On the Consequences of Eliminating Capital Tax Differentials

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Abstract

In the United States structure and equipment capital are effectively taxed at different rates. Recently, President Obama joined the group of policy makers and economists who propose to eliminate these differentials. This paper analyzes the consequences of such a reform using an incomplete markets model with equipment-skill complementarity. We find that the reform increases average welfare by approximately 0.1%. Importantly, we find that the reform does not involve the usual efficiency vs. equality trade-off: it improves both.

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Keywords: Uniform capital tax reform, equipment capital, structure capital, equipment-skill complementarity.

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1 Introduction

In the current U.S. tax code different types of capital are taxed at different rates effectively.[1] Recently, President Obama's administration has proposed to eliminate these differentials in a budget neutral way.[2] This paper analyzes the aggregate and distributional consequences of such a reform using an incomplete markets model with two types of capital and equipment-skill complementarity. We find that this reform creates economy-wide productive efficiency gains by reallocating capital from low to high return capital. In addition, by decreasing the skill premium, eliminating capital tax differentials redistributes from the rich to the poor. Therefore, the reform does not suffer from the usual efficiency vs. equality trade-off: it improves both.

Specifically, we build an infinite horizon model with heterogeneous agents with the following features. First, agents are either skilled or unskilled, and the skill type is permanent. Second, both skilled and unskilled agents are subject to idiosyncratic labor productivity shocks. Third, there are two types of capital, structure capital and equipment capital, and the production function features a higher degree of complementarity between equipment capital and skilled labor than between equipment capital and unskilled labor, as documented empirically for the U.S. economy by Krusell, Ohanian, Ríos-Rull, and Violante (2000).[3] Finally, the government uses linear taxes on capital income and consumption and a non-linear labor income tax schedule to finance government consumption and repay debt. We solve for the stationary competitive equilibrium of this model and calibrate the model parameters to the U.S. economy. The main objective of the paper is to use the calibrated model to evaluate the long-run effects of a uniform capital tax reform, which equalizes the tax rates on the two types of capital while keeping the rest of the fiscal policies intact.

Gravelle (2011) estimates that the U.S. effective corporate tax rate on equipment capital is 26% and that on structure capital is 32%.[4] Combining the 15% flat capital income tax rate that consumers face with the differential capital tax rates at the corporate level, the overall effective tax rate on equipment capital is 37.1% while the overall effective tax rate on structure capital is 42.2%. We find that in our model a budget neutral tax reform that eliminates capital tax differentials equates the tax rates on both types of capital at 39.66%.

This uniform tax reform has two effects on the economy. First, it improves productive

[1] Bilicka and Devereux (2012) document differences in effective tax rates across different types of capital assets for a large set of countries including major European countries such as Germany, France, Italy and the U.K.

[2] See the 2011 U.S. President's State of the Union Address at http://www.whitehouse.gov/the-press-office/2011/01/25/remarks-president-state-union-address. For more details, see also the President's Framework for Business Tax Reform (2012).

[3] Flug and Hercowitz (2000) provide evidence for equipment-skill complementarity for a large set of countries.

[4] Effective tax rates across capital types differ because of differences between tax depreciation allowances and actual economic depreciation rates. For more details, see Gravelle (1994).

efficiency by reallocating a given capital stock more efficiently between the two types of capital. Intuitively, by taxing structure capital at a higher rate than equipment capital, the current tax code distorts firms' capital decisions in favor of using more equipment capital. As a result, the marginal return to equipment capital is lower than the marginal return to structure capital under the current tax system. By eliminating the tax differentials, the proposed reform eliminates this distortion through capital reallocation from the capital type with lower returns, equipment capital, to the capital type with higher returns, structure capital. This way capital reallocation brings the economy closer to its production possibilities frontier. Second, due to equipment-skill complementarity, a lower level of equipment capital in the new steady state decreases the skill premium, indirectly redistributing from the skilled to the unskilled. This implies a more egalitarian distribution of resources between skilled and unskilled agents (a decline in 'between-group' inequality). We conclude that the proposed reform does not involve the usual equality vs. efficiency trade-off: eliminating capital tax differentials not only increases efficiency, but also improves equality.

Next, we measure the steady-state welfare consequences of the uniform capital tax reform. We first consider a Utilitarian social welfare function that puts equal weights on all agents. Using this measure, we find that the overall steady-state welfare gains of the reform are 0.09% in terms of lifetime consumption. We also compute the welfare gains for skilled and unskilled agents separately. We find that on average the unskilled agents' welfare increases by 0.15% whereas the skilled agents' welfare decreases by less than 0.01%. We interpret these findings as follows. By increasing productive efficiency, the reform increases average welfare for both types of agents. Through indirect redistribution, the reform increases welfare of unskilled agents and decreases welfare of skilled agents. It turns out that, in our benchmark model, the efficiency gains and redistribution losses almost fully offset each other for the skilled agents, resulting in a small welfare loss. For the unskilled agents, the redistributive gains plus the efficiency gains sum up to a significant welfare gain.

The government could also accompany the capital tax reform by a modification of the labor tax code in order to distribute the efficiency gains across agents in a different way. In particular, the government could distribute a larger share of the efficiency gains to the skilled agents to ensure that they are not worse off. The important point is that the uniform capital tax reform makes it possible to increase the average steady state welfare of one type of agents without decreasing it for the other type. An example of such a reform is provided in Section 4.3.

In this paper, we compare welfare across steady states. In an environment similar to ours, Domeij and Heathcote (2004) shows that a government that ignores short-run welfare finds it optimal to choose very low capital tax rates in order to encourage long-run capital accumulation, and that such a reform has dire welfare consequences in the short-run. Our reform does not suffer as much from this critique, because average capital taxes are almost unaffected by the tax reform, and there is only a small, 0.4%, increase in the aggregate capital stock from the initial to the new steady state. To check the validity of this argument, we analyze another tax reform, in which the uniform capital tax is set so that the aggregate capital stock remains unchanged across steady states. This requires uniform capital taxes to be slightly higher than in the benchmark reform. We find that the welfare gains of this reform are over two thirds of the benchmark welfare gains, validating our argument.

We also conduct a number of robustness checks and find that the main quantitative conclusions are robust to alternative preference specifications, the degree of labor tax progressivity, and the degree of idiosyncratic wage risk. Assuming that the United States is an open economy that faces a fixed world interest rate does not change the main results substantially either. As expected, the reform is more redistributive when we assume a higher degree of equipment-skill complementarity.

In all our quantitative exercises, we find that the uniform capital tax reform improves productive efficiency. In Section 5, we analytically show that, under our parametric specification of the labor disutility function, productive efficiency indeed calls for taxing capital uniformly. We solve a Ramsey problem for a version of our economy with a representative agent - and hence, without redistribution and insurance concerns - and find that it is optimal to tax capital uniformly. This result suggests that a reform towards equalizing capital taxes should improve productive efficiency, thereby providing a theoretical justification for our quantitative findings.[5] We also show that if the government has a redistributive goal, then it might be optimal to tax equipments at a higher rate than structures. This result suggests that a reform from the status quo, in which equipments are taxed at a lower rate than structures, towards uniform capital taxation should improve equality, which is indeed what we find in the quantitative analysis.

Related Literature. This paper is related to a set of papers that evaluate the consequences of eliminating capital tax differentials. The closest to this paper is Auerbach (1989), who computes the welfare gains associated with eliminating the capital tax differentials that existed prior to the U.S. Tax Reform Act of 1986. Because he is not interested in the distributional consequences of his reform, he uses a model without heterogeneity. Modeling heterogeneity is crucial for our paper, however, because our main message is that a uniform capital tax reform improves not only efficiency, but also equality. Auerbach (1983) and Gravelle (1994) both compute the deadweight loss of misallocation of capital that is created by differential

[5]Some authors have argued that investment in equipment capital might create positive externalities. Under that assumption, efficiency would call for taxing equipments at a lower rate than structures. Thus, a reform that increases equipment capital taxes and decreases structure capital taxes would be bad for efficiency. Auerbach, Hassett, and Oliner (1994) point out, however, that it is hard to support the existence of such positive externalities on empirical grounds. The current paper abstracts from externalities.

taxation of capital and find losses that are in the range of 0.10 to 0.15 % of U.S. GNP assuming Cobb-Douglas production technologies. Recently, Gravelle (2011) evaluates the implications of reforming the tax depreciation rules present in the U.S. tax code for the effective tax rates on different types of capital and for the corporate tax revenues. Unlike our paper, she is not interested in the economy-wide implications and the welfare consequences of her reform.

There is a related literature that analyzes optimal capital tax policy in environments with multiple types of capital. The productive efficiency result of Diamond and Mirrlees (1971) implies that, in an environment with multiple capital types, all capital should be taxed at the same rate. Auerbach (1979) shows that in an overlapping generations environment it might be optimal to tax capital differentially if the government is exogenously restricted to a narrower set of fiscal instruments than in Diamond and Mirrlees (1971). Similarly, Feldstein (1990) proves the optimality of differential capital taxation in a static model in which the government is restricted to set the tax rate on one type of capital equal to zero. Slavík and Yazici (2014) shows that a government that has redistribution and incentive provision goals generically finds it optimal to tax capital differentially if the production technology features equipment-skill complementarity. Conesa and Dominguez (2013) considers an economy with tangible and intangible capital where capital is taxed twice: first, through a corporate income tax, and second, at the consumer level, through a dividend tax. They find that the optimal long-run policy features zero corporate taxes and positive dividend taxes. These papers are normative, whereas the current paper is a positive analysis of the aggregate and distributional consequences of the uniform capital tax reform recently proposed by the Obama administration.

Finally, this paper is related to a growing literature, which analyzes the quantitative effects of tax reforms using incomplete markets models with heterogeneous agents, such as Domeij and Heathcote (2004), Conesa and Krueger (2006), Conesa, Kitao, and Krueger (2009) and Heathcote, Storesletten, and Violante (2012), among others. Our paper is most closely related to Domeij and Heathcote (2004) in the sense that both papers provide positive analyses of capital tax reforms. While Domeij and Heathcote (2004) focuses on the consequences of capital tax cuts, we analyze an environment with multiple capital and focus on a policy reform which (by equalizing capital tax rates) changes the mix between equipment and structure capital taxation, but leaves the overall level of capital taxation virtually unaffected. We believe that understanding the consequences of the uniform capital tax reform is very relevant given the current policy debates fueled by the Obama proposal. Methodologically, we contribute to this literature by analyzing tax reforms in an environment with equipment-skill complementarity. Modelling equipment-skill complementarity is important as it allows us to take into account the effect of the tax reform on the wage distribution.

The rest of the paper is organized as follows. In Section 2, we lay out the model. Section 3 discusses calibration. Section 4 provides our main quantitative findings and Section 5 provides theoretical insights for the quantitative results by analyzing Ramsey optimal tax problems. Finally, Section 6 concludes.

2 Model

We consider an infinite horizon growth model with two types of capital (structures and equipments), two types of labor (skilled and unskilled), consumers, firms, and a government.

Endowments and Preferences. There is a continuum of measure one of agents who live for infinitely many periods. In each period, they are endowed with one unit of time. Ex-ante, they differ in their skill levels: they are born either skilled or unskilled, $i \in \{u, s\}$. Skilled agents can only work in the skilled labor sector and unskilled agents only in the unskilled labor sector. The skill types are permanent. The total mass of the skilled agents is denoted by π_s, the total mass of the unskilled agents is denoted by π_u. In the quantitative analysis, skill types correspond to educational attainment at the time of entering the labor market. Agents who have college education or above are classified as skilled agents and the rest of the agents are classified as unskilled agents.

In addition to heterogeneity between skill groups, we model heterogeneity within each skill group by assuming that agents face idiosyncratic labor productivity shocks over time. The productivity shock, denoted by z, follows a type-specific Markov chain with states $Z_i = \{z_{i,1}, ..., z_{i,I}\}$ and transitions $\Pi_i(z'|z)$. An agent of skill type i and productivity level z who works l units of time produces $l \cdot z$ units of effective i type of labor. As a result, her wage per unit of time is $w_i \cdot z$, where w_i is the wage per effective unit of labor in sector i. The modeling of productivity shocks, and hence within-group inequality, is important as it allows us to analyze the effects of uniform capital tax reform on within-group inequality.

Preferences over sequences of consumption and labor, $(c_{i,t}, l_{i,t})_{t=0}^{\infty}$, are defined using a separable utility function

$$E_i\Big(\sum_{t=0}^{\infty} \beta_i^t u(c_{i,t}) - v(l_{i,t})\Big),$$

where β_i is the time discount factor which is allowed to be different across skill types.[6] For each skill type, unconditional expectation, E_i, is taken with respect to the stochastic processes governing the idiosyncratic labor shock. There are no aggregate shocks.

[6]Attanasio, Banks, Meghir, and Weber (1999) provide empirical evidence for differences in discount factors across education groups. In our quantitative analysis, we calibrate the discount factors so as to match the observed difference in wealth between skilled and unskilled agents. We perform a version of our benchmark quantitative exercise in which we assume that the discount factors are equal for the two types of agents. As we report in Section 4.5, our main quantitative results are robust to this modification.

Technology. There is a constant returns to scale production function: $Y = F(K_s, K_e, L_s, L_u)$, where K_s and K_e refer to aggregate structure capital and equipment capital and L_s and L_u refer to aggregate effective skilled and unskilled labor, respectively. We also define a function $\tilde{F}$ that gives the total wealth of the economy: $\tilde{F} = F + (1-\delta_s)K_s + (1-\delta_e)K_e$, where δ_s and δ_e are the depreciation rates of structure and equipment capital, respectively.

The key feature of the technology is equipment-skill complementarity, which means that the degree of complementarity between equipment capital and skilled labor is higher than that between equipment capital and unskilled labor. This implies that an increase in the stock of equipment capital decreases the ratio of the marginal product of unskilled labor to the marginal product of skilled labor. In a world with competitive factor markets, this implies that the skill premium, defined as the ratio of skilled to unskilled wages, is increasing in equipment capital. Structure capital, on the other hand, is assumed to be neutral in terms of its complementarity with skilled and unskilled labor. These assumptions on technology are in line with the empirical evidence provided by Krusell, Ohanian, Ríos-Rull, and Violante (2000). Letting $\partial F/\partial m$ be the partial derivative of function F with respect to variable m, we formalize these assumptions as follows.

Assumption 1. $\frac{\partial F/\partial L_s}{\partial F/\partial L_u}$ *is independent of* K_s.

Assumption 2. $\frac{\partial F/\partial L_s}{\partial F/\partial L_u}$ *is strictly increasing in* K_e.

There is a representative firm which, in each period, hires the two types of labor and rents the two types of capital to maximize profits. In any period t, its maximization problem reads:

$$\max_{K_{s,t},K_{e,t},L_{s,t},L_{u,t}} \quad F(K_{s,t}, K_{e,t}, L_{s,t}, L_{u,t}) - r_{s,t}K_{s,t} - r_{e,t}K_{e,t} - w_{s,t}L_{s,t} - w_{u,t}L_{u,t},$$

where $r_{s,t}$ and $r_{e,t}$ are the rental rates of structure and equipment capital, and $w_{u,t}$ and $w_{s,t}$ are the wages rates paid to unskilled and skilled effective labor in period t.

Asset Market Structure. There is a single risk free asset which has a one period maturity. Consumers can save using this asset but are not allowed to borrow. Every period total savings by consumers must be equal to total borrowing of the government plus the total capital stock in the economy.

Government. The government uses linear consumption taxes every period $\{\tau_{c,t}\}_{t=0}^{\infty}$ and linear taxes on capital income net of depreciation. The tax rates on the two types of capital are allowed to be different. Let $\{\tau_{s,t}\}_{t=0}^{\infty}$ and $\{\tau_{e,t}\}_{t=0}^{\infty}$ be the sequences of tax rates on structure and equipment capital. It is irrelevant for our analysis whether capital income is taxed at the consumer or at the corporate level. We assume without loss of generality that all capital income taxes are paid at the consumer level. The government taxes labor income using a sequence of possibly non-linear functions $\{T_t(y)\}_{t=0}^{\infty}$, where y is labor income and $T_t(y)$ are the taxes paid by the consumer. The government uses taxes to finance a stream of expenditure $\{G_t\}_{t=0}^{\infty}$ and repay government debt $\{D_t\}_{t=0}^{\infty}$.

In our quantitative analysis we focus on the comparison of stationary equilibria. For that reason, instead of giving a general definition of competitive equilibrium, here we only define stationary recursive competitive equilibria. The formal definition of non-stationary competitive equilibrium is relegated to Appendix A. In order to define a stationary equilibrium, we assume that policies (government expenditure, debt and taxes) do not change over time.

Before we define a stationary equilibrium formally, notice that, in the absence of aggregate productivity shocks, the returns to saving in the form of the two capital types are certain. The return to government bond is also known in advance. Therefore, in equilibrium all three assets must pay the same after-tax return, i.e., $R = 1 + (r_s - \delta_s)(1 - \tau_s) = 1 + (r_e - \delta_e)(1 - \tau_e)$, where R refers to the stationary return on the bond holdings. As a result, we do not need to distinguish between saving through different types of assets in the consumer's problem. We denote consumers' asset holdings by a.

Stationary Recursive Competitive Equilibrium (SRCE). SRCE is two value functions V_u, V_s, policy functions $c_u, c_s, l_u, l_s, a'_u, a'_s$, the firm's decision rules K_s, K_e, L_s, L_u, government policies $\tau_c, \tau_s, \tau_e, T(\cdot), D, G$, two distributions over productivity-asset types $\lambda_u(z, a), \lambda_s(z, a)$ and prices w_u, w_s, r_s, r_e, R such that

1. The value functions and the policy functions solve the consumer problem given prices and government policies, i.e., for all $i \in \{u, s\}$:

$$\begin{aligned} V_i(z, a) = \max_{(c_i, l_i, a'_i) \geq 0} \quad & u(c_i) - v(l_i) + \beta_i \sum_{z'} \Pi_i(z'|z) v_i(z', a'_i) \qquad \text{s.t.} \\ (1 + \tau_c)c_i + a'_i \quad \leq \quad & w_i z l_i - T(w_i z l_i) + Ra, \end{aligned}$$

   where $R = 1 + (r_s - \delta_s)(1 - \tau_s) = 1 + (r_e - \delta_e)(1 - \tau_e)$ is the after-tax asset return.

2. The firm solves:

$$\max_{K_s, K_e, L_s, L_u} \quad F(K_s, K_e, L_s, L_u) - r_s K_s - r_e K_e - w_s L_s - w_u L_u.$$

3. The distribution λ_i is stationary for each skill type, i.e. for all $i : \lambda'_i(z, a) = \lambda_i(z, a)$. This means:

$$\lambda_i(\bar{z}, \bar{a}) \quad = \quad \int_z \int_{a: a'_i(z,a) = \bar{a}} \lambda_i(z, a) \cdot da \cdot d\Pi_i(\bar{z}|z).$$

4. Markets clear:

$$\begin{aligned}
\sum_i \pi_i \int_z \int_a a \cdot d\lambda_i(z,a) &= K_s + K_e + D, \\
\pi_s \int_z \int_a z l_s(z,a) \cdot d\lambda_s(z,a) &= L_s, \\
\pi_u \int_z \int_a z l_u(z,a) \cdot d\lambda_u(z,a) &= L_u, \\
C + G + K_s + K_e &= \tilde{F}(K_s, K_e, L_s, L_u),
\end{aligned}$$

where $C = \sum_{i=u,s} \pi_i \int_z \int_a c_i(z,a) \cdot d\lambda_i(z,a)$ denotes aggregate consumption.

5. Government budget constraint is satisfied.

$$RD + G = D + \tau_c C + \tau_e (r_e - \delta_e) K_e + \tau_s (r_s - \delta_s) K_s + T_{agg},$$

where $T_{agg} = \sum_{i=u,s} \pi_i \int_z \int_a T(w_i z l_i(z,a)) \cdot d\lambda_i(z,a)$ denotes aggregate labor tax revenue.

We explain how we solve for the SRCE in Appendix B.

3 Calibration

To calibrate model parameters, we assume that the SRCE defined in the previous section - computed using the current U.S. tax system - coincides with the current U.S. economy. We first fix a number of parameters to values from the data or from the literature. These parameters are summarized in Table 1. We then calibrate the remaining parameters so that the SRCE matches the U.S. data along selected dimensions. Our calibration procedure is summarized in Table 2.

We assume that the period utility function takes the form

$$u(c) - v(l) = \frac{c^{1-\sigma}}{1-\sigma} - \phi \frac{l^{1+\gamma}}{1+\gamma}.$$

One period in our model corresponds to one year. In the benchmark case, we use $\sigma = 2$ and $\gamma = 1$. These are within the range of values that have been considered in the literature. We calibrate ϕ to match the average labor supply.

We further assume that the production function takes the same form as in Krusell, Ohanian, Ríos-Rull, and Violante (2000):

$$Y = F(K_s, K_e, L_s, L_u) = K_s^{\alpha} \left(\nu \left[\omega K_e^{\rho} + (1-\omega) L_s^{\rho} \right]^{\frac{\eta}{\rho}} + (1-\nu) L_u^{\eta} \right)^{\frac{1-\alpha}{\eta}}. \quad (1)$$

Table 1: Benchmark Parameters

Parameter	Symbol	Value	Source
Preferences			
Relative risk aversion parameter	σ	2	
Inverse Frisch elasticity	γ	1	
Technology			
Structure capital depreciation rate	δ_s	0.056	GHK
Equipment capital depreciation rate	δ_e	0.124	GHK
Share of structure capital in output	α	0.117	KORV
Measure of elasticity of substitution between equipment capital K_e and unskilled labor L_u	η	0.401	KORV
Measure of elasticity of substitution between equipment capital K_e and skilled labor L_s	ρ	-0.495	KORV
Relative supply of skilled workers	p_s/p_u	0.778	U.S. Census
Productivity			
Productivity persistence of skilled workers	ρ_s	0.9408	KL
Productivity volatility of skilled workers	$var(\varepsilon_s)$	0.1000	KL
Productivity persistence of unskilled workers	ρ_u	0.8713	KL
Productivity volatility of unskilled workers	$var(\varepsilon_u)$	0.1920	KL
Government polices			
Labor tax progressivity	τ_l	0.18	HSV
Overall tax on structure capital income	τ_s	0.422	Gravelle (2011)
Overall tax on equipment capital income	τ_e	0.371	Gravelle (2011)
Consumption tax	τ_c	0.05	KL
Government consumption	G/Y	0.16	NIPA
Government debt	D/Y	0.6	St. Louis FED

This table reports the benchmark parameters that we take directly from the literature or the data. The acronyms GHK, KORV, HSV, and KL stand for Greenwood, Hercowitz, and Krusell (1997), Krusell, Ohanian, Ríos-Rull, and Violante (2000), Heathcote, Storesletten, and Violante (2012), and Krueger and Ludwig (2013), respectively. NIPA stands for the National Income and Product Accounts.

Krusell, Ohanian, Ríos-Rull, and Violante (2000) estimate α, ρ, η, and we use their estimates, but they do not estimate ω and ρ. We calibrate these parameters to U.S. data, as we explain in detail below.

As for government policies, we assume that the government consumption-to-output ratio equals 16%, which is close to the average ratio in the United States during the period 1980 – 2012, as reported in the National Income and Product Accounts (NIPA) data. To approximate the progressive U.S. labor tax code, we follow Heathcote, Storesletten, and Violante (2012) and assume that tax liability given labor income y is defined as:

$$T(y) = \bar{y}\left[\frac{y}{\bar{y}} - \lambda\left(\frac{y}{\bar{y}}\right)^{1-\tau_l}\right],$$

where $\bar{y}$ is the mean labor income in the economy, $1-\lambda$ is the average tax rate of a mean income individual, and τ_l controls the progressivity of the tax code. Using 2005 CPS data, Heathcote, Storesletten, and Violante (2012) estimate $\tau_l = 0.18$. We use their estimate and calibrate λ to clear the government budget.

Gravelle (2011) documents that because of differences in tax depreciation rates, the effective tax rates on structure capital and equipment capital differ at the firm level. Specifically, the effective corporate tax rate on structure capital is 32% and the effective corporate tax rate on equipment capital is 26%. We assume that the capital income tax rate at the consumer level is 15%, which approximates the U.S. tax code. This implies an overall tax on structure capital of $\tau_s = 1 - 0.85 \cdot (1 - 0.32) = 42.2\%$ and an overall tax on equipment capital of $\tau_e = 1 - 0.85 \cdot (1 - 0.26) = 37.1\%$. These numbers are in line with a 40% tax, which Domeij and Heathcote (2004) report for aggregate capital stock. We follow Krueger and Ludwig (2013) and assume that the consumption tax $\tau_c = 5.0\%$.[7] Finally, we assume a government debt of 60% of GDP, as in the U.S. between 1990 and the last recession.

We set the ratio of skilled to unskilled agents to be consistent with the 2011 US Census data. We cannot identify the mean levels of the idiosyncratic labor productivity shock z for the two types of agents separately from the remaining parameters of the production function and therefore set $E[z] = 1$ for both skilled and unskilled. This assumption implies that w_i corresponds to the average wage rate of agents of skill type i. Thus, skill premium in the model economy is given by w_s/w_u. We assume that the processes for z differ across the two types of agents. Specifically, we assume that for all $i \in \{u, s\} : \log z_{t+1} = \rho_i \log z_t + \varepsilon_{i,t}$. Following Krueger and Ludwig (2013), we pick $\rho_s = 0.9408, var(\varepsilon_s) = 0.0100$, $\rho_u = 0.8713$, $var(\varepsilon_u) = 0.0192$. We approximate these processes by finite number Markov chains using the Rouwenhorst method described in Kopecky and Suen (2010).

There are still six parameter values left to be determined: these are the two production function parameters, ω and ν, which govern the income shares of equipment capital, skilled labor and unskilled labor, the labor disutility parameter ϕ, the discount factors β_s and β_u, and the parameter governing the overall level of taxes in the tax function, λ. We calibrate ω and ν so that (i) the labor share equals 2/3 (approximately the average labor share in 1980 – 2010 as reported in the NIPA data) and (ii) the skill premium w_s/w_u equals 1.8 (as reported

[7] For the details on the computation of this number, see Mendoza, Razin, and Tesar (1994).

Table 2: Benchmark Calibration Procedure

Parameter	Symbol	Value	Target	Data and SRCE	Source
Production function parameter	ω	0.4800	Labor share	2/3	NIPA
Production function parameter	ν	0.6551	Skill premium $\frac{w_s}{w_u}$	1.8	HPV
Disutility of labor	ϕ	123.7	Labor supply	1/3	
Skilled discount factor	β_s	0.9825	Capital-to-output ratio	2.9	NIPA, FAT
Unskilled discount factor	β_u	0.9807	Relative wealth	2.680	Census
Tax function parameter	λ	0.8423	Gvt. budget balance		

This table reports our benchmark calibration procedure. The production function parameters ν and ω control the income share of equipment capital, skilled and unskilled labor in output. The tax function parameter λ controls the labor income tax rate of the mean income agent. Relative wealth refers to the ratio of the average skilled to average unskilled agents' asset holdings. The acronym HPV stands for Heathcote, Perri, and Violante (2010). NIPA stands for the National Income and Product Accounts, and FAT stands for the Fixed Asset Tables.

by Heathcote, Perri, and Violante (2010) for the 2000s). We choose ϕ so that the aggregate labor supply in steady state equals 1/3 (as is commonly assumed in the macro literature). We calibrate β_s and β_u so that (i) the capital-to-output ratio equals 2.9 (approximately the average of 1980 – 2011 as reported in the NIPA and Fixed Asset Tables data), and (ii) the asset holdings of an average skilled agent are 2.68 times those of an average unskilled agent (as in the 2010 Census). Finally, following Heathcote, Storesletten, and Violante (2012), we choose λ to clear the government budget constraint in equilibrium. Table 2 summarizes our calibration procedure.

4 Consequences of the Reform

In this section, we use the model calibrated in Section 3 to analyze the aggregate and distributional consequences of a budget neutral capital tax reform that equates the tax rates on structure and equipment capital. We are interested in measuring the effects of reforming the capital tax system alone. Therefore, we keep other government policies intact. Specifically, the government needs to finance the pre-reform level of expenditure and debt in the new steady state. The labor income tax code is not modified either.[8] To sum up, the reform finds the uniform tax rate on the two types of capital that clears the overall government's budget given that no other government policy instruments change.

[8]Observe that labor income tax revenue might still change since the capital tax reform affects prices and people's labor supply in equilibrium.

We first analyze the effects of the budget neutral uniform capital tax reform on prices and macroeconomic quantities. Second, we show that the reform improves both productive efficiency and (between-group) equality. Third, we compute how the reform affects aggregate and individual welfare.

A brief summary of our results is as follows. We find that the reform equates capital taxes at 39.66%. This means that the tax rate on equipment capital increases by about 3 percentage points whereas the tax rate on structure capital decreases by approximately the same amount. The change in the average capital tax is negligible. As a result of the tax changes, the amount of equipment capital decreases and the amount of structure capital increases. Interestingly, the average return to capital stays almost the same even though the total capital stock increases and the supply of both types of labor decline in the new steady state. This is due to the fact that the reform improves the productive efficiency of the economy by reallocating capital from low return capital to high return capital. In addition, we find that the reform gives rise to a more egalitarian distribution of consumption and hours worked between skilled and unskilled agents. The average steady-state welfare increases by 0.09%. This is a substantial welfare gain given that the reform involves a relatively modest change to the capital tax system.

Finally, our results are robust to alternative preference specifications, labor tax progressivity, and the degree of individual wage risk. Interpreting the United States as an open economy which faces a fixed interest rate does not change the main results substantially either. The reform is more redistributive for a higher degree of equipment-skill complementarity.

4.1 Macroeconomic Variables

Taxes and Prices. The first two rows of Table 3 display the current capital taxes as well as the uniform tax rate implied by the tax reform. We find that the uniform tax rate that applies to both types of capital and satisfies government's budget given the status quo labor income taxes, debt, and spending policies is 39.66%. Importantly, as we report in the third row of the table, the average tax on capital is almost unchanged. This implies that the reform we analyze changes the mix between equipment and structure capital taxes, but leaves overall capital taxation virtually unaffected.[9]

The three rows in the middle of Table 3 report the pre-tax returns to capital net of depreci-

[9] The tax numbers reported in the table are cumulative in the sense that they accumulate the capital income taxes that are paid at the firm level and at the consumer level. Alternatively, we can deduct the 15% flat capital income tax that consumers face to get the effective corporate capital income tax rates. In that case, while the pre-reform corporate effective tax rate on equipments is 26% and that on structures is 32%, the post-reform uniform effective corporate tax rate is 29.01%. This number could be interpreted as the statutory corporate tax rate if the equality of the effective tax rates on equipment and structure capital was achieved by setting the depreciation allowances for the two types of capital equal their actual economic depreciation rates.

Table 3: Taxes and Prices Before and After the Reform

Variable	Status Quo	Reform	Change
τ_s	42.20%	39.66%	-6.0%
τ_e	37.10%	39.66%	6.9%
avg. τ	39.62%	39.66%	0.1%
$r_s - \delta_s$	2.56%	2.45%	-4.1%
$r_e - \delta_e$	2.35%	2.45%	4.3%
avg. $r - \delta$	2.46%	2.45%	-0.04%
w_s/w_u	1.8	1.7964	-0.20%
w_s	0.5495	0.5491	-0.09%
w_u	0.3053	0.3056	0.11%

This table reports the effects of the uniform capital tax reform on steady-state taxes and prices. τ_s denotes the tax on structure capital, τ_e denotes the tax on equipment capital, avg. τ denotes the average tax on capital. $r_s - \delta_s$ denotes the pre-tax return on structure capital net of depreciation δ_s, $r_e - \delta_e$ denotes the pre-tax return on equipment capital net of depreciation δ_e, and avg. $r - \delta$ denotes the average return on capital net of depreciation. w_s denotes the average wage of the skilled agents, w_u denotes the average wage of the unskilled agents, and w_s/w_u denotes ratio of skilled to unskilled wages, i.e. the skill premium.

ation before and after the reform. The fourth and the fifth rows show that after the reform the return to structure capital declines while the return to equipment capital increases until they are equalized.[10] This is because the reform gives rise to an increase in the level of structure capital and a decrease in the level of equipment capital, as reported in Table 4. The sixth row of Table 3 displays the average returns to aggregate capital where the average is computed by weighing the return to each type of capital by its amount. We see, maybe somewhat surprisingly, that the average return to capital does not change much even though the aggregate capital stock increases and the level of both types of labor inputs decline (see Table 4 below). With diminishing returns to capital and capital-labor complementarity, one would expect the observed changes in aggregate capital stock and labor supply to decrease the average return to capital further. However, because of the uniform capital tax reform, there is capital reallocation from the capital with lower returns, equipment capital, towards the capital with higher returns, structure capital. This reallocation prevents the average return to capital from decreasing further. The fact that the average return to capital remains virtually unchanged suggests that the reform improves productive efficiency. We quantify and discuss the improvement in productive efficiency in detail in Section 4.2.

Finally, the last three rows of Table 3 report the effects of the reform on wages. First, the skill premium, w_s/w_u, decreases. This is a direct implication of the decline in the stock of equipment

[10] The non-arbitrage condition implies that after-tax returns to the two types of capital must be equal. After the reform, the capital tax is uniform and, thus, the pre-tax returns to the two types of capital are equal as well.

capital and the assumptions on technology. We also find that the average wage of the skilled agents, w_s, decreases whereas the average wage of the unskilled agents, w_u, increases. This is because the reform increases the amount of structure capital in the new steady state. This implies that wages of both types of agents increase by the same proportion since, by Assumption 1, the complementarity between structure capital and the two types of labor is the same. The reform also decreases the level of equipment capital which depresses the wages of both types of agents. However, because of equipment-skill complementarity formalized in Assumption 2, the impact on skilled wages is larger. Quantitatively, we find that the cumulative effect is negative for skilled wages and positive for unskilled wages.

Allocations. Table 4 displays the effects of the tax reform on aggregate allocations. The left panel shows how factors of production and total output are affected. The right panel shows how net after-tax capital income and consumption for the two groups of agents change.

The lower tax rate on structure capital gives rise to an increase in its level in the new steady state. In contrast, the higher tax rate on equipment capital results in a lower level of equipment capital. Overall, we find that the reform increases the steady state level of total capital stock by 0.40%. The increase in the total capital stock is due to the fact that the reform increases the average productivity of a given amount of capital by eliminating the distortion in capital allocation. We find that skilled labor supply decreases by 0.01% and unskilled labor supply decreases by 0.05%. The reason for the labor supply changes is as follows. As reported in Table 3, wages decrease for the skilled agents. A decline in wages pushes labor supply up due to an income effect and down due to a substitution effect. When $\sigma > 1$, as in our benchmark parameterization, the income effect dominates, which implies that skilled labor supply should increase with wage.[11] In addition, skilled labor supply is pushed down because of an income effect related to the increase in the skilled agents' average net after-tax capital income $(R-1) \cdot A_s$ (see the right panel of the Table 4). It turns out that these effects almost offset each other and the skilled labor supply decreases only slightly. In contrast, as reported in Table 3, unskilled wages increase, and this pushes unskilled labor supply down since $\sigma > 1$. In addition, average unskilled capital income $(R-1) \cdot A_u$ increases which decreases their labor supply. In the end, unskilled labor supply decreases more than skilled labor supply. Overall, we find that changes in the levels of factors of production lead to an increase in output.

The first two rows of the right panel of Table 4 report the average consumption levels of the skilled agents, C_s, and the unskilled agents, C_u. We find that consumption of the skilled agents decreases. The reason is that the negative effect of the decrease in the skilled agents' wages on their consumption dominates the positive effect of the increase in their capital income.

[11] This comparative statics result holds exactly in a static model without wealth. The presence of positive wealth weakens the income effect.

Table 4: Changes in Allocations due to the Reform

Variable	Change	Variable	Change
K_s	1.34%	C_s	-0.02 %
K_e	-0.53%	C_u	0.07%
K	0.40%	C	0.02%
L_s	-0.01%	$(R-1)\cdot A_s$	0.50 %
L_u	-0.05%	$(R-1)\cdot A_u$	0.42 %
L	-0.04%		
Y	0.02%		

This table reports the effects of the uniform capital tax reform on allocations. "Change" refers to the change between the pre-refom and post-reform steady state. K_e denotes equipment capital, K_s denotes structure capital, K denotes aggregate capital, L_s denotes the average supply of skilled labor, L_u denotes the average supply of unskilled labor, Y denotes output, and Net Output denotes $Y - \delta_s K_s - \delta_e K_e$. C_s denotes the average skilled consumption, C_u denotes the average unskilled consumption, and C denotes average (aggregate) consumption. $(R-1)\cdot A_s$ denotes the average after-tax return to skilled agents' asset holdings, and $(R-1)\cdot A_u$ denotes the average after-tax return to unskilled agents' asset holdings.

Unskilled consumption, on the other hand, increases, because both wages and capital income of the unskilled agents increase.

4.2 Productive Efficiency and Equality

In this section, we discuss the efficiency and (between-group) equality consequences of the uniform capital tax reform. We find that the reform improves productive efficiency and increases the degree of equality between skilled and unskilled agents.

Productive Efficiency. Productive efficiency measures how efficient the economy is in turning inputs into output. The productive efficiency result of Diamond and Mirrlees (1971) suggests that the differential tax treatment of the two types of capital might create inefficiencies by distorting capital accumulation decisions. Indeed, as we prove in Section 5, a Ramsey planner that does not have redistributive concerns finds it optimal to tax equipments and structures at the same rate. This implies that uniform capital tax reform should improve productive efficiency. Intuitively, in the steady state before the reform, the pre-tax return to structure capital is higher than the pre-tax return to equipment capital. A reform towards uniform capital taxation creates capital reallocation from the capital with lower returns, equipment capital, towards the capital with higher returns, structure capital. This reallocation, then, brings the economy closer to its production possibility frontier increasing output for a given level of capital.

To measure the productive efficiency gains of the reform, we compare the before and after reform net output, i.e. $Y - \delta_s K_s - \delta_e K_e$. We focus on net output, because it takes into account the depreciation costs of using different types of capital and thus measures the available resources in the economy after depreciated capital has been replaced. Obviously, the tax reform does not only affect the way aggregate capital is allocated across the two capital types but it also changes the level of aggregate capital and the supply of both types of labor. In fact, in our benchmark analysis, the supply of both types of labor decrease, putting a downward pressure on net output. Similarly, the increase in total capital stock puts a upward pressure on net output. In order to isolate the change in net output that is due to capital reallocation, we decompose the total change in net output as follows.

We define an auxiliary interim allocation in which the aggregate capital and the labor supplies of both skilled and unskilled agents are kept constant at the pre-reform steady-state levels. In this interim allocation, capital is allocated across the two capital types in a way that equates their net marginal returns (i.e., $r_s - \delta_s = r_e - \delta_e$), which is what happens when taxes on the two types of capital are equalized. The change in net output from the pre-reform steady state to the interim allocation measures the gains of allocating the pre-reform amount of aggregate capital across the two types of capital more efficiently.[12] We call these gains *reallocation gains.*[13] We find that the reallocation gains amount to 0.0016% of net output. We also consider the average net return to capital as an alternative measure of how efficiently capital is used in production. We find that the average net return to capital is 1.35% higher in the interim allocation than in the pre-reform steady state.

Between-Group Equality. Next, we discuss the consequences of the reform for the degree of equality between the skilled and the unskilled agents. The discussion of how the reform affects within-group inequality is postponed to Section 4.4. Before the reform unskilled agents work more than skilled agents. As reported in Table 4, after the reform, both labor supplies decline but unskilled labor supply declines more, implying a more equal distribution of hours worked across agents. Second, skilled consumption declines and unskilled consumption increases, which makes the consumption distribution more equal. This is especially interesting given the fact that the labor tax code is kept intact. There is more equality after the uniform capital tax reform because the reform redistributes *indirectly* from the skilled to the unskilled agents by decreasing the skill premium. By increasing the tax rate on equipment capital, the reform decreases the level of equipment capital. Under the assumption of equipment-skill complementarity, the decline in equipment capital then decreases the skill premium as shown in the last row of Table 3.[14]

[12]Observe that with a fixed total capital stock and fixed labor supplies, net output is maximized when the net marginal returns are equalized across the two types of capital, i.e. at the interim allocation.

[13]The tax reform affects the allocation of labor as well. Our measure of reallocation gains, however, measures the gains coming from a better allocation of capital alone and deliberately ignores the gains arising from a better allocation of labor by not optimizing over labor in the interim allocation. In this sense, our measure provides a lower bound for total reallocation gains.

[14]In fact, in Section 5 we show that a planner who wants to redistribute from skilled to unskilled agents might

It is important to realize that eliminating capital tax differentials always improves efficiency in our model. Because of the nature of the pre-reform capital tax differentials in the U.S. tax code (structure capital tax rate being higher than equipment capital tax rate), the uniform capital tax reform that we consider also increases equality. As a result, this reform does not suffer from the efficiency vs. equality tradeoff, which is present in many other tax reforms. In the next section, we evaluate how the efficiency and equality improvements of the reform manifest themselves in terms of welfare.

4.3 Welfare Gains

In this section, we analyze the steady state welfare gains of switching from the current capital tax system to a system with a uniform 39.66% capital tax rate. Our measure of welfare gains and losses is standard. The welfare gains of allocation x relative to allocation y are defined as a fraction by which the consumption in allocation y would have to be increased in each date and state in order to make its welfare equal to the welfare of allocation x.

First, we want to evaluate the effect of the reform on aggregate welfare. To do so, we consider a Utilitarian social welfare function which puts an equal weight on every agent. This welfare criterion also represents the ex-ante welfare of an individual who has a probability p_s of being born as a skilled agent and a probability p_u of being born as an unskilled agent. The welfare gains of the uniform capital tax reform are 0.09% in consumption equivalent units. Then, we evaluate the welfare consequences of the reform for skilled and unskilled agents separately. We find that the average skilled welfare declines by 0.01% whereas the average unskilled welfare increases by 0.15%. We interpret these findings as follows. The reform, by abolishing tax differentials, increases productive efficiency, which increases welfare for both types of agents. However, the reform also depresses equipment capital accumulation, and hence, decreases the skill premium, which implies indirect redistribution from the skilled agents to the unskilled agents. It turns out that, under our benchmark parameterization, the efficiency gains and redistribution losses incurring to the skilled agents almost fully offset each other, resulting in a small welfare loss. For the unskilled agents, the redistributive gains plus the efficiency gains sum up to a significant welfare gain of 0.15%.

A Reform where No Type Loses. We also analyze a uniform capital tax reform, in which the government distributes a larger share of the efficiency gains to the skilled agents to ensure that the average skilled welfare is unchanged. In this reform, the uniform tax rate on capital is slightly below the benchmark reform level. This implies a smaller decrease in the skill premium and, hence, less indirect redistribution from the skilled to the unskilled. To make up

find it optimal to go beyond uniform capital taxation and tax equipments at a higher rate than structures.

for the capital tax revenue loss, the government decreases λ, which increases the overall level of labor taxes, but does not change labor tax progressivity. In particular, the parameter τ_l, which controls labor tax progressivity, is kept constant. As a result of this reform, unskilled welfare goes up by 0.15% while skilled welfare does not change by construction (more precisely, the unskilled gain 0.147% in this reform while they gain 0.150% in the benchmark reform). More generally, by choosing a lower level of the uniform capital tax rate and the appropriate λ, the government can distribute a positive share of the overall welfare gains of the reform to the skilled agents.[15]

A Reform that Switches the Tax Differential. Given the success of increasing equipment taxes as a redistributive policy, we analyze another budget-neutral tax reform in which we switch the two capital tax rates, i.e., set $\tau_e = 42.2\%$ and $\tau_s = 37.1\%$. The labor tax parameter λ is set so that the government budget is balanced. This reform decreases skilled welfare by 0.05% and increases unskilled welfare by 0.30% relative to status quo. Compared to the benchmark uniform capital tax reform, the current reform redistributes more from the skilled to the unskilled because increasing the tax on equipment capital further decreases the skill premium further (namely by 0.4% relative to the 0.2% in the benchmark reform). Relative to uniform capital taxation, this reform hurts productive efficiency by creating distortions in allocation of total capital across equipments and structures. In fact, this reform implies small reallocation losses (of -0.0001%) relative to the status quo. Nevertheless, the overall welfare gains of this reform, 0.17%, are substantially larger than that of the benchmark uniform capital tax reform, 0.09%, because redistribution gains dominate productive efficiency losses.[16]

A Reform with Fixed Aggregate Capital Stock. Our benchmark reform increases the total capital stock by 0.4% from the initial to the the new steady state. Comparing welfare across steady states with different capital levels can be problematic since raising capital from one steady state to the next might be costly over the transition. Therefore, to abstract from the steady-state welfare gains coming from the increase in aggregate capital, we consider an alternative reform, in which we choose the uniform capital tax rate so that the aggregate capital stock is constant across steady states.[17] We find that aggregate welfare increases by 0.06%,

[15]For instance, there is a reform that distributes all the gains of uniform capital taxation to the skilled agents subject to keeping unskilled welfare unchanged. In this reform, skilled welfare increases by 0.48%. The welfare gains of this reform should be taken with caution though. The reason is that, unlike the benchmark reform, in which average taxes on capital remain almost unchanged, this alternative reform decreases the average taxes on capital significantly, to 36.61%, and increases labor tax collection substantially. As a result, steady-state level of total capital stock increases much more than it does in the benchmark reform, namely by 1.7%, which makes steady-state welfare comparison less desirable.

[16]We do not analyze reforms in which equipment capital tax is increased substantially because in such cases the skill premium would decrease significantly and this might affect people's education attainment decisions. As a result, analyzing such reforms would require modeling education decisions explicitly which goes beyond the scope of this paper.

[17]The uniform capital tax rate that keeps the aggregate capital at the pre-reform steady-state level is 40.55%,

Table 5: Pre-tax Labor Earnings Inequality

Variable	Status Quo	Reform	Change
Skilled Pre-tax Labor Earnings			
Variance of logs	0.1271	0.1273	0.17%
Gini	0.1963	0.1965	0.07%
Unskilled Pre-tax Labor Earnings			
Variance of logs	0.1319	0.1320	0.07%
Gini	0.2015	0.2016	0.03%

This table reports the effects of the benchmark uniform capital tax reform on steady-state measures of labor earnings inequality.

which implies that two thirds of the welfare gains in the benchmark reform are coming from a more efficient allocation of the pre-reform amount of aggregate capital between equipment and structure capital.

4.4 Within-Group Inequality

In this section, we perform two distinct exercises. First, we investigate the effects of uniform capital tax reform on within-group inequality by comparing the variances of log labor earnings and the Gini coefficients of labor earnings for each skill group before and after the reform. Second, we analyze whether the quantitative consequences of the reform are robust to the degree of labor productivity risk.

Effects of the Reform on Within-Group Inequality. In Section 4.2, we explain how the uniform capital tax reform affects the distribution of resources between skilled and unskilled agents but remain silent on the effects of the reform on the distribution of resources within each skill group. This is what we explore in this section. There are two channels through which the reform might affect within-group earnings inequality. First, by increasing (decreasing) the marginal product of labor of a skill group, it scales up (down) the variance of wages. Second, this change in wages affects people's labor supply responses. However, since both the Gini coefficient and the variance of logs are scale independent inequality measures, they do not capture the first channel. Therefore, the changes in within-group labor earnings inequality reported in Table 5 are due to endogenous labor supply responses. The last column of the table shows that within-group earnings inequality increases for both skill groups slightly.

Role of Labor Productivity Risk. Our benchmark calibration does not replicate the wage inequality that we observe in the United States: Heathcote, Perri, and Violante (2010) report

which is higher than the one in the benchmark reform, 39.66%. This creates extra revenues for the government, and thus, to keep the government budget balanced, we decrease the total labor tax revenue by increasing λ.

Table 6: Pre-tax Labor Earnings Inequality with Calibrated Wage Risk

Variable	Status Quo	Reform	Change
Skilled Pre-tax Labor Earnings			
Variance of logs	0.4398	0.4405	0.16%
Gini	0.3521	0.3523	0.05%
Unskilled Pre-tax Labor Earnings			
Variance of logs	0.4524	0.4527	0.08%
Gini	0.3583	0.3584	0.03%

This table reports the effects of the uniform capital tax reform on steady-state measures of labor earnings inequality in an environment with calibrated wage risk. In particular, we increase the variance of the idiosyncratic shocks so as to match the variance of logged wages of 0.45 as reported by Heathcote, Perri, and Violante (2010).

that the variance of log wages equals 0.45 in the data whereas in our benchmark calibration this number is 0.17. The question then is: Are our results regarding the effects of the reform on within-group inequality biased by the fact that the model does not match the observed wage inequality in the first place? To answer this question, we provide an alternative calibration of the model in which we also match the variance of log wages in the model to the U.S. data. We do so by scaling up the variances of the productivity shocks of the two skill types by a common factor κ so that the variance of log wages equals 0.45 as in the data. The persistence parameters ρ_s and ρ_u are kept the same as in the benchmark calibration. We conduct the uniform capital tax reform for this environment. The last column of Table 6 shows that the implications of the reform on within-group inequality are quantitatively similar to the benchmark calibration.

We also analyze the robustness of our main results to variations in productivity risk. We compare the efficiency and (between-group) equality consequences of the reform under the "Calibrated risk" parameterization specified above to the one under benchmark calibration by comparing the first and second columns of Table 7. We find that higher productivity risk does not produce results significantly different from the benchmark results. To render a more complete robustness check, we also evaluate the consequences of the reform for a version of our economy where we decrease the variance of productivity shocks substantially (namely by setting $\kappa = 0.1$).[18] The results of this exercise are reported in the column called "Low risk" in Table 7. Again, we find that our results are robust. To the extent that one interprets the wage risk as the residual uninsured wage risk, we can read these results as saying that the consequences of the uniform capital tax reform are robust to the degree of market incompleteness.

[18] In the limit, decreasing the variance of the skill shocks further would lead to a representative agent environment. We chose not to go all the way to the representative agent environment, because with two representative agents, the steady state equilibrium is not unique and the comparison to the benchmark reform is not straightforward.

Table 7: Role of Wage Risk

	Benchmark	Calibrated risk	Low risk
uniform tax	39.66%	39.67%	39.66%
reallocation gains	0.0016%	0.0016%	0.0016%
w_s/w_u	-0.20%	-0.22%	-0.19%
welfare gains	0.09%	0.08%	0.09%
skilled gains	-0.01%	-0.01%	-0.01%
unskilled gains	0.15%	0.15%	0.15%

This table reports the results of the reform for different levels of wage risk. The column "Benchmark" refers to the benchmark quantitative analysis. The column "Calibrated risk" refers to an exercise in which we increase the variance of the idiosyncratic shocks so as to match the variance of logged wages of 0.45 as reported by Heathcote, Perri, and Violante (2010). The column "Low risk" refers to an exercise in which we decrease the variance of the idiosyncratic shocks ten times, bringing the model closer to a model with two representative agents. In both exercises, we keep the skill persistence parameters unchanged. Regarding the rows of the table, "uniform tax" refers to the uniform tax on equipment and structure capital that leaves steady state government budget balanced, "reallocation gains" refers to the change in net output associated with a better allocation of capital, "w_s/w_u" denotes the ratio of skilled to unskilled wages. "Welfare gains" denote the aggregate steady state welfare gains of the reform, while "skilled gains" ("unskilled gains") refers to the skilled (unskilled) agents' steady state welfare gains.

4.5 Sensitivity Analysis

In this section, we analyze the sensitivity of our quantitative results to the parameters that control preferences, technology, and progressivity of the labor tax code. Specifically, we perform the following exercise. We change the parameter of interest and keep all other parameters that we do not calibrate fixed. We recalibrate the model under this new parameterization. Then, we conduct the uniform capital tax reform, and evaluate the changes in macroeconomic aggregates and welfare in the new steady state. The results are summarized in Table 8 and Table 9. In addition, we also analyze an open economy version of the model. These results are reported in Table 10. The main finding is that our main results are quite robust to changes in parameters and openness of the economy.

In particular, we find that the reform decreases the steady state level of equipment capital and increases the steady state level of structure capital in all the sensitivity exercises we conduct. Similarly, it is always the case that consumption and hours worked become more equally distributed between skilled and unskilled agents. In Table 8, Table 9 and Table 10, we do not report these results, but instead focus on the efficiency and equality consequences of the reform.

Sensitivity to Preference Parameters. Table 8 shows that our main results are robust to changes in preference parameters. First, the uniform capital tax rate is very close to the benchmark value in all the exercises. Second, as in the benchmark case, there are productive efficiency gains from capital reallocation as shown in the row called "reallocation gains." Third,

Table 8: Sensitivity to Preference Parameters

	Benchmark			Benchmark		
σ	**1**	2	**4**	2	2	2
γ	1	1	1	**0.5**	1	**2**
uniform tax	39.62%	39.66%	39.68%	39.67%	39.66%	39.64%
reallocation gains	0.0016%	0.0016%	0.0017%	0.0016%	0.0016%	0.0016%
w_s/w_u	-0.15%	-0.20%	-0.27%	-0.21%	-0.20%	-0.18%
welfare gains	0.04%	0.09%	0.12%	0.09%	0.09%	0.09%
skilled gains	0.01%	-0.01%	-0.07%	-0.02%	-0.01%	-0.001%
unskilled gains	0.06%	0.15%	0.21%	0.16%	0.15%	0.14%

This table reports the sensitivity of our main quantitative results to preference parameters. Each column reports the results for a particular combination of σ (the curvature of utility from consumption) and γ (the curvature of disutility of labor). We always change a particular parameter and leave the rest of the parameters that are not calibrated unaffected. Regarding the rows of the table, "uniform tax" refers to the uniform tax on equipment and structure capital that leaves steady state government budget balanced, "reallocation gains" refers to the change in net output associated with a more efficient allocation of capital, "w_s/w_u" denotes the ratio of skilled to unskilled wages. "Welfare gains" denote the aggregate steady state welfare gains of the reform, while "skilled gains" ("unskilled gains") refers to the skilled (unskilled) agents' steady state welfare gains.

the skill premium decreases by roughly 0.2% in all the exercises, which implies a more equal distribution of consumption and labor across agents. We conclude that our finding that the uniform capital tax reform improves both efficiency and equality is robust to preference parameters. Aggregate steady state welfare gains are around 0.1%, they increase with σ, but do not change much with γ. Typically, skilled agents lose and unskilled agents gain from the reform. An interesting case is the one with $\sigma = \gamma = 1$, in which both types of agents gain. The reason is that in this parameterization, the decline in skilled consumption is very modest and is more than offset by the decrease in skilled labor supply. This case shows that the uniform capital tax reform increases both types' average welfare for a set of reasonable parameter values, even without being accompanied by a modification of the labor tax code.

Additional Sensitivity. Each column in Table 9, starting with the second one, reports an additional sensitivity exercise. The first row of the table shows that, in all these exercises, the uniform capital tax rate is very close to the benchmark value. In addition, the second and third rows of Table 9 show that the reform improves both efficiency and equality as in the benchmark case. Next, we discuss each robustness exercise reported in Table 9 in more detail.

Elasticity of Substitution. The column denoted by "$\eta = 0.79$" reports the consequences of the reform in an economy with higher elasticity of substitution between equipment capital and

Table 9: Additional Sensitivity

	Benchmark	$\eta = 0.79$	$\rho = -1$	$\tau_l = 0.15$	$\tau_l = 0.21$	Uniform β
uniform tax	39.66%	39.67%	39.65%	39.65%	39.67%	39.29%
reallocation gains	0.0016%	0.0017%	0.0014%	0.0016%	0.0016%	0.0016%
w_s/w_u	-0.20%	-0.28%	-0.24%	-0.20%	-0.20%	-0.17%
welfare gains	0.09%	0.10%	0.10%	0.09%	0.09%	0.10%
skilled gains	-0.01%	-0.05%	-0.02%	-0.01%	-0.01%	-0.02%
unskilled gains	0.15%	0.19%	0.18%	0.15%	0.15%	0.17%

This table reports additional sensitivity results. Each column reports the results for a particular parameter specification. We always change a particular parameter and leave the rest of the parameters that are not calibrated unaffected. "$\eta = 0.79$" refers to an exercise in which we increase the elasticity of substitution between equipment capital and unskilled labor by increasing η from its benchmark value of 0.401 to 0.79. The column "$\rho = -1$" refers to an exercise in which we decrease the elasticity of substitution between equipment capital and skilled labor by increasing ρ from its benchmark value of -0.495 to -1. The columns "$\tau_l = 0.15$" and "$\tau_l = 0.21$" refer to exercises in which we change the labor progressivity parameter τ_l from its benchmark value of 0.18. The column "Uniform β" refers to an exercise in which all agents are assumed to have the same discount factor. Regarding the rows of the table, "uniform tax" refers to the uniform tax on equipment and structure capital that leaves steady state government budget balanced, "reallocation gains" refers to the the change in net output associated with a better allocation of capital, "w_s/w_u" denotes the ratio of skilled to unskilled wages. "Welfare gains" denote the aggregate steady state welfare gains of the reform, while "skilled gains" ("unskilled gains") refers to the skilled (unskilled) agents' steady state welfare gains.

unskilled labor.[19] Specifically, we set $\eta = 0.79$ instead of the benchmark value of $\eta = 0.401$.[20] For this elasticity value, we observe that skilled welfare decreases more and unskilled welfare increases more relative to the benchmark parameterization. This is intuitive: higher η means a lower degree of complementarity between equipment capital and unskilled labor. In that case, a decline in equipment capital decreases unskilled wages less and, therefore, depresses the skill premium more relative to the benchmark case, as reported in the third row of Table 9. This means that there is a higher degree of indirect redistribution from the skilled to the unskilled. The average welfare gain is higher relative to the benchmark case because of the concavity of the utility function.

The column denoted by "$\rho = -1$" reports the consequences of the reform in an economy with a lower elasticity of substitution between equipment capital and skilled labor (in the benchmark $\rho = -0.495$). The results are similar to those in the previous exercise, which is intuitive: lower ρ means a higher degree of complementarity between equipment capital and skilled labor, which, in relative terms, is similar to a lower degree of complementarity between equipments and unskilled

[19]Given the aggregate production function specified in (1), the elasticity of substitution between unskilled labor and equipment capital is $1/(1-\eta)$, while the elasticity of substitution between skilled labor and equipment capital is $1/(1-\rho)$.

[20]This value has been used, for example, in He and Liu (2008), who use the same production function as ours, and comes from an empirical study by Duffy, Papageorgiou, and Perez-Sebastian (2004).

labor. To conclude, in a world with a higher degree of equipment-skill complementarity, a greater portion of the gains of the uniform capital tax reform accrues to the unskilled people, thereby increasing the egalitarian nature of the reform.

Labor Tax Progressivity. We take the estimate of the progressivity of the labor income tax code τ_l from a study by Heathcote, Storesletten, and Violante (2012). It is likely that different approaches would yield different results. We, therefore, report the sensitivity of our results to the progressivity of the labor tax code in the next two columns of Table 9. We decrease and increase τ_l by 1/6 to $\tau_l = 0.15$ and $\tau_l = 0.21$, respectively, and find that our results are robust to this change.

Uniform Discount Factors. Next, we calibrate a version of our model in which all agents have the same discount factor. The results of the uniform tax reform are reported in Table 9 in the column entitled "Uniform β". We find that our main quantitative results do not depend on the assumption of heterogenous discount factors. With a uniform discount factor, however, the calibrated model is not able to match the observed wealth distribution across skilled and unskilled agents.

Open Economy. The United States is not literally a closed economy, but following the literature, we consider that scenario a useful benchmark. In this section, we consider the polar opposite case and analyze the consequences of the uniform capital tax reform assuming that the United States is a small open economy that faces a fixed world interest rate. This exercise illustrates to what extent the implications of the uniform capital tax reform depend on the degree of openness. In particular, we calibrate the after-tax interest rate so that in the steady state, the net foreign asset position is -20% of GDP, which is approximately the number for the U.S. economy as reported by the Bureau of Economic Analysis.

The results of the uniform capital tax reform for this environment are reported in Table 10. We find that the reallocation gains and the skill premium changes are very similar to the closed economy benchmark. For both agents, the welfare gains are smaller in the open economy exercise. The reason is the following. Recall that equating capital taxes increases the return to capital due to the reallocation from less to more productive capital. In our benchmark closed economy, this encourages asset accumulation of domestic agents towards the new steady-state, which increases their steady-state welfare. In an open economy, higher return to capital implies an immediate capital inflow from abroad until the (after-tax) return to capital is equal to its pre-reform level. As a result, the asset holdings of the domestic agents stay largely unchanged across the steady states. This implies that the change in aggregate capital stock across steady states does not contribute to the welfare gains of domestic agents. Therefore, not surprisingly, the overall welfare gains in the open economy exercise are almost the same as the welfare gains in a closed economy in which we abstract from the effect of capital accumulation on aggregate

Table 10: Role of Openness

	Benchmark	Open Economy
uniform tax	39.66%	39.59%
reallocation gains	0.0016%	0.0016%
w_s/w_u	-0.20%	-0.20%
welfare gains	0.09%	0.06%
skilled gains	-0.01%	-0.05%
unskilled gains	0.15%	0.14%

This table reports the implications of the uniform capital tax reform for different degrees of openness. The column "Benchmark" refers to the benchmark closed economy exercise. The column "Open Economy" refers to an exercise in which the after-tax interest is kept fixed. Regarding the rows of the table, "uniform tax" refers to the uniform tax on equipment and structure capital that leaves steady state government budget balanced, "reallocation gains" refers to the change in net output associated with a more efficient allocation of capital, "w_s/w_u" denotes the ratio of skilled to unskilled wages. "Welfare gains" denote the aggregate steady state welfare gains of the reform, while "skilled gains" ("unskilled gains") refers to the skilled (unskilled) agents' steady state welfare gains.

welfare by keeping aggregate capital fixed. We discuss the details of this exercise in Section 4.3.

5 Theoretical Insights from Ramsey Taxation

The aim of this section is to provide a theoretical justification for why the uniform capital tax reform enhances productive efficiency. To abstract away from redistribution and insurance roles of taxation, we consider a version of the economy outlined in Section 2 in which a representative agent provides both skilled and unskilled labor and faces no productivity shocks. We analyze a Ramsey tax problem and show that, in such an environment without redistribution and insurance concerns, it is optimal to tax equipments and structures uniformly. This finding suggests that uniform capital tax reform should be improving productive efficiency. Through an example, we also show that, if a planner cares sufficiently about the unskilled agents, then taxing equipments at a higher rate can be optimal. This result suggests that a move from status quo towards uniform capital taxes might be improving between-group equality.

Now, we lay out the representative agent economy. There is a representative consumer who supplies skilled and unskilled labor. The representative consumer has the following preferences over consumption c_t, skilled labor $l_{s,t}$, and unskilled labor $l_{u,t}$:

$$u(c_t) - v(l_{s,t}) - v(l_{u,t}).$$

This is the main distinction of the representative agent economy from the model economy outlined in Section 2. Another distinction is that, following the Ramsey tax tradition, we assume that labor taxes are linear, i.e. $T_t(y_t) = \tau_t \cdot y_t$.[21]

Representative Agent's Problem. Taking prices and government policy as given, the representative agent solves:

$$\begin{aligned}
\max_{\{c_t, l_{s,t}, l_{u,t}, a_{t+1}\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v(l_{s,t}) - v(l_{u,t}) \right] \qquad \text{s.t. for all } t \geq 0 \\
c_t + a_{t+1} \quad \leq \quad & (1 - \tau_t) \sum_{i=s,u} l_{i,t} w_{i,t} + R_t a_t, \\
& c_t, l_{s,t}, l_{u,t} \geq 0, \qquad a_0 > 0 \text{ given.}
\end{aligned}$$

where

$$R_t = [1 + (1 - \tau_{s,t})(r_{s,t} - \delta_s)] = [1 + (1 - \tau_{e,t})(r_{e,t} - \delta_e)] \tag{2}$$

has to hold in equilibrium.

The asset market clearing condition is:

$$K_{s,t} + K_{e,t} + D_t = a_t. \tag{3}$$

Finally, the goods market clears (government's budget constraint is satisfied by Walras' Law):

$$c_t + G_t + K_{s,t+1} + K_{e,t+1} = \tilde{F}(K_{s,t}, K_{e,t}, L_{s,t}, L_{u,t}). \tag{4}$$

There are also two labor market clearing conditions given by $l_{i,t} = L_{i,t}$ for all $t \geq 0$ and $i \in \{s, u\}$. In the rest of this section, we impose labor market clearing conditions implicitly by using $l_{i,t}$ and $L_{i,t}$ interchangeably.

The definition of competitive equilibrium is very similar to the one provided for an environment with productivity shocks and two types of agents in Appendix A, and therefore, is omitted for the sake of brevity.

Characterization of Equilibrium. Letting p_t be the Lagrange multiplier on period t household budget constraint, the first-order optimality conditions of the representative agent

[21] In addition, we assume that the measure of each type of agents is one, agents have identical discount factors, and the government does not tax consumption. These assumptions are made only for expositional purposes and do not affect the theoretical insights provided in this section.

are:[22]

$$(c_t): \qquad \beta^t u'(c_t) = p_t, \tag{5}$$

$$(l_{i,t}): \qquad \beta^t v'(l_{i,t}) = p_t(1-\tau_t)w_{i,t}, \text{ for } i = s, u, \tag{6}$$

$$(a_{t+1}): \qquad p_t = p_{t+1}R_{t+1}. \tag{7}$$

These conditions together with the budget constraint holding with equality every period characterize the representative agent's decisions given policy and prices.

The following pricing conditions follow from the solution to the representative firm's problem.

$$r_{s,t} = F_{1,t}, \qquad r_{e,t} = F_{2,t}, \qquad w_{s,t} = F_{3,t}, \qquad w_{u,t} = F_{4,t}. \tag{8}$$

Ramsey Problem. The government chooses a tax policy such that the resulting equilibrium allocation maximizes the welfare of the representative agent.[23] Following the Ramsey taxation literature (see Chari and Kehoe (1999)), we use the primal approach and transform the optimal tax problem above to a problem in which the government chooses allocations directly. In order to do that, we will show that the equilibrium is characterized by three fairly simple conditions. The first condition is the resource constraint of the economy (4).

The second condition is an implementability constraint which is derived using the agent's budget constraints along with first-order conditions and a transversality condition:

$$\sum_{t=0}^{\infty} \beta^t \left[u'(c_t)c_t - v'(l_{s,t})l_{s,t} - v'(l_{u,t})l_{u,t}\right] = u'(c_0)R_0 a_0. \tag{9}$$

The third constraint follows from the assumption that labor income taxes cannot depend on labor type.[24] With this assumption, the first order conditions with respect to two labor types, summarized in equation (6), together with the factor pricing conditions in (8), imply that in any equilibrium:

$$\frac{v'(l_{s,t})}{v'(l_{u,t})} - \frac{F_{3,t}}{F_{4,t}} = 0. \tag{10}$$

[22] These first order conditions assume that the non-negativity constraints on consumption and labor are not binding. This is true if u and v satisfy appropriate Inada conditions, which are satisfied by the functions we consider in the quantitative exercises.

[23] We restrict the set of policies available to the government by fixing the initial tax rates on both types of capital to some constant numbers, $\tau_{s,0}$ and $\tau_{e,0}$. This is a standard assumption in the Ramsey taxation literature. Without this assumption, the government would raise all necessary revenue through taxing initial capital, which is non-distortionary.

[24] The case when labor taxes are allowed to depend on skill types is theoretically interesting, but not relevant in practice. In the U.S. tax code, labor taxes are functions of labor income only, which is what we pursue in this paper, both in this section as well as in the quantitative analysis of the uniform capital tax reform.

The next proposition states formally that these three conditions fully characterize the equilibrium allocation of consumption, labor, and capital.

Proposition 1. *The consumption, labor, and capital allocations and period zero capital tax rates and return on debt in any competitive equilibrium satisfy conditions* (4), (9), *and* (10). *Moreover, if any consumption, labor, and capital allocations and period zero capital tax rate and return on debt satisfy these conditions, then one can find a tax policy, a price system, and debt holdings that, together with the given allocations and period zero prices, constitute a competitive equilibrium.*

Proof. Relegated to Appendix C. ∎

The Ramsey problem can then be formulated as follows.

$$\max_{\{c_t, l_{s,t}, l_{u,t}, K_{s,t+1}, K_{e,t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v(l_{s,t}) - v(l_{u,t})\right] \qquad \text{s.t.} \qquad (4), (9), \text{ and } (10).$$

Proposition 2. *Suppose that* $v(l) = \frac{l^{1+\gamma}}{1+\gamma}$. *Then, equipment capital and structure capital are taxed at a uniform rate in the solution to the Ramsey problem.*

Proof. First, we will verify that constraint (10) does not bind by checking that it is satisfied at the solution to a relaxed problem which does not include constraint (10).

Consider the relaxed problem where we do not include (10) as a constraint. Letting the multipliers on the feasibility constraint be λ_t and the multiplier on the implementability constraint be μ, the first-order optimality conditions with respect to type i labor for $i = s, u$ and all $t \geq 0$ are

$$(l_{i,t}) \quad : \quad \beta^t v'(l_{i,t}) + \mu\beta^t[v''(l_{i,t})l_{i,t} + v'(l_{i,t})] = \lambda_t w_{i,t},$$

which under the assumption on the disutility function can equivalently be written as

$$\beta^t v'(l_{i,t})[1 + \mu(1+\gamma)] = \lambda_t w_{i,t}. \tag{11}$$

Dividing (11) for skilled labor by the same equation for unskilled labor and using $w_{s,t} = F_{3,t}$ and $w_{u,t} = F_{4,t}$, we get that, for all $t \geq 0$,

$$\frac{v'(l_{s,t})}{v'(l_{u,t})} - \frac{F_{3,t}}{F_{4,t}} = 0.$$

This implies that (10) is slack.

Therefore, the first order conditions with respect to the two capital types are:

$$\begin{aligned}(K_{s,t+1}) &: \gamma_t = \gamma_{t+1}\left[F_{s,t+1} + (1-\delta_s)\right],\\(K_{e,t+1}) &: \gamma_t = \gamma_{t+1}\left[F_{e,t+1} + (1-\delta_e)\right].\end{aligned}$$

Combining the two equations above and using the factor pricing conditions in (8) gives

$$F_{s,t+1} + (1-\delta_s) = F_{e,t+1} + (1-\delta_e). \tag{12}$$

Comparing (12) with (2), which has to hold in any equilibrium, proves the result. ∎

Proposition 2 establishes that a government that is not concerned about redistribution and insurance finds it optimal to tax equipments and structures at the same rate.[25] We interpret this result as providing a theoretical justification for the productive efficiency gains of uniform capital tax reform we find in Section 4.

Example with Redistribution. Now we consider the same environment but with two types of agents, skilled and unskilled, to gain insights into the effects of uniform capital tax reform on equality.

Representative Agents' Problem. Taking prices and government policy as given, the skilled and unskilled representative agents (i.e., $\forall i \in \{s,u\}$) solve:

$$\begin{aligned}\max_{\{c_{i,t},l_{i,t},a_{i,t+1}\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty}\beta^t\left[u(c_{i,t}) - v(l_{i,t})\right] \qquad \text{s.t. for all } t \geq 0\\ c_{i,t} + a_{i,t+1} \quad \leq \quad & (1-\tau_t)l_{i,t}w_{i,t} + R_t a_{i,t},\\ & c_{i,t}, l_{i,t} \geq 0, \qquad a_{i,0} > 0 \text{ given,}\end{aligned}$$

where $R_t = [1 + (1-\tau_{s,t})(r_{s,t} - \delta_s)] = [1 + (1-\tau_{e,t})(r_{e,t} - \delta_e)]$ has to hold in equilibrium.

As in the previous section, we use the primal approach and transform the optimal tax problem to a problem in which the government chooses allocations directly. The three constraints,

[25] In doing so, Proposition 2 also contributes to the literature on the optimality of uniform capital taxation. The productive efficiency result of Diamond and Mirrlees (1971) implies that it is optimal to tax different types of capital at the same rate in order to avoid production distortions. Auerbach (1979) and Feldstein (1990) show that, when there are restrictions on the taxation of different inputs to production, similar to the restriction that different types of labor cannot be taxed at different rates in our environment imposed by constraint (10), then the Diamond and Mirrlees (1971) result does not hold any more: it might be optimal to tax different types of capital differently. We recover optimality of uniform capital taxation because, under our functional form assumption regarding the labor disutility function, labor supply elasticities of skilled and unskilled labor are constant and identical. This implies that there is no need to tax the two types of labor differently, which is why constraint (10) does not bind, and hence, we are back in the Diamond and Mirrlees (1971) world.

equations (13) − (15) in the Ramsey problem below, fully characterize equilibrium allocations that can be achieved by the taxes available. The first constraint is the resource constraint of the economy whereas the second constraint consists of an implementability constraint for each agent. The third constraint follows from the restriction that labor taxes cannot depend on skill type.

Ramsey Problem.

$$\max_{\{(c_{i,t},l_{i,t})_{i=s,u},K_{s,t+1},K_{e,t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \sum_{i=s,u} \Delta_i [u(c_{i,t}) - v(l_{i,t})] \qquad \text{s.t.}$$

$$\text{for all } t \geq 0, \qquad G_t + \sum_i c_{i,t} + K_{s,t+1} + K_{e,t+1} = \tilde{F}(K_{s,t}, K_{e,t}, L_{s,t}, L_{u,t}), \tag{13}$$

$$\text{for all } i = s, u, \qquad \sum_{t=0}^{\infty} \beta^t \left[u'(c_{i,t})c_{i,t} - v'(l_{i,t})l_{i,t}\right] = u'(c_{i,0})R_0 a_{i,0}, \tag{14}$$

$$\text{for all } t \geq 0, \qquad \frac{v'(l_{s,t})}{v'(l_{u,t})}\frac{u'(c_{u,t})}{u'(c_{s,t})} - \frac{F_{3,t}}{F_{4,t}} = 0. \tag{15}$$

Now we will show through an example that if the society cares sufficiently about unskilled agents, then this redistributive motive will imply that it is optimal to tax equipments at a higher rate then structures. In this example, following the parametric assumptions in all our quantitative exercises, we assume $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ and $v(l) = \frac{l^{1+\gamma}}{1+\gamma}$. Furthermore, suppose $\sigma < 1$.[26] To provide an analytical result, we consider an extreme case in which $\Delta_s = 0$, meaning that the society only cares about the unskilled agents.

Letting λ_t, μ_i, and ψ_t be the Lagrange multipliers on the three constraints of the Ramsey problem respectively and assuming constraint (15) binds with a greater than or equal to sign (we will verify this by proving the the Lagrange multiplier $\psi_t > 0$), the first-order optimality conditions with respect to the two capital types are:

$$(K_{s,t+1}) \quad : \quad \lambda_t = \lambda_{t+1}\tilde{F}_{s,t+1}, \tag{16}$$

$$(K_{e,t+1}) \quad : \quad \lambda_t = \lambda_{t+1}\tilde{F}_{e,t+1} - \psi_{t+1}\frac{\partial\left(\frac{F_{3,t+1}}{F_{4,t+1}}\right)}{\partial K_{e,t+1}}. \tag{17}$$

The asymmetry in the first-order conditions arises from the equipment-skill complementarity assumption: structure capital does not affect the wage ratio whereas equipment capital is more complementary with skilled labor than it is with unskilled labor, i.e., $\partial\left(\frac{F_{3,t+1}}{F_{4,t+1}}\right)/\partial K_{e,t+1} > 0$. Therefore, differential taxation of capital is optimal as long as $\psi_{t+1} \neq 0$. In the case with

[26] The assumption on the coefficient of relative risk aversion is made for analytical convenience. The results of this section continue to apply in the case of $\sigma \geq 1$, but the analysis becomes significantly more involved.

a representative agent and without redistribution, we showed that the version of constraint (15) for representative agent economy, constraint (10), is slack, and hence, it is optimal to tax capital uniformly. We will show that constraint (15) binds when we have two agents and there is a sufficiently strong redistribution motive. Suppose by way of contradiction that constraint (15) does not bind. Then, the first-order optimality conditions of the Ramsey problem for consumption and labor are:

$$\begin{array}{rrcl} (c_{i,t}): & \beta^t u'(c_{i,t})(\Delta_i + \mu_i(1-\sigma)) & = & \lambda_t \\ (l_{i,t}): & \beta^t v'(l_{i,t})(\Delta_i + \mu_i(1+\gamma)) & = & \lambda_t w_{i,t}. \end{array}$$

Using the above first-order conditions together with $\Delta_s = 0$ implies:

$$\frac{v'(l_{s,t})}{v'(l_{u,t})} \frac{u'(c_{u,t})}{u'(c_{s,t})} \frac{[1/(1-\sigma) + \mu_u]}{[1/(1+\gamma) + \mu_u]} = \frac{F_{3,t}}{F_{4,t}},$$

which, since $\gamma > 0$, implies that in the absence of constraint (15),

$$\frac{v'(l_{s,t})}{v'(l_{u,t})} \frac{u'(c_{u,t})}{u'(c_{s,t})} < \frac{F_{3,t}}{F_{4,t}}. \tag{18}$$

This establishes that constraint (15) binds in the solution to the Ramsey problem, and in particular in the direction assumed, i.e., $\psi_t > 0$.

Now we explain intuitively why constraint (15) binds in this direction. Using firm's optimality conditions and rearranging (18), we get

$$\frac{v'(l_{s,t})}{u'(c_{s,t}) w_{s,t}} < \frac{v'(l_{u,t})}{u'(c_{u,t}) w_{u,t}}. \tag{19}$$

Using agent's intratemporal optimality condition in equilibrium, we can define the intratemporal wedge for agent of type i as:

$$1 - \tau_{i,t} = \frac{v'(l_{i,t})}{u'(c_{i,t}) w_{i,t}}.$$

Using this definition, we can rewrite (19) as:

$$1 - \tau_{s,t} < 1 - \tau_{u,t}.$$

Intuitively, since society cares about unskilled agents only, the government wants to redistribute from skilled agents to unskilled agents. One way to achieve this goal is to tax skilled labor at a higher rate than unskilled labor. This is why constraint (15) binds in the $(1 - \tau_{s,t}) \geq (1 - \tau_{u,t})$ direction.

Finally, (16) and (17) under $\psi_{t+1} > 0$ imply it is optimal to set $\tilde{F}_{s,t+1} < \tilde{F}_{e,t+1}$. This, together with the non-arbitrage condition which has to hold in any equilibrium, establishes that it is optimal to tax equipments at a higher rate than structures, i.e., $\tau_{e,t+1} > \tau_{s,t+1}$ for all $t \geq 0$. Intuitively, the government wants to tax skilled labor at a higher rate than unskilled labor due to redistributive concerns. However, this is not allowed by the labor tax code. Therefore, the government taxes equipment capital at a higher rate, thereby indirectly achieving a higher tax rate on the labor that is more complementary with equipment capital, skilled labor.[27]

This example highlights the redistributive role of differential taxation of capital when labor income taxes cannot depend on people's skill types.[28] In particular, taxing equipments at a higher rate helps to redistribute from skilled to unskilled people. This implies that the uniform capital tax reform that we consider in this paper - one that starts from a status quo where equipments are taxed at a lower rate - might improve equality. That is indeed what we find in all our quantitative exercises.

6 Conclusion

This paper analyzes the aggregate and distributional consequences of a reform that eliminates capital tax differentials. We find that such a reform leads to improvements in productive efficiency. We also find that by decreasing the skill premium, the reform increases the degree of equality in the economy. This implies that the reform does not suffer from the usual efficiency vs. equality trade-off. As result of the reform, skilled agents' steady-state welfare decreases by 0.01%, while unskilled agents' welfare increases substantially by 0.15%, resulting in aggregate welfare gains of approximately 0.1%.

[27] It is straightforward to prove that $\psi_t < 0$ and constraint (15) binds in the opposite direction when $\Delta_s = 1$, which implies it is optimal to set $\tau_{s,t} > \tau_{e,t}$. That is, a government that wants to redistribute from unskilled to skilled agents finds it optimal to tax structures at a higher rate than equipments.

[28] We prove a related result in Slavík and Yazici (2014) for a dynamic Mirrleesian environment with labor productivity shocks, in which the government is able to choose a tax schedule which is arbitrarily non-linear in agents' labor earnings, but cannot depend on agents' types directly.

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Appendix

A Definition of Competitive Equilibrium

First, denote the partial history of productivity shocks up to period t by $z^t \equiv (z_0, ..., z_t)$. Also, denote the unconditional probability of z^t for agent of skill type i by $P_{i,t}(z^t)$. For each agent type, this unconditional probability is achieved by applying the transition probability matrix $\Pi_i(z'|z)$ recursively. We denote by Z_i^t the set in which z^t lies for an agent of type i.

Equilibrium. A competitive equilibrium consists of a policy $(\tau_{c,t}, T_t(\cdot), \tau_{s,t}, \tau_{e,t}, D_t, G_t)_{t=0}^{\infty}$, an allocation $((c_{i,t}(z^t), l_{i,t}(z^t),\, a_{i,t+1}(z^t))_{z^t \in Z_i^t, i}, K_{s,t}, K_{e,t}, L_{s,t}, L_{u,t})_{t=0}^{\infty}$, and a price system $(r_{s,t}, r_{e,t}, w_{s,t}, w_{u,t}, R_t)_{t=0}^{\infty}$ such that:

1. Given the policy and the price system, the allocation $((c_{i,t}(z^t), l_{i,t}(z^t), a_{i,t+1}(z^t))_{,z^t})_{t=0}^{\infty}$ solves each consumer i's problem, i.e.,

$$\max_{\{(c_{i,t}(z^t), l_{i,t}(z^t), a_{i,t+1}(z^t))_{z^t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \sum_{z^t \in Z_i^t} P_{i,t}(z^t)\beta_i^t u(c_{i,t}(z^t)) - v(l_{i,t}(z^t)) \qquad s.t.$$

$$\forall t \geq 0, z^t, \qquad (1+\tau_{c,t})c_{i,t}(z^t) + a_{i,t+1}(z^t) \leq l_{i,t}(z^t)w_{i,t}z_t - T_t(l_{i,t}(z^t)w_{i,t}z_t) + R_t a_{i,t}(z^{t-1}),$$

$$\forall t \geq 0, z^t, \qquad c_{i,t}(z^t) \geq 0, a_{i,t+1}(z^t) \geq 0, l_{i,t}(z^t) \geq 0,$$

$$\text{given } a_0^i > 0,$$

where $R_t = [1 + (1-\tau_{s,t})(r_{s,t} - \delta_s)] = [1 + (1-\tau_{e,t})(r_{e,t} - \delta_e)]$ is the after-tax return to savings via holding bonds, structure capital, or equipment capital.

2. In each period $t \geq 0$, taking factor prices as given, $(K_{s,t}, K_{e,t}, L_{s,t}, L_{u,t})$ solves the following firm's problem:

$$\max_{K_{s,t}, K_{e,t}, L_{s,t}, L_{u,t}} \qquad F(K_{s,t}, K_{e,t}, L_{s,t}, L_{u,t}) - r_{s,t}K_{s,t} - r_{e,t}K_{e,t} - w_{s,t}L_{s,t} - w_{u,t}L_{u,t}.$$

3. Markets for assets, labor, and goods clear: for all $t \geq 0$,

$$K_{s,t} + K_{e,t} + D_t = \sum_{i=u,s} \pi_i \sum_{z^{t-1} \in Z_i^{t-1}} P_{i,t-1}(z^{t-1}) a_{i,t}(z^{t-1}), \text{ where } z^{-1} \text{ is the null history,}$$

$$L_{i,t} = \pi_i \sum_{z^t \in Z_i^t} P_{i,t}(z^t) l_{i,t}(z^t) z_t \text{ for } i = u, s,$$

$$G_t + \sum_{i=u,s} \pi_i \sum_{z^t \in Z_i^t} P_{i,t}(z^t) c_{i,t}(z^t) + K_{s,t+1} + K_{e,t+1} = \tilde{F}(K_{s,t}, K_{e,t}, L_{s,t}, L_{u,t}).$$

4. The government's budget constraint is satisfied every period: for all $t \geq 0$,

$$G_t + R_t D_t = D_{t+1} + \sum_{j=s,e} \tau_{j,t}(r_{j,t} - \delta_j) K_{j,t} + \sum_{i=u,s} \pi_i \sum_{z^t \in Z_i^t} P_{i,t}(z^t) \Big(T_t(l_{i,t}(z^t) w_{i,t} z_t) + \tau_{c,t} c_{i,t}(z^t) \Big).$$

B Numerical Method

In this section, we outline our solution method.

General Solution Method for the Calibration Exercise. Traditionally, one would solve for a SRCE for a fixed set of parameters, including those that will be calibrated (i.e., $\omega, \nu, \phi, \beta_s, \beta_u$). This is done in the following way.

1. Start with an initial guess on prices w_s, w_u, R, government expenditure G and transfers T.

2. **Inner loop:** Solve for the policy functions given these parameters. The details of this computation are explained below.

3. Find the stationary distributions λ_s, λ_u implied by the policy functions.

4. Compute aggregate capital and labor supplies along with output. Given these, compute prices w_s, w_u, R implied by demand, government expenditure G as a fraction of output. Compute the implied transfers T that clear government budget.

5. Check if these prices and government policies coincide with the initial guesses. If not, update the guesses on prices, transfers and government expenditure and iterate.

One would normally solve this problem for each set of parameters during the calibration procedure, in which we are calibrating $\omega, \nu, \phi, \beta_s, \beta_u$ to hit a selected set of targets. We found it useful to include the calibration procedure directly into the loop above. Our procedure that combines solving for the SRCE with the calibration procedure is as follows.

1. Start with an initial guess on $w_s, w_u, R, G, T, \omega, \nu, \phi, \beta_s, \beta_u$.

2. Repeat steps 2. - 4. from above.

3. Check if the prices and government policies coincide with the initial guesses. Check if the aggregate labor supply, the skill premium, the labor share, the capital-to-output ratio and the relative asset holdings match the targets (see Table 2). If not, update the guesses on $w_s, w_u, R, G, T, \omega, \nu, \phi, \beta_s, \beta_u$ and iterate.

Solving the Inner Loop. Next, we briefly outline our version of the endogenous grid method (EGM) for the incomplete markets model with endogenous labor, i.e. how we solve the 'inner loop' above. The policy function iteration version of the standard EGM with fixed labor and income shocks captured by shocks to y_t can be summarized as follows (we find is useful to use time indices, although this method can be used for stationary problems as well):

1. With an initial guess on a_{t+2} and a fixed a_{t+1}, we use the Euler equation to recover a_t

$$\begin{aligned} u'(c_t) &= \beta R_{t+1} E_t \left[u'(c_{t+1})\right] \\ u'(R_t a_t + y_t - a_{t+1}) &= \beta R_{t+1} E_t \left[u'(R_{t+1} a_{t+1} + y_{t+1} - a_{t+2})\right] \\ a_t &= \frac{1}{R_t} \left\{ (u'^{-1} \left(\beta R_{t+1} E_t \left[u'(R_{t+1} a_{t+1} + y_{t+1} - a_{t+2})\right]\right) - y_t + a_{t+1} \right\} \end{aligned} \tag{20}$$

2. Once we have $a_t(a_{t+1}, y_t)$ we 'invert' it to get $a_{t+1}(a_t, y_t)$. We also recover $c_t(a_t, y_t)$. Then we go backward starting in the last period in a finite horizon problem. We iterate until $a_{t+1} = a_{t+2}$ in an ∞ problem.

Our Method with Endogeneous Labor. In our model the complication is that income is endogeneous, because the labor choice is endogeneous, so that labor income of type $i \in \{s, u\}$ agent in period t is: $y_{i,t} = w_{i,t} z_t \cdot l_{i,t}$ with $l_{i,t}$ endogenous. To take care of endogenous labor (and consumption taxes, which were not included in the discussion above), we need to take into account the intratemporal optimality condition (dropping the index i for type for simplicity):

$$\lambda(1 - \tau_l)(w_t z_t l_t)^{-\tau_l} w_t z_t u'(c_t) = -(1 + \tau_c) v'(l_t) \tag{21}$$

Therefore the system we need to solve is:

$$\begin{aligned} u'(c_t) &= \beta R_{t+1} E_t \left[u'(c_{t+1})\right] \\ \lambda(1 - \tau_l)(w_t z_t l_t)^{-\tau_l} w_t z_t u'(c_t) &= -(1 + \tau_c) v'(l_t) \\ (1 + \tau_c) c_t + a_{t+1} &= \lambda (w_t z_t l_t)^{1-\tau_l} + R_t a_t \end{aligned}$$

The intratemporal optimality condition is non-linear and thus costly to solve numerically. We therefore solve the non-linear intratemporal optimality condition only occasionally, similarly to the method proposed by Barillas and Fernandez-Villaverde (2007). In our model, we assume that government debt is a given fraction of output. Transfers are included in the labor tax function. We also need to take into account that the tax function takes mean income $\bar{y}$ as an argument. Our method can thus be summarized as follows:

1. **Loop in labor policies:** Fix an initial guess on policy $l_t(a_t, z_t)$ and labor disutility parameter ϕ.

2. **Loop in prices and calibrating the parameters:** Fix $w_s, w_u, R, B, \omega, \nu, \beta_s, \beta_u, \lambda, \bar{y}$.

   (a) We use $y_t = \lambda(w_t z_t l_t)^{1-\tau_l}$ and solve for policies c_t and a_{t+1} as if y_t was exogenous using equation (20). Observe that to use equation (20), we need to express the labor policy as $l_t(a_{t+1}, z_t)$ rather than $l_t(a_t, z_t)$. We use $l_t(a_t, z_t)$ and $a_t(a_{t+1}, z_t)$ to get $l_t(a_{t+1}, z_t)$. This approach is in fact very similar similar to the original endogeneous grid idea.

   (b) We find the stationary distributions λ_s, λ_u implied by the policy functions.

   (c) We then compute aggregate capital and labor supplies along with output. Observe that while labor policies are constant in this loop, labor supply will depend on the stationary asset distributions. Given these, we compute prices w_s, w_u, R implied by demand, mean labor income $\bar{y}$, government debt B and government expenditure G as a given fraction of as a fraction of output.

   (d) We then check if the prices coincide with the initial guesses. We check if the new $\bar{y}$ and B coincide with the guesses and whether the government budget balances (given that G is a given fraction of output). We check if the aggregate labor supply, the skill premium, the labor share, the capital-to-output ratio and the relative asset holdings match the targets (see Table 2). If not, we update the guesses on $w_s, w_u, R, B, \omega, \nu, \beta_s, \beta_u, \lambda, \bar{y}$ and iterate.

3. Given the policy c_t we find the labor policy $\hat{l}_t$ that satisfies the intratemporal first order condition (21). We set ϕ so that aggregate labor hits the target.

4. We then use $\alpha l_t + (1-\alpha)\hat{l}_t$ (with $\alpha \in (0,1)$) as a new guess for the labor policy and iterate until convergence. While we have no theorem that guarantees convergence, we find that the procedure performs well in our model.

Solution Method for the Reform Exercise. We use the same method as just outlined with one difference. In Step 2., we keep $B, \omega, \nu, \beta_s, \beta_u, \lambda, \bar{y}$ fixed and search for equilibrium w_s, w_u, R, as well as for $\tau_s = \tau_e$ that clear government budget.

C Proof of Proposition 1

First, we prove that any competitive equilibrium has to satisfy equations (4), (9), and (10). In any equilibrium, (4) has to be satisfied since it is the goods market equilibrium condition.

Now we prove that any equilibrium allocation has to satisfy (9). Remember that a consumption, labor and asset allocation, $(c_t, l_{i,t}, a_{t+1})_{t=0}^{\infty}$ solves the agent's problem if and only if it satisfies the first-order optimality conditions (5) − (7), the budget constraints and the transversality condition

$$\lim_{t\to\infty} p_t a_{t+1} = 0.$$

Now, multiply each period t budget constraint of the agent with the Lagrange multiplier associated with it and sum over time. Using (7) and the transversality condition above, this expression becomes

$$\sum_{t=0}^{\infty} p_t[c_t - (1-\tau_t)\sum_{i=s,u} l_{i,t}w_{i,t}] = p_0 R_0 a_0. \tag{22}$$

Substituting p_t out using (5) and (6), we get (9).

Finally, (10) follows from the comparison of (6) for the two types of labor.

Second, suppose that a consumption, labor, and capital allocation and period 0 capital tax rates and return on government bond satisfy conditions (4), (9), and (10). Now, we construct an equilibrium.

First, for all $t \geq 0$, define factor prices using (8). Under these prices, capital and labor allocations satisfy firm's optimality conditions by construction.

Second, define the interest rate on government bonds: for all $t \geq 0$,

$$R_{t+1} = \frac{u'(c_t)}{\beta u'(c_{t+1})}. \tag{23}$$

The taxes on the two types of capital are defined using the arbitrage condition, equation (2), for all $t \geq 1$.

Now, define labor taxes: for all $t \geq 0$,

$$\frac{u'(c_t)}{v'(l_{u,t})} = \frac{1}{(1-\tau_t)}\frac{1}{w_{u,t}}. \tag{24}$$

Define asset holdings in any period $t \geq 1$ as follows. Multiply period t budget constraint by $\beta^t u'(c_t)$, sum it over all periods following t, and using (23) and (24) with the transversality condition, we get:

$$a_t = \sum_{m=t}^{\infty} \beta^{\tau-t+1}[u'(c_t)c_t + \sum_{i=s,u} v'(l_{i,t})l_{i,t}]/u'(c_{t-1}). \tag{25}$$

Under this construction of taxes and prices, the consumption, labor, and asset allocations satisfy agent's first-order conditions $(5)-(7)$ and period budget constraint in every period $t \geq 0$. Thus, agent's optimality is also satisfied.

Good's market clearing condition is satisfied by condition (4).

Finally, define government's bond holdings D_t in any period $t \geq 0$ to satisfy the asset market clearing condition (3).

To complete the proof, we have to verify that D_t defined as above satisfies government's budget every period. To see this, use (3) to plug in for a_t in agent's period t budget constraint. This gives:

$$c_t + K_{s,t+1} + K_{e,t+1} + D_{t+1} = (1-\tau_t) \sum_{i=s,u} L_{i,t} w_{i,t} + R_t (K_{s,t} + K_{e,t} + D_t).$$

Using the second part of (23) and rearranging, we get:

$$D_{t+1} = \sum_{i=s,u} (1-\tau_t) L_{i,t} w_{i,t} + \sum_{j=s,e} K_{j,t} [1 + (1-\tau_{j,t})(r_{j,t} - \delta_j)] + R_t D_t - c_t - K_{s,t+1} - K_{e,t+1},$$

or equivalently:

$$D_{t+1} + \tau_t L_{i,t} w_{i,t} + \sum_{j=s,e} K_{j,t} (r_{j,t} - \delta_j)(1-\tau_{j,t}) = \tilde{F}(K_{s,t}, K_{e,t}, L_{s,t}, L_{u,t}) - c_t - K_{s,t+1} - K_{e,t+1} + R_t D_t,$$

which, using (4), proves that the government budget constraint holds with equality. ∎