Reward-to-Risk Ratios of Fund of Hedge Funds

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ABSTRACT

This chapter examines whether the fund of hedge fund portfolios dominate the U.S. equity and bond markets based on alternative measures of reward-to-risk ratios. Standard deviation is used to measure total risk and both nonparametric and parametric value-at-risk is used to measure downside risk when the reward-to-risk ratios are constructed. We find that the fund of funds index has higher reward-to-risk ratios compared to several stock and bond market indices. This result is especially strong when the risk measures are calculated from the most recent year's data and is robust as the measurement window is extended to four years.

1. INTRODUCTION

Investors base their portfolio asset allocation decisions on the interactions between risks and returns of available financial securities. The assumption of risk aversion implies that securities with greater risk should demand greater return. Although the trade-off between risk and return is well-established in financial economics, the ability to generate higher expected returns per unit risk can vary from one security to another. This chapter compares various reward-to-risk ratios for the Fund of Hedge Fund (FoHF) index with those of several bond and stock market indices.

Traditional risk measures used in portfolio performance measurement assume that returns are normally distributed and therefore the standard deviation of the empirical return distribution is a good estimate of risk only if the underlying return distribution is close to normal. The first measure of reward-to-risk that we use is the Sharpe ratio (1966) which is equal to the ratio of the mean excess return of a portfolio to its standard deviation. The Sharpe ratio is the most common measure of how well the return of a portfolio compensates the investor for the risk taken. However, a common criticism is that it is too broad since it includes the total risk of a portfolio in its denominator. Another potential issue regarding the calculation of Sharpe ratios for the FoHF index is the non-normality of hedge fund return distributions.

The hedge fund literature provides evidence that distributions of hedge fund returns tend to deviate from normality. Malkiel and Saha (2005) report that the distribution of hedge fund returns generally have high kurtosis and negative skewness. The documented deviation from normality can be traced to the unique investment strategies that hedge funds follow. Fung and Hsieh (1997) observe that hedge fund managers are flexible to choose among a diverse set of asset classes and they can use dynamic trading strategies that involve short sales, leverage and derivatives. Such strategies have the potential to induce option-like payouts and exposure to tail events for hedge funds. In a follow-up study, Fung and Hsieh (2001) focus on hedge funds that use trend-following strategies. They construct several trend-following factors that can replicate key features of hedge fund returns such as skewness and positive returns during extreme market movements. Mitchell and Pulvino (2001) investigate merger arbitrage strategies and conjecture that returns to risk arbitrage are related to market returns in a nonlinear way. Their results indicate that merger risk arbitrage is similar to writing uncovered index put options. Agarwal and Naik (2004) find that nonlinear payoff structures exist for a wide range of hedge fund strategies including equity-oriented positions. They state that ignoring the downside risk of hedge funds can result in significantly higher losses during large market downturns. Brown, Gregoriou and Pascalau (2009) look at the diversification effect of investing in FoHFs and find that the magnitude of skewness is an increasing function of diversification offered by FoHFs. Their finding suggests that downside risk exposure may not be diversifiable. Finally, Bali, Gokcan and Liang (2007) and Liang and Park (2007) provide direct evidence that downside risk measures such as value-at-risk, expected shortfall and tail risk can explain the cross-section of hedge fund returns.

Downside risk is a function of the higher order moments of a return distribution and even without the existence of nonlinear payoffs, higher order moments such as skewness and kurtosis have been found to play an important role in asset pricing. The mean-variance portfolio theory of Markowitz (1952) has been extended by Arditti (1967) and Kraus and Litzenberger (1976) to incorporate the effect of skewness. These studies present three-moment asset pricing models with investors that hold concave preferences and prefer positive skewness. The main implication of these models is that assets that increase a portfolio's skewness are more desirable and should command lower expected returns. Harvey and Siddique (2000) extend these unconditional pricing models and incorporate conditional co-skewness. Again, the implication is that risk-averse investors prefer positively skewed assets to negatively skewed assets. As far as the fourth-moment is concerned, Dittmar (2002) builds on the theoretical works of Kimball (1993) and Pratt and Zeckhauser (1987) and finds preference for lower kurtosis. Asset distributions with lower probability mass in their tails are preferred and therefore assets that increase a portfolio's kurtosis are less desirable and should command higher expected returns.

Downside risk increases with kurtosis and decreases with skewness (Cornish and Fisher (1937)). Given the importance of these return moments for asset pricing and the prevalence of downside risk in hedge fund returns, we place special emphasis on the concept of downside risk in our reward-to-risk analysis. To investigate how much expected return each index commands per unit of downside risk, we use both a nonparametric and parametric measure of value-at-risk in the construction of the alternative reward-to-risk ratios. For the nonparametric VaRSharpe ratio, the denominator is the absolute value of the minimum index return over various past sample windows. For the parametric reward-to-downside risk measure (PVaRSharpe), the denominator is based on the lower tail of Hansen's (1994) skewed *t*-density.

The results indicate that the FoHF index outperforms the bond and stock market indices based on traditional Sharpe ratios on average. Although the Sharpe ratios decrease for every index as the sampling window for the calculation of standard deviation is extended and this decline is most pronounced for the FoHF index, it has the highest Sharpe ratio regardless of the sampling window. When we take downside risk into account through nonparametric and parametric value-at-risk, the results are similar. The FoHF index has higher downside risk-adjusted Sharpe ratios compared to all bond and stock market indices and this result is especially strong at shorter sampling windows for value-at-risk measurement.

The chapter is organized as follows. Section 2 discusses the methodology for calculating the reward-to-risk ratios. Section 3 explains the data and presents the summary statistics. Section 4 discusses the empirical results. Section 5 concludes.

2. METHODOLOGY

We estimate three reward-to-risk ratios that differ from each other based on the risk measure used in the denominator. The first of these ratios is the standard Sharpe ratio:

$$Sharpe_{i,t} = \frac{R_{i,t} - R_{f,t}}{StDev_{i,t}} \tag{1}$$

where $R_{i,t}$ denotes the month t return on the fund of funds, bond or stock market index i and R_f is the risk-free rate as measured by the 1-month Treasury bill return. The standard deviation for index i is computed using the squared deviations of monthly returns from their means. For each month t and index i, past k months are used to compute the standard deviation where k takes the alternative values of 12, 24, 36 or 48. Specifically,

$$StDev_{i,t} = \sqrt{\frac{1}{k-1}\sum_{j=0}^{k}\left(R_{i,t-j} - \overline{R}_i\right)^2} \tag{2}$$

In order to take downside risk into account, we first use a nonparametric measure of value-at-risk which measures how much the value of a portfolio could decline in a fairly extreme outcome. In our analysis, we use the minimum index returns observed during past k months of daily data where k again takes the alternative values of 12, 24, 36 or 48. These original value-at-risk measures are multiplied by -1 before the construction of the reward-to-risk ratios so that higher magnitudes of the measure correspond to greater downside risk. After we calculate nonparametric value-at-risk measures each month using rolling windows, Sharpe ratios that incorporate these nonparametric value-at-risk estimates are computed. Specifically, VaRSharpe is defined as:

$$VaRSharpe_{i,t} = \frac{R_{i,t} - R_f}{VaR_{i,t}} \tag{3}$$

where $VaR_{i,t}$ is the nonparametric value at risk.

Finally, for the parametric measure of value-at-risk, we use the skewed *t*-density, which accounts for skewness and excess kurtosis in the data. Hansen (1994) introduces a generalization of the Student *t*-distribution where asymmetries may occur, while maintaining the assumption of a zero mean and unit variance. This skewed *t* (ST) density is given by:

$$f(z_t;\mu,\sigma,v,\lambda)=\begin{cases} bc\left(1+\dfrac{1}{v-2}\left(\dfrac{bz_t+a}{1-\lambda}\right)^2\right)^{-\frac{v+1}{2}} & \text{if } z_t<-a/b \\ bc\left(1+\dfrac{1}{v-2}\left(\dfrac{bz_t+a}{1+\lambda}\right)^2\right)^{-\frac{v+1}{2}} & \text{if } z_t\geq -a/b \end{cases} \tag{4}$$

where $z_t=\dfrac{R_t-\mu}{\sigma}$ is the standardized excess market return and the constants *a*, *b*, and *c* are given by

$$a=4\lambda c\left(\frac{v-2}{v-1}\right)b^2=1+3\lambda^2-a^2,\ c=\frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi(v-2)}\Gamma\left(\frac{v}{2}\right)} \tag{5}$$

The parametric approach to calculating value-at-risk is based on the lower tail of the ST distribution. Specifically, we estimate the parameters of the ST density (μ, σ, υ, λ) using the past 12, 24, 36 or 48 months of return data and then find the corresponding percentile of the estimated distribution. Assuming that $R_t=f_{v,\lambda}(z)$ follows an ST density, parametric value-at-risk is the solution to

$$\int_{-\infty}^{\Gamma_{ST}(\Phi)} f_{v,\lambda}(z)dz = \Phi \tag{6}$$

where $\Gamma_{ST}(\Phi)$ is the value-at-risk threshold based on the ST density with a loss probability of Φ. Sharpe ratios that incorporate parametric value-at-risk are defined as:

$$PVaRSharpe_{i,t} = \frac{R_{i,t} - R_f}{PVaR_{i,t}} \tag{7}$$

3. DATA AND DESCRIPTIVE STATISTICS

We gather the data for the FoHF index returns from the Hedge Fund Research (HFR) database. The database reports monthly index values for various hedge fund strategies beginning from January 1990 and the sample period used in the following analysis extends until December 2011. HFR indices are broken down into four main strategies, each with multiple sub-strategies. These strategies include equity hedge (equity market neutral, quantitative directional, short-bias, etc.), event driven (distressed / restructuring, merger arbitrage, etc.), macro (commodity, currency, etc.) and relative value (convertible arbitrage, fixed-income corporate, etc.). HFR also reports a Fund of Funds Composite index which includes over 650 constituent funds. FoHFs invest with multiple managers through funds or managed accounts and their main benefit is designing a diversified portfolio of managers to reduce the risk of investing with an individual manager. Fund of Funds Composite index is an equally-weighted index and it is commonly used by hedge fund managers as a performance benchmark. A fund needs to report monthly gross returns and returns net of all fees to be included in the index. Moreover, the assets need to be reported in US dollars and the fund needs to have at least \$50 million under management or have been actively trading for at least twelve months. Funds are included in the composite index the month after their addition to the database.

We also collect data for various bond and stock market indices for comparison purposes. Specifically, we collect price data for indices that track Treasury bonds with maturities of 5, 10, 20 and 30 years. For equities, we focus on the S&P 500 index and the NYSE/AMEX/NASDAQ index with distributions. All the data for the bond and stock market indices come from the Center for Research in Security Prices (CRSP). The yield for the 1-month Treasury bill which is used to proxy for the risk-free rate is downloaded from Kenneth French's online data library.

Table 1 reports the descriptive statistics for all indices. A comparison of means shows that the NYSE/AMEX/NASDAQ index has the highest monthly return (0.78%), however the S&P 500 index has not generated as high an average return (0.58%). This difference can be explained by the greater returns generated by small stocks historically. The mean returns on the bond indices increase by time to maturity with the 5-year bond index delivering 0.56% per month and the 30-year bond index delivering 0.73% per month. In terms of means, the FoHF index sits somewhere in the middle in this picture with a monthly mean return of 0.61%. The medians tell a similar story with the biggest difference being that both stock market indices have generated higher median returns than all other indices. NYSE/AMEX/NASDAQ index had a median return of 1.34% over the sample period whereas S&P 500 index had a median return of 1.01%. Again, the median returns for the bond indices increase by time to maturity and vary from 0.58% to 0.89%. The FoHF index still positions itself in the middle with a median return of 0.77%.

Table 1. Descriptive Statistics for Fund of Hedge Funds, Bond and Equity Indices

This table presents descriptive statistics for the returns of various fund of hedge funds, bond and equity indices in the US. The four bond market indices are based on 5-year, 10-year, 20-year and 30-year maturity Treasury bonds. The two equity indices are the S&P 500 index and the NYSE/AMEX/NASDAQ Composite index. The descriptive statistics that are presented in the table are the mean, standard deviation, minimum, 25th percentile, median, 75th percentile, maximum, skewness and kurtosis.

	Mean	St Dev	Min	25th	Median	75th	Max	Skew	Kurtosis
Fund of Funds	0.0061	0.0171	-0.0747	-0.0021	0.0077	0.0159	0.0685	-0.6718	6.7061
5-year bond	0.0056	0.0128	-0.0338	-0.0020	0.0058	0.0145	0.0452	-0.1755	3.3092
10-year bond	0.0063	0.0203	-0.0668	-0.0058	0.0071	0.0190	0.0854	-0.0719	4.0789
20-year bond	0.0077	0.0286	-0.1059	-0.0084	0.0087	0.0244	0.1445	0.0619	5.7720
30-year bond	0.0073	0.0290	-0.1474	-0.0134	0.0089	0.0270	0.1741	0.2930	6.7463
S&P 500	0.0058	0.0439	-0.1694	-0.0195	0.0101	0.0340	0.1116	-0.5630	3.9987
NYSE/AMEX/ NASDAQ	0.0078	0.0455	-0.1846	-0.0189	0.0134	0.0385	0.1153	-0.6827	4.2297

With respect to the standard deviations, we find that the stock market indices are generally more volatile compared to the bond market indices. NYSE/AMEX/NASDAQ and S&P 500 indices have monthly standard deviations of 4.55% and 4.39%, respectively. The standard deviations of the bond market indices increase from 1.28% for the 5-year bond index to 2.90% for the 30-year bond index. This finding is in line with the higher interest rate sensitivities associated with bonds of longer durations. The FoHF index has the second lowest standard deviation which is equal to 1.71%.

The patterns for standard deviations also manifest themselves when we look at the maximum and minimum returns. The highest (lowest) maximum (minimum) returns belong to the equity indices and the bond indices with longer times to maturity. For example, there has been a month in which the NYSE/AMEX/NASDAQ index has gained

11.53% in value and the 30-year bond index has gained 17.41% in value. Similarly, there has been a month during which the NYSE/AMEX/NASDAQ index has lost 18.46% of its value and the 30-year bond index has lost 14.74% of its value. The extreme returns for the FoHF index are milder with a minimum monthly return of -7.47% and a maximum monthly return of 6.85%. This finding is consistent with the diversification effects inherent in fund of funds strategies as argued in Fung and Hsieh (2000).

Finally, we compare the higher order moments of the indices. The FoHF index has the second most negative skewness statistic (-0.67) after the NYSE/AMEX/NASDAQ index (-0.68). The other stock market index, S&P 500, also has negative skewness (-0.56). This is consistent with earlier findings in the literature that the tails of the hedge fund and equity return distributions are longer on the left side compared to the right side. The negative skewness associated with these indices was also foreshadowed by their higher medians compared to the means. For the bond market indices, the skewness statistic increases with time to maturity. The 5-year bond index has a skewness statistic of -0.18 whereas the 30-year bond index distribution is positively skewed with a statistic of 0.29.

The kurtosis of the FoHF index is again substantial and equal to 6.71. In other words, the FoHF return distribution has more mass on its tails compared to the normal distribution and thus, is leptokurtic. Kurtosis again increases with time to maturity for the bond market indices from 3.31 to 6.75. The kurtosis for stock market indices lie somewhere in the middle among the bond market indices with a kurtosis statistic for the NYSE/AMEX/NASDAQ (S&P 500) index equal to 4.00 (4.23).

4. EMPIRICAL RESULTS

Table 2 presents the traditional Sharpe ratios that incorporate the standard deviation of a portfolio in its denominator. We calculate these monthly Sharpe ratios in a rolling window fashion and use different sampling windows to calculate the standard deviations. The length of the sampling windows ranges from 12 to 48 months. We present both the time-series mean and the standard deviations of the reward-to-risk ratios for all indices.

Table 2. Standard Deviation-Based Sharpe Ratios for Fund of Hedge Funds, Bond and Equity Indices

This table presents the standard deviation-based Sharpe ratios for various fund of hedge funds, bond and equity indices in the US. The four bond market indices are based on 5-year, 10-year, 20-year and 30-year maturity Treasury bonds. The two equity indices are the S&P 500 index and the NYSE/AMEX/NASDAQ Composite index. The numerator of the standard deviation-based Sharpe ratio is equal to the monthly return of the index minus the risk-free rate. The denominator is equal to the standard deviation of monthly returns over the past 12, 24, 36 or 48 months. Each row reports the means of each ratio and the standard deviations are presented in parentheses.

	Sharpe12		Sharpe24		Sharpe36		Sharpe48	
Fund of Funds	0.3516	(0.5182)	0.2979	(0.3166)	0.2629	(0.2363)	0.2357	(0.1517)
5-year bond	0.2223	(0.3709)	0.2127	(0.2823)	0.2088	(0.2338)	0.2037	(0.1857)
10-year bond	0.1801	(0.3278)	0.1676	(0.2080)	0.1630	(0.1592)	0.1561	(0.1123)
20-year bond	0.1949	(0.3024)	0.1766	(0.1639)	0.1724	(0.1204)	0.1674	(0.0821)
30-year bond	0.1397	(0.3072)	0.1252	(0.1576)	0.1231	(0.1119)	0.1179	(0.0744)
SP500	0.1653	(0.3601)	0.1292	(0.2472)	0.1104	(0.1988)	0.1054	(0.1719)
NYSE/AMEX/ NASDAQ	0.2324	(0.3703)	0.1904	(0.2443)	0.1679	(0.1967)	0.1583	(0.1656)

When the standard deviation is calculated from the most recent year's data, the FoHF index generates the highest excess return per unit risk. The Sharpe ratio for FoHF is equal to 0.352 which implies that the index demands extra 35 basis points of expected return per 1% increase in standard deviation. The comparison between the bond and stock market indices does not present any clear pattern. Although the NYSE/AMEX/NASDAQ index has a superior Sharpe ratio (0.232) compared to all the bond indices, the S&P index lags behind most of the bond indices with a Sharpe ratio of 0.165. There is a declining pattern for the bond indices with Sharpe ratios of 0.222 for the 5-year bond index and 0.140 for the 30-year bond index. Another point to note is that the FoHF index also has the highest variation in Sharpe ratios. We observe this pattern for the other ratios as well.

Extending the sampling window for calculating standard deviations to 24 months does not dramatically alter the results. The Sharpe ratio of the FoHF index declines to 0.298 from 0.352, but it is still the index that generates the highest excess return per unit risk. Note that the reduction in the Sharpe ratio is mechanical due to the positive relation between standard deviation and time horizon and this reduction is encountered for all indices. The NYSE/AMEX/NASDAQ index has a greater Sharpe ratio (0.190) compared to all bond indices except the 5-year bond index (0.213). On the other hand, the S&P 500 index has a smaller Sharpe ratio (0.129) compared to all bond indices except the 30-year bond index (0.125). For sampling windows of 36 and 48 months, the results are similar except that S&P 500 now has the lowest Sharpe ratios and the 20-year bond index begins to outperform the NYSE/AMEX/NASDAQ index. Most importantly, the FoHF index has the highest Sharpe ratio regardless of the sampling window for the standard deviation.

One final point is that the decrease in the Sharpe ratios as the sampling window increases is sharper for the FoHF index compared to the other indices. For the 12-month window, the Sharpe ratio of the FoHF index exceeds its closest follower by 0.120 (0.352 vs. 0.232) whereas the difference is reduced to 0.038 (0.236 vs. 0.204) for the 48-month window.

These results collectively suggest that the FoHF index generates a higher excess return per unit risk when risk is measured by standard deviation. However, there is enough evidence in the literature to believe that the standard deviation is an incomplete measure of risk for hedge fund returns whose distribution deviates from normality. This is also evidenced by the negatively skewed and leptokurtic behavior of the FoHF index returns in Table 1. Therefore, to take the nonlinearities hedge fund returns into account, we calculate alternative Sharpe ratios based on nonparametric and parametric value-at-risk.

Table 3 presents Sharpe ratios that are based on nonparametric value-at-risk. These VaRSharpe ratios scale expected excess returns by the absolute value of the minimum return of a portfolio during a recent sample window where the length of the window varies between 12 and 48 months. When we focus on *VaRSharpe12*, we find that the FoHF index generates the highest excess return per unit downside risk. The ratio for the FoHF index is equal to 2.207 and exceeds those of the other indices multiple-fold. We again note that the time-series standard deviation of the VaRSharpe measure is the greatest for the FoHF index. In other words, although the FoHF index easily outperforms the other indices based on this particular metric, this outperformance seems to be variable through time. *VaRSharpe12* for the NYSE/AMEX/NASDAQ index is equal to 0.254) and greater than those of all bond market indices except the 5-year bond index which has a *VaRSharpe12* of 0.320. We observe that the downside risk-adjusted Sharpe ratio has a declining pattern for the bond market indices as the time to maturity increases and the 30-year bond index has a *VaRSharpe12* of 0.239. The S&P 500 index has a similar performance with a *VaRSharpe12* of 0.245. To summarize, the FoHF index is the superior performer based on *VaRSharpe12* and neither the bond nor the stock market indices clearly dominate each other.

Table 3. Nonparametric Value at Risk-Based Sharpe Ratios for Fund of Hedge Funds, Bond and Equity Indices

This table presents the nonparametric value at risk-based Sharpe ratios for various fund of hedge funds, bond and equity indices in the US. The four bond market indices are based on 5-year, 10-year, 20-year and 30-year maturity Treasury bonds. The two equity indices are the S&P 500 index and the NYSE/AMEX/NASDAQ Composite index. The numerator of the nonparametric value at risk -based Sharpe ratio is equal to the monthly return of the index minus the risk-free rate. The denominator is equal to the minimum monthly index return over the past 12, 24, 36 or 48 months. Each row reports the means of each ratio and the standard deviations are presented in parentheses.

	VaRSharpe12		VaRSharpe24		VaRSharpe36		VaRSharpe48	
Fund of Funds	2.2073	(5.8254)	0.9947	(3.9006)	0.2004	(0.2793)	0.1306	(0.1267)
5-year bond	0.3198	(0.6192)	0.1565	(0.2003)	0.1260	(0.1352)	0.1104	(0.0973)
10-year bond	0.2187	(0.4049)	0.1128	(0.1402)	0.0936	(0.0974)	0.0779	(0.0612)
20-year bond	0.2473	(0.8470)	0.1055	(0.1091)	0.0825	(0.0676)	0.0738	(0.0415)
30-year bond	0.2387	(1.4856)	0.0778	(0.1037)	0.0585	(0.0631)	0.0497	(0.0364)
SP500	0.2452	(0.6728)	0.0831	(0.1398)	0.0608	(0.1050)	0.0513	(0.0825)
NYSE/AMEX/ NASDAQ	0.2544	(0.4572)	0.1206	(0.1473)	0.0897	(0.1124)	0.0742	(0.0874)

When we extend the sampling window to calculate nonparametric value-at-risk, the VaRSharpe ratios again decline mechanically. The reason is that the absolute value of the minimum return during the last 48 months has to be equal to or greater than that during the last 12 months. Analyzing the longer horizon VaRSharpe ratios makes some patterns apparent. First, the FoHF index continues to be the best performer regardless of the sampling window. Second, the 5-year bond index continues to have the highest VaRSharpe ratio after the FoHF index and for the 36-month and 48-month horizons, the

10-year bond index also outperforms the NYSE/AMEX/NASDAQ index. Third, the S&P 500 index continues to have the lowest excess return per unit downside risk after the 30-year bond index. Finally, similar to the results from the traditional Sharpe ratio analysis, the margin by which the VaRSharpe ratio of the FoHF index exceeds those of the other indices declines as the sampling window increases. For example, *VaRSharpe12* of the FoHF index is seven times as much as that of the 5-year bond index which its closest follower. However, as the sampling window is extended to 48 months, the difference between the VaRSharpe ratios decreases substantially. This is due to the fact that the reduction in the VarSharpe ratios is much steeper for the FoHF index compared to the other indices. *VaRSharpe48* measures for the FoHF and the 5-year bond indices are equal to 0.131 and 0.110, respectively.

Next, we investigate the reward-to-risk ratios that have parametric value-at-risk based on Hansen's (1994) skewed t-density in their denominators. Table 4 presents the results. The inference from the analysis of PVaRSharpe ratios corroborates the findings from Table 3. When we focus on the 12-month sampling horizon for the construction of the parametric downside risk measure, we find that the FoHF index again has the highest reward-to-risk ratio with a *PVaRSharpe12* of 1.104. One can also see that the 10-year bond index also performs well for this metric with a *PVaRSharpe12* of 0.643. The stock market indices, namely the NYSE/AMEX/NASDAQ and S&P 500 indices have PVaRSharpe ratios of 0.167 and 0.131, respectively. These values are lower than those of all bond market indices with the exception of the 30-year bond index. The extension of the sampling window again reduces the reward-to-risk ratios for all indices. The FoHF index continues to be the best performer regardless of the length of the sampling window. However, as observed for the traditional and nonparametric value-at-risk based Sharpe ratios, the decline in the PVaRSharpe ratio is steeper than the other indices. For example, the ratio of *PVaRSharpe12* of the FoHF index to that of the 5-year bond index is more than 4 when the 12-month sampling window is used whereas for the 48-month sampling window, the FoHF and 5-year bond indices have PVaRSharpe ratios of 0.137 and 0.108, respectively. A closer look at the results reveals that the bond market indices generally outperform the stock market indices and there is a downward trend in the reward-to-risk ratios among the bond market indices especially for longer sampling windows.

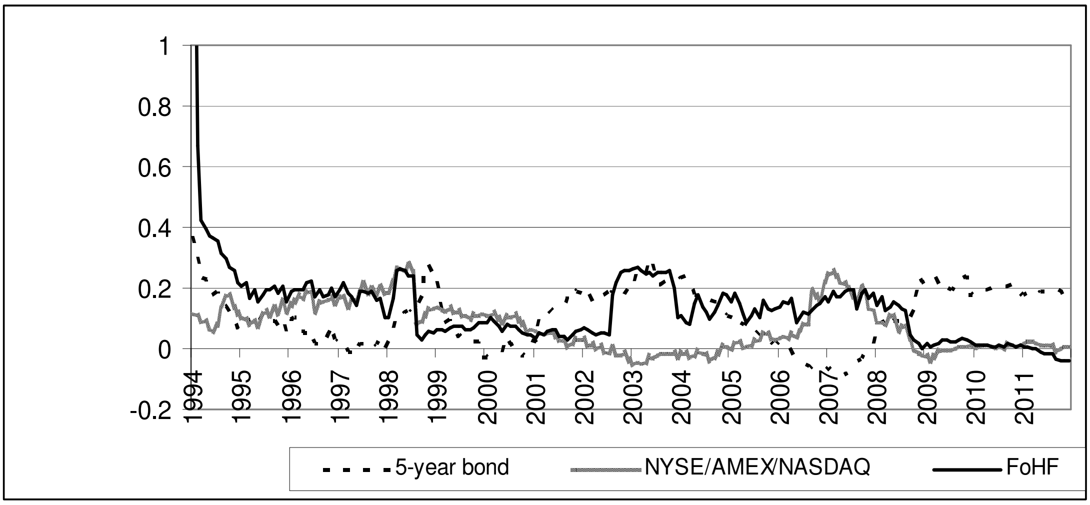
Table 4. Parametric Value at Risk -Based Sharpe Ratios for Fund of Hedge Funds, Bond and Equity Indices

This table presents the parametric value at risk-based Sharpe ratios for various fund of hedge funds, bond and equity indices in the US. The four bond market indices are based on 5-year, 10-year, 20-year and 30-year maturity Treasury bonds. The two equity indices are the S&P 500 index and the NYSE/AMEX/NASDAQ Composite index. The numerator of the parametric value at risk-based Sharpe ratio is equal to the monthly return of the index minus the risk-free rate. The denominator is equal to the first percentile of Hansen's (1994) skewed t-density estimated using the monthly returns from over the past 12, 24, 36 or 48 months. Each row reports the means of each ratio and the standard deviations are presented in parentheses.

	PVaRSharpe12		PVaRSharpe24		PVaRSharpe36		PVaRSharpe48	
Fund of Funds	1.1037	(4.3839)	0.6146	(1.7954)	0.2264	(0.4187)	0.1365	(0.1729)
5-year bond	0.2456	(0.7375)	0.1326	(0.1711)	0.1168	(0.1265)	0.1082	(0.0956)
10-year bond	0.6431	(7.6812)	0.0971	(0.1222)	0.0877	(0.0916)	0.0782	(0.0597)
20-year bond	0.2614	(1.4548)	0.1058	(0.1848)	0.0840	(0.0739)	0.0761	(0.0422)
30-year bond	0.1306	(0.2904)	0.0924	(0.3949)	0.0606	(0.0755)	0.0525	(0.0409)
SP500	0.1314	(0.2936)	0.0722	(0.1218)	0.0554	(0.0942)	0.0490	(0.0769)
NYSE/AMEX/ NASDAQ	0.1666	(0.2965)	0.1035	(0.1253)	0.0833	(0.0993)	0.0726	(0.0799)

Figures 1 and 2 present plots of traditional and nonparametric value-at-risk based Sharpe ratios, respectively. For these figures, we choose only one bond market and one stock market index to show the relative performance of the FoHF index to keep the exposition clean. To be conservative, we focus on the 5-year bond and NYSE/AMEX/NASDAQ indices which have proved to be the bond and stock market indices that have performed the best over the sample period. Moreover, we present the graphs for the reward-to-risk measures that use standard deviation and nonparametric value-at-risk calculated from a 48-month sampling window since the superior performance of the FoHF index becomes less pronounced as the sampling window is extended.

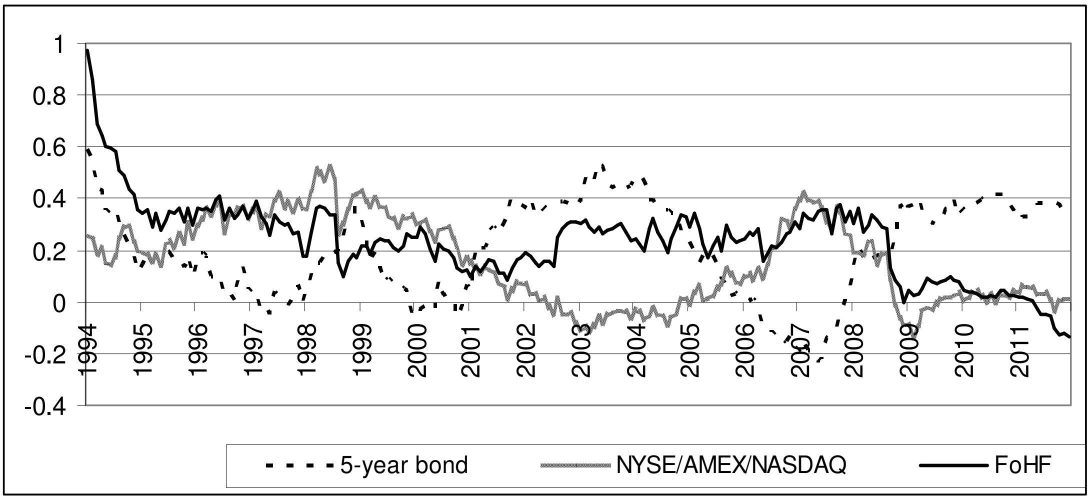
Figure 1. Standard Deviation-Based Sharpe Ratios



This figure plots the standard deviation-based Sharpe ratios for the fund of hedge funds, 5-year bond and NYSE/AMEX/NASDAQ indices between January 1994 and December 2011. The numerator of the standard deviation-based Sharpe ratio is equal to the monthly return of the index minus the risk-free rate. The denominator is equal to the standard deviation of monthly returns over the past 48 months.

The figures show that the FoHF index had a superior performance at the beginning of the sample period based on both reward-to-risk metrics, but the Sharpe ratios dropped to the level of the NYSE/AMEX/NASDAQ index by 1996. We see that the superior performance of the FoHF index is not uniform through time. This observation is consistent with the large volatility associated with the reward-to-risk ratios of the FoHF index uncovered in the earlier analysis. There have been periods in which either the 5-year bond or the NYSE/AMEX/NASDAQ or both have outperformed the FoHF index. One such period is the period after the recent global financial crisis and it can clearly be seen that the reward-to-risk ratios took a downward turn in the second half of 2008. During this period, the performance of the stock market has also been dismal and the 5-year bond index has generated higher returns per unit risk. Both figures also capture the stock market crash of the early last decade after the internet bubble burst as evidenced by the steep decline in the reward-to-risk ratios of the NYSE/AMEX/NASDAQ after 2000.

Figure 2. Nonparametric Value at Risk-Based Sharpe Ratios



This figure plots the parametric value at risk-based Sharpe ratios for the fund of hedge funds, 5-year bond and NYSE/AMEX/NASDAQ indices between January 1994 and December 2011. The numerator of the parametric value at risk-based Sharpe ratio is equal to the monthly return of the index minus the risk-free rate. The denominator is equal to the first percentile of Hansen's (1994) skewed t-density estimated using the monthly returns from over the past 48 months.

CONCLUSION

We investigate whether the fund of hedge fund portfolios outperform various bond and stock market indices in terms of being able to generate higher returns per unit risk. Due to the potential non-normality associated with hedge fund returns, we give special emphasis to the concept of downside risk in our analysis. Consequently, apart from the traditional Sharpe ratio, we also construct reward-to-risk ratios that use non-parametric or parametric measures of value-at-risk in their denominator for various indices. Our main finding is that the FoHF index has superior reward-to-risk ratios compared to all bond and stock market indices. Although this superior performance is more pronounced when the risk measures are calculated using data from the last 12 months, the ability of the FoHF index to generate higher returns per unit risk is robust regardless of the sampling window. We also find that the documented outperformance is not a phenomenon that has been observed consistently through time and there have been periods in which the FoHF index has lagged behind the other indices.

REFERENCES

Agarwal, V. and Naik, N.Y. (2004). Risks and Portfolio Decisions Involving Hedge Funds. *Review of Financial Studies,* 17(1): 63-98.

Arditti, F.D. (1967). Risk and the Required Return on Equity. *Journal of Finance,* 22(1): 19-36.

Bali, T.G., Gokcan, S. and Liang, B. (2007). Value at Risk and the Cross Section of Hedge Fund Returns. *Journal of Banking and Finance*, 31(4): 1135-1166.

Brown, S.J., Gregoriou, G. and Pascalau, R. (2012). Is It Possible to Overdiversify? The Case of Funds of Hedge Funds. *Review of Asset Pricing Studies*, forthcoming.

Cornish, E.A. and Fisher, R.A. (1937). Moments and Cumulants in the Specification of Distributions. In: La Revue de l'Institute International de Statistique, 4. Reprinted in Fisher, R.A. (1950). In: Contributions to Mathematical Statistics. Wiley, New York, NY.

Dittmar, R.F. (2002). Nonlinear Pricing Kernels, Kurtosis Preference, and Evidence from the Cross Section of Equity Returns. *Journal of Finance,* 57(1): 369-403.

Fung, W. and Hsieh, D.A. (1997). Empirical Characteristics of Dynamic Trading Strategies: The Case of Hedge Funds. *Review of Financial Studies,* 10(2): 275-302.

Fung, W. and Hsieh, D.A. (2000). Performance Characteristics of Hedge Funds and CTA Funds: Natural versus Spurious Biases. *Journal of Financial and Quantitative Analysis,* 35(3): 291-307.

Fung, W. and Hsieh, D.A. (2001). The Risk in Hedge Fund Strategies: Theory and Evidence from Trend Followers. *Review of Financial Studies,* 14(2): 313-341.

Hansen, B.E. (1994). Autoregressive Conditional Density Estimation. *International Economic Review,* 35(3): 705-730.

Harvey, C.R. and Siddique, A. (2000). Conditional Skewness in Asset Pricing Tests. *Journal of Finance,* 55(3): 1263-1295.

Kimball, M. (1993). Standard Risk Aversion. *Econometrica,* 61(3): 589-611.

Kraus, A. and Litzenberger, R.H. (1976). Skewness Preference and the Valuation of Risk Assets. *Journal of Finance,* 31(4): 1085-1100.

Liang, B. and Park, H. (2007). Risk Measures for Hedge Funds: A Cross-sectional Approach. *European Financial Management,* 13(2): 333-370.

Malkiel, B.G. and Saha, A. (2005). Hedge Funds: Risk and Return. *Financial Analysts Journal*, 61(6): 80-88.

Markowitz, H. (1952). Portfolio Selection. *Journal of Finance,* 7(1): 77-91.

Mitchell, M. and Pulvino, T. (2001). Characteristics of Risk and Return in Risk Arbitrage. *Journal of Finance,* 56(6): 2135-2175.

Pratt, J. and Zeckhauser, R. (1987). Proper Risk Aversion. *Econometrica,* 55(1): 143-154.

Sharpe, W.F (1966). Mutual Fund Performance. *Journal of Business,* 39(1): 119-138.