Modification to Fuzzy Extent Analysis Method and its performance Analysis

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***Abstract*—Analytic Hierarchical Process (AHP) is one of the most popular Multi-Criteria Decision Making (MCDM) techniques while fuzzy set theory is extensively incorporated into original AHP in order to address vagueness in human judgments. There are a number of algorithms proposed for Fuzzy AHP (FAHP), however, Fuzzy Extent Analysis (FEA) is one of the most frequently used model. This study evaluates the performance of this model against a modified FEA method which utilizes centroid defuzzification. This study shows that modified FEA method performs significantly better than its original model and thus can lead to more effective decision making.**

I. Introduction

In a Multiple-Criteria Decision Making (MCDM) process, prioritizing and assigning weights to each criteria with reference to a set of available alternatives is key to effective decision making. Analytic Hierarchy Process (AHP) proposed by Thomas L. Saaty [1] is one such technique used in MCDM through which experts provide pairwise comparisons and this information is processed in a comparison matrix in order to calculate priority vector.

One of the major concerns regarding to the original AHP is, transforming human judgments, which are communicated usually by means of linguistic phrases such as "significantly more", "slightly more" etc, into a 1-9 numerical scale, due to the inherent uncertainty in human language. Disregarding the vagueness of human language in the decision analysis process may lead to wrong decisions [2].

Since it is introduced by Zadeh [3] Fuzzy Set Theory has received extensive attention from researchers from variety of discipline in the past five decades. As opposed to the dichotomous (i.e., conventional) crisp set theory which assumes an object either belongs to a set or not, fuzzy set theory represents the belongingness with a degree of membership value. This approach allowed a more realistic representation of the nature and has been successfully applied in various fields such as control theory [4], health care [5], system modeling/data mining [6], [7], etc.

Fuzzy AHP (FAHP) is introduced as an extension of fuzzy set theory in the context of MCDM, where the linguistic variables obtained from the decision makers during the comparison matrix elicitation phase are represented with *fuzzy numbers* as opposed to the original crisp numbers of the infamous 1-9 scale of Saaty [1]. Utilization of the fuzzy numbers, enabled the analysts to incorporate the inherent vagueness of the linguistic variables to the decision making process.

There are number of different techniques proposed over the years which prioritize and rank the available criteria based on comparison ratios represented by fuzzy numbers [8][9][10] and a review of these techniques is provided by Buyukozkan [11]. Fuzzy Extent Analysis (FEA) proposed by Chang [12] is one of the most frequently used FAHP algorithm [13]. In this study, we propose a modification to this model and evaluate the performance of the proposed modification with an experimental analysis.

The rest of the paper is organized as follows. In section II, we will provide an extensive overview of the FEA model. In secion III, we will present the proposed modification to the FEA model and discuss the details of the experimental setup in order to evaluate the performance of the proposed modification. In Section IV, the results of the experimental analysis will be demonstrated. The paper will finalize with some concluding remarks Section V.

II. Fuzzy Extent Analysis

Before providing a review of FEA model, we first provide a brief overview of the fuzzy logic and fuzzy arithmetic. Fuzzy sets can record the imprecision arising in human judgments which are neither random nor stochastic [14]. Instead of a single value, fuzzy number represents a set of possible values each having its own membership function between zero and one. A triangular fuzzy number is represented by *[lower value, mean value, upper value]*, i.e., $[l \;\; m \;\; u]$ with membership functions μ_M given by;

$$\mu_M(x) = \begin{cases} \frac{x}{m-l} - \frac{l}{m-l}, & x \in [l \; m] \\ \frac{x}{m-u} - \frac{u}{m-u}, & x \in [m \; u] \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

The same is graphically illustrated in Figure 1.

Let $(l_1 \;\; m_1 \;\; u_1)$ and $(l_2 \;\; m_2 \;\; u_2)$ then the basic fuzzy arithmetic operations are summarized as follows;

- Addition:
  $(l_1 \; m_1 \; u_1) \oplus (l_2 \; m_2 \; u_2) = (l_1 + l_2 \quad m_1 + m_2 \quad u_1 + u_2)$

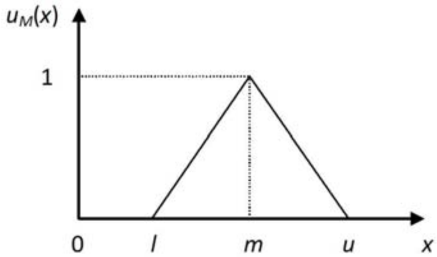

Fig. 1: Membership function of Triangular Fuzzy Number

- Multiplication:
  $(l_1\ m_1\ u_1) \odot (l_2\ m_2\ u_2) = (l_1.l_2 \quad m_1.m_2 \quad u_1.u_2)$
- Scalar Multiplication:
  $(\lambda\ \lambda\ \lambda) \odot (l_1\ m_1\ u_1) = (\lambda.l_1 \quad \lambda.m_1 \quad \lambda.u_1)$
- Inverse:
  $(l_1\ m_1\ u_1)^{-1} \approx (1/u_1 \quad 1/m_1 \quad 1/l_1)$

One of the most popular FAHP technique was proposed by Chang [12] which uses Fuzzy Extent Analysis in order to calculate the crisp weights from fuzzy comparison matrices. In the original Extent Analysis method, provided we have $X = \{x_1, x_2, \cdots, x_n\}$ as an object set and $G = \{g_1, g_2, \cdots, g_n\}$ as a goal set, then for each object, extent analysis for each goal g_i is performed. Applying this theory in fuzzy comparison matrix, one can calculate the value of fuzzy synthetic extent with respect to the i^{th} object as follows;

$$S_i = \sum_{j=1}^{m} M_{g_i}^{j} \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^{j} \right]^{-1} \tag{2}$$

Where

$$\sum_{j=1}^{m} M_{g_i}^{j} = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j \right) \tag{3}$$

In case of a crisp comparison matrix, final weights can be obtained as a result of the process explained above. However, for the case where fuzzy triangular numbers are utilized in the judgment scale, the result would be a fuzzy triangular weight value as indicated in Equation 3.

Later in the decision making process (i.e. choosing the best alternative) a crisp weight from these fuzzy triangular weights should be determined. A naive approach would be just using the means (i.e., mean of each fuzzy weight obtained from Equation 2). However, as opposed to the straight forward ordering of crisp numbers, ordering of the fuzzy numbers is not that simple and one should be more careful. Chang [12] suggests utilizing the concept of comparison of fuzzy numbers in order to determine crisp weights from the fuzzy weights.

In the original approach, for each fuzzy weight, a pair wise comparison with the other fuzzy weights are conducted, and the degree of possibility of being greater than these fuzzy weights are obtained. The minimum of these possibilities are used as the overall score for each criterion i.

Finally these scores are normalized (i.e. so that they sum up to 1), and the corresponding normalized scores are used as the weights of the criteria. That is to say by applying the comparison of the fuzzy numbers, the degree of possibility is obtained for each pair wise comparison as follows:

$$V(M_2 \geq M_1) = hgt(M_1 \cap M_2) = \mu_{M_2}(d) =$$
$$\begin{cases} 1, & \text{if } m_2 \geq m_1 \\ 0, & \text{if } l_1 \geq u_2 \\ \frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & \text{otherwise.} \end{cases}$$

The same is illustrated in the Figure 2.

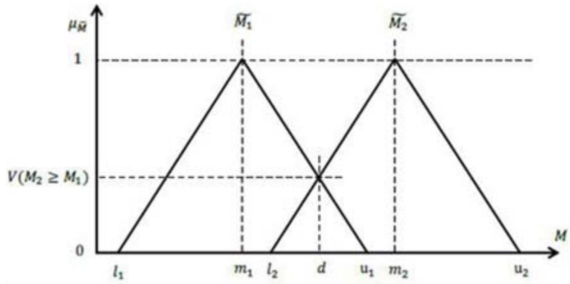

Fig. 2: Degree of possibility

Note that, degree of possibility for a convex fuzzy number to be greater than k convex fuzzy numbers is given by;

$$V(M \geq M_1, M_2, \cdots, M_k) = V[(M \geq M_1) \text{ and } (M \geq M_2), \cdots, (M \geq M_k)]$$

$$= \min V(M \geq M_i), \quad i = 1, 2, \cdots, k$$

Assuming that $w_i' = \min V(M_i \geq M_k)$ then weight vector is given by

$$W' = w_1', w_2', \cdots, w_n'$$

Normalizing the above weights gives us the final priority vector $w_1, w_2, \cdots, w_n$.

Wang et.al. [15] review the normalization processes in fuzzy systems and proposed various improvements and modification. In case of FEA, row sums are normalized in order to calculate fuzzy synthetic extent values as given by Equation 2. Wang et.al. [15] proposed following modification to this formula.

$$S_i = \frac{\sum_{j=1}^{n} l_{ij}}{\sum_{j=1}^{n} l_{ij} + \sum_{k=1,k\neq i}^{n} \sum_{j=1}^{n} u_{kj}}, \frac{\sum_{j=1}^{m} m_{ij}}{\sum_{k=1}^{n} \sum_{j=1}^{n} m_{kj}}, \frac{\sum_{j=1}^{n} u_{ij}}{\sum_{j=1}^{n} u_{ij} + \sum_{k=1,k\neq i}^{n} \sum_{j=1}^{n} l_{kj}} \tag{4}$$

In addition, Wang et.al. [15] criticized FEA technique and through an example showed that this method cannot estimate true weights from fuzzy comparison matrix. The main criticism revolves around the fact that this method may assign a zero as criterion weight which disturbs the whole decision making hierarchy.

The basis of extent analysis theory is that it provides a degree to which one fuzzy number is greater than another fuzzy number, and this degree of greatness is considered as criterion weights. Therefore, if two fuzzy numbers do not intersect then the degree of greatness of one fuzzy number to the other is 100 percent and therefore it will assign 1 as weight to that criterion while the other criteria will be assigned as zero weight.

In light of the above discussion, Wang et.al [15] summarized the main problems with this method as under;

- Once a criteria is assigned a zero weight, it will not be considered in the decision making process.
- This method may lose some useful information in the form of judgment ratios in the fuzzy comparison matrices as some of the criterion are assigned zero weight.
- It was shown that weights calculated through this method may not represents the true relative importance of that criteria.
- This method might select the worst decision alternative as the best one and thus leads to wrong decision making

III. Modified Fuzzy Extent Analysis

Model proposed by Chang [12] is often mistakenly categorized as Fuzzy Extent Analysis method which is just the first part of this model. The other part is the defuzzication and/or ranking of weights which is carried out through principal of comparison of fuzzy numbers based on degree of possibility.

Review of the existing literature shows that there is no generally accepted method to rank fuzzy numbers [16] and there are numerous articles written on this subject [17], [18] [19], [20], [21], [22]. In this study, instead of degree of possibility, we use centroid defuzzication originally proposed by Ross [23] which is one of the most popular technique for defuzzication [24].

In order to compare original model with the modified model, we generate set of random matrices and apply the selected algorithms on these matrices. Methodology to generate fuzzy comparison matrices is given as follows;

We randomly generate crisp weights $w_1, w_2, \cdots, w_n$ and normalize them. A perfectly consistent matrix is formed as follows;

$$W = \begin{pmatrix} w_1/w_1 & w_1/w_2 & \cdots & w_1/w_n \\ w_2/w_1 & w_2/w_2 & \cdots & w_2/w_n \\ \vdots & \vdots & \ddots & \vdots \\ w_n/w_1 & w_n/w_2 & \cdots & w_n/w_n \end{pmatrix}$$

Once the comparison matrix is generated, each element of the matrix is converted into a triangular fuzzy number $[l\ m\ u]$ with a fuzzification parameter α such that $l = w_i/w_j - \alpha$, $m = w_i/w_j$ and $u = w_i/w_j + \alpha$.

We use three different values of fuzzication parameter, i.e. 0.05, 0.10, 0.15 and four different matrix sizes i.e. 3, 7, 11 and 15. For each one combinations we generate total of hundred random matrices and thus total data set contains 1200 matrices. Three algorithms namely FEA [12], FEA with modified normalization [15] and FEA with centroid defuzzification will be used to calculate weights from these randomly generated matrices.

IV. Results and Discussion

In order to evaluate performance of above mentioned selected algorithms, average error terms are calculated as follows;

Fuzzy comparison matrices are constructed through n randomly generated weights and corresponding n weights are calculated from these fuzzy comparison matrices by utilizing selected algorithms. Afterwards, error term is calculated by taking a difference between initial weights and calculated weights. For each instance, average of n error terms is taken so that for each instance there is one error term.

There are total of 12 combinations of fuzzification parameter and matrix size and for each combination there are 100 fuzzy comparison matrices. In order to graphically present the results, error terms are averaged for each of 12 combination and these results are graphically presented in Figure 3

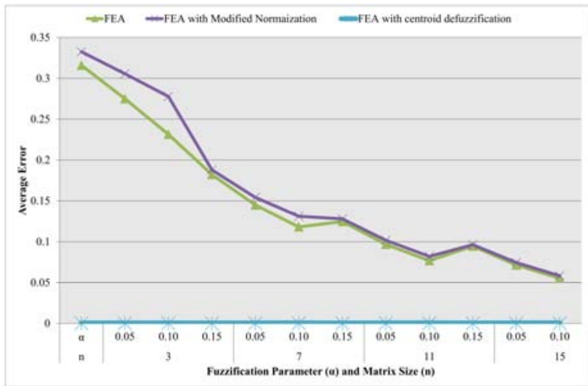

Fig. 3: Average Errors

As illustrated above mean average error is significantly lower for FEA with centroid defuzzification as compared to the original FEA model [12] as well as when compared with FEA with modified normalization[15]. In addition, through Anova test, the difference of error terms between the original FEA model and modified FEA model is found to be significant.

Note that as the size of the matrix is increased, error terms tend to decrease. However, this phenomenon is observed due to the fact that as the size of the matrix is increased, the values of the starting normalized weights is also decreased and hence the corresponding error terms decreases with increase in matrix size.

V. Example

We use the same example as was presented by Chang [12], therefore, to view the given fuzzy comparison matrices, original article can be referred. Criterion weights calculated from both the original FEA model as well as modified FEA method is tabulated in Table I;

TABLE I: Criteria Weights

Criteria Weights	FEA	Modified FEA
C1	0.13	0.19
C2	0.41	0.32
C3	0.03	0.16
C4	0.43	0.33

As we have four criteria and three alternatives, therefore at second level, we compare each alternative with respect to each criterion. These weights are tabulated in Table II and III

TABLE II: Criteria Weights (Modified FEA Model)

Criterion	A1	A2	A3
C1	0.29	0.26	0.44
C2	0.43	0.29	0.28
C3	0.33	0.33	0.33
C4	0.27	0.39	0.33

TABLE III: Criteria Weights (Original FEA Model)

Criterion	A1	A2	A3
C1	0.28	0.21	0.51
C2	0.66	0.16	0.19
C3	0.35	0.33	0.32
C4	0.22	0.42	0.36

Based on these weights, overall score of each alternative with respect to each algorithm is tabulated in Table IV

TABLE IV: Overall Score

Alternatives	FEA	Modified FEA
A1	0.41	0.34
A2	0.28	0.33
A3	0.31	0.34

Based on modified FEA model, each alternative is given equal importance while based on original FEA model, alternative 1 would have been preferred. Therefore, care must be taken by decision makers while ranking the alternatives in order to be more effective in their decision making process.

VI. Conclusion

In this study, we modified the original FEA method to derive priority vector from fuzzy comparison matrices by replacing degree of possibility with centroid defuzzication method. We evaluated these two algorithms through a data set of 1200 matrices and found that accuracy of the weights is improved significantly.

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