. Assumption 5a states that the fraction of agents without ideas is large enough relative to the fraction of agents with ideas that it is feasible to finance the total amount of subsidies agents with ideas should receive to attain full efficiency outcome even if we only take p units from each agent without an idea. Assumption 5b states that the total wealth of agents without ideas is large enough to finance a period one transfer of $\bar{k} - p$ to all agents with ideas. These two parts together guarantee that, if the storage technology is sufficiently inefficient (return to storage A is below a threshold level), then the economy attains the full information efficient level of investment for poor agents with ideas. Neither Assumption 5a nor Assumption 5b are necessary for our main subsidy result; they are made merely for expositional purposes. In the Appendix, I explain in detail how Proposition 4 and its proof should be modified if Assumption 5 is dropped.

Assumption 5. (a) $\eta_1 \bar{\Delta} \leq \eta_0 p$; **(b)** $\eta_1 [\bar{k} - p] \leq \eta_0 \sum_w \zeta_w w$.

Proposition 4. *Suppose Assumptions 4 and 5 hold. Then, in the constrained efficient allocation:*

1. $k_1^*(r, 1) = \bar{k}$, $k_1^*(w, 0) = 0$, *for all* $w \in W$, *and*

$$
\begin{aligned}
k_1^*(p, 1) &= \bar{k}, \text{ if } A \leq \bar{A}; \\
&< \bar{k}, \text{ and is the unique solution to } A = \frac{\eta_0 f(k_1^*(p, 1), \theta_l)}{k_1^*(p, 1) - p - \beta \eta_1 f(k_1^*(p, 1), \theta_l)}, \text{ if } A > \bar{A},
\end{aligned}
$$

where

$$
\bar{A} \equiv \frac{\eta_0 f(\bar{k}, \theta_l)}{\bar{k} - p - \eta_1 \beta f(\bar{k}, \theta_l)} \in (0, \beta^{-1}).
$$

2. *For all* $w \in W$, $\delta_1^*(w, 1) = k_1^*(p, 1) - p$, $\delta_2^*(w, 1) = -f(k_1^*(p, 1), \theta_l)$, *and* $(\delta_1^*(w, 0), \delta_2^*(w, 0))$ *is chosen to satisfy* $\Delta^*(w, 0) = -\frac{\eta_1}{\eta_0} \Delta^*(w, 1)$ *and (1)-(2), with non-negative consumption for all agents.*

Proof. Relegated to the Appendix. □

The intuition for why the NPV of transfers going to poor agents with ideas is related to the returns to storage is simple. We know that productive efficiency calls for subsidizing poor agents with ideas. Since rich agents with ideas can always mimic to be poor, they have to get the same subsidies as poor agents with ideas. As a result, all agents with ideas, potential start-ups, receive the same subsidy. To finance the subsidies going to agents with ideas, the planner has to tax agents without ideas in net present value terms. Consequently, individuals without ideas end up getting strictly negative NPV of resources. But what is the extent of the subsidies that the planner can give without violating incentive compatibility of the agents without ideas? The answer depends on the returns to the storage technology, A. The reason is that period two transfers of agents with ideas is strictly negative, and hence if agents without ideas want to pretend to have ideas, they have to pay back to the society in period two. For agents without ideas, the only way to carry resources into period two is via the storage technology.

If $A = 0$, then it is impossible for agents without ideas to carry resources to period two. In that case, they cannot pretend to have ideas; therefore, planner can transfer $\bar{\Delta}$ to agents with ideas and attain full information efficiency. In other words, the incentive constraint of the agents without ideas is not binding in the constrained efficient allocation. As A increases, storing resources instead of consuming in period one becomes less wasteful. There is a threshold level of the return to storage technology, $\bar{A}$, such that above this level, the benefit of lying to have an idea (not financing but enjoying the subsidy) exceeds the cost of doing so for the full information NPV of transfers, $\bar{\Delta}$. As a result, when $A > \bar{A}$, the incentive constraint of those agents without ideas start binding and agents with ideas cannot be subsidized $\bar{\Delta}$, which implies poor agents with ideas cannot invest at the full information level, $\bar{k}$.

Nonetheless, as long as $A < \beta^{-1}$, some subsidy is still incentive compatible since $A < \beta^{-1}$ implies that it is costly to store resources which then implies it is costly for agents without ideas to pretend to have ideas. In this case, the amount of subsidy going to agents with ideas is determined by equating the benefit and cost to the agents without ideas of lying to have ideas.

Corollary 5 below uses the calculation of constrained efficient allocation provided by Proposition 4 to establish that agents with ideas should receive transfers with strictly positive NPV of transfers.

Corollary 5. *In the constrained efficient allocation, society transfers a strictly positive NPV of resources from agents without ideas to agents with ideas, i.e.,* $\Delta^*(w, 1) > 0$.

Proof. Relegated to the Appendix. □

Proposition 4 shows that under some parameters, the society attains the full information efficient allocation, even under the informational constraints. This result is an artifact of risk neutrality and hence will vanish if more general utility functions are assumed. On the other hand, the main result of Proposition 4, that due to informational problems productive efficiency requires transferring resources from agents without ideas to ones with them, holds with risk-averse preferences as well.

Proposition 4 also points to an interesting property of the model economy: the distribution of wealth affects the constrained efficient distribution of productive activity in the economy. To see this, remember that Proposition 4 tells that when $A > \bar{A}$, the constrained efficient level of investment for a poor agent with an idea depends on her wealth level, p. Now, consider another wealth distribution with ζ_p fraction of agents having initial wealth $p + \epsilon$ and ζ_r fraction having $r - \epsilon\frac{\zeta_p}{\zeta_r}$, where $\epsilon > 0$ and small. This new wealth distribution is a perturbation of the old one in a way that preserves the mean. By Proposition 4, in the economy with the perturbed wealth distribution, $k_1^*(r,1) = \bar{k}$ and investment level for a poor agent with an idea is given by $A = \frac{\eta_0 f(k_1^*(p,1),\theta_l)}{k_1^*(p,1)-(p+\epsilon)-\eta_1\beta f(k_1^*(p,1),\theta_l)}$.[13] This means that in the current model, when Assumptions 4 and 5 hold and $A > \bar{A}$, the constrained efficient distribution of productive activity depends on how initial wealth is distributed across agents. This result is summarized in the following corollary.[14]

Corollary 6. *Suppose Assumptions 4 and 5 hold, and $A > \bar{A}$. The distribution of productive activity in the constrained efficient allocation depends on the distribution of wealth.*

It is important to note that this result crucially depends on private information assumptions. In the full information efficient allocation, investment levels for both agents with ideas is $\bar{k}$, independent of how a total of $\sum_w \zeta_w w$ units is distributed across agents. The intuition for why the constrained efficient level of productive activity depends on the distribution of wealth is as follows. Under private information, the marginal social cost of investment is not only equal to its resource cost. For a given distribution of initial wealth, increasing the investment level of poor agents with ideas tightens some incentive compatibility conditions. Thus, there is an incentive cost of increasing investment in addition to the resource cost. Changing the wealth distribution changes this incentive cost of investment while leaving the resource cost untouched. Consequently, between two otherwise identical economies with different distributions of wealth, the resource cost of investment is the same, which implies that the full information efficient allocation is the same. However, the incentive costs in these two economies are potentially different, making the social marginal costs of investment different, which results in different distributions of constrained efficient productive activity.

3.2 Discussion of Assumptions

This section discusses the role of informational assumptions on the subsidy result. I also provide a brief discussion of how the analysis would have to be modified under risk-aversion.

Informational assumptions. The assumption that θ, the returns to a start-up, is unobservable is the sole cause of the subsidy result. To see this, consider a version of the model in which, for each agent, θ is realized publicly. Assume that initial type, (w, i), and actions are still private information. In that case, the planner can attain full information efficiency without subsidizing any agent, even under Assumption 4. This result is shown in Proposition 7 below.

[13] Here, I abuse the notation, hoping this does not cause any confusion. In the original economy, p denotes two things: poor agents and their wealth level. In the perturbed economy, p denotes poor agents, whereas $p + \epsilon$ denotes their wealth level. The same is true for r.

[14] Indeed, one can show how such a perturbation affects the constrained efficient allocation precisely. It is possible to show that, if $\epsilon > 0$ and small, meaning wealth distribution becomes marginally more equal, then $k_1^*(p,1)$ and $\Delta^*(p,1)$ both increase. Intuitively, thanks to an increase in p, with the same amount of NPV of transfers, poor agents with ideas can have higher investment, $k_1^*(p,1)$. This increases the amount that agents with ideas can transfer back to the planner in the second period. This means that agents without ideas have to save more through the inefficient storage technology if they want to pretend to have ideas. This, in turn, relaxes the incentive constraint, and allows the planner to transfer a higher NPV of resources to agents with ideas. This, then allows poor agents with ideas to invest more. So, in fact, one can show that increasing p increases $k_1^*(p,1)$ by more than one for one.

Proposition 7. *Suppose that θ is observable for each agent. Then, in the constrained efficient allocation:*

1. *For all $w \in W$, $k_1^*(w,1) = \bar{k}$ and $k_1^*(w,0) = 0$.*
2. *For all $w \in W$, $\delta_1^*(p,1) = \bar{k} - p$, $\delta_2^*(p,1,\theta_l) = -f(\bar{k},\theta_l)$, $\delta_2^*(p,1,\theta_h)$ is such that $\Delta^*(p,1) = 0$, and $(\delta_1^*(w,i), \delta_2^*(w,i))_{(w,i)\neq(p,1)}$ is chosen to satisfy $\Delta^*(w,i) = 0$, and (1)-(2), with non-negative consumption for all agents.*

Proof. Relegated to the Appendix. □

The intuition is simple. When θ is observable, the planner can make period two transfers depend on the realization of θ. Therefore, even if the low state return, θ_l, is very low (Assumption 4), the agent can still pay back to the society the future value of resources transferred to her in period one, $\bar{k} - p$, by paying a sufficiently high amount in the high state. Proposition 7 precisely establishes that the only reason in the model why the society has to subsidize agents with ideas is because start-up returns are private information.

The assumptions that initial type and actions are observable imply that the planner has to respect incentive compatibility conditions when subsidizing poor agents with ideas. Consider, for instance, a model that is identical to the baseline model introduced in Section 2, except that initial type, (w,i), is publicly known at no cost. As long as θ is unobservable, the society still has to make $\bar{\Delta}$ units of transfers to poor agents with ideas. However, now it is trivial to make this transfer since the planner knows exactly the agents who have ideas but lack resources to invest in them.

Similarly, if investment is assumed to be observable, keeping the rest of the model the same as the baseline model, subsidizing agents with ideas would be trivial. It is not beneficial for an agent without an idea to lie to have one and get the subsidy since she has to invest it, and hence cannot consume it. The exercise in which everything else is kept the same but storage is assumed to be observable is the same as assuming there is no storage technology, or $A = 0$. From Proposition 4, it follows that in this case, the planner can make transfers with NPV that is sufficient to attain full information efficiency.

Risk aversion. The assumption of risk-neutrality that I maintain throughout the paper is not a substantial assumption in the sense that productive efficiency would call for a subsidy towards poor agents with ideas even people were risk-averse. To see this, observe that, due to dimisnishing marginal returns, the planner would like poor agents with ideas to invest in the full information efficient allocation independent of the social weight assigned to them. As long as Assumption 4 holds, poor agents with ideas invest less than the full information efficient level. To increase their investment towards the efficient level, the planner needs to subsidize them. This is the sense in which productive efficiency still requires subsidizing poor agents with ideas. Of course, under risk aversion, the constrained efficient allocation would also be shaped by redistribution (depending on the social weights on different individuals) and insurance motives. Thus, who should be subsidized (if anyone) then becomes a matter of who we care about in the economy and how that redistribution motive trades off with the subsidy required by productive efficiency.

4 Implementation

This purpose of this section is to provide an implementation of the constrained efficient allocation via a subsidy program like the U.S. SBA's Business Loan Program. The SBA is the major government institution in the United States assisting business start-ups in particular and small businesses in general. One can consider that the subsidy program that implements the constrained efficient allocation provides a justification for the subsidies that the SBA's Business Loan Program hands out to start-up firms.

I first show that laissez-faire markets cannot carry out the required subsidy, and hence, cannot implement the constrained efficient allocation. Then, I provide an implementation of the constrained efficient allocation via a subsidy program, and finally I compare that subsidy program to the SBA's Business Loan Program.

The physical and informational environment is the same as described in Section 2. The main difference is that there is an incomplete asset market that allows agents to competitively trade risk-free bonds in period one which pay a gross interest return of R in period two. I assume that bond contracts are fully enforced, meaning people have to pay back their debts. Agents can still save via their private storage technology. There are three differences between using the bond market and storage. First, it is possible to borrow in the bond market whereas people cannot store a negative amount. Second, the interest rate R is determined in equilibrium while the return to storage is an exogenous parameter. Third, unlike storage, individual actions in the bond market are public information.[15]

There are two institutions: a government and an institution of the government that aids start-up businesses. The government taxes all individuals in the society lump-sum, by an amount T, and transfers these funds to its institution. Any individual can apply to this institution for a subsidy. The institution asks the agent to report her wealth, business idea, and investment plan, w', i', and k_1', respectively. Then, after observing the amount borrowed (or lent) and the reports, the institution decides whether or not to provide the subsidy, $\tau(b_1, w', i', k_1')$.

Taking the tax-subsidy system (T, τ) and the interest rate R as given, an agent (w, i) who decided to apply to the institution for a subsidy solves the following problem:

Agent's problem.

$$\max_{c_1, c_2, k_1, s_1, b_1, w', i', k_1'} u(c_1) + \beta \sum_{\theta} \mu_\theta u(c_{2\theta}) \tag{5}$$

[15] The assumption that trading in the bond market is publicly observable is a reasonable assumption, at least for the United States and for most European countries. The reason is that trading bonds requires the services of financial intermediaries in most cases, as a result of which individual information regarding such trades is officially recorded in intermediaries' accounts. Then, in the United States, people are asked to declare their interest income, and hence bond holdings, for tax purposes. Individuals can lie about their bond holdings (in a similar way they can lie about their labor earnings) but, since there are official records of their transactions and the Internal Revenue Service audits people randomly about the information they provide for tax purposes, people would state their bond holdings correctly as long as the punishment for misreporting is large enough. This makes bond trading via financial intermediaries de facto observable. The storage technology, on the other hand, is similar to keeping money under the pillow which does not enter any official records.

s.t.

$$c_1 + k_1 + s_1 + b_1 \leq w - T + \tau(b_1, w', i', k_1'),$$
$$c_{2\theta} \leq f(k_1, \theta)i + Rb_1,$$
$$k_1, s_1 \geq 0.$$

An agent who does not apply for a subsidy ($a = 0$ agent) would solve a very similar problem. The only difference is there would be no $\tau(b_1, w', i', k_1')$ in that agent's problem, and hence there would not be any w' and k_1' choice. However, since in the current setup there is no cost of applying for a subsidy, without loss of generality, assume that all agents apply.

Below is the definition of the incomplete market equilibrium with a tax-subsidy system.

Market Equilibrium with Taxes. *Given* (T, τ)*, a market equilibrium is* $X(w, i)_{w \in W, i \in I}$ *where* $X(w, i) \equiv (c_1(w, i), \{c_2(w, i, \theta)\}_{\theta \in \Theta}, k_1(w, i), s_1(w, i), b_1(w, i), w'(w, i), i'(w, i), k_1'(w, i))$ *and interest rate* R *such that:*

1. *Given* R*,* $X(w, i)$ *solves (5) for each agent* (w, i)*,*
2. *Bond market clears:* $\sum_{(w,i) \in W \times I} \zeta_w \eta_i b_1(w, i) = 0,$
3. *Government budget balances:* $\sum_{(w,i) \in W \times I} \zeta_w \eta_i \tau(b_1(w, i), w'(w, i), i'(w, i), k_1'(w, i)) = T.$

An allocation (c, k, s, δ) is *implementable in the market with a tax-subsidy system* (T, τ) if, given (T, τ), (c, k, s, b, w', i', k') with some interest rate R constitute a market equilibrium.

4.1 Market Equilibrium under Laissez-Faire

Before providing an actual tax-subsidy system that implements constrained efficient allocation, I first analyzes what happens under no government intervention, i.e., $(T, \tau) = 0$.

Proposition 8. *Suppose Assumptions 4 and 5 hold. Then, the constrained efficient allocation cannot be attained in the market equilibrium under laissez-faire.*

Proof. Relegated to the Appendix. □

Proposition 4 already proved that the constrained efficient allocation involves transferring strictly positive NPV of resources from agents without ideas to those with ideas. Proposition 8 then follows since markets cannot make such transfers on their own.[16] A separate entity, like a government, should intervene and make the necessary transfers between agents. It is important to note that laissez-faire market equilibrium is ex post Pareto efficient. However, it is not output maximizing (ex ante Pareto efficient), and that is the reason why transfers are optimal from the perspective of a government that cares about aggregate consumption.

4.2 Optimal Tax-Subsidy System

This section provides an actual tax-subsidy system that implements the constrained efficient allocation.

[16] The fact that I restrict attention to incomplete markets from the outset is without loss of generality. It is easy to show that under given informational assumptions and the assumption that agents cannot write contracts ex ante (before (w, i) is realized), agents cannot reach an allocation with higher total output than the incomplete markets equilibrium.

In that regard, define

$$T = \frac{\eta_1}{\eta_0}\Delta^*(w,1) \quad \text{and} \quad \tau(b_1, w', i', k_1') = \begin{cases} \frac{1}{\eta_0}\Delta^*(w,1), & \text{if } b_1 \leq -\beta f(k_1^*(p,1), \theta_l); \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

Proposition 9. *Suppose Assumptions 4 and 5 hold. Then, the market equilibrium with the tax-subsidy system defined in* (6) *implements the constrained efficient allocation.*

Proof. Relegated to the Appendix. □

First, observe that the function that describes optimal subsidies, $\tau(\cdot)$, does not actually depend agents' reports about their unobservable characteristics and actions: wealth levels, ideas, or investment levels. This is due to the fact that these reports people send are just cheap talk. If we allowed the government to (partially) monitor agents at some cost, then the reports could have information value in which case the optimal subsidy function would depend on these reports.

The way the implementation works is as follows. An agent who borrows above the threshold gets a net subsidy of $-\frac{\eta_1}{\eta_0}\Delta^*(p,1) + \Delta^*(p,1)/\eta_0 = \Delta^*(p,1)$. Remember that this is exactly the amount of the NPV of transfers agents with ideas get in the planner's problem. Therefore, agents with ideas borrow at the threshold level, get the subsidy, and invest at the constrained efficient level. Agents without ideas would like to do the same; however, for them, the only way to pay back in period two is to save through the storage technology, s_1, which is costly since s_1 is wasteful. The threshold amount of borrowing required to get the subsidy is chosen such that this cost is weakly higher than the benefit of getting the subsidy. Therefore, only agents with ideas get the subsidy, and hence the budget of the agency balances.

4.3 Comparing the Model's Implementation to the SBA's Business Loan Program

A comparison between the implementation provided above and the actual Business Loan Program is in order. The implementation provided in Section 4.2 is similar to the actual system in the United States in the sense that in both, the government taxes all citizens and transfers some of its tax revenue to the agency that deals with start-ups, with the intention of subsidizing potential start-ups that are financially constrained.[17] A more important similarity is that, in the model, the government agency uses borrowing and lending activities of agents as a device for screening agents with ideas, and subsidizes only those agents who borrow above a threshold. The loan program of the SBA follows a similar strategy: instead of directly subsidizing people who claim to have ideas, the SBA subsidizes only those who get loans from commercial banks.

However, since the model is very simple, there are also significant differences between the paper's implementation and the actual system in the United States. Here, I stress two of those discrepancies and what causes them.

[17] The fact that in the paper's implementation all the tax revenue goes to the agency dealing with start-ups is immaterial. One can add exogenous government to the model, G, and then only $T - G$ units would be transferred to the agency. As long as this G is also subtracted from the right-hand side of the aggregate feasibility condition in the planner's problem, all the analysis goes unchanged.

First, there is no default in the model economy; agents only sign non-state-contingent that they have to honor by assumption. This creates a discrepancy between the model and the actual program because the actual loan program does not give out direct subsidies but rather provides loan guarantees to qualified borrowers. These guarantees ensure the lenders that in case of default the SBA will pay back a certain percentage of the loan. This, in turn, causes the interest rate on SBA backed loans to be lower relative to loans that are not backed by SBA, thereby effectively subsidizing borrowers.

Second, in real life it is possible to monitor some features of start-ups at some cost, while the model abstracts away from any sort of monitoring. As a result, the actual Business Loan Program takes applicants reports about, say, their ideas, more seriously and spends some resources (labor) to determine whether or not the ideas are worth subsidizing. In the model, once an agent sends a report to the agency saying she has an idea, there is no way to check whether she is lying or not. Therefore, in the model's implementation, the function τ, which determines who gets subsidized and which potentially depends on agents' reports, does not actually depend on agents' reports.

5 Conclusion

This paper provides a rationale for governments to subsidize agents who have ideas (potential start-ups) but do not have enough resources to invest in them. If we accept that returns to start-up firms are privately observed by the owners of the firms, then constrained efficiency calls for subsidizing poor agents with ideas. If society knew who has ideas but lacks resources to invest in them, then it is simple to implement the subsidy. However, I assume here that people's wealth levels, whether they have ideas or not, and how they use their resources are unobservable to others. These additional private information assumptions imply that the delivery of the subsidies should be crafted carefully in order to ensure that only those with productive ideas self-select into applying for the subsidies.

The paper also provides an implementation of the constrained efficient allocation similar to the U.S. SBA's Business Loan Program. Even though the main idea behind both the implementation in the model and the actual Business Loan Program are the same, that is to subsidize financially constrained individuals with productive ideas in an incentive compatible manner, there are still significant discrepancies between the model's implementation and the actual program. This is due mainly to the fact that the model economy is very simple. Introducing default and/or a costly monitoring technology may bring the model close enough to reality that the implementation of the model may allow us to analyze the efficiency of the details of the SBA's actual loan program and similar government programs in the rest of the world. This may be an interesting direction for future work.

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