USING EMISSION FUNCTIONS IN MODELING ENVIRONMENTALLY SUSTAINABLE TRAFFIC ASSIGNMENT POLICIES

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Abstract. Transport systems play a crucial role for sustainable development, and hence, sustainable urban transportation has recently become a major research area. Most of the existing studies propose evaluation methods that use simulation tools to assess the sustainability of different transportation policies. Although there are some recent studies, considering the sustainability dimension and the resulting policies through mathematical programming models is still an open research area. In this study, we focus on controlling the gas emissions for the environmental sustainability and propose several mathematical programming models that incorporate the measurements of gas emissions over a traffic network. The proposed models both reflect the route choice decisions of the network users and the decisions of the transportation managers that aim at making the transport systems more sustainable. We define the emission functions in terms of the traffic flow so that the accumulated emission amounts can be modeled accurately, particularly in case of congestion. Using the proposed emission functions, we introduce alternate objective functions into our optimization models and incorporate several policies which are based on the well-known toll pricing and capacity enhancement. We conduct a computational study on a well-known testing network and present numerical results to evaluate the proposed alternate models.

1. Introduction. In the last few decades the sustainable development issues have raised a significant interest due to the adverse effects of the considerable increase in urban population. Having many potential negative externalities such as, congestion, high energy consumption and air pollution, the urban transport systems play a very crucial role in maintaining sustainability. The literature includes many definitions of sustainable transport [28]. In a very compact way, a sustainable transportation system should respond to mobility needs, but at the same time should attend to the habitat, the equity in the society and the economic advancement in the present as well as in the future [14]. Basically, the main issues in sustainable transportation

2000 Mathematics Subject Classification. Primary: 90B20, 90C33; Secondary: 90B10.
Key words and phrases. transport, traffic assignment, mathematical programs with complementarity constraints, emission functions, toll pricing, capacity enhancement.
may be classified into three categories: economical, social and environmental [35]. Economical issues involve business activities, employment and productivity. Some of the social issues are equity, human health, and public involvement. Environmental issues, on the other hand, consist of pollution prevention, climate protection and habitat preservation.

According to the world and city population estimates prepared by the United Nations [52], the population of large cities (metropolitan areas with a population of 750,000 or more) was 1.3 billion in 2005, accounting for 20% of the total population of 6.5 billion, and is expected to reach 1.8 billion (22.5% of an estimated 8 billion) in 2025. As the population grows in urban areas, so does the number of motor vehicles. In a recent study, it is estimated that the total vehicle stock (only road vehicles with at least four wheels) will increase from about 800 million in 2002 to over 2 billion units in 2030 and by this time, 56% of the world’s vehicles will be owned by non-OECD (Organization for Economic and Co-operation and Development) countries, as opposed to just 24% in 2002 [12]. Compared to the developed countries, the traffic congestion poses a more serious and pressing problem for developing countries, where the road networks cannot cope with the rapid growth in traffic. The increase in traffic volume has certainly had dramatic and potentially irreversible effects on the environment. The environmental impacts of the transportation include air pollution, water pollution, noise generation, landscape degradation (loss of the ecologically productive lands, fragmentation of the habitat, hydrologic disruptions), depletion of nonrenewable resources, heat island effects and wildlife deaths from collisions [34]. Among those impacts, the mostly addressed one in urban transportation literature is the air pollution, since private cars, trucks and buses are the main contributors to the air pollution in the urban areas. Unfortunately, the urban transportation system is responsible for the emission of many pollutants such as carbon monoxide (CO), nitrogen oxides (NOx), hydrocarbons (HC), particulate matters (PM), benzene, carbon dioxide (CO2), and so on. All of these pollutants have several adverse health effects on living organisms, land, crops, water, and air [37]. Accordingly, we concentrate on reducing the air pollution caused by the urban transportation to support the environmental sustainability in our study.

Several strategies are proposed in the literature to improve the performance of the transport systems in terms of the environmental issues. These strategies involve the vehicle and fuel technology changes, road operational improvements and demand management (see also [14]). All these strategies have their advantages and drawbacks. The question is how effective would the alternate strategies be in reducing congestion, cutting the fuel use, and hence, lowering the pollution. Basically, the main goal of the related studies is to alleviate congestion and transport emissions through the use of different policies. In our study, we propose alternate optimization models that involve sustainability measures based on the gas emission amounts. We base our discussion on two major policies under elastic demand: toll pricing and capacity enhancement. Traffic management problems involving such policies are generally modeled using bilevel programming. In these models, an upper (system) level involves the decisions about a certain policy to achieve a predetermined objective and the lower (user) level reflects the decisions of the rational network users and their reactions to the upper level decisions [43]. In this study we also consider such a bilevel structure and focus on introducing different emission related objective functions to the the upper level problem. It is important to point out that
the emission concentrations are calculated using the emission functions, which we define in terms of the traffic flow in order to reflect the accumulations mainly in case of congestion. To define the proposed emission functions, we use the functions of emission amounts versus vehicle speeds provided by the European Environment Agency.

Next we present some of the studies, which also focus on reducing the emission amounts in transport systems. Most of these studies use simulation tools like TREMOVE [13] to evaluate the environmental sustainability of different transportation policies. There are also several studies that exploit mathematical programming approaches. Tzeng and Chen [51] propose multi-objective traffic assignment models and solve them using nonlinear programming techniques to obtain various solutions that lead to low CO emissions. Rilett and Benedek [6, 48] investigate an equitable traffic assignment model with environmental cost functions. They analyze the impacts of CO emissions when user and system optimum traffic assignments are applied to various networks. These studies make use of a simple macroscopic CO emission model found from the TRANSYT 7F software. Sugawara and Niemeier [50] discuss an emission-optimized traffic assignment model, which uses the average speed CO emission factors developed by the California Air Resources Board [10]. They report that the emission-optimized assignment is the most effective when the network is under low to moderately congested conditions. Meanwhile, emission factors are usually determined as the average values per vehicle kilometer for each vehicle category. In the literature, several mathematical models and simulation tools using emission factors are proposed to minimize emission [13, 47]. Nagurney [42] introduces the term of emission pricing, which is based on setting the toll prices to satisfy the predetermined emission levels. Nagurney [40, 41] also proposes sustainable urban transportation models with basic emission factors and emission constraints. Following Nagurney’s work, subsequent studies use the average emission factors for the sake of computational simplicity. The emission factors that are determined by several institutions give reasonable approximations of the real emission amounts in relatively less congested networks. However, in case of high congestion the amount of emission committed by the vehicles fluctuates considerably in time mainly due to the emission during engine start and stop. Therefore, using emission factors may not be sufficient to reflect the real situation especially for highly congested networks. To this end, emission functions in terms of the traffic flow may provide a different angle to evaluate different policies. Along this line, Yin and Lawphongpanich [56] propose an emission function in terms of traffic flow, where the coefficients are equivalent to those in TRANSYT 7F (see also [48]). In their work, Yin and Lawphongpanich consider a biobjective model, where the objectives are the minimization of the congestion as well as the minimization of the total emission through toll pricing. In this regard, their model has a similar structure as one of the models that we propose in this study. Nonetheless, Yin and Lawphongpanich [56] consider only the minimization of total network emission using the toll pricing policy, while in this study we focus on various other additional emission based objectives and management policies.

In this paper we introduce sustainability measures based on the emission amounts that depend on the traffic flow. This approach reflects the accumulated emission amounts more accurately in case of congestion. From a policy maker’s point of view, we propose several mathematical programming models that could be used for achieving, in a sense, sustainable traffic assignment. To illustrate the effects of the
proposed models and associated policies, we provide a computational study on a well-known test network and then discuss our observations.

2. Emission Functions. Emission modeling is a wide research area. In one of the early studies, Guensler and Sperling [25] show that vehicle emissions are highly dependent on the vehicle speed. Many researchers have studied the relation between transport emissions and vehicle types, speeds, driving styles, weather or several other factors [18, 23, 24, 30]. Akçelik [2, 3] has performed extensive studies to show that there is a direct relationship between the vehicle speed and the traffic flow on the link. In this study, we consider emission functions and express them in terms of the traffic flow. First, we express the emission of a specific pollutant in terms of the speed. Then, using the mathematical relationship between the traffic flow and the average vehicle speed, we obtain a single composite function of the pollutant emission with respect to the traffic flow.

European Environment Agency (EEA) is a major information source for those involved in developing, adopting, implementing and evaluating environmental policies. In the framework of the activities of the European Topic Centre for Air and Climate Change, EEA has financed COPERT 4, a software tool used world-wide to calculate air pollutant and greenhouse gas emissions from road transport. Vehicle emissions are expressed as a function of average speed for pre-EURO and EURO class vehicles in COPERT 4. Accordingly, the emission of pollutant $p$ in grams per kilometer of an EURO (European emission standards) class vehicle is expressed as

$$ e^p(v) = \frac{(a^p + e^p v + f^p v^2)}{(1 + b^p v + d^p v^2)}, \tag{1} $$

where $a^p$, $b^p$, $c^p$, $d^p$ and $f^p$ are parameters that are specific to vehicle and pollutant types, and $v$ corresponds to the vehicle speed (kilometers per hour). Figure 1(a) and Figure 1(b) show the relation between the vehicle speed and the emission of CO and NOx pollutants for a EURO3 gasoline vehicle, respectively.

![NOx emission](image1)

(a) NOx emission

![CO emission](image2)

(b) CO emission

Figure 1. Emissions of two major pollutants with respect to the average speed per vehicle.

We use the well-known travel time (cost) function defined by Bureau of Public Roads [8] as given in relation (2). To introduce this function and our notation, let us denote the set of nodes by $\mathcal{N}$ and the set of arcs by $\mathcal{A}$ for a transportation network. An arc (or a link) of the network is designated by $(i, j) \in \mathcal{A}$, $i, j \in \mathcal{N}$. 
If we also denote the flow on link \((i,j)\) in vehicles per hour by \(f_{ij}\), then the travel time or cost in hours is given by

\[
c_{ij}(f_{ij}) = \alpha_{ij} \left( 1 + 0.15 \left( \frac{f_{ij}}{\beta_{ij}} \right)^4 \right).
\]

(2)

Here \(\alpha_{ij}\) is the free flow travel time in hours and \(\beta_{ij}\) is the capacity given in vehicles per hour. Then, we express the average speed in kilometers per hour on link \((i,j) \in A\) as a function of the flow amount

\[
v_{ij}(f_{ij}) = \frac{l_{ij}}{c_{ij}(f_{ij})},
\]

(3)

where \(l_{ij}\) designates the length of link \((i,j)\) given in kilometers. Using the emission–vehicle speed function (1) and the vehicle speed–traffic flow function (3), we construct a composite function to express the total emission in terms of the traffic flow. Basically, we estimate the total emission of pollutant \(p\) in grams per hour on a particular link \((i,j)\) with

\[
e_{ij}^p(f_{ij}) = f_{ij} \times l_{ij} \times e^p(v_{ij}(f_{ij})).
\]

(4)

It is expected that when the road capacity is reached and congestion occurs, vehicles start to follow stop/go pattern which decreases the average vehicle speed and increases the total emission significantly. Such a behavior is illustrated in Figure 2 for the total emission function of NOx.

![Figure 2. NOx emission per vehicle with respect to the flow/capacity ratio.](image)

NOx is known to be one of the major pollutants emitted during the traffic congestion. In fact, the transportation sources are reported to be responsible for a considerable amount of NOx emissions in the US. Moreover, almost half of all NOx emissions result from the road traffic in the UK [17] and NOx emissions show an increasing trend in the recent years [26, 45]. Leveraging on this, we also focus on NOx emission and omit superscript \(p\) from equation (4) in the subsequent part of this work. However, we note that other pollutants can easily be incorporated into the proposed models. As the emission amounts from different pollutants usually have large differences in magnitude, the total emission on a link can be calculated
by introducing proper scaling coefficients to equation (4) in case of multiple pollutants.

3. Proposed Mathematical Models with Emission Functions. The solution of the traffic assignment problem yields the optimum flow on the transportation network and is obtained when a stable pattern of travelers’ choice is reached. This is called the user equilibrium [54]. There are two different formulations of the traffic assignment problem [15]. The path formulation incorporates predetermined routes having specific order of links and this requires the enumeration of all possible paths which can be prohibitive even for moderate problem instances. In the multi-commodity formulation, the modeling structure is based on the numbers of users that are headed to each destination on each link. Though the general multi-commodity formulation is based on the origin-destination (O-D) pairs, the special structure of this transportation problem enables to distinguish the flows based only on the destination points [15]. In this computationally efficient formulation, a commodity is associated with each destination. Thus if we denote \( D \) as the set of destination points in the network, then we consider the decision variable \( x^s_{ij} \), denoting the flow of commodity \( s \in D \) on link \((i, j) \in A\) in the multi-commodity formulation.

The network is managed based on the peak-hour demand which is assumed to be variable, or more commonly addressed as elastic. For elastic demand, the number of trips from an origin to a destination depends on the minimum travel time between them. Traditionally, it is assumed that the travel demand decreases as the travel time increases. This relation is represented by a demand function denoted by \( g_{is}(w_{is}) \) with \( w_{is} \) being the travel time between O–D pair \((i, s)\). To the best of our knowledge, in literature two types of travel demand functions [5] are mainly used: exponential and linear. In this study, we use the widely-applied linear demand function

\[
g_{is}(w_{is}) = \mu_{is}w_{is} + \nu_{is}, \tag{5}\]

where \( \mu_{is} \) and \( \nu_{is} \) are network specific parameters. Consequently, if we denote the travel demand between O–D pair \((i, s)\) by \( d^s_{is} = g_{is}(w_{is}) \), then \( w_{is} = g_{is}^{-1}(d^s_{is}) \). The link flows that satisfy the user-equilibrium can be obtained by solving the following mathematical programming formulation:

\[
\text{REG : minimize } \sum_{(i,j) \in A} \int_{0}^{f_{ij}} c_{ij}(y) \, dy - \sum_{i \in \mathcal{N}} \sum_{s \in D} \int_{0}^{d^s_{is}} g_{is}^{-1}(v) \, dv, \tag{6a}\]

subject to \( \sum_{j: (i, j) \in A} x^s_{ij} - \sum_{j: (j, i) \in A} x^s_{ji} = d^s_{is}, \quad i \in \mathcal{N}, s \in D, \tag{6b}\)

\( \sum_{s \in D} x^s_{ij} = f_{ij}, \quad (i, j) \in \mathcal{A}, \tag{6c}\)

\( x^s_{ij} \geq 0, \quad (i, j) \in \mathcal{A}, s \in \mathcal{D}, \tag{6d}\)

\( d^s_{is} \geq 0, \quad i \in \mathcal{N}, s \in \mathcal{D}. \tag{6e}\)

Here the set of constraints (6b) is for the flow conservation and constraints (6c) link the total flow on an arc to the flows resulting from individual destination points. Constraints (6d) and (6e) ensure that the link flows and travel demands are nonnegative. Problem \((\text{REG})\) given in (6) is a convex programming problem and
its first order optimality conditions are
\[ x_{ij}^s [c_{ij}(f_{ij}) - \lambda_s^i + \lambda_s^j] = 0, \quad (i, j) \in \mathcal{A}, s \in \mathcal{D}, \quad (7a) \]
\[ c_{ij}(f_{ij}) - \lambda_s^i + \lambda_s^j \geq 0, \quad (i, j) \in \mathcal{A}, s \in \mathcal{D}, \quad (7b) \]
\[ d_s^i [\lambda_s^i - g_s^{-1}(d_s^i)] = 0, \quad i \in \mathcal{N}, s \in \mathcal{D}, \quad (7c) \]
\[ \lambda_s^i - g_s^{-1}(d_s^i) \geq 0, \quad i \in \mathcal{N}, s \in \mathcal{D}, \quad (7d) \]
\[ \sum_{j:(i,j)\in \mathcal{A}} x_{ij}^s - \sum_{j:(j,i)\in \mathcal{A}} x_{ji}^s = d_s^i, \quad i \in \mathcal{N}, s \in \mathcal{D}, \quad (7e) \]
\[ \sum_{s \in \mathcal{D}} x_{ij}^s = f_{ij}, \quad (i, j) \in \mathcal{A}, \quad (7f) \]
\[ x_{ij}^s \geq 0, \quad (i, j) \in \mathcal{A}, s \in \mathcal{D}, \quad (7g) \]
\[ d_s^i \geq 0, \quad i \in \mathcal{N}, s \in \mathcal{D}, \quad (7h) \]

where \( \lambda_s^i i \in \mathcal{N}, s \in \mathcal{D} \) are the dual variables associated with constraints (6b). At optimality \( \lambda_s^i \) gives the minimum travel time between O–D pair \( (i, s) \).

Bilevel programming is a branch of hierarchical mathematical optimization. In a bilevel model, the objective is to optimize the upper level objective while simultaneously optimizing the lower level problem. In a typical bilevel traffic equilibrium problem, the upper level problem involves the decisions about a certain policy (like toll pricing or capacity enhancement) to achieve a predetermined objective (like reducing the congestion or the investment cost). In the lower level we model the traffic equilibrium reflecting the decisions of the rational network users and their reactions to the upper level decisions. In other words, given an upper level decision, the lower level problem leads to the traffic assignment problem given in (6). A common approach to solve bilevel models is to reformulate the lower level problem in terms of its optimality conditions. In our case, these optimality conditions are given by (7). Due to the constraints (7a) and (7c), the resulting nonlinear programming problems are referred to as mathematical programs with complementarity constraints (MPCCs) [7, 36].

In the following subsections, we discuss several mathematical modeling models in the form of typical MPCCs. In all these models, the objectives involve alternate sustainability measures based on the proposed emission functions, and the constraints involve the optimality conditions of the user equilibrium problem.

3.1. Total Network Emission. In this section, we propose models with the objective of minimizing the total network emission. We try to achieve this objective via two policies: (i) toll pricing and (ii) capacity enhancement.

Toll Pricing. As mobility increases, not only each new driver pays a higher congestion cost compared to previously present drivers, but he/she also reduces the road space available to other drivers. This cost is external to the marginal driver. Thus, a road user’s marginal private cost is lower than her marginal social cost [31, 46, 49, 53]. It is important to note that the concept of road pricing emerged from this idea. Toll pricing policies have recently become more practical due to the advent of electronic tolling, and hence, received significant attention from transportation planners and researchers. The first-best toll pricing problem assumes that all roads on the network can be tolled [4]. There exist several first-best toll pricing models with various objective functions: minimizing the total tolls collected, minimizing the largest nonnegative toll to be collected, minimizing the total tolls
collected while constraining this total to be zero and allowing negative tolls (allowing users to collect a payment on some links and pay a toll on others) and minimizing the number of toll booths [27]. Nonetheless, the first-best toll pricing framework can hardly be applied in real life. Alternatively, it has been proposed to allow a subset of the roads to be tolled and the resulting problem is known as the second-best toll pricing problem [9, 29, 32, 33, 44]. Here, we focus on this latter problem and use toll prices as disincentives to discourage travelers from choosing more congested links, and consequently, to reduce the emissions.

Let $\bar{A}$ be the set of tollable links and $t_{ij}$ be the toll price on link $(i, j) \in A$. We assume that $t_{ij}$ cannot exceed a prescribed upper bound $t_{ij}^{\max}$, where $t_{ij}^{\max} > 0$ if $(i, j) \in \bar{A}$ and $t_{ij}^{\max} = 0$ otherwise. Our optimization model for minimizing the total emission is given as

$$TTE: \text{minimize } \sum_{(i,j) \in A} e_{ij}(f_{ij}), \quad (8a)$$

subject to

$$\sum_{(i,j) \in \bar{A}} t_{ij} f_{ij} \geq \gamma_1 R^{\max}, \quad (8b)$$

$$0 \leq t_{ij} \leq t_{ij}^{\max}, \quad (i, j) \in A, \quad (8c)$$

$$c_{ij}(f_{ij}) + t_{ij} - \lambda_i^s + \lambda_j^s \geq 0, \quad (i, j) \in A, s \in D, \quad (8d)$$

$$c_{ij}(f_{ij}) + t_{ij} - \lambda_i^s + \lambda_j^s \geq 0, \quad (i, j) \in A, s \in D, \quad (8e)$$

$$k_{ij} z_{ij}^2, \quad (7c) - (7h), \quad (8f)$$

where $R^{\max}$ denotes the maximum revenue that can be received from enforcing tolls and $\gamma_1 \in [0, 1]$ is a parameter specified by the decision makers to represent a certain fraction of the maximum revenue. The parameter $R^{\max}$ can be obtained by solving the traditional toll pricing problem with the objective of revenue maximization (see also Section 4). Constraint (8b) ensures that the collected revenue is above a fraction of the maximum possible revenue. Constraints (8d)-(8f) are similar to the optimality conditions (7) with the addition of toll $t_{ij}$ to the travel cost $c_{ij}(f_{ij})$ in equations (8d) and (8e).

**Capacity Enhancement.** Network design problems (NDPs) in transportation context deal with decisions about (re)structuring the underlying networks. Under budgetary constraints, discrete NDPs usually focus on decisions related to the link or lane additions, whereas continuous NDPs are limited to decisions on network improvements that can be modeled using continuous variables such as the lane and lateral clearance changes and also other enhancements that produce incremental changes in capacities. Due to the intrinsic complexity of the model formulation, NDP has been recognized as one of the most challenging problems in the literature [1, 11, 20, 38, 39, 55]. As we are interested in introducing new models by mainly focusing on alternate objective functions based on emission amounts for environmental sustainability, we restrict our attention to the continuous case. However, we note that the proposed modeling approaches can also be applied for discrete network design problems.

We assume that the investment and operating cost function associated with the capacity enhancement on link $(i, j)$ is given by $k_{ij} z_{ij}^2$, where $z_{ij}$ represents the capacity enhancement and $k_{ij}$ the associated cost coefficient [1]. Note that this type of quadratic cost functions are frequently used in the literature (i.e. [22, 57]), but other types can easily be incorporated into the proposed models.
enhancement naturally affects the travel time on link \((i, j)\) and leads to
\[ c_{ij}(f_{ij}, z_{ij}) = \alpha_{ij} \left( 1 + 0.15 \left( f_{ij}/(\beta_{ij} + z_{ij}) \right)^4 \right). \] (9)

We next denote the set of link capacities that could be enhanced by \(\bar{A}_2\) and the maximum capacity enhancement on link \((i, j)\) by \(z_{ij}^{\text{max}}\). Then, \(z_{ij}^{\text{max}} > 0\), if \((i, j) \in \bar{A}_2\) and \(z_{ij}^{\text{max}} = 0\), otherwise. Using this new notation, our capacity enhancement model with the objective of minimizing the total emission is given by

\[ \text{CTE} : \quad \text{minimize} \sum_{(i,j)\in A} e_{ij}(f_{ij}, z_{ij}), \] (10a)

\[ \text{subject to} \quad \sum_{(i,j)\in \bar{A}_2} k_{ij} z_{ij}^2 \leq \gamma_2 B_{\text{max}}, \] (10b)

\[ 0 \leq z_{ij} \leq z_{ij}^{\text{max}}, \quad (i,j) \in A, \quad (i,j) \in A, \quad s \in D, \] (10c)

\[ x_{ij}^s \left[ c_{ij}(f_{ij}, z_{ij}) - \lambda_i^s + \lambda_j^s \right] = 0, \quad (i,j) \in A, \quad s \in D, \] (10d)

\[ c_{ij}(f_{ij}, z_{ij}) - \lambda_i^s + \lambda_j^s \geq 0, \quad (i,j) \in A, \quad s \in D, \] (10e)

\[ (7c) - (7h). \] (10f)

Here \(B_{\text{max}}\) is the maximum budget that can be allocated for capacity enhancement and \(\gamma_2 \in [0, 1]\) is a prespecified parameter to represent a certain fraction of the maximum budget. Constraint (10b) ensures that the total cost of enhancing the network is below the specified fraction of the budget. The parameter \(B_{\text{max}}\) can be calculated by solving model \(\text{CTE}\) after relaxing constraints (10b); see also Section 4. Constraints (10d)-(10f) are the optimality conditions of the traffic assignment problem as presented in (7), where (10d) and (10e) are obtained by replacing the travel cost \(c_{ij}(f_{ij})\) by \(c_{ij}(f_{ij}, z_{ij})\).

Simultaneous Toll Pricing and Capacity Enhancement. To observe the combined effect of the toll pricing and the capacity enhancement strategies, we develop a model which incorporates these traffic management policies simultaneously. Then, the mathematical programming model becomes

\[ \text{TCTE} : \quad \text{minimize} \sum_{(i,j)\in A} e_{ij}(f_{ij}, z_{ij}), \] (11a)

\[ \text{subject to} \quad (8b), (8c), (10b), (10c), \] (11b)

\[ x_{ij}^s \left[ c_{ij}(f_{ij}, z_{ij}) + t_{ij} - \lambda_i^s + \lambda_j^s \right] = 0, \quad (i,j) \in A, \quad s \in D, \] (11c)

\[ c_{ij}(f_{ij}, z_{ij}) + t_{ij} - \lambda_i^s + \lambda_j^s \geq 0, \quad (i,j) \in A, \quad s \in D, \] (11d)

\[ (7c) - (7h). \] (11e)

The parameters \(R_{\text{max}}\) and \(B_{\text{max}}\) are the same as in the models \(\text{TTE}\) and \(\text{CTE}\), respectively. The underlying idea in developing this model is similar to the one that defines simultaneous positive and negative tolls: encouraging the users by enhancing the capacity of some links and discouraging them by collecting toll on some other links. If the traffic authority follows the strategy to toll only those links, of which the capacities are enhanced, this can be interpreted as the intent to recover the capacity enhancement costs by collecting tolls.

3.2. Emission Dispersion. Directing the vehicle flow to other parts of the transportation network through the toll pricing policy may lead to high emission accumulations in the wider area of the network. Therefore, it may be preferable to disperse
the emission rather than minimizing the total emission. In this regard, we propose alternate models under the toll pricing and capacity enhancement policies, where we focus on the pollutant concentration in different areas of the network instead of the total emission amount. We refer to these models as the emission dispersion models. The emission concentration is defined as the emission amount per unit link length, and by using equation (4) the concentration of pollutant $p$ on link $(i,j)$ is given as

$$\bar{e}_{ij}^p(f_{ij}) = f_{ij}e^p(v_{ij}(f_{ij})).$$

(12)

Basically, $\bar{e}_{ij}^p$ is measured in grams per kilometer and hour. As mentioned in Section 2, for ease of exposition, we focus on a single pollutant and omit superscript $p$ from equation (12).

We start with two new models that are obtained by modifying the objective function (8a) of the toll-pricing model ($TTE$). The main difference between these models is their scope of evaluating the emission concentration. The objective of the first model is based on minimizing the maximum link emission concentration over the whole network. The first model then becomes

$$TED_1 : \min \left\{ \max_{(i,j) \in A} \bar{e}_{ij}(f_{ij}) : (8b) - (8f) \right\}.$$  (13)

With this objective, the solution of the model is biased towards policies, which may lead to a more balanced concentration over the entire network. The objective of the second model is differentiating the emission concentrations in different sections of the network. Traffic flows with reasonable emission levels in a highly populated section of a network may sum up to excessive amounts in that section. Due to the land use characteristics (such as; residential, commercial, and so on), the network management authorities may determine upper limits on the emission amount at certain sections of the network. Let $\zeta_{ij}$ denote the threshold on the emission concentration level for link $(i,j)$. The product of this amount with the link length gives the threshold on the emission level for that link. As the public health is at stake, it would be natural to set different levels of restrictions on the emission amounts for different parts of the network. For example, one may enforce smaller concentration levels for harmful pollutants in highly populated areas. Note that in practice the decision makers may specify a threshold for each section (zone) of the network and consider the same zone-based threshold for each link belonging to that specific zone. With this dispersion type of objective, we penalize the amount of emission on each link that exceeds the specified upper limit. This discussion leads to our second model as

$$TED_2 : \min \left\{ \sum_{(i,j) \in A} \max \left\{ e_{ij}(f_{ij}) - \zeta_{ij}l_{ij}, 0 \right\} : (10b) - (10f) \right\}.$$  (14)

The dispersion of the emission throughout the network may also be attained by capacity enhancement. Similar to the toll pricing models as described above, we modify the capacity enhancement model ($CTE$) by incorporating the proposed types of objective functions. The corresponding capacity enhancement models then become

$$CED_1 : \min \left\{ \max_{(i,j) \in A} \bar{e}_{ij}(f_{ij}, z_{ij}) : (10b) - (10f) \right\}.$$
and

$$CED_2 : \min \left\{ \sum_{(i,j) \in A} \max \{ e_{ij}(f_{ij}, z_{ij}) - \zeta_{ij}l_{ij}, 0 \} : (10b) - (10f) \right\},$$

respectively.

Finally, by replacing the objective function of the model (TCTE) we obtain the simultaneous toll pricing and capacity enhancement models with the emission dispersion based objectives as

$$TCED_1 : \min \left\{ \max_{(i,j) \in A} \bar{e}_{ij}(f_{ij}, z_{ij}) : (11b) - (11e) \right\},$$

and

$$TCED_2 : \min \left\{ \sum_{(i,j) \in A} \max \{ e_{ij}(f_{ij}, z_{ij}) - \zeta_{ij}l_{ij}, 0 \} : (11b) - (11e) \right\},$$

respectively.

In the next section, we elaborate on how the solutions provided by the total emission and emission dispersion models perform in terms of the resulting emission amounts.

4. Computational Study. We conduct a computational study to analyze the effects of the proposed models on the emission amounts, and evaluate the toll pricing and capacity enhancement policies with respect to the specified sustainability measures. The main difficulty of solving the proposed models come from the complementarity constraints, since these constraints induce a nonconvex feasible region [36]. Fortunately, there exists a meta-solver, namely NLPEC, to handle MPCCs automatically. When called, NLPEC reformulates the complementarity constraints of a MPCC model with a user specified reformulation option. NLPEC calls a user-specified nonlinear programming solver to solve the reformulated model. The results from the nonlinear programming solver are then translated back into the original MPCC model and the complementarity constraints are checked for violation. Among all available solvers, CONOPT [16] performed the best in our experiments. All the results we present are obtained using the following options: reftype mult, initmu 1, numsolves 5, finalmu 0. For several combinations of reformulations and option files, we refer the reader to [19] and the current version of NLPEC manual1. Note that, NLPEC solver is accessible through GAMS modeling language [21].

In our study, we use the well-known Sioux Falls network (see Figure 4) which consists of 24 nodes, 76 links and 552 O-D pairs2. Its trip table is nearly symmetric, all the connections are bi-directional and represented by two arcs each of which has identical characteristics. It is important to note that the presented map is not to scale, so the length of links is not related to the free flow time between pairs of nodes. The original Sioux Falls network data includes the fixed peak hour demand for O-D pairs. To obtain the problem instances of our models under the elastic demand, we generate parameters of the linear demand function given in (5) as follows: We first solve the model (REG) with the original fixed demand data to optimality by

1http://www.gams.com/dd/docs/solvers/nlpec.pdf (last accessed on November 2011)
2The data of this model can be reached at http://www.bgu.ac.il/~bargera/tntp (last accessed on November 2011)
omitting the second term in the objective function (6a). With the optimal link flow values at hand, we then calculate the associated travel time for each link. In the next step, the path(s) with minimum travel time are identified for each O–D pair. Denoting this minimum travel time as \( \lambda^*_{is} \) and the original fixed demand for O–D pair \((i,s)\) as \( d^*_{is} \), the parameters of the elastic demand function in (5) are calculated from the linear interpolation of points \((\lambda^*_{is}, d^*_{is})\) and \((\delta \lambda^*_{is}, d^*_{is}/\delta)\), where \(\delta\) is a random number generated from the uniform distribution on the interval \((2,3)\). We also use the optimal solution of the modified \((REG)\) model to calculate the threshold value \(\zeta_{ij}\) on the emission concentration for each link \((i,j) \in A\). For this optimal solution, we calculate the total emission in each zone and divide it by the total length of the links in that zone to estimate the zone emission concentration. We scale these emission concentrations by zone dependent coefficients to determine the zone based threshold values. The zone coefficients are specified as inversely proportional to the corresponding population density. We assume that the population density decreases in the following order of zones: residential, commercial, industrial and non-urban. In particular, the coefficients are selected as 0.7, 0.9, 1.1 and 1.3, respectively. Then the threshold value of each link is set equal to the corresponding zone based threshold value. Notice that the zone coefficients indicate our preferences with respect to the concentration levels associated with the optimal solution of the modified \((REG)\), which can be considered as a reference solution. Basically, we would like to obtain a new solution which performs better than the reference one with respect to decision makers’ preferences. When a zone coefficient is less than 1, this indicates that the decision makers prefer a solution with lower emission concentrations in that zone with respect to the reference solution. In our implementation, we assume that it is more preferable to reduce the concentration levels in residential and commercial zones, and therefore, we set the corresponding coefficients to be less than 1. To achieve the desired improvements in the selected zones, we compromise on the concentration levels in the other less dense zones by assigning zone coefficients which are larger than 1.

We choose the following arcs to be tolled: \((6,8), (8,6), (10,15), (11,4), (14,11), (15,10), (15,22) (22,15)\). The same set of arcs is also considered for capacity expansion. In the subsequent figures, all these arcs are also marked with appropriate symbols depending on the problem solved (toll pricing (T), capacity enhancement (C), both strategies (X)). We note that the results obtained by the proposed models depend on the arcs to be tolled and/or whose capacities to be enhanced. To determine the maximum revenue parameter \(R^{\text{max}}\), we solve an auxiliary model that is obtained from \((TTE)\) by relaxing the inequality (8b) and replacing the objective (8a) by the maximization of \(\sum_{(i,j) \in A_1} t_{ij} f_{ij}\). The optimum objective function value of this auxiliary model provides the value of the parameter \(R^{\text{max}}\). In a similar fashion, the model \((CTE)\) is solved without inequality (10b), and the total capacity enhancement cost associated with its optimum solution is used to set the maximum budget parameter \(B^{\text{max}}\). In all our experiments, we consider the accumulated emission for a single pollutant, namely NOx. The variation of the total NOx emission with respect to \(\gamma_1\) and \(\gamma_2\) values are plotted in Figures 3(a) and 3(b), respectively. Based on these figures, we set \(\gamma_1\) to 0.70 and \(\gamma_2\) to 0.80. For the interested readers, this data set is available online.

The optimum link emissions are illustrated on the graphical representations of the Sioux Falls network in Figures 4-7, and some comparative emission statistics

\[\text{http://people.sabanciuniv.edu/sibirbil/emission/data_index.html}\]
(a) Emission versus ratio of the maximum revenue.  
(b) Emission versus ratio of the allocated budget.

**Figure 3.** The experiments conducted to determine parameters $\gamma_1$ and $\gamma_2$.

**Table 1.** Statistics for models with the objective of minimizing the total emission

<table>
<thead>
<tr>
<th>Model</th>
<th>Value</th>
<th>Change</th>
<th>Value</th>
<th>Change</th>
<th>Value</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>REG</td>
<td>378.556</td>
<td>-8.2%</td>
<td>374.488</td>
<td>-1.1%</td>
<td>344.529</td>
<td>-9.0%</td>
</tr>
<tr>
<td>TTE</td>
<td>347.668</td>
<td>-8.2%</td>
<td>1.107</td>
<td>-1.1%</td>
<td>1.097</td>
<td>-9.0%</td>
</tr>
<tr>
<td>CTE</td>
<td>0.368</td>
<td>-36.7%</td>
<td>0.382</td>
<td>-3.9%</td>
<td>0.225</td>
<td>-38.7%</td>
</tr>
<tr>
<td>TCTE</td>
<td>2.802</td>
<td>-22.5%</td>
<td>2.663</td>
<td>-5.0%</td>
<td>2.244</td>
<td>-19.9%</td>
</tr>
</tbody>
</table>

are provided in Tables 1-3. In all of the figures, the network is colored such that the least emission values are observed on green links, whereas very high emission amounts are observed on red links. All other colors represent intermediate values. The average concentration is calculated by dividing the total network emission by the total length of links. The average vehicle emission is calculated as the total network emission divided by the total number of trips. In all of the tables, for each model the values of various criteria and their relative differences with respect to those of the model (REG) are presented in columns “Value” and “Change”, respectively.

As the model (REG) corresponds to the case where there is no intervention from a traffic authority, its optimal solution is used as a benchmark and the associated emission amounts are depicted in Figure 4. As it is common for many cities, we observe that most of the NOx emission is concentrated around the city center. We use these benchmark amounts to analyze the efficiency of applying different policies that we propose in this study.

First we investigate the results associated with the solutions of three models aiming to minimize the total network emission: (TTE), (CTE) and (TCTE). Emission amounts corresponding to the optimum solutions of these models are illustrated in Figure 5, and the statistics about emission amounts are provided in Table 1. The main conclusion is that toll pricing based policies are more effective in reducing the total emission. From the total network emission row of Table 1, it can be
observed that models (TTE) and (TCTE) achieve an emission decrease of about 8.2% and 9.0% respectively compared to (REG). Meanwhile, only 1.1% decrease was achieved with the capacity enhancement model (CTE). A close examination shows that the success of toll pricing policies can be attributed to their potential for reducing the number of trips. As the demand is assumed to be variable and depending on the travel time, pricing type policies direct some of the trips to alternative transportation means, which in turn leads to a reduction in total emission level. On the other hand, the enhancement type policies generate additional demand due the increased capacity. For example, the total number of trips at the optimal solution of the model (CTE) is 2.6% higher than the one obtained by the model (REG) as given in Table 1. This behavior limits their effectiveness in decreasing the total emission. Meanwhile, the model (CTE) is only superior in terms of the average vehicle emission criterion as the total network emission slightly decreases and the total number of trips increases when compared against the model (REG). As the demand decrease is restricted while the emission decrease is substantial, the solution associated with the mix strategy considered in the model (TCTE) seems to be the most efficient one.

Next, we contrast the models (TED1), (CED1) and (TCED1), which have the common objective of minimizing the maximum emission concentration. The optimum solutions are illustrated in Figure 6 and the corresponding outcomes are summarized in Table 2. Inferences similar to those made for the models minimizing the total emission are also valid here. First of all, the maximum link emission concentrations are significantly lower for all three models due to their objective functions. The model (TED1) provides a solution with the least total emission, and also the least number of trips and the highest average vehicle emission. The solution of the model (CED1) results in a total emission and demand almost equal to those of (REG). Moreover, it can be noticed from the results that (CED1)
Table 2. Statistics for models with the objective of minimizing the maximum emission concentration

<table>
<thead>
<tr>
<th>Model</th>
<th>Value</th>
<th>Change</th>
<th>Value</th>
<th>Change</th>
<th>Value</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Network Emission</td>
<td>378.556</td>
<td>-7.6%</td>
<td>349.941</td>
<td>-7.6%</td>
<td>381.123</td>
<td>+0.7%</td>
</tr>
<tr>
<td>Average Concentration</td>
<td>1.206</td>
<td>-7.6%</td>
<td>1.114</td>
<td>-7.6%</td>
<td>1.214</td>
<td>+0.7%</td>
</tr>
<tr>
<td>Minimum Concentration</td>
<td>0.368</td>
<td>-66.8%</td>
<td>0.122</td>
<td>-66.8%</td>
<td>0.412</td>
<td>+12.0%</td>
</tr>
<tr>
<td>Maximum Concentration</td>
<td>2.802</td>
<td>-23.7%</td>
<td>2.138</td>
<td>-23.7%</td>
<td>2.472</td>
<td>-11.8%</td>
</tr>
<tr>
<td>Total Number of Trips</td>
<td>360,608</td>
<td>-9.8%</td>
<td>325,325</td>
<td>-9.8%</td>
<td>365,614</td>
<td>+1.4%</td>
</tr>
<tr>
<td>Average Vehicle Emission</td>
<td>1.050</td>
<td>+2.5%</td>
<td>1.076</td>
<td>+2.5%</td>
<td>1.042</td>
<td>-0.7%</td>
</tr>
</tbody>
</table>

Table 3. Statistics for models with the objective of minimizing the maximum emission concentration

<table>
<thead>
<tr>
<th>Model</th>
<th>Value</th>
<th>Change</th>
<th>Value</th>
<th>Change</th>
<th>Value</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole Network</td>
<td>378.556</td>
<td>-5.8%</td>
<td>356.686</td>
<td>-5.8%</td>
<td>378.659</td>
<td>+0.0%</td>
</tr>
<tr>
<td>Residential</td>
<td>73.907</td>
<td>+0.1%</td>
<td>73.951</td>
<td>+0.1%</td>
<td>74.452</td>
<td>+0.7%</td>
</tr>
<tr>
<td>Commercial</td>
<td>124.636</td>
<td>-17.9%</td>
<td>102.332</td>
<td>-17.9%</td>
<td>119.131</td>
<td>-4.4%</td>
</tr>
<tr>
<td>Industrial</td>
<td>140.079</td>
<td>+1.5%</td>
<td>142.239</td>
<td>+1.5%</td>
<td>136.837</td>
<td>-2.3%</td>
</tr>
<tr>
<td>Non-urban</td>
<td>39.934</td>
<td>-4.4%</td>
<td>38.164</td>
<td>-4.4%</td>
<td>48.239</td>
<td>+20.8%</td>
</tr>
</tbody>
</table>

requires concentration increase on some links to reduce the concentration of others, which is not really a desirable outcome. Finally, the solution provided by the mix strategy model (TCED1) is moderate in terms of the total emission and the demand decrease, and also leads to a higher decrease in the maximum emission amount.

Finally, we compare the remaining models (TED2), (CED2) and (TCED2) based on the results given in Figure 7 and Table 3. In terms of both the total emission and total excess emission, the strategy incorporated into the model (TCED2) is the most efficient. It seems that by successfully diverting the actual traffic, the undesirable excess emission in a relatively populated commercial zone is dramatically reduced and shifted to non-urban areas. Additionally, excess emission is moderately reduced in residential and industrial zones. The model (TED2) produces quite similar outcomes as the model (TCED2) but it is less efficient. The last model (CED2) provides similar results with (REG) in terms of the total emission amount. Moreover, both the total and excess emissions are highly increased for the non-urban areas, and the excess emission is significantly reduced in the commercial area. To summarize, the capacity enhancement is not as efficient as the pricing strategies but accomplishes its emission dispersion mission when compared against the do-nothing strategy of solving the model (REG).
5. Conclusion. In this study we propose several new optimization models to support the management of urban transportation networks with environmental sustainability concerns. We derive emission functions in terms of the traffic flow in order to reflect the emission amounts in the congested networks more accurately. Based on the proposed emission functions, we also introduce alternate objective functions into the optimization models. We investigate two main policies: toll pricing and capacity enhancement. The proposed models based on the toll pricing strategy provide good results in terms of the emission amounts as tolling reduces the total demand on the network. We have also observed that under the capacity enhancement strategy, the increased capacity of a link decreases the travel time on that specific link, and hence, increases the associated travel demand and the emission. This limits the capacity enhancement policy, but still some improvement could be achieved even if the demand increases. The best results are obtained by applying toll pricing and capacity enhancement simultaneously.

Note that determining the set of arcs to be tolled and/or enhanced is a significant issue to obtain effective policies. As a future research, decisions on selecting the arcs to be tolled and/or enhanced can also be incorporated into the proposed models. As the users of a transportation network drive different types of vehicles or commute by means of public transport, the proposed models could be extended with considering the multi-modal nature of the problem. This shall also increase the accuracy of the models in terms of accumulated emissions, since different vehicles have different emission profiles. Moreover, the road types, such as belt lines, highways, and so on, could also have an impact on the emission profiles. Finally, we intend to investigate fast solution methods that utilize the special structure of the proposed models to solve the large scale real-life problems efficiently.

Acknowledgments. This work has been partially supported by The Scientific and Technological Research Council of Turkey (TÜBİTAK) under grant 109M137, and also by Galatasaray University Research Fund.
(a) Toll pricing ($TTE$).
(b) Capacity enhancement ($CTE$).
(c) Toll pricing and capacity enhancement ($TCTE$).

Figure 5. Pictorial representation of link emissions for models aiming to minimize the total emission.
(a) Toll pricing ($TED_1$).

(b) Capacity enhancement ($CED_1$).

(c) toll pricing and capacity enhancement ($TCED_1$).

Figure 6. Pictorial representation of link emissions for models aiming to minimize the maximum emission concentration.
Figure 7. Pictorial representation of link emissions for models aiming to minimize the zonal excess emission.
REFERENCES.


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