A hierarchical model for cash transfer system design problem

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Abstract. This paper presents a hierarchical model that incorporates strategic, tactical, and operational decisions of cash transfer management system of a bank. The aim of the model is to decide on the location of cash management centers, the number and routes of vehicles, and the cash inventory management policies to minimize the cost of owning and operating a cash transfer system while maintaining a pre-defined service level. Owing to the difficulty of finding optimal decisions in such integrated models, an iterative solution approach is proposed in which strategic, tactical, and operational problems are solved separately via a feedback mechanism. Numerical results show that such an approach is quite effective in reaching at greatly improved solutions with just a few iterations, making it a very promising approach for similar models.

Keywords: Location; vehicle routing; VRP; cash transfer; inventory management

1 Introduction

A major function of a bank is to act as an intermediary between its clients by collecting deposits from some while dispensing cash to the others. Typically, banks perform these operations via their branches and automated banking machines located across a geographical market. Since such transactions are naturally uncertain, cash positions at branches change randomly throughout the day.

Effective management of cash positions at branches is critical. Banks do not prefer to carry extra cash due to opportunity cost is associated with it. Any extra cash can be deposited to central banks or loaned to other banks for overnight interest. Furthermore, extra cash makes banks susceptible to theft, fraud, and so on. Falling short of necessary cash, however, is also undesirable and perhaps, even more harmful. Client requests for withdrawal should be met immediately or with very little delay as failure to do so may have severe negative consequences such as loss of goodwill or even loss of confidence.

Therefore, banks need to transfer cash in and out of branches to manage these inventories in a rational way. A cash transfer system has two main sets of cost

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items. The first set includes operational costs of managing a system such as fixed and variable costs related to the cash management centers (CMCs) and armored vehicles (AVs) while the second set includes opportunity and shortage costs of having too much or too little cash at the branches. An effective management of a cash transfer system should trade-off these costs when making design and operational decisions in the system.

In this paper we present a hierarchical model and an iterative solution approach for a cash transfer management system design problem. Our model falls into the general class of integrated location-routing-inventory models. The literature on integrated models for any pair of these decisions is relatively well developed. A comprehensive review on integrated location-routing, location-inventory, and inventory-routing problems is provided by Shen (2007). Other papers, such as Melo, Nickel, and Saldanha-da-Gama (2009) as well as Klibi, Martel, and Guitouni (2010) give a review of supply chain design models that include integrated facility location models. In cash logistics context, recently there are few papers for inventory-routing problems (Wagner 2010 and van Anholt et al. 2013). However, the literature that consider all three levels of decisions is scant and those that present such integrated models report a very limited results.

In one of the earlier works Ambrosino and Scutella (2005) study a detailed location-routing model in a four-layer distribution network. A more concise model that considers inventories in a similar manner is presented in Hiassat and Diabat (2011). However, neither papers gave any numerical experimentation of a reasonable detail but rather solved only a very small instance. Shen and Qi (2007) is perhaps the first study that explicitly considers cycle and safety inventory costs and routing in a location problem. However, they do not consider detailed routing decisions; instead they develop an approximation for the routing cost and incorporate it in a nonlinear program.

The model closest to ours is given in Javid and Azad (2010), in which authors study a capacitated facility location problem that explicitly considers routing and inventory decisions. They first formulate the problem as a mixed-integer convex problem, which is solved by LINDO for small sized problems. The largest problem that was solved to optimality within 12 hours by LINDO had three potential DCs, two vehicles, and nine customers. For larger size problems they utilize a hybrid Tabu Search and Simulated Annealing heuristic method. They solved problems as large as 50 potential DCs, 65 vehicles, and 400 customers.

Our paper introduces a novel addition to this growing literature. First, the very few models that explicitly include routing and inventory decisions along with location decisions consider a “traditional” inventory management setting. We on the other hand deal with a cash inventory setting, which is perhaps the first considered in a network design problem here. Even though cash inventory management is an area that has not received enough attention in logistics literature, with the continuing general economic climate of low-interest borrowing, logistics and other operational costs have become more important determinants of the profitability of financial institutions. Our work offers a step towards addressing this research gap in the literature.

Secondly, we propose an iterative solution method for the problem, instead of dealing with approximate integrated models or meta-heuristics as most past works do. We obtain promising results on the performance of an iterative approach that solves a series of simpler problems with updated parameters. Finally, we generate problem instances based mostly on the real-life data we obtained from a commercial bank. Some of this data can be used in future studies.

In the rest of this paper we first present the model and the iterative approach. Section 3 contains the description of parameter estimation and instance generation followed by reporting of the set of numerical experiments. We conclude the paper with few remarks and avenues for future research in Section 4.

2 The model

Our approach consists of a series of well-known problems for strategic, tactical, and operational decisions and an iterative solution method to reach an optimal or near-optimal solution. We consider a location problem at the strategic level, a vehicle routing problem at the tactical level, and a cash management problem at the operational level. The fourth problem, termed as a vehicle number determination problem, is a peculiar tactical problem that ties these three problems in the iterative solution approach we developed. Each of the following subsections is devoted to the exposition of one problem and its relationship to the other problems as well as to the overall iterative procedure.

2.1 Strategic problem: An uncapacitated facility location problem (UFLP)

The strategic problem is an extension of the standard UFLP that determines the CMC locations and CMC-branch assignments so as to minimize the total cost. In addition to the location and assignment variables, we defined a third set of decision variables for the number of vehicles to take transportation capacity and cost of the AVs into account, albeit approximately. These extensions add a bit more realism to the fixed and variable cost structures and are expected to lead to better location decisions.

We consider an uncapacitated case, as it is usually easy to increase capacity at these centers by acquiring additional machinery or hiring new personnel. The vehicles also have annual fixed costs such as tax, insurance, as well as personnel costs. Transportation costs are incorporated approximately through CMC-branch assignment as typical in facility location problems.

Let the index $i \in I = \{1, 2, \ldots, m\}$ represent the alternative CMC locations and the index $j \in J = \{1, 2, \ldots, n\}$ represent the branches. The decision variables used in our version of UFLP are:

$$y_i = \begin{cases} 1, \text{ if CMC at location } i \text{ opened,} \\ 0, \text{ otherwise,} \end{cases}$$

$$z_i = \text{The number of AVs assigned to CMC at location } i.$$

$$x_{ij} = \begin{cases} 1, \text{ if the branch } j \text{ is assigned to CMC at location } i, \\ 0, \text{ otherwise.} \end{cases}$$

The parameters are given as:
f_i : Fixed cost of opening a CMC at location i,
g_i : Fixed cost of an AV assigned to CMC at location i,
a_{ij} : Direct trip cost between locations i and j,
s_{ij} : Direct travel time between the CMC at location i and the branch at j,
S : Maximum total time an AV can be used in a year.

Our iterative approach requires some of these parameters to be revised at each iteration. To facilitate it, we also define the following iteration parameters:
α^{t-1} : System-wide ratio of total route length to total direct travel (including backhaul),
k_j^{t-1} : The number of cash transfer requests made by branch j.

These parameters are revised at each iteration of the algorithm and the superscript $(t-1)$ refers to the values obtained in the previous iteration. Although the impact of α and k are combined in the formulation, we chose to separately represent them since they are obtained from different problems. Given all the variables and parameters, the strategic problem can be formulated as follows:

$$\text{Minimize} \quad \sum_{i \in I} f_i y_i + \sum_{i \in I} g_i z_i + \sum_{i \in I} \sum_{j \in J} \alpha^{t-1} k_j^{t-1} a_{ij} x_{ij} \tag{1}$$

$$\text{subject to} \quad \sum_{i \in I} x_{ij} = 1, \text{ for all } j \in J, \tag{2}$$

$$x_{ij} \leq y_i, \text{ for all } i \in I, j \in J, \tag{3}$$

$$\sum_{i \in J} \alpha^{t-1} k_j^{t-1} s_{ij} x_{ij} \leq S z_i, \text{ for all } i \in I, \tag{4}$$

$$y_i \text{ and } x_{ij} \in \{0, 1\}, \text{ for all } i \in I \text{ and } j \in J, \tag{5}$$

$$z_i \in \{0, 1, \ldots\} \text{ for all } i \in I. \tag{6}$$

Objective function (1) includes the fixed costs of CMCs and AVs and the direct transportation cost, which is adjusted with a routing factor and the approximate number of trips to a particular branch. Constraints (2) and (3) are the standard constraints in UFLP which ensure that each branch is assigned to one open CMC. Constraints (4) ensure that there are enough vehicles that cover the anticipated total travel time at each CMC. This constraint provides a rough-cut estimate of the minimum number of vehicles without considering the detailed routing issues. Finally, (5) and (6) define the binary and general integer variables.

2.2 Tactical problem: A Vehicle Routing Problem (VRP)

Our tactical model is a version of VRP, in which the location of CMCs and CMC-branch assignments from the UFLP are given as input. A VRP is solved for each CMC to determine the number of vehicles and their routes that minimize the total vehicle and traveling costs. Let n_i denote the number of branches assigned to CMC i and the set $J_i = \{1, \ldots, n_i\}$ denote those branches. Indices $j = 0$ and $j = n_i + 1$ are used to indicate the start and the end of the tours, i.e., the CMC location. For notational conciseness we also define new sets as $\underline{J}_i = J_i \cup \{0\}$ and $\overline{J}_i = J_i \cup \{n_i + 1\}$. Furthermore, let

$$u_{jk} = \begin{cases} 1, \text{ if the branch } k \text{ is visited immediately after branch } j, \\ 0, \text{ otherwise, and} \end{cases}$$

$$t_j = \text{The time branch } j \text{ starts receiving the service.}$$

The problem parameters are defined as:
s_{jk} : Trip time from branch j to branch k,
h : Average service time at a branch,
B : The length of a shift,
M : A large number.

We also define an iterative parameter β^{t-1} as the average aggregate trip frequency of routes. Our VRP can be modeled as:

$$\text{Minimize} \quad g_i \sum_{j \in J_i} u_{0j} + \beta^{t-1} \sum_{j \in J_i} \sum_{k \in \overline{J}_i} a_{jk} u_{jk} \tag{7}$$

$$\text{subject to} \quad \sum_{j \in \underline{J}_i, j \neq k} u_{jk} = 1, \text{ for all } k \in J_i, \tag{8}$$

$$\sum_{k \in \overline{J}_i, k \neq j} u_{jk} = 1, \text{ for all } j \in J_i, \tag{9}$$

$$t_j \geq s_{0j} - M(1 - u_{0j}), \text{ for all } j \in J_i, \tag{10}$$

$$t_k \geq t_j + s_{jk} + h - M(1 - u_{jk}), \text{ for all } j \in J_i \text{ and } k \in \overline{J}_i, \tag{11}$$

$$t_{n_i+1} \leq B, \tag{12}$$

$$u_{jk} \in \{0, 1\}, \text{ and } t_j \geq 0 \text{ for all } j \in J_i \text{ and } k \in \overline{J}_i. \tag{13}$$

The objective function consists of the fixed vehicle costs and the total trip costs. Constraints (8-11) are standard constraints in VRPs. Constraints (8) ensure that each branch is visited by a vehicle and (9) ensure that each vehicle also leaves a branch it has visited. Constraints (10) and (11) ensure that vehicles are given sufficient time for service and travel between visiting branches. Finally, the condition on the shift length is given in (12) and the non-negativity and binary restrictions are given in (13).

While CMC locations and assignments to branches are given as input from the strategic model, here the number of vehicles for each CMC is found by re-optimizing a more detailed representation of the vehicle and trip costs.

2.3 Operational problem: A cash management problem under uncertainty (CMPU)

At the operational level, we solve a cash management problem at each branch where cash transactions are uncertain. In CMPU, the cash positions of the branches are continuously reviewed and at any period one has to decide if any action of cash transfer to or from the branch should take place and if so, what should be the transfer amounts. In general, each of these actions incur fixed and variable costs, which are defined in our context as:

K_1 : Fixed cost of transferring money from a branch to a vehicle,
K_2 : Fixed cost of transferring money from a vehicle to a branch,
k_1 : Unit variable cost of transferring money from a branch to a vehicle,
k_2 : Unit variable cost of transferring money from a vehicle to a branch.

Hence, when the cash position of a branch changes from x to y, the transfer cost can be written as:

$$A(x, y) = \begin{cases} K_1 + k_1(x - y), & \text{if } y < x, \\ 0 & \text{if } y = x, \\ K_2 + k_2(y - x), & \text{if } y > x. \end{cases}$$

Under a variety of conditions, several authors, such as Girgis (1968), Porteus (1972), Porteus and Neave (1972), and Constantinides and Richard (1978), show that a "two-sided" generalization of (s, S) policy from inventory management, i.e., (u, U, D, d), is an optimal policy for the cash management problem. The four parameters defining this policy suggest that if the cash position falls to or below u, enough cash is obtained to raise the cash position up to U and if the cash position rises to or above d, enough cash is removed from the branch to bring the cash position down to D.

While the form of the optimal policy is known, it is rather challenging to compute the optimal policy parameters. The problem, however, can be somewhat simplified for our setting. First, it would not be a very strong assumption to take the fixed costs of cash transfers between the branch and the vehicle as equal, i.e., $K_1 = K_2$, since in both cases, the vehicles take similar routes and similar actions are taken at the time of transfer such as counting money, approvals, and so on. Second, the variable portion of the cash transfer can be assumed negligible as compared to the fixed costs, i.e., $k_1 = k_2 = 0$, because much of the personnel cost is already sunk in our setting. Under these conditions, Milbourne (1983) has shown that a (u, z, d) policy would be optimal. This policy is a special case of the two-sided policy described above, with $z = U = D$.

Despite these simplifications, however, computation of policy parameters remains a challenge under general net transaction distributions. Furthermore, while it is relatively easy to estimate the cost of holding excess cash (such as the overnight interest rate), estimating the cost of cash shortage is not straightforward. Shortage cost includes the cost of borrowing at the interbank interest rate, but sometimes it might not be feasible or desirable to borrow from another institution. Moreover, one also needs to account for the cost of lost goodwill. Therefore, rather than finding all three policy parameters with an estimated cost of shortage, we assume that the management sets a "service-level" that restricts the probability of cash shortage when a cash transfer to the branch is awaited. This service level helps us to set the lower threshold u independent of other parameters and then compute the remaining two parameters, z and d. Our treatment is analogous to approximating (s, S) policy parameters with an

Economic Order Quantity - ReOrder Point (EOQ-ROP) approach in inventory management.

The model we have chosen towards this end is that of Miller and Orr (1966). Their results are based on two key assumptions: cash transfers are immediate and cash movements at a branch have zero mean, i.e., deposits and withdrawals cancel each other, on average. Under these conditions Miller and Orr has calculated the optimal decisions as

$$z^* = \sqrt{\frac{3Kn_t\mu_t^2}{4r}} \text{ and } d^* = 3z^*,$$

where
μ_t : Mean cash transaction size,
K : Fixed cost of transferring money to or from a branch,
n_t : Mean number of cash transactions on a day, and
r : Daily interest rate.

While in our case most branches have nonzero net transaction average, we nonetheless use Miller and Orr's model as an approximation. We find the (z^*, d^*) as described above and then add u^* to these values to obtain the triple policy parameters $(u^*, z^* + u^*, d^* + u^*)$, where u^* is computed by using the service level and the next transaction distribution. After calculation of these parameters we find the average cash levels and the average number of transactions via a simulation.

2.4 Integrating problem: The vehicle number determination problem (VNDP)

The last problem acts as an integrating problem among UFLP, VRP, and CMPU and it resulted from the practice of the bank from which this study derives. While both UFLP and VRP take the number of vehicles into account, their treatments are based on simplifying assumptions: UFLP determines the minimum number of vehicles based on "adjusted" direct distance and VRP, on the other hand assumes that each branch will be visited every day. VNDP offers a correction via a probabilistic analysis of branches' transfer requests. It uses VRP and CMPU results from the previous iteration while modifying the parameters to be used in UFLP and VRP in the next iteration.

Now suppose that VRP produces k_i routes for CMC i. Let there be n_{ik} branches represented by the set $J_{ik} = \{1, 2, \ldots, n_{ik}\}$ on each route $k \in K_i = \{1, 2, \ldots, k_i\}$. Also, let p_j^{ik} denote the probability that a branch $j \in J_{ik}$ requests a cash transfer on a given day. These probabilities are assumed to be independent across branches. The probability that at least one branch on route $k \in K_i$ requests a transfer can be expressed as:

$$P_{ik} = 1 - \prod_{j \in J_{ik}} (1 - p_j^{ik}),$$

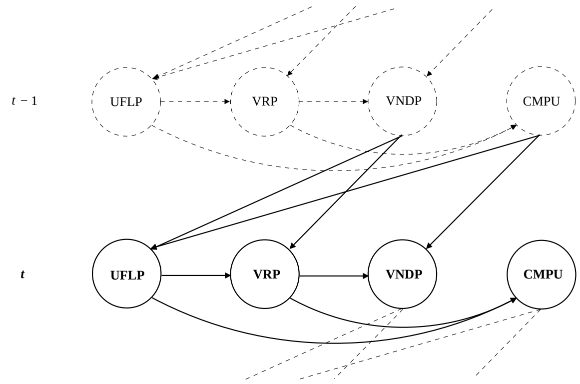

Fig. 1. The structure of the iterative algorithm.

where probabilities (p_j^{ik}) are calculated via simulation of the CMPU. Then, the probability distribution of the number of routes used on a day would be a multinomial distribution for each CMC i with probabilities, $P_{i1}, P_{i2}, \ldots, P_{ik_i}$. Hence, for CMC i, the probability that all routes need to be served on a particular day is given as $q_{ik_i} = \prod_{k=1}^{k_i} P_{ik}$. Similarly, the probability that $(k_i - 1)$ routes to be used is given as $q_{i(k_i-1)} = \sum_{k'=1}^{k_i}(1 - P_{ik'}) \prod_{k \neq k'} P_{ik}$, and so on. The expected number of vehicles is then found by enumerating all probabilities q_{ik} for $1, 2, \ldots, k_i$ routes and then taking the expectation, i.e.

$$NoV[\text{VNDP}] = \sum_{k=0}^{k_i} k q_{ik}. \tag{14}$$

This calculation is based on the fact that whether a single branch or all the branches in a route requests a service, one vehicle is used. Also, note that this particular setup is based on the practice of the bank where routes are determined and fixed before any daily operations take place (more on this later). Total distance however, would depend on how many branches (an which branches) are visisted on a particular day. This is taken into account in finding the expected total distance, which is given as:

$$Dist[\text{VNDP}] = \sum_{i=1}^{m} P_{ik} U_{ik}, \tag{15}$$

where U_{ik} be the distance of the kth route of CMC i. This quantity is used to revise the distance correction factor used in UFLP as well as to compute the total transportation cost.

In our solution method, these four problems are solved iteratively with some parameters updated at each iteration. Figures 1 and 2 depict the structure of the algorithm at different levels of detail. In nutshell, UFLP uses CMPU results

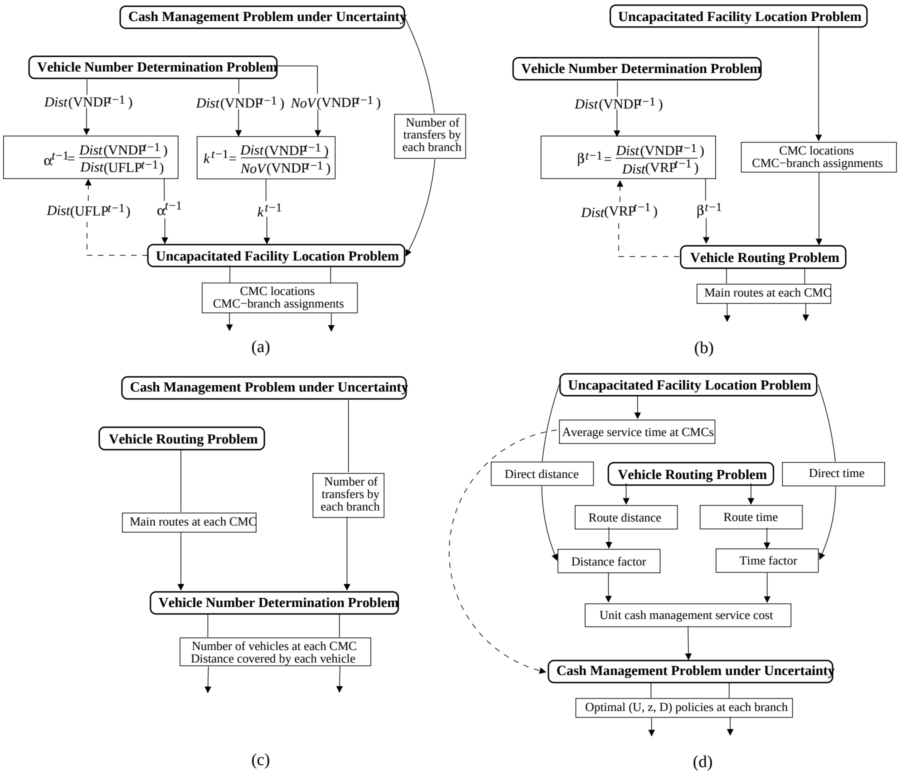

Fig. 2. The components of the iterative algorithm: (a) Uncapacitated Facility Location Problem, (b) Vehicle Routing Problem, (c) Vehicle Number Determination Problem, and (d) Cash Management Problem under Uncertainty.

to determine the expected cash transfer requests at each branch, while using the VNDP's results to adjust the transportation cost and time parameters. VRP uses UFLP results on the CMC locations and CMC-branch assignments and VNDP's results to adjust the distance parameter. VNDP uses VRP results to obtain the routes and CMPU's results to obtain the number of transfers, which are used to calculate the branches' transfer request probabilities. Finally, CMPU uses the results of UFLP and VRP to find the transfer request fixed costs at each branch and hence the policy parameters, which in turn determine the service request frequencies on of the branches.

3 A Numerical Study

The main purpose of this study is to investigate the convergence properties of our approach. We are also interested in how much an iterative approach would likely to improve the system over a piece-meal approach, i.e., solving strategic, tactical, and operational problems only once without any feedback mechanism. In our implementation, both UFLP and VRP are modeled using GAMS and solved using the commercial optimizer LINDO. Other computational steps including the simulation are performed in MS Excel.

Our instances are partially based on data obtained from a bank that operates in Turkey. It is the largest private bank in Turkey with its 1,300 branches. However, for illustrative purposes, we only considered a region where the bank had 86 branches. We would like to point out that our approach would work for much larger instances as well because in the VRP, the most computationally demanding part of our approach, the problem sizes tend to be stable due to the constraint on working hours in a day.

We obtained the addresses of each of the 86 branches from the bank and then generated the distance matrices in both time and length using Google Maps, which uses the actual road network. Whenever the travel time from a CMC candidate location to a branch and back exceeds the nine-hour work day, we eliminated the assignment variables, so the branch would not assigned to the CMC.

Table 1 summarizes all the cost estimations in the local currency Turkish Lira and US Dollars at the exchange rate at that time. A portion of fixed cost of CMCs is the leasing cost, which we have estimated using real-estate pages. We came up with an average figure that is used at all locations. For other components of fixed costs such as cleaning, heating, lighting, etc. we just added a fixed percentage of the leasing cost. The second cost is that of the AVs. According to the information provided by the bank, AVs are regular commercial vehicles that are modified for certain security requirements. AVs can be operational for an average of 10 years. The straight-line depreciation of the price of an AV and an verage maintenance cost is used as the AV annual cost. The variable AV cost is essentially the mileage cost due to gas.

Above estimates do not include labor cost. Under the current operations all labor costs are fixed. There are four types of personnel employed in the cash transfer system: drivers, security guards, clerks, and supervisors. To calculate the cost of these personnel we used the minimum wage as a basis. First we have estimated how much a minimum wage personnel cost to a company including health and pension benefits, vacation pay, and so on. Roughly, drivers and security guards cost twice as much as a minimum wage personnel. The clerks and supervisors cost three and four times that amount, respectively.

In the current operations, each AV is assigned four personnel; one of each. This number might seem excessive, but in the current mode of operation, drivers are required to be in the vehicle at all times and not to perform any other task. According to Turkish regulations, security personnel is also barred from performing any other function. A clerk is needed to perform all the transactions and exchanges and a supervisor is to oversee all the operations. A CMC is typically staffed with three personnel; also one of each, except a driver. These personnel costs are added to costs of CMCs and AVs to obtain the fixed costs.

Cost type	TL	USD
CMC annual cost	77,581	43,100
AV annual cost	5,104	2,835
AV variable cost (per km)	0.68	0.38
Minimum wage	11,088	6,160
Driver or guard cost	22,176	12,320
Clerk cost	33,264	18,480
Supervisor cost	44,352	26,640
Total CMC annual fixed cost	177,373	98,540
Total AV annual fixed cost	127,071	90,595

Table 1. Cost estimations

We now turn to the estimation of the cash transaction parameters at the branches, which include the nature of cash movements at each branch, fixed cost of cash transfers, and the daily interest rate. The last one is perhaps the easiest one to estimate (we have simply divided the interbank borrowing rate, which was 8% at the time, by 365 days). We are also aware that this is particularly high; in most developed economies this rate is extremely low.

While in most cash management literature fixed cost is usually readily available (as transferring money from one investment option to another entails some fees) in our case, it includes the fixed effort of transferring the cash from a vehicle to a branch (or, vice versa) as well as the transportation cost portion that can be allocated to the branch. For the first portion, we have estimated the average times the branch and vehicle personnel spend in the money exchange and multiply it with hourly wage rates to estimate a fixed labor cost. The transportation cost portion is rather more crude; we have identified the total route transportation cost and divided it among the branches based on a weight computed from the direct distances to CMC. Hence, farther branches received a larger share of the transportation cost while closer branches received a smaller share.

We now move on with the description of demand parameters. We need to introduce some notation to facilitate the exposition. Let,

$\mu_w, \sigma^2_{\mu w}$: Mean and variance of the size of withdrawals,
n_w, σ^2_{nw} : Mean and variance of the number of daily withdrawals,
$\mu_d, \sigma^2_{\mu d}$: Mean and variance of the size of deposits,
n_d, σ^2_{nd} : Mean and variance of the number of daily deposits,

The number of transactions at a branch is then $n_t = n_w + n_d$ and the weighted average cash movement size is $\mu_t = (n_w\mu_w + n_d\mu_d)/n_t$. Although Miller and Orr (1966) model assume roughly equal sized withdrawals and deposits, in our cases we have observed that on average there are roughly three to four times more withdrawals than deposits, while the average deposit size is roughly three to four times that of withdrawals.

To compute the lower threshold u^* of the triple policy we need to estimate the lead-time and lead-time cash withdrawal (or deposit) demand. The mean of daily cash demand is simply $\mu = n_w\mu_w - n_d\mu_d$ and its variance is $\sigma^2 =$

$n_w\sigma^2_{\mu w} + \mu^2_w\sigma^2_{nw} + n_d\sigma^2_{\mu d} + \mu^2_d\sigma^2_{nd}$. In our numerical experiments, we have also observed that normal distribution is a fairly good approximation for the net daily transactions, although the actual distribution have fatter tails. Finally we have set the lead time as the one fourth of the route travel time between a branch and its CMC and assumed that it is same for all branches at a route. The rationale is that the most a particular branch is away from the CMC is about half the route and we used the half of that time (the average) as the approximation of the lead-time. Finally, we set the risk of running out of cash during cash transfer lead-time as 1% (i.e., 99% service level).

We have tested our approach on 12 problem instances, each of which has the same parameters except the demand parameters. The instances are only differentiated with respect to the cash demand parameters. To estimate the means and variances of the sizes and numbers of deposits and withdrawals, we used monthly data for a 12-month period, each month corresponding to an instance. We refer to these instances as “January”, “February”, etc.

To initialize the procedure, we start with solving a CMPU for each branch assuming that each branch is also a CMC. In this case the fixed cost of transferring money consists only of the labor portion. We expected branches put a substantially high number of transfer requests since the cost of doing so is rather low at the beginning. Based on those results we generate average transfer requests at each branch and start “Iteration 1” by solving a UFLP considering only the CMC costs and the direct transportation cost. Subsequently, we solve VRPs and VNDPs for each CMC and then CMPU at each branch to end the first iteration. We then move to the second iteration and continue until all three iteration parameters (α, β, and k_j) converge.

Tables 2 and 3 report our results at varying in details. We choose to give only the January’s results in detail (Table 2) as other instances were similar. As expected, there are substantially high number of cash transfer requests and low cash levels at the initialization stage (Iteration 0). However, the solution quickly moves to a converging pattern and by the fourth iteration it converges to a solution. One of the decisions, the number of vehicles, shows a noteworthy pattern. This decision is found in UFLP, VRP, and VNDP. Although UFLP is not able to capture the impact of this variable sufficiently, it does not present an obstacle for the convergence of the algorithm. We also observed that the impact of VNDP on revising the number of vehicles is basically negligible (refer to Equation (14)). However its impact on the distance correction is important (see Equation (15)).

In Table 3 we only reported those of the summary results (marked with “*” in Table 2). The results are somewhat similar to those of January instance. Eight of the other instances took four iterations; one took five, and two took three iterations. Each instance’s iterative pattern is also similar where the problems quickly converge to a lower number of CMC locations and AVs and mostly settle there. From this table we also observe that the iterative solution approach improves greatly upon a piece-meal approach. For example, in January, the total cost at the end of Iteration 1 (would be the result of a piece-meal approach)

Problem	Iteration (t)	0	1	2	3	4
UFLP	Number of CMCs*	86	14	10	10	10
UFLP	Number of vehicles		11.8	6.9	8.3	10.0
UFLP	Total distance		1,622,953	1,141,654	942,083	942,083
VRP	Number of vehicles		18	17	17	17
VRP	Total route length		3,803	4,190	4,190	4,190
VRP	Total distance		959,679	1,057,302	1,057,302	1,057,302
VNDP	Number of vehicles*		18	17	17	17
VNDP	Total distance*		957,457	947,365	946,300	946,300
CMPU	Total number of requests	28,428	17,724	16,704	16,704	16,704
CMPU	Total average cash level*	9,618,084	24,898,274	27,535,662	27,535,662	27,535,662
	Iteration parameters					
UFLP	Distance correction (α^t)	0.67	0.59	0.83	1.00	1.00
VRP	Distance correction (β^t)	0.67	1.00	0.90	0.90	0.90
	Costs					
UFLP	CMC fixed costs	15,262,196	2,484,544	1,774,674	1,774,674	1,774,674
VNDP	AV fixed costs		2,655,043	2,507,541	2,507,541	2,507,541
VNDP	Travel cost		652,961	646,079	645,353	645,353
CMPU	Cash holding cost	769,447	1,991,862	2,202,853	2,202,853	2,202,853
	Total cost*	16,031,643	7,784,410	7,131,147	7,130,421	7,130,421

Table 2. Detailed results of the January instance.

is 7,784,410. It is reduced to 7,130,421 as a result of the iterative approach, which corresponds to about 8.4% improvement. The average improvement over 12 instances is 7.8%, which indicates that a substantial improvement in total cost can be obtained by an iterative approach.

In summary, both the convergence and the improvement results indicate that an iterative approach such as ours presents a great promise as a viable solution approach in the improvement of such design and operational decisions. One final noteworthy result is that most of the improvement in these instances came at the end of Iteration 2, while the algorithm uses the remaining steps to ensure convergence of the algorithm. Knowing that such great improvements can be obtained in a few iterations is a further credit to an iterative approach.

4 Concluding remarks

This paper presents a novel addition to the scant but growing literature of cash logistics operations. The overall problem is potentially very difficult to solve and even to model in an integrated fashion. Therefore, an iterative approach such as ours that solves a series of location, routing, and cash inventory problems is an attractive one.

Our approach and the model are open to several improvements and extensions. Some of our modeling choices came from the particular practices we observed at the bank, which might not be valid or desirable in other cases. Firstly, the cash management problem that we model here is based on a number of assumptions that might not hold in other banking environments. For example, in general, some branches have net withdrawals while the others have net deposits. Hence, the cash management policy parameters must be found by observing these differences among the branches and the corresponding vehicle routing problem could also be revised to take advantage of such differences among the branches. Secondly, the vehicle routing part can also be made more dynamic. However, that would necessitate a different and potentially a much more difficult version of the vehicle routing problem. Thirdly, an integrated approach could also be developed albeit possibly with more simplifying assumptions. Finally, a real-time control and decision support system can be developed for some of the tactical and operational decisions.

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Instance	**Iteration (t)**	**0**	**1**	**2**	**3**	**4**	**5**
Jan	Number of CMCs	86	14	10	10	10	
	Number of vehicles		18	17	17	17	
	Total distance		957,457	947,365	946,300	946,300	
	Total average cash level	9,618,084	24,898,274	27,535,662	27,535,662	27,535,662	
	Total cost	16,031,643	7,784,410	7,131,147	7,130,421	7,130,421	
Feb	Number of CMCs	86	13	10	10	10	
	Number of vehicles		18	17	17	17	
	Total distance		966,150	902,489	904,540	904,540	
	Total average cash level	7,954,135	22,532,743	24,567,775	24,567,775	24,567,775	
	Total cost	15,898,527	7,423,629	6,863,111	6,864,511	6,864,511	
Mar	Number of CMCs	86	14	10	10	10	10
	Number of vehicles		20	17	17	17	17
	Total distance		994,626	1,005,494	997,635	1,001,523	1,001,523
	Total average cash level	11,903,928	28,726,610	33,080,877	33,080,877	33,080,877	33,080,877
	Total cost	16,214,510	8,411,030	7,614,407	7,609,047	7,611,699	7,611,699
Apr	Number of CMCs	86	14	9	9	9	
	Number of vehicles		19	19	19	19	
	Total distance		960,376	977,459	972,778	972,778	
	Total average cash level	8,340,658	23,916,725	27,049,245	27,049,245	27,049,245	
	Total cost	15,929,449	7,855,380	7,230,294	7,227,102	7,227,102	
May	Number of CMCs	86	14	11	11	11	
	Number of vehicles		19	18	18	18	
	Total distance		977,670	856,824	854,915	854,915	
	Total average cash level	9,548,230	25,731,865	30,164,435	30,164,435	30,164,435	
	Total cost	16,026,055	8,012,385	7,604,672	7,603,370	7,603,370	
Jun	Number of CMCs	86	14	11	11		
	Number of vehicles		19	19	19		
	Total distance		973,696	928,029	928,029		
	Total average cash level	9,548,230	24,757,850	27,745,785	27,745,785		
	Total cost	16,026,055	7,931,754	7,607,242	7,607,242		
Jul	Number of CMCs	86	14	10	10		
	Number of vehicles		19	17	17		
	Total distance		977,670	955,171	955,171		
	Total average cash level	8,828,137	24,122,858	27,498,010	27,498,010		
	Total cost	15,968,447	7,883,664	7,133,459	7,133,459		
Aug	Number of CMCs	86	13	10	10	10	
	Number of vehicles		19	17	16	16	
	Total distance		971,967	1,021,075	1,020,822	1,020,822	
	Total average cash level	9,691,987	27,062,211	30,421,063	30,421,063	30,421,063	
	Total cost	16,037,555	7,937,456	7,412,247	7,264,573	7,264,573	
Sep	Number of CMCs	86	15	12	12	12	
	Number of vehicles		19	18	18	18	
	Total distance		960,033	902,765	901,199	901,199	
	Total average cash level	9,008,141	26,093,475	28,959,204	28,959,204	28,959,204	
	Total cost	15,982,847	8,206,753	7,717,051	7,715,984	7,715,984	
Oct	Number of CMCs	86	14	10	10	10	
	Number of vehicles		19	17	17	17	
	Total distance		977,645	958,301	957,318	957,318	
	Total average cash level	8,673,301	28,051,644	32,771,237	32,771,237	32,771,237	
	Total cost	15,956,060	8,197,950	7,557,451	7,556,781	7,556,781	
Nov	Number of CMCs	86	14	10	10	10	
	Number of vehicles		19	17	16	16	
	Total distance		967,954	919,875	923,733	923,733	
	Total average cash level	8,996,093	25,429,554	29,259,249	29,165,969	29,165,969	
	Total cost	15,981,884	7,981,574	7,250,286	7,097,953	7,097,953	
Dec	Number of CMCs	86	14	10	10	10	
	Number of vehicles		19	17	17	17	
	Total distance		977,670	996,579	995,629	995,629	
	Total average cash level	9,560,877	27,679,537	31,601,283	31,601,283	31,601,283	
	Total cost	16,027,066	8,168,198	7,489,960	7,489,312	7,489,312	

Table 3. Summary results on 12 instances.