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Railway crew capacity planning problem with connectivity of schedules

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ABSTRACT

We study a tactical level crew capacity planning problem in railways which determines the minimum required crew size in a region while both feasibility and connectivity of schedules are maintained. We present alternative mathematical formulations which depend on network representations of the problem. A path-based formulation in the form of a set-covering problem along with a column-and-row generation algorithm is proposed. An arc-based formulation of the problem is solved with a commercial linear programming solver. The computational study illustrates the effect of schedule connectivity on crew capacity decisions and shows that arc-based formulation is a viable approach.

1. Introduction

Crew is one of the most crucial resources in railways. In most systems, crew-related costs outweigh even the dominating energy expenditures. For instance, they constitute more than one third of general expenditures of the Turkish State Railways (Turkish State Railways, 2004, 2008, 2010). Dutch Railways report a share of more than 30% of their total operational expenses due to wages in both 2011 and 2012 (Dutch Railways, 2012). In 2012, Association of American Railroads has reported that 21.3% of the total operational expenses was spent for wages which was second in the ranking right after fuel expenses (Association of American Railroads, 2012). Hence, effective crew management is one of the critical planning issues.

For effective planning and management, railway systems are districted into multiple crew regions; home/base station of a region is responsible for its own crew while a centralized authority is responsible for the coordination and synchronization of regions. The operational level regional planning problems have attracted the utmost attention from both practice and academia; they consider assignment of crew to train duties which includes the tasks for a train service specified with a starting time and location and an ending time and location. This planning level is usually prescribed as crew scheduling. A common understanding considers a two-phase approach for this level: crew pairing and crew rostering. The pairing phase constructs sequences of train duties (by sequentially pairing duties with each other) spanning a finite planning horizon; if the pairings are feasible with respect to rest periods, total work time, total rest time, etc., they constitute feasible crew schedules. Rostering phase is concerned with the assignment of individual crew members to crew schedules.

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At the strategic level, system/company-wide decisions that would vastly affect the level and allocation of crew resources over all regions are made. Some of these issues are related to establishment of new crew regions or new crew exchange stations as well as high level adjustment of company practice such as re-distribution of inter-regional workload (assignment of duties to regions). At the tactical level, effective individual capacities of crew regions are the main planning issues; given the train timetable under the responsibility of a particular crew region, the minimum number of crew members required to operate these train services need to be determined. Based on the crew capacities of the regions, crew scheduling (crew pairing and crew rostering) is done at the operational level.

In hierarchical planning frameworks where a top-to-bottom approach is used, adjustments at the lower level plans are common and usual. Reflecting more concerns related with lower level plans into higher level plans is desirable as it keeps the level of such adjustments reasonable. In this respect, tactical planning decisions that take into account the operational level considerations as much as possible are considered more effective. In this study, we consider the tactical level crew capacity planning problem that involves finding the minimum number of crews in a region required to operate a predetermined set of train duties and take into account a particular set of new planning issues which mostly affects the pairing phase.

In most systems, train service schedules are periodic (e.g. weekly, bi-weekly, monthly); however, the recurrence of crew schedules and rosters over the periodic train service schedules are overlooked at the pairing phase. When the recurrences are considered, the last duty on every feasible schedule in a period would be connected to the first duty of one of the feasible schedules in the next period. This phenomenon shall be considered as schedule connectivity. In this respect, connectivity between two duties is governed by the same restrictions of a feasible pairing between two duties in the same schedule, i.e. there should be sufficient rest time after the end of the earlier duty and before the beginning of the later.

We may easily exemplify shortcomings of not considering the connectivity between schedules even on a small example with two crew members only: let us assume that the end of the period is Sunday midnight and one crew member returns back to the base at 11 pm on Sunday while the other member returns back at 11:30 pm conducting their last duties at the end of the week. In the case that the minimum home rest requirement is 16 h, none of these crew will be allowed to work before 3 pm on Monday. However, the earliest duty at the beginning of the week may start much earlier, for instance at 8 am on Monday. In essence, there should be at least 3 crew members (or 3 pairings) at this base station so that the pairings can feasibly be connected from one period to the next. This small example shows that the pairings may fail to honor some of the regulations in the second week of operations although we have feasible pairings for the planning period of one week only.

In practice, infeasibilities regarding the connectivity of the schedules are tackled at the operational level; either the managers resort to patching or the schedules are manually modified to guarantee connectivity so that the same schedules can be repeated from one period to the next. Thus, maintaining periodically connected schedules is a challenging task; the availability of crew should be guaranteed with respect to the periodic recurrence of the planning horizon.

In this study, the feasibility of crew schedules and their connectivity are integrated within a tactical level planning problem to find a set of schedules that satisfy the operational rules and regulations. The problem minimizes the crew size (i.e. number of crew members) required in a region. Our contributions shall be summarized as follows:

- Based on a well-known network representation of the problem, we develop a path-based formulation in the form of a set covering problem.
- We present an enhanced version of the network representation which enables an arc-based formulation in the form of a minimum cost network flow problem.
- We compare the computational performance of the two formulations with data sets acquired from Turkish Railways.
- We show that the decisions on regional crew capacities ignoring connectivity of the schedules might significantly differ from those where connectivity of schedules are integrated into the planning process.

Following a review of the literature on crew-related railway problems in Section 2, we propose two alternative formulation approaches and develop solution methods in Section 3. Section 4 presents the results of a computational study that compares the two solution approaches. In Section 5, we wrap up our findings and suggest further research issues.

2. Literature review

Crew planning problems at railways have been studied for various environments considering particular railway companies, and nation-wide or region-wide systems. For instance, Caprara et al. (1997, 1999, 2001) focus on the Italian case; Freling et al. (2001), Morgado and Martins (1998), Kroon and Fischetti (2001) and Abbink et al. (2005) focus on Dutch railways; Vaidyanathan et al. (2007) focus on the North American railways; Jutte et al. (2011) study the largest European railways DB Schenker; and Şahin and Yüceoğlu (2011) focus on Turkish railways. Although the problem environment is different from one system to the other, several features including universally accepted rules as well as company and legislative regulations are common. In this respect, the literature is compact as this line of research has improved steadily mostly by adding and integrating new and more challenging issues to the problem environment at different planning stages.

With respect to the problem domain, our study is closest to Ernst et al. (2001) and Şahin and Yüceoğlu (2011); they both consider minimizing the crew size required to operate the trains under the responsibility of a region. Ernst et al. (2001) consider the problem in two phases; the number of crew members is determined at the planning phase and the connectivity of the crew schedules is maintained at the operational (rostering) phase. Their heuristic two-stage approach minimizes the number of crew members in the first stage and tries to satisfy the connectivity of rosters in the second stage. Our approach considers the connectivity of schedules integrated within the pairing phase while minimizing the number of crew members. Şahin and Yüceoğlu (2011) study the planning stage by enriching the problem environment with the day-off planning issue which is usually considered as an operational planning concern. They focus on optimally minimizing the number of crew members required in the region to cover the duties. They develop a network representation of the problem and solve the corresponding network flow problem with a solver. Different than Şahin and Yüceoğlu (2011), we not only consider an additional operational planning issue as the connectivity of schedules, but also enhance the existing network representation in order to develop two alternative formulations of the new problem.

Crew planning and associated problems are generally studied with two mainstream formulation approaches: minimum cost network flow formulations and set covering/partitioning type formulations. While research on crew planning problems with a network flow formulation is limited (Vaidyanathan et al., 2007; Şahin and Yüceoğlu, 2011), set partitioning and set covering formulations are more frequently used (Caprara et al., 1997, 1999, 2001; Ernst et al., 2001; Jutte et al., 2011; Shen and Chen, 2014; Jutte and Thonemann, 2015). The number of studies using set covering type formulations has increased steadily over the years. As researchers attempt to solve real life problems of practical size, the scalability of the solution methods become even more critical. Column-generation based methods carry this potential for set covering type formulations. As exemplified in Caprara et al. (1999), Jutte et al. (2011), Jutte and Thonemann (2012), and Shen and Chen (2014) using set-covering type formulations and column generation in one way or the other, good quality near-optimal solutions are found for large-scale real-life problems in reasonable computational times, indicating satisfactory computational performance. On the other hand, no direct comparison of alternative solution approaches is found in the literature.

From a methodological point of view, we intend to test the effectiveness of both mainstream approaches for handling the new connectivity issue in crew schedules: (i) the path based formulation in the form of a set-covering problem is inspired by the serial work in Caprara et al. (1997, 1999, 2001) with the adaptation of a novel methodology in Muter et al. (2013) which was also used in a primitive version of the same problem in Suyabatmaz and Şahin (2012) and (ii) the arc-based formulation follows the footsteps of Vaidyanathan et al. (2007) and Şahin and Yüceoğlu (2011) by adapting their network representations. As a result, we study a new version of the crew capacity planning problem and develop alternate formulations of this problem using two competitive approaches, and test the computational performance of these approaches using appropriate solution methods.

3. Mathematical formulations and solution methods

Although the interpretation of nodes and arcs may differ from one study to another, both formulation approaches in the literature depend on a space–time network representation of the problem. The formulation in the form of a set-covering problem promoted in Caprara et al. (1997) relies on identifying paths on the network each of which corresponds to a feasible crew schedule; the path-based formulation selects a subset of such schedules sufficient enough to cover all duties. The arc-based formulation in Vaidyanathan et al. (2007) and Şahin and Yüceoğlu (2011) is essentially a minimum-cost network flow problem with lower bounds on the arcs and/or demand and supply to be satisfied at some nodes. For both of the formulation approaches, we present the corresponding network representations, mathematical programming formulations and appropriate solution methods.

3.1. Path-based formulation

For the path-based formulation, the feasible crew schedules are extracted as source-sink paths of a network representing the problem data. In our interpretation of this network, nodes represent events and have two attributes: space and time, respectively representing the place (i.e. the station) and the time of the event. Each duty is defined with two nodes: on-duty node represents the beginning of the duty and tie-up node represents the end of the duty in both space and time. The source node marks the beginning of the schedule in time while the sink node marks the end. Arcs represent the engagement of crew with various activities in the space–time network as follows:

- *source arcs* from the source node to an on-duty node correspond to the first duty in a schedule;
- *sink arcs* from a tie-up node to the sink node represent the last duty in a schedule;
- *duty arcs* from an on-duty node to its corresponding tie-up node represent the engagement with the duty;
- *rest arcs* from a tie-up node to an on-duty node correspond to the rest period between the duty of the tie-up node and the duty of the on-duty node;
- *deadhead arcs* from a tie-up node to another tie-up node represents the deadheading of the crew member;
- *direct connection arcs* from a tie-up node at a non-base station to another on-duty node at the same station represent the connection of the crew from one duty to another without a rest.

An example with two feasible schedules of a problem with single day-off requirement on a two-layer network is shown in Fig. 1. A source-sink ($s-t$) path corresponds to a crew schedule representing a sequence of events and activities the crew is engaged with. Since the network is generated according to the rules and regulations, the flow on an $s-t$ path corresponds to

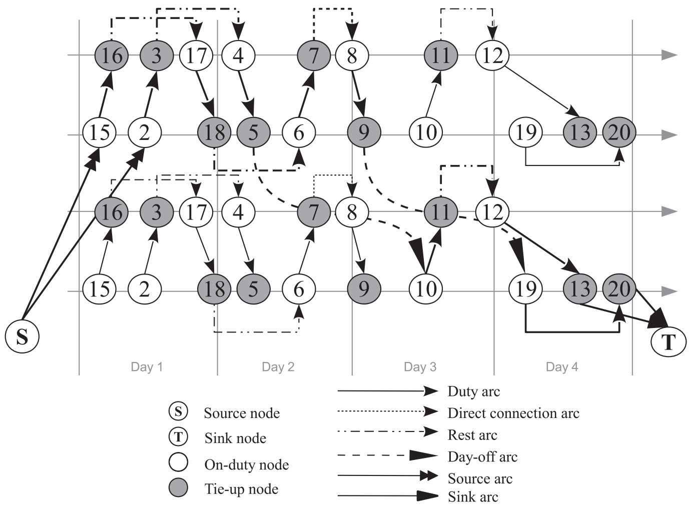

Fig. 1. An illustration of two sample schedules.

a feasible schedule. Beginning of a schedule is marked with a flow on a source arc which is connected to an on-duty node (i.e. first duty in the schedule). After covering a set of on-duty nodes and tie-up nodes (i.e. set of duties) and using different types of arcs between them, the flow reaches the sink node. The last tie-up node in the path before reaching the sink node indicates the end of the crew schedule. In this example, two distinct s—t paths (i.e. crew schedules) are marked with bold arcs. One of the schedules starts with a source arc (S,15) and ends with a sink arc (20,T), the other starts with a source arc (S,2) and ends with a sink arc (13,T).

For the mathematical formulation, let I denote the set of duties (i.e. duty arcs in the network) and J denote the set of feasible schedules (i.e. s—t paths). A binary parameter a_{ij} indicates that duty $i \in I$ is included in schedule $j \in J$ when $a_{ij} = 1$. The minimum number of crew members required to cover duty i is represented by c_i and $l_{jj'}$ indicate the connectivity relationship between schedules as follows:

$$l_{jj'} = \begin{cases} 1, & \text{if schedule } j \text{ can be connected to schedule } j'; \\ 0, & \text{otherwise.} \end{cases}$$

Decision variables are defined as

$$x_j = \begin{cases} 1, & \text{if schedule } j \text{ is selected/included in solution;} \\ 0, & \text{otherwise,} \end{cases}$$

and

$$y_{jj'} = \begin{cases} 1, & \text{if schedule } j \text{ is connected to schedule } j'; \\ 0, & \text{otherwise.} \end{cases}$$

Then, the path-based formulation of the regional crew capacity planning problem with schedule connectivity (RCCP-C) becomes:

$$[\text{RCCP-C}]_p \qquad \text{minimize} \sum_{j \in J} x_j \tag{1}$$

$$\text{subject to} \sum_{j \in J} a_{ij} x_j \geqslant c_i, \quad i \in I, \tag{2}$$

$$\sum_{j' \in J, j' \neq j} l_{jj'}\, y_{jj'} - x_j = 0, \quad j \in J, \tag{3}$$

$$\sum_{j' \in J, j' \neq j} l_{j'j}\, y_{j'j} - x_j = 0, \quad j \in J, \tag{4}$$

$$x_j \in \{0, 1\}, \quad j \in J, \tag{5}$$

$$y_{jj'} \in \{0, 1\}, \quad j, j' \in J. \tag{6}$$

Objective function (1) minimizes the number of selected schedules (i.e. number of crew members as each schedule may be associated with one crew). Constraints (2) ensure that each duty is covered by at least the required number of schedules.

Constraints (3) guarantee that each selected schedule follows (is connected to) another selected schedule in the solution. Likewise, constraints (4) guarantee that a selected schedule is being followed by (connects to) another selected schedule in the solution. Constraints (5) and (6) represent the binary nature of decision variables.

Solving $[\text{RCCP-C}]_p$ directly is not practical due to the size of set J, which contains all possible schedules as it requires enumerating all $s-t$ paths. Due to constraints (3) and (4), $[\text{RCCP-C}]_p$ belongs to the class of problems with column-dependent-rows (Muter et al., 2013); existence of connectivity constraints (3) and (4) (i.e. rows of the formulation) depends on the existence of columns (represented by x_j) in the problem. For this class of problems, Muter et al. (2013) prescribe a column-and-row generation (CRG) algorithm.

CRG algorithm starts with a restricted master problem (RMP) that includes only a selected subset of schedules (decision variables), and then iteratively adds new schedules to RMP to improve its objective function value. At each iteration, RMP is solved to optimality and the optimal dual values from RMP are used to solve the pricing subproblem (PSP) which allows generating new schedules and their associated connectivity constraints simultaneously by appropriately estimating the dual values of the missing connectivity constraints. In this respect, an accurate estimate of the dual values corresponding to the missing constraints is crucial. In essence, addition of a new schedule induces new constraints to be added to RMP. However, when this new schedule is to be generated by PSP, the dual information associated with the constraints that are connecting the new schedule to the existing schedules is missing from the solution of the most recent RMP. Hence, a traditional column generation algorithm would not suffice to solve this problem. In order to correctly compute the reduced cost of a column (with PSP), we need to know the associated dual variables of the missing constraints (rows) a priori.

In order to mathematically describe PSP that generates a new variable representing a feasible schedule, let J_c be the set of existing schedules in RMP, and the set of remaining feasible schedules be $\overline{J_c}$. Let u_i, v_j and w_j be the dual variables corresponding to, respectively, constraints $i \in I$ in (2) and constraints (3) and (4) for only $j \in J_c$. The dual of the LP relaxation of RMP can then be formulated as

$$\text{maximize} \sum_{i \in I} u_i \tag{7}$$

$$\text{subject to} \sum_{i \in I} a_{ij} u_i - v_j - w_j \leqslant 1, \quad j \in J_c, \tag{8}$$

$$l_{jj'}(v_j + w_{j'}) \leqslant 0, \quad j, j' \in J_c,\ j \neq j', \tag{9}$$

$$u_i \geqslant 0, \quad i \in I, \tag{10}$$

$$v_j, w_j \ \text{u.i.s} \quad j \in J_c. \tag{11}$$

PSP is to find a schedule $j \in \overline{J_c}$ that has a negative reduced cost (i.e. violates the corresponding dual constraint in (8)). If we search for the schedule that is expected to make the most improvement in the objective function (1) value of RMP, the pricing problem becomes the following two-stage problem to find

$$j^* = \arg \min_{j \in \overline{J_c}} \left\{ 1 - \sum_{i \in I} a_{ij} u_i + v_j + w_j \right\} \tag{12}$$

$$\text{minimize } v_j \tag{13}$$

$$\text{subject to } l_{jj'}(v_j + w_{j'}) \leqslant 0, \quad j \in \overline{J_c},\ j' \in J_c, \tag{14}$$

$$\text{at least one constraint in (14) is tight;} \tag{15}$$

$$\text{minimize } w_j \tag{16}$$

$$\text{subject to } l_{j'j}(v_{j'} + w_j) \leqslant 0, \quad j' \in J_c,\ j \in \overline{J_c}, \tag{17}$$

$$\text{at least one constraint in (17) is tight.} \tag{18}$$

where problems (13)–(15) and (16)–(18) impose the connectivity relations between the new schedule $j \in \overline{J_c}$ and the existing schedules in J_c.

Constraints (14) and (17) impose that the dual constraints (9) are not violated as we try to find the schedule with the largest violation in dual constraint (8). The new schedule, represented by x_{j^*}, is expected to enter the optimal basis of RMP (i.e. the new schedule is included in the optimal solution) at the next iteration of the CRG algorithm. We know that when x_{j^*} enters the basis, the connectivity constraints in (3) and (4) are to be honored with two new basic connectivity variables $y_{j^*j'} = 1, j' \in J_c$, and $y_{j'j^*} = 1, j' \in J_c$. Optimal solution of (13)–(15) indicates which $y_{jj'}$ variable is to enter the basis (takes a value of 1) in (3), corresponding to a tight constraint in (14). This is indeed imposed by the complementary slackness condition associated with constraint (15). Similarly, the optimal solution of (16)–(18) indicates which $y_{j'j}$ variable is to enter the basis (takes a value of 1) in (4), corresponding to a tight constraint in (17) with respect to the complementary slackness condition associated with constraint (18).

We may exploit the solution of PSP in (12)–(18) as follows:

- For each schedule $j \in \overline{J_c}$, if the schedule can be connected to the existing schedule $j' \in J_c$, (i.e. $l_{jj'} = 1$, $j \in \overline{J_c}$, $j' \in J_c$) the corresponding constraint (14) appears as $v_j \leqslant -w_{j'}$. Therefore, for each $j \in \overline{J_c}$, the solution of (13)–(15) becomes

$$v_j = \min_{j' \in J_c, l_{jj'}=1} (-w_{j'}) = \max_{j' \in J_c, l_{jj'}=1} (w_{j'}). \tag{19}$$

- For all existing schedules $j' \in J_c$ that can connect to the new schedule j (i.e. $l_{j'j} = 1$, $j' \in J_c$, $j \in \overline{J_c}$) the corresponding constraint (17) appears as $w_j \leqslant -v_{j'}$ and for each $j \in \overline{J_c}$, the solution of (16)–(18) becomes

$$w_j = \min_{j' \in J_c, l_{j'j}=1} (-v_{j'}) = \max_{j' \in J_c, l_{j'j}=1} (v_{j'}). \tag{20}$$

Then, the two stage problem (12)–(18) can be reformulated as

$$j^* = \arg \min_{j \in \overline{J_c}} \left\{ 1 - \sum_{i \in I} a_{ij} u_i + \max_{j' \in J_c, l_{jj'}=1} (w_{j'}) + \max_{j' \in J_c, l_{j'j}=1} (v_{j'}) \right\} \tag{21}$$

As in the column generation algorithm, the CRG algorithm achieves optimality when the objective function value (21) is non-negative (i.e. no column exists with negative reduced cost). Consequently, if

$$1 - \sum_{i \in I} a_{ij^*} u_i + v_{j^*} + w_{j^*} < 0,$$

x_{j^*} is added to RMP and RMP is augmented along with one new connectivity constraint of type (3) and one new linking constraint of type (4). Then, the correct termination condition for the CRG algorithm that optimally solves the linear programming (LP) relaxation of $[\text{RCCP-C}]_p$ may be formalized with the following theorem.

Theorem 1. *Let $u_i, i \in I$, $v_j, j \in J_c$, and $w_j, j \in J_c$ be the optimal dual solution corresponding to the optimal basis $\mathscr{B}$ of the current RMP. The primal solution associated with $\mathscr{B}$ is optimal for the LP relaxation of $[\text{RCCP-C}]_p$ if*

$$1 - \sum_{i \in I} a_{ij} u_i + \max_{j' \in J_c, l_{jj'}=1} (w_{j'}) + \max_{j' \in J_c, l_{j'j}=1} (v_{j'}) \geqslant 0 \tag{22}$$

for every $j \in \overline{J_c}$.

The proof of this theorem follows from the analysis of the CRG algorithm developed in Muter et al. (2013).

The optimal solution of LP relaxation can be obtained with a CRG algorithm when the termination condition in Theorem 1 is reached. From a methodological and computational point of view, there might be potential obstacles in the CRG algorithm:

- To initialize the CRG algorithm, an initial feasible solution to RMP ($[\text{RCCP-C}]_{sc}$) should be known. A trivial procedure would be to produce schedules each of which cover a single train duty. However, such schedules would be infeasible with respect to rest period restrictions; the maximum length of a rest period is not honored when there is only one duty in a feasible schedule.
- The two stage PSP (21) is solved at each iteration. In order to find a feasible solution to the problem, one could develop a constructive algorithm to produce a set of schedules which together cover all the duties in the schedule. To find a new schedule that is expected to improve the solution to RMP, one could calculate the reduced costs of all remaining feasible schedules ($\forall j \in \overline{J_c}$). Yet, this would first of all require knowing all feasible schedules, and it would also be computationally cumbersome to calculate the reduced costs for all of them.

Next, we discuss how our algorithm handles these obstacles.

3.1.1. Initial solution procedure

An initial feasible solution to $[\text{RCCP-C}]_p$ is required to construct the initial RMP. A feasible solution requires all duties to be covered (i.e. corresponding duty arcs in the network are included in selected paths) and each schedule is connected to two schedules from both ends. Recall that the objective is to attain the minimum number of schedules. Therefore, a path with more duty arcs is preferred. In order to find such a set of schedules, we develop a greedy search algorithm which iteratively covers additional duty arcs by finding a new s—t path in each iteration.

The algorithm is based on the idea of finding a path including the largest number of duty arcs that are yet uncovered at an iteration. This is achieved by appropriately setting the lengths of the uncovered duty arcs. In order to maintain connectivity of schedules throughout the iterations, we use an ordered list of connected schedules as follows:

- The first schedule on the list does not necessarily follow any other schedule found so far while the new path to be found is expected to connect to the last schedule on the list. In order to achieve this, we modify the length of the source arcs at the

beginning of the iteration in such a way that the first duty of the new path to be found is connected to the last duty of the path found in the previous iteration.
- At the end of the iteration, we check if the new path may connect to the first schedule on the list. If this is the case, then the current list of schedules makes a closed circuit of connected schedules, and we initiate a new list at the next iteration. Otherwise, we again modify the source arcs at the beginning of the next iteration.

We terminate the algorithm when all duty arcs are covered in some path. Flowchart of the algorithm is given in Fig. 2. At any iteration, the new path may not cover any previously uncovered duty arc. In this case, we modify this path such that it can connect to the first path in the active list of connected schedules, thus forming a closed circuit of connected schedules.

3.1.2. Solution method for PSP

PSP in (21) is seemingly a bi-level optimization problem. However, it can be formulated to find a path on the network where the arc lengths are arranged in such a way that the total length of an $s-t$ path represents the reduced cost of the corresponding new variable. A solution to PSP at any iteration of the CRG algorithm is found using the dual values (u_i, v_j and w_j) obtained from the optimal solution of the most recent RMP. At each iteration of the algorithm arc lengths are arranged as follows:

- the dual price of each duty ($-u_i$) is introduced as the arc length of the corresponding duty arcs (in a multi-layer network all copies of the same duty arc are updated);
- the length of a source arc (s, i) is set to $\max_{j \in J_c^{s(i)}}(v_j)$ where $J_c^{s(i)}$ is the set of schedules in the current RMP which can connect to a schedule that starts with duty i, and
- the length of a sink arc (i, t) is set to $\max_{j \in J_c^{e(i)}}(w_j)$ where $j \in J_c^{e(i)}$ is the set of schedules in the current RMP which can be connected by a schedule that ends with duty i.

Consequently, solving a shortest $s-t$ path problem on this network corresponds to finding a path that represents a schedule j^* with a total length of $c_{si_{j^*}} - \sum_{i \in I} a_{ij^*} u_i + c_{i'_{j^*}t}$ where $c_{si_{j^*}}$ is the length of the source arc that enters the on-duty node of the first duty in schedule j^* denoted by i_{j^*} and $c_{i'_{j^*}t}$ is the length of the sink arc that emanates from the tie-up node of the last dusty in schedule j^* denoted by i'_{j^*}. If

$$1 + c_{si_{j^*}} - \sum_{i \in I} a_{ij^*} u_i + c_{i'_{j^*}t} < 0,$$

the corresponding schedule (variable) is added to RMP; otherwise, the termination criteria is satisfied as no schedule with negative reduced cost is found.

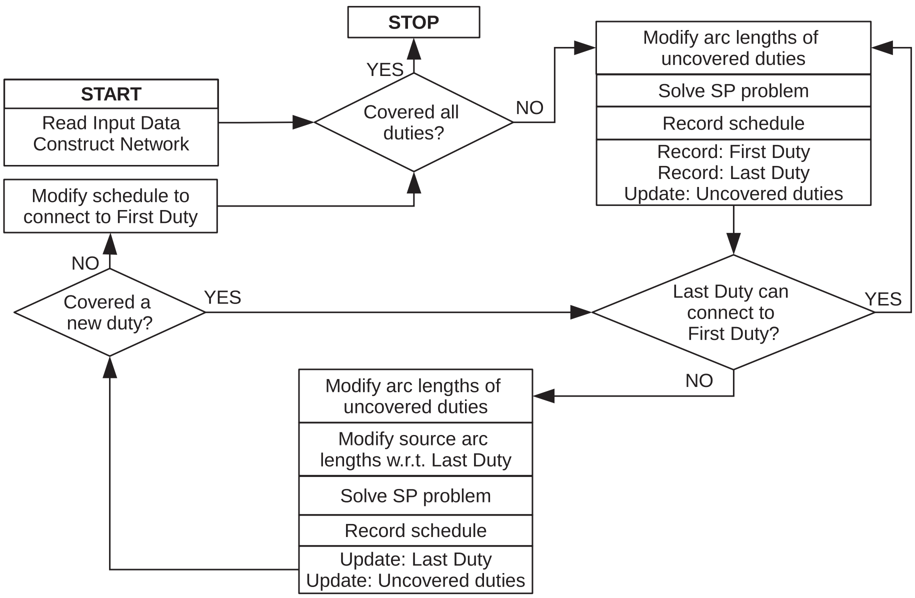

Fig. 2. The flow of the initial solution procedure.

3.1.3. Procedure for the column-and-row generation algorithm

CRG algorithm that solves the LP relaxation of $[\text{RCCP-C}]_p$ has three main components: initial solution, RMP and PSP. The iterative mechanism of the algorithm is depicted in Fig. 3.

The optimal solution to LP relaxation may not be integer feasible; to obtain an integer feasible solution, we solve (1)–(6) as an IP problem with the schedules generated by the CRG algorithm until the termination. Since the columns generated by the algorithm represents a limited solution space, the optimal solution of (1)–(6) with the columns in terminating RMP generates an integer feasible upper-bounding solution to the problem. Alternatively, an optimal solution can be obtained by implementing a branch-and-bound algorithm.

3.2. Arc-based formulation

Considering the network representation in Fig. 1, connectivity of the schedules corresponds to connecting the s—t paths from their each end in such a way that a flow may circulate on these paths. To represent this circulating flow, we devise an enhanced version of this representation. A new arc type is defined; a *connectivity arc* emanates from a tie-up node that may mark the end of a schedule to an on-duty node at home station that may mark the beginning of a schedule. Unlike any other arc in the network, time attribute of the tail node will be later/greater than the time attribute of the head node; connectivity arc goes from the end of the planning horizon to the beginning, representing the recurrence of the planning horizon. In other

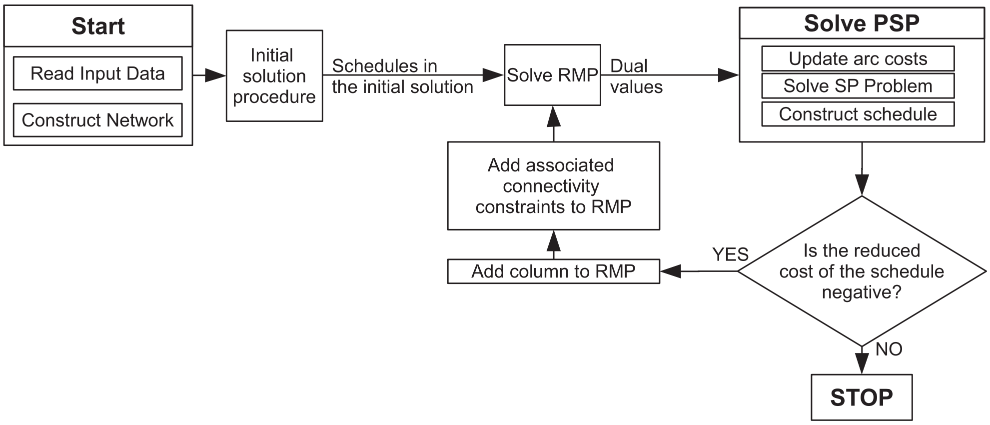

Fig. 3. The flow of the column-and-row generation algorithm.

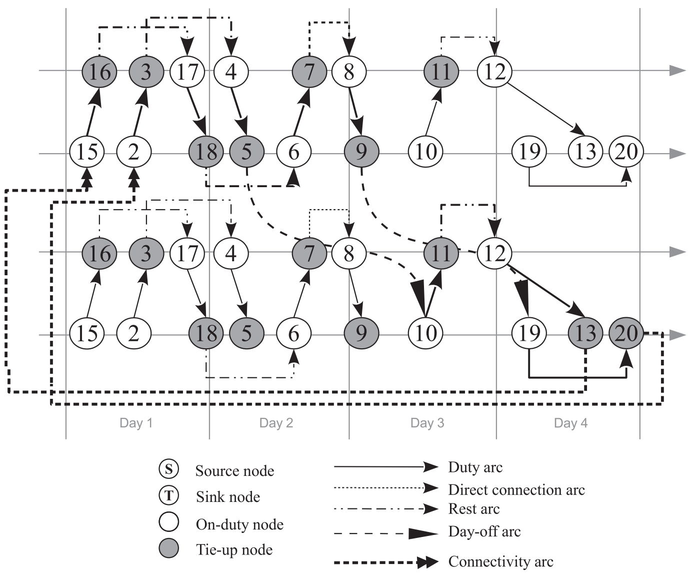

Fig. 4. An illustration of two sample schedules with connectivity arcs.

Table 1
Notation for the arc-based formulation.

N	Set of nodes in the network
L	Set of layers in the network
A	Set of all arcs in the network
A_d	Set of duty arcs in the network
A_c	Set of connectivity arcs in the network
A_{n^+}	Set of outgoing arcs at node n
A_{n^-}	Set of incoming arcs at node n

words, head node of a connectivity arc corresponds to a duty that will be covered in the next recurrence of planning period. The flow on this arc represents the rest period of a crew member that is between her last duty (i.e. tail node) in a period and the first duty (i.e. head node) in the next period. In order to represent the rest activity in-between, time difference of the head node and the tail node has to honor resting constraints; connectivity arcs are created between such nodes.

A unit flow on a connectivity arc connects two schedules of a crew member by marking the end of one and the beginning of the latter. The tail node of a connectivity arc defines the end of a path that corresponds to the last duty of that schedule, and the head node defines the first duty of a path that corresponds to the first duty of another schedule. Consequently, source and sink nodes that represent the beginning and the end of the planning horizon are no longer needed in the enhanced network representation. Moreover, source arcs and sink arcs are naturally excluded as there are no source and sink nodes.

We use the example in Fig. 1 to demonstrate the use of connectivity arcs and modifications on the network representation. In the modified example in Fig. 4, connectivity arcs (13,15) and (20,2) represents the connection between the two schedules from one period to the next. The crew following these schedules continue working without violating rules and regulations through the end of the first period into the second.

In the modified network, total flow on connectivity arcs corresponds to number of schedules linked with each other (i.e. number of crew members). Thus, the problem is to minimize the total flow on connectivity arcs. Notation for the network and the mathematical formulation of the minimum flow problem is given in Table 1.

In addition, c_a denotes required number of crew members to cover the duty represented by arc a. Decision variable $\bar{x}_a^l$ denotes the amount of flow on copy of duty arc $a \in A_d$ on layer $l \in L$. Then, the arc-based formulation in the form of a minimum-cost network flow problem is as follows:

$$[\text{RCCP-C}]_a \qquad \text{minimize} \sum_{a \in A_c} \bar{x}_a \tag{23}$$

$$\text{subject to} \sum_{a \in A_{n^+}} \bar{x}_a = \sum_{a \in A_{n^-}} \bar{x}_a, \quad \forall n \in N, \tag{24}$$

$$\sum_{l \in L} \bar{x}_a^l \geqslant c_a, \quad \forall a \in A_d, \tag{25}$$

$$\bar{x}_a \in \mathbb{Z}_+, \quad \forall a \in A. \tag{26}$$

The objective function (23) minimizes the total amount of flow on connectivity arcs, which corresponds to minimizing the number of crew members required to operate the given duties in the region. Constraints (24) are the flow-balance constraints. The coverage constraint (25) for a duty guarantees that the total amount of flow on all copies of the duty arc is at least as much as the required amount, c_a, to ensure that duties are covered by the required number of crew schedules. Constraints (26) represent the domain of the decision variables.

4. Computational study

We perform our computational study with the following goals in mind: (i) empirically observe both efficiency and effectiveness of the proposed solution methods and compare them against each other and (ii) observe if the optimal decision on a regional crew size differs when connectivity of schedules is considered.

We implement the formulations and the algorithms in C++ using ILOG Optimization Studio 12.4 as the LP and IP solver on a PC with Intel Core i7 @3.40 GHz CPU and 12 GB RAM. Test problems represent three different crew regions in Turkish Railways system; instances are generated with a planning horizon of one week and two weeks using various appropriate day-off requirements. Ankara district operates 35 train duties originating from the home-base station during a week, Haydarpasa and Eskisehir operate 44 each. Some of these train services run longer than the maximum length of a duty period which may require assigning two crew members rather than one.

In order to observe the change in the optimal crew size (i.e. the objective function) when the connectivity of schedules is enforced, we, first, need to reproduce the results in Şahin and Yüceoğlu (2011). As a side product of this effort, we have succeeded to improve their results. In Appendix A, we discuss modifications for the network representation which led to significant improvements in the solution quality. The optimal objective function values of the test problems ignoring the schedule connectivity are found as a result of this preliminary computational study.

We first investigate the quality of the solutions obtained with $[\text{RCCP-C}]_p$ formulation using the CRG algorithm and the heuristic idea to obtain integer feasible solutions. In Table 2, we present results of the CRG algorithm when the initial RMP is constructed by the procedure in Section 3.1.1. For the LP relaxation of the problem, "Itr" shows the number of column generation iterations until termination which also corresponds to the number of variables (schedules) added to the problem. LP-OFV corresponds to the objective function value of the optimal solution of the LP relaxation; LP Time is the time (in seconds) it takes to terminate. For the IP heuristic solution as described in Section 3.1.3, IP-OFV shows the objective function value of the integer feasible solution where IP Time indicates the time required by CPLEX solver to solve the IP problem (bounded by one hour).

Noting that the problem size increases when the planning horizon is longer and the number of days-off is larger, we may summarize the results as follows:

- CRG algorithm is capable of solving the LP relaxation to optimality within reasonable computational time (3600 s) when the problem is smaller. As the problem gets larger, the 1-h time limit is reached before the termination condition is satisfied.
- With the schedules added to the problem until termination or the end of 1-h limit, we can find an integer feasible solutions. For, 9 out of 18 test problems, the integer optimality cannot be verified within 1 h. We may not necessarily explain the unverified optimality with the size of the problem or the number of schedules in the problem at the terminal iteration of the CRG algorithm.

Although the CRG algorithm does not show superior performance, it may still be considered as a viable option when combined with the IP heuristic even for larger problems as integer feasible solutions can be obtained within reasonable computation time.

The arc-based formulation $[\text{RCCP-C}]_a$, on the other hand, is capable of solving not only the LP relaxation but also the integer version of the problem in reasonable time. In Table 3, LP-OFV corresponds to the optimal objective function value of the LP relaxation of $[\text{RCCP-C}]_a$ while OPT indicates the optimal objective function value of the IP problem and Time shows the time required (in s) to solve the problem to IP optimality. The last column RCCP indicates the objective function value of the optimal solution to the problem when connectivity of schedules is ignored (as shown in Appendix A). We observe the following results:

- The arc-based formulation is capable of solving all of the test problems to optimality within 2 min of computational time.
- When the optimal objective function values of the problems (OPT for the one with schedule connectivity and RCCP for the one ignoring schedule connectivity), we observe significant differences in the optimal crew size ranging from 2% to 15%.

From a methodological point of view, both solution approaches, path-based formulation using CRG algorithm and arc-based formulation, are capable of generating integer feasible solutions. Yet, arc-based formulation is better off not only by solution quality but also with respect to computational time. Thus, it seems to be better off for the problem with schedule connectivity.

From a practical point of view, the results showing the differences in the optimal crew size between the two versions of the problem are crucial. For example, in Eskisehir region with a planning horizon of one week and 2 days-off, the optimal

Table 2
Results for the path-based formulation.

Region	Weeks	Days-off	Itr	LP-OFV	LP time	IP-OFV	IP time
Ankara	1	0	143	54.00	28	57	5
	1	1	317	59.80	276	66	3600
	1	2	322	70.80	253	75	3600
	2	0	338	54.00	327	56	21
	2	1	735	54.89	3600	65	3600
	2	2	702	59.40	3600	69	3600
Haydarpasa	1	0	310	39.00	231	41	15
	1	1	394	44.50	412	49	3600
	1	2	360	53.40	440	59	3600
	2	0	660	39.21	3600	41	224
	2	1	648	41.98	3600	47	3600
	2	2	648	45.55	3600	50	1700
Eskisehir	1	0	160	69.00	45	73	8
	1	1	454	72.20	615	78	3600
	1	2	476	86.20	800	96	3600
	2	0	401	69.00	613	72	72
	2	1	545	69.00	1556	73	482
	2	2	631	72.88	3600	77	598

Table 3
Results for the arc-based formulation.

Region	Weeks	Days-off	LP-OFV	OPT	Time	RCCP	Diff (%)
Ankara	1	0	54.00	54	1	50	8.00
	1	1	59.80	60	1	55	9.09
	1	2	70.80	71	2	66	7.58
	2	0	54.00	54	1	50	8.00
	2	1	54.15	55	14	52	5.77
	2	2	58.66	59	63	57	3.51
Haydarpasa	1	0	39.00	39	1	38	2.63
	1	1	44.50	45	15	43	4.65
	1	2	53.40	54	16	52	3.85
	2	0	39.00	39	1	38	2.63
	2	1	41.00	41	37	41	0.00
	2	2	44.41	45	102	44	2.27
Eskisehir	1	0	69.00	69	1	65	6.15
	1	1	72.20	73	2	67	8.96
	1	2	86.20	87	4	76	14.47
	2	0	69.00	69	1	65	6.15
	2	1	69.00	69	16	65	6.15
	2	2	70.91	71	143	68	4.41

crew size jumps from 76 to 87 when the schedule connectivity is considered. In practice, such a deficiency may not be resolved with reserve crew members or roster patching at the operational level. The results clearly confirm that ignoring the connectivity of schedules may even lead to infeasibility at the regional level. Hence, the schedule connectivity should evidently be taken into consideration during the tactical level planning process.

5. Conclusions and future research

Effective crew management is a critical planning problem in railways. In the planning process, tactical decisions are made by considering a finite planning horizon that repeats periodically; but the recurrence of the planning period is overlooked, which may lead to crew schedules that are not implementable in practice. We extend the previous work on tactical crew capacity planning to consider a new operational issue that is concerned with the connectivity of schedules from one period to the other.

Our numerical experiments show that the arc-based formulation is capable of solving all real-life cases to optimality within reasonable computational time. On the other hand, CRG algorithm fails to solve even the LP relaxation within reasonable computational time due to convergence problems. Both solution methods are capable of generating integer feasible solutions but arc-based formulation outweighs not only by solution quality but also with respect to computational time. In essence, these two approaches can be considered as competitors or at least alternatives to each other while the literature has given more attention to the path-based formulation for its advantages due to scalability. In this battle for a new version of the problem with schedule connectivity, arc-based formulation seems better off despite lack of attention in earlier studies. From the decision-maker's point of view, the results clearly show that the decisions on regional crew capacities ignoring the connectivity of the schedules might significantly differ from those where connectivity of schedules are integrated into the problem at the planning/pairing phase.

A further refinement concerning schedule connectivity shall be considered with respect to the pattern through which the crew schedules maintain connectivity. In a cyclic roster over a planning horizon of a fixed number of periods (equal to the number of crew schedules or crew members), each crew member completes the same cyclic sequence of crew schedules while each crew member starts her cycle with a different schedule. Cyclic rostering allows the workload distribution to be inherently fair (Ernst et al., 2001). Indeed, cyclic rosters provide perfect balance over several recurrences of the planning period in the long term. Constructing a non-cyclic roster is easier compared to a cyclic one, but the resulting uneven workload distribution in the short-term planing horizon is a drawback. In this study, we do not necessarily enforce a cyclic connectivity; it requires a more elaborate formulation with both approaches. In this respect, we consider cyclic connectivity of schedules as the next step in this research avenue which should also contribute to the domain of workload balancing in general.

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Appendix A. Improvements in the network representation

Şahin and Yüceoğlu (2011) have discussed two special cases regarding the availability of crew to cover duties that are too close to either the beginning or the end of the planning horizon; they propose to modify network generation procedure slightly to handle these special cases.

It is assumed that the crew is located at the home station at the beginning of the planning horizon and she has to return to the home station at the end. However, this assumption may not hold for two cases:

- If the on-duty node at an away station is too close to the beginning of the planning horizon, there may not be any incoming arcs from the source node to this on-duty node. It is moved to the end of the planning horizon as an *away early on-duty* node with an updated time attribute.
- If the tie-up node at an away station is too close to the end of the planning horizon, the crew member performing such a duty may not return back to the home station before the end of the planning horizon. *Pseudo sink arcs* are introduced to connect these duties to the sink node supposing that the crew member comes back home through deadheading or performing a duty at the beginning of the next planning horizon.

Şahin and Yüceoğlu (2011) have not discussed any particular order for creating the *away early on-duty* nodes and the *pseudo sink arcs*. By processing *away early on-duty* nodes before processing *pseudo sink arcs*, we introduce less number of *pseudo sink arcs* while increasing the likelihood of an s-t path to cover more duties. If the *away early on-duty* nodes are moved to the end of the planning horizon before introducing *pseudo sink arcs* from tie-up nodes at an away station, there might exist other types of arcs between these two nodes since the time attributes of the *away early on-duty* nodes will be updated. If such tie-up nodes can be connected to the *away early on-duty* nodes, it would allow a crew member engaging with those duties to come back to home station at the end of the planning horizon. Hence, some pseudo sink arcs are eliminated consequently.

Our second modification is concerned with a more practical issue. The rests periods that are close to the either the beginning or the end of the planning horizon may lead to unnecessarily longer rest periods. In the original network representation, such cases are not prohibited. With our modification, we remove the "unnecessary" source and sink arcs from the network regarding these practical considerations as follows:

- A source arc exists from the source node to an on-duty node if the time attribute of that on-duty node is within the maximum home rest period that starts at the beginning of the planning horizon. In a multi-layer network (where the day-off requirement is considered) since the first layer represents the duties before taking a day-off, same rule should be applied.
- A sink arc exists from the tie-up node to a sink node if time attribute of that tie-up node is within the maximum home rest period that ends at the end of the planning horizon. In a multi-layer network (where the day-off requirement is considered) since the last layer represents the duties after taking all necessary day-offs, same procedure should be applied.

The computational study is performed on a data set that is representative of three different crew regions in the Turkish State Railways system, namely Ankara, Haydarpasa and Eskisehir. We implement the network flow formulation in C++ using ILOG Optimization Studio 12.2 as the LP and IP solver on a PC with Intel Core i7 @3.20 GHz CPU and 24 GB RAM. Results are reported in Table A.1; OPT column corresponds to the results from Şahin and Yüceoğlu (2011); LP, OPT* and Time columns

Table A.1
Results for railway crew capacity planning using the arc-based formulation.

Region	Weeks	Days-off	OPT	LP	OPT*	Time (s)
Ankara	1	0	–	50.00	50	1
	1	1	57	54.80	55	1
	1	2	67	65.20	66	1
	2	0	–	50.00	50	1
	2	1	54	52.00	52	8
	2	2	58	56.33	57	49
Haydarpasa	1	0	–	38.00	38	1
	1	1	46	42.66	43	6
	1	2	53	51.20	52	8
	2	0	–	38.00	38	1
	2	1	46	40.15	41	52
	2	2	46	43.50	44	77
Eskisehir	1	0	–	65.00	65	1
	1	1	79	66.20	67	2
	1	2	94	75.33	76	3
	2	0	–	65.00	65	1
	2	1	76	65.00	65	35
	2	2	81	67.07	68	112

correspond to the optimal objective function value of the LP relaxation, optimal integer objective function value and computational time required in seconds, respectively.

With the computational study, we observe the impact of the improvements on the network representation upon the performance of the arc-based formulation and the solution quality. This has two dimensions (i) possible improvement in the solution quality, and (ii) reduction in computational effort. In Şahin and Yüceoğlu (2011) the existence of so-called "unnecessary" source arcs and sink arcs does not impact the solution quality. As the objective function minimizes the required number of crew members, the optimal solution implicitly prohibits such unnecessary home rest periods. Yet, we should also note that removing these arcs decreases the size of the network and may affect the computational effort. The results indicate a significant decrease in the number of required crew members (i.e. decrease in the objective function value). We observe 2.6% to 5.9% improvement on average in Ankara and Haydarpasa; but, the most striking ones are for Eskisehir where improvements vary between 15% and 25%. For example, with a planning horizon of 1-week and 2-day-off requirement, we find the optimal number of crew members as 76 which is 18 less than that reported in Şahin and Yüceoğlu (2011).

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