Filling Position Incentives in Matching Markets

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Abstract

One of the main problems in the hospital-doctor matching is the maldistribution of doctor assignments across hospitals. Namely, many hospitals in rural areas are matched with far fewer doctors than what they need. The so called "Rural Hospital Theorem" (Roth (1984)) reveals that it is unavoidable under stable assignments. On the other hand, the counterpart of the problem in the school choice context—low enrollments at schools— has important consequences for schools as well. In the current study, we approach the problem from a different point of view and investigate whether hospitals can increase their filled positions by misreporting their preferences under well-known Boston, Top Trading Cycles, and stable rules. It turns out that while it is impossible under Boston and stable mechanisms, Top Trading Cycles rule is manipulable in that sense.

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1 Introduction

Initiated by Gale and Shapley (1962), matching theory has been fruitful in both theory and practice. It has influenced the design many matching markets including entry-level

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labor markets and student placement systems in U.S. Stability has been the central notion in matching theory. Besides its theoretically appealing requirement, its practical importance is also documented in the literature (e.g., see Roth (1991)).

While both theoretical and practical evidences demonstrate that stability is a key factor for the well-working of matching markets, there are still important problems of stable allocations. In the paradigm of the hospital-doctor (intern) matching problem, one of the critical issues is the maldistribution of doctor assignments across hospitals. Specifically, the number of doctors assigned to many hospitals in rural areas through the centralized National Residency Matching Program (NRMP) is much lower than what it is needed. Talbott (2007) indeed reports a striking statistics that while there are 280 doctors for every 100.000 people in U.S., there are only 103 doctors for every 100.000 people in the 18-county area of the Mississippi Delta. Similar doctor assignment problems exist in other countries including Japan, United Kingdom and Australia.[1] Moreover, the counterpart of this problem in the school choice context—low enrollments at schools—is important as well. It is well known that schools with lower enrollments receive less funds from government,[2] yet, a harsher consequence is the closure of such schools. For instance, Hatfield et al. (2012) give examples of Chicago Public School System and once-venerable Jamaica High School in New York City for the latter consequence of the lower enrollment.

An immediate question then turns out to be whether the problem can be overcome through changing centralized matching institutions. The matching theory literature, however, reveals that such a change would not work. Specifically, the so called "Rural Hospital Theorem" shows that the number of filled slots at a hospital is the same across all stable outcomes (Roth (1984)).[3] Given this result, in this paper, we approach the problem from a different point of view and address the question of whether hospitals can increase their

[1]See Nambiar and Bavas (2010), Shallcross (2005), and Kamada and Kojima (2010).

[2]Thornton and Ashley (2012) indeed document a case (based on Washington State Office of the Auditor's report) reporting a committed fraud by a state university in US to increase its enrollments to obtain more state funds.

[3]Indeed, Roth (1986) demonstrates that a hospital with an empty position not only fills the same exact number of its slots across all stable matchings, but also it matches with the exact same group of interns.

number of filled positions through misreporting their preferences under well-known Boston, Top Trading Cycles (hereafter, TTC), and stable rules.

In the formal analysis, we let hospitals have substitutable preferences, which is a more general preference class than the usually assumed responsive preferences. In order to model the filling position incentives of hospitals, we assume that each hospital prefers larger group of interns to smaller ones irrespective of interns in the groups as long as the former does not exceed its capacity. We take interns' preferences common knowledge, whereas, hospitals' preferences are their private information and have to be elicited. In this environment, hospitals might benefit from misreporting preferences through either filling more positions or matching with a more preferred group of interns while having the same number of assigned interns. As the current work is primarily interested in the position filling motive of hospitals, we say that a mechanism is strongly manipulable if there exists a problem instance and hospital with false preference such that it fills more positions under the false preference profile. On the other hand, a mechanism is said to be weakly manipulable if it is either strongly manipulable at some problem or there exists a problem instance at which some hospital is better off by misreporting its preference without increasing its number of filled positions.

We show that even though Boston and stable mechanisms are not strongly manipulable, they turn out to be vulnerable in the weak sense. On the contrary, TTC rule is strongly (hence, weakly) manipulable even in the class of responsive preferences. A policy related lesson to be taken from these results is that while it is impossible to avoid weak manipulations under the well-known rules, social planner should prefer Boston and stable mechanisms in order to avoid strong manipulations. On the other hand, it is known that Boston mechanism is vulnerable to preference misreporting by interns. Therefore, if we also take interns' manipulations into account as well, then the intern-optimal stable mechanism seems to a better choice as it is both strategy-proof for interns and not manipulable at least in the above strong sense by hospitals.

While the paper presents the results in the intern-hospital matching context, it has implications in the school choice context as well. School choice is generally considered as one-sided since schools' priorities are determined according to certain criteria imposed by law, hence, they might not represent their actual preferences. Yet, there are student placement systems in which schools form their own priorities (preferences). For instance, The New York City public school system, which is the largest one in the country, is two-sided rather than one sided in that sense (Abdulkadiroglu et al. (2005)). We, therefore, can consider schools as strategic and take their manipulation possibilities into account. Each of the considered mechanisms in this paper has being extensively used by school districts,[4] and there has been an ongoing debate about which one should be preferred. This paper adds another dimension to that debate by identifying an important disadvantage of TTC against Boston and especially the student (intern)-optimal stable mechanism.

There is an extensive manipulation line of research in the literature. While the intern-optimal mechanism is strategy-proof for interns, no stable rule is immune to preference manipulations by hospitals (Dubins and Freedman (1981) and Roth (1982)). The current paper sheds more light on that result by revealing that the way a hospital is better off through misreporting is not due to filling more positions, yet, matching with more preferred group of interns. Sönmez (1997, 1999) demonstrates that stable mechanisms are vulnerable to capacity and pre-arrangement manipulations, respectively. As opposed to the stable rules, Kesten (2012) shows that Boston and TTC are immune to capacity manipulations. Some of these negative results are recovered in large markets. Kojima and Pathak (2009) prove that, under some regularity conditions, the scope of profitable preference and capacity manipulations diminishes under the student-optimal stable mechanism as the market gets large. Some other papers regarding manipulations in matching markets include Kojima (2011), Ergin (2002), Sönmez and Pathak (2013), Afacan (2012b,a), Konishi and Unver (2006), and Kojima (2006).

[4]The student (intern)-optimal stable mechanism is in use in the New York City and Boston school districts (see Abdulkadiroglu et al. (2005) and Abdulkadiroglu et al. (2006)). Minneapolis and Lee Country of Florida use Boston mechanism (see Abdulkadiroğlu and Sönmez (2003)). New Orleans Recovery School District adopted TTC for the first time in 2012 (Vanacore (2012)).

2 Model

A matching problem consists of a tuple (H, I, R, q). The first two components are the sets of hospitals and interns, respectively. Each intern $i \in I$ has a preference relation R_i over the set of hospitals and being unassigned (denoted by $\emptyset$). The last component $q = (q_h)_{h \in H}$ is the capacity profile of hospitals.

Similar to the interns, each hospital $h \in H$ has a preference relation R_h over the groups of interns. The choice of hospital h among group of interns $I' \subseteq I$ is defined as follows:

$$C_h(I', R_h, q_h) = \{I'' \subseteq I' : \ |I''| \leq q_c \ \& \ I'' R_h \tilde{I} \text{ for any } \tilde{I} \subseteq I' \text{ such that } |\tilde{I}| \leq q_h\}.$$

The preference profile of both hospitals and interns is $R = (R_k)_{k \in I \cup H}$. For a given pair of hospitals h, h' and intern i, we write hP_ih' if hR_ih' and $h \neq h'$ (similarly for hospitals). Hospitals' preferences are *substitutable*, that is, for any hospital h and pair of groups of interns $I', I'' \subseteq I$ such that $I' \subset I''$, $C_h(I'', R_h, q_h) \cap I' \subseteq C_h(I', R_h, q_h)$. In the current paper, we are interested in the filling position incentives of hospitals. To this end, we assume that hospitals always prefer larger group of interns irrespective of the interns in the group. Formally, we make the following assumption:

For any $h \in H$ and pair of groups of interns $I', I'' \subseteq I$ such that $q_h \geq |I'| > |I''|$, $I' R_h I''$.

A *matching* μ is an assignment of interns to hospitals such that no intern is assigned more than one hospital, and no hospital is assigned to more interns than its quota. We write μ_k for the assignment of intern (hospital) $k \in H \cup I$ under μ. A matching μ is *individually rational* if $\mu_i R_i \emptyset$ for any $i \in I$, and $\mu_h = C_h(\mu_h, R_h, q_h)$ for any hospital $h \in H$. Matching μ is *blocked* by an intern-hospital pair $(i, h) \in I \times H$ if $hP_i\mu_i$ and $i \in C_h(\mu_h \cup i, R_h, q_h)$. A matching μ is *stable* if it is individually rational and is not blocked by any pair $(i, h) \in I \times H$. For ease of notation, in the rest of the paper, we just write (R, q) to denote a given matching problem.

A *mechanism* ψ is a systematic way of assigning a matching for every problem (R, q). A mechanism ψ is stable if $\psi(R, q)$ is stable for every (R, q).

In our analysis, there are two ways for a hospital to be better off by misreporting its preference: (i) increasing its number of filled positions or (ii) matching with a more preferred group of interns but having the same number of filled positions. As hospitals are assumed to be primarily interested in the former, we separate these two motives behind preference misreporting and give more emphasis to the former. The definitions below formalize our manipulation notions.

Definition 1.

(i) Mechanism ψ is strongly manipulable at a problem instance (R, q) if there exist a hospital h and R'_h such that $|\psi_h(R'_h, R_{-h}, q)| > |\psi_h(R, q)|$.[5]

(ii) Mechanism ψ is weakly manipulable at a problem instance (R, q) if it is strongly manipulable at the problem or there exist a hospital h and R'_h such that $|\psi_h(R'_h, R_{-h}, q)| = |\psi_h(R, q)|$ and $\psi_h(R'_h, R_{-h}, q) P_h \psi_h(R, q)$.

We say that mechanism ψ is immune to (weak) strong manipulations if it is not (weakly) strongly manipulable at any problem.

Before proceeding to the results, we need to introduce some further notations. Given two groups of interns I' and I'' such that $I' \subset I''$, $C_h(I'', R_h, q_h|I')$ is the choice set of hospital h out of I'' given that I' has to be in this set. That is, $I' \subseteq C_h(I'', R_h, q_h|I')$ and $C_h(I'', R_h, q_h|I') R_h \tilde{I}$ for any $\tilde{I}$ such that (i) $|\tilde{I}| \leq q_h$ and (ii) $I' \subseteq \tilde{I} \subseteq I''$.

In the rest of the paper, we investigate the vulnerability of three well-known mechanisms.

[5] R_{-h} stands for the preference profile of hospitals and interns except hospital h.

2.1 Boston Mechanism

Kojima (2007) outlines the Boston mechanism for substitutable preferences. For the sake of completeness, we define the algorithm below.

Step 1. Each intern applies to his best choice. Let I_h^1 be the set of interns applying to hospital h in this step. Then, each hospital h accepts the set of interns $C_h(I_h^1, R_h, q_h)$ and rejects the rest of them.

In general,

Step k. Each rejected intern in the previous round applies to his next best hospital. Let I_h^k denote the set of interns applying to hospital h in this step. Let A_h^{k-1} be the set of accepted interns by hospital h in the previous rounds. Then, in this step, hospital h accepts the set of interns $C_h(I_h^k \cup A_h^{k-1}, R_h, q_h | A_h^{k-1})$ and rejects the rest of them.

The algorithm terminates whenever every intern is matched with a hospital (including the "null hospital", denoted by $\emptyset$, representing being unassigned). Boston mechanism is highly criticized due to the lack of strategy-proofness. Nevertheless, Kesten (2012) shows that it is non-manipulable via capacity underreporting. Below we show that it is immune to strong manipulations, yet, manipulable in the weak sense.

Theorem 1. *Boston mechanism is immune to strong manipulations, nevertheless, it is weakly manipulable.*

Proof. See Appendix.

□

2.2 Top Trading Cycles Mechanism

Below we outline the Top Trading Cycles mechanism (attributed to David Gale) (see Shapley and Scarf (1974) and Abdulkadiroğlu and Sönmez (2003)).

Step 1. Assign a counter to each hospital keeping the track of the number of available positions. Initially set the counters equal to the capacities of hospitals. Each intern points to his favorite hospital. Each hospital h points to the intern who would be chosen if the hospital would have only one available position. That is, it points to the intern in $C_h(I, R_h, 1)$. As everything is finite, there exists at least one cycle. Every intern in a cycle is assigned to the hospital which he is pointing to and then removed. The counters of hospitals belonging a cycle are decreased by one while keeping those of other hospitals same.

In general,

Step k. Each unassigned intern in the previous step points to his favorite hospital having left available position. For any hospital h, let A_h^{k-1} be the group of interns assigned to hospital h in the previous rounds. Moreover, we write A^{k-1} for the set of interns assigned to some hospital in the previous rounds (this set includes the interns assigned to the null hospital in the previous rounds). Each hospital h with an available position points to the intern who would be chosen among remaining ones given the set of assigned interns up to the current step A_h^{k-1}. Formally, it points to intern in $C_h(I \setminus (A^{k-1} \setminus A_h^{k-1}), R_h, |A_h^{k-1}| + 1|A_h^{k-1})$. As everything is finite, there exists at least one cycle. Every intern in a cycle is assigned to the hospital which he is pointing to and then removed. The counters of hospitals belonging to a cycle are decreased by one while keeping those of other hospitals same.

The algorithm terminates when all interns are assigned to some hospital (including the null one). Abdulkadiroğlu and Sönmez (2003) defines TTC for responsive preferences (Roth (1985)) and the above version is the generalization of TTC for substitutable priorities. Even though TTC is strategy-proof for interns and immune to capacity manipulations (Kesten (2012)), the result below demonstrates that it is strongly manipulable even in the class of responsive preferences.

Theorem 2. *TTC is strongly (weakly) manipulable even in the class of responsive preferences.*

Proof. Let us consider a problem instance where $I = \{i_1, i_2, i_3\}$ and $H = \{h_1, h_2\}$ with

$q_{h_1} = 2$ and $q_{h_2} = 1$. The preference profile of interns and hospitals is given below (the preference of hospital h_1 is responsive):

$$R_{i_1}: \; h_2, \emptyset, h_1; \;\; R_{i_2}: \; h_1, \emptyset, h_2; \;\; R_{i_3}: \; h_2, h_1, \emptyset.$$

$$R_{h_1}: i_2, i_1, i_3, \emptyset; \;\; R_{h_2}: \; i_2, i_3, i_1, \emptyset.$$

It is easy to verify that the TTC outcome at the above problem (R, q) is as given below:

$$TTC(R, q) = \begin{pmatrix} i_1 & i_2 & i_3 \\ \emptyset & h_1 & h_2 \end{pmatrix}$$

Let us consider false preference for hospital h_1: $R'_{h_1}: \; i_1, i_2, i_3, \emptyset$. Then, the TTC outcome at false preference profile $R' = (R'_{h_1}, R_{h_2})$ is given below.

$$TTC(R', q) = \begin{pmatrix} i_1 & i_2 & i_3 \\ h_2 & h_1 & h_1 \end{pmatrix}$$

As we can see, $|TTC_{h_1}(R, q)| < |TTC_{h_1}(R', q)|$ showing that TTC is strongly (weakly) manipulable even in the class of responsive preferences.

□

2.3 Stable Mechanisms

The following intern-proposing deferred acceptance algorithm (Gale and Shapley (1962)) produces the intern-optimal stable matching, which is the anonymously preferred stable assignment by interns to all other stable outcomes.

Step 1. Each intern applies to his favorite hospital. Let I_h^1 be the set of interns applying to hospital h in this step. Then, each hospital h tentatively accepts the interns in the set $\tilde{I}_h^1 = C_h(I_h^1, R_h, q_h)$ and rejects the rest.

In general,

Step k. Each rejected intern in the previous step applies to his next best hospital. Let I_h^k be the set of interns applying to hospital h in this step. Each hospital h then tentatively accepts the ones in $\tilde{I}_h^k = C_h(\tilde{I}_h^{k-1} \cup I_h^k, R_h, q_h)$.

The algorithm terminates when every intern is tentatively assigned to a hospital (including the null hospital). The intern-optimal stable mechanism, denoted by ψ^I, gives the intern-optimal stable matching for every problem. The hospital proposing version of the deferred acceptance algorithm (denoted by ψ^H) gives the hospital-optimal stable matching, which is the anonymously preferred stable outcome by hospitals to all other stable matchings.

Both ψ^I and ψ^H are widely used in entry-level labor markets. For instance, while the former is in use in NRMP, The Veterinary Internship and Residency Matching Program (VIRMP), and The Dietetic Internship Matching Program (DIMP) have been using the latter.[6] On the other hand, the former is very popular in placing students to schools. The two largest school districts: New York City and Boston employ the student (intern)-optimal stable mechanism in assigning students to schools.

In what follows, we show that while stable mechanisms are immune to strong manipulations, they are weakly manipulable, We first need to prove two auxiliary results that are of interests on their own.

Alkan (2002) introduces the cardinal monotonicity notion, then, Hatfield and Milgrom (2005) adopt it to matching with contracts setting and introduce the law of aggregate demand condition. Formally, the preference of hospital h satisfies *law of aggregate demand* if, for any two groups of interns I' and I'' such that $I' \subset I''$, $|C_h(I', R_h, q_h)| \leq |C_h(I'', R_h, q_h)|$. As hospitals prefer larger groups of interns, the true preferences of hospitals satisfy law of aggregate demand.

[6] For a detailed information about the programs: http://www.virmp.org and http://www.eatright.org/ACEND/content.aspx?id=186.

Lemma 1. *The preferences of hospitals satisfy law of aggregate demand.*

In the more general matching with contract environment, Hatfield and Milgrom (2005) show that the hospitals hire the same number of doctors at any stable matching if the hospitals' preferences are substitutable and satisfy law of aggregate demand (so called "Rural Hospital Theorem" (Roth (1984)). Due to our substitutable supposition along with the above lemma, this result carries over to our setting.

Corollary 1 (Rural Hospital Theorem)**.** *The number of filled positions at every hospital is the same across stable matchings.*

Remark 1. A recent paper by Aygun and Sönmez (2012) reveals that Hatfield and Milgrom (2005) implicitly assume a condition they call "irrelevance of rejected contracts" (IRC). Moreover, they demonstrate that law of aggregate demand and substitutability imply IRC. Hence, IRC is not an issue for the above corollary that we will use in the proof of the main theorem of this section.

Theorem 3. *Stable mechanisms are immune to strong manipulations, nevertheless, they are weakly manipulable.*

Proof. See Appendix. □

3 Conclusion

While stability has proved very critical for the well-working of matching markets, it still has problems. One of the most important of them is the doctor shortages in rural area hospitals. In the school-choice context, it corresponds to low enrollments at some schools, which has important consequences as well. While the Rural Hospital Theorem (Roth (1984)) reveals the impossibility of overcoming the problem under stable allocations, this paper asks the question of whether hospitals can increase their filled positions through misreporting their preferences under well-known mechanisms. It turns out that hospitals can not increase their filled positions under Boston and stable rules through misreporting, yet, there is a room for that under Top Trading Cycles mechanism.

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Appendix

Proof of Theorem 1. For ease of notation, we write BM for the mechanism. Assume for a contradiction that BM is strongly manipulable. This means that there exist a problem instance (R, q), hospital h, and R'_h such that $|BM_h(R, q)| < |BM_h(R'_h, R_{-h}, q)|$. As hospitals prefer larger group of interns, it implies that the set of interns applying to hospital h in the course of BM at problem (R, q) is less than its total capacity. Given this observation, if hospital h does not reject any applying intern in the course BM at false preference profile, then $BM_h(R, q) = BM_h(R'_h, R_{-h}, q)$, contradicting our starting supposition.

Let us assume that hospital h rejects some applying intern at false preference profile. For ease of notation, we write $R' = (R'_h, R_{-h})$. Let k be the earliest step in the course of BM in which hospital h rejects an intern i. As the assignments up to step k are the same across both true and false preference profiles, there is an extra unfilled position at hospital h in step k as it rejects intern i, which is not the case at the true preference profile. On the other hand, intern i now applies to other hospitals which might cause some other interns to be rejected. These rejected interns in turn might apply to hospital h. However, in order to increase its number of filled positions, there has to be a rejected intern by hospital h initiating a rejection chain causing more than one intern to apply hospital h. This, however, is impossible, that is, a rejected intern by hospital h might cause at most one other intern to apply to hospital h in the course of BM. This shows that $|BM_h(R, q)| \geq |BM_h(R', q)|$, contradicting our starting supposition, hence, BM is immune to strong manipulations.

In order to see the manipulability of Boston mechanism in the weak sense, let us consider a problem instance consisting of $I = \{i_1, i_2, i_3\}$ and $H = \{h_1, h_2, h_3\}$ with $q_{h_1} = q_{h_2} = q_{h_3} = 1$. The preference profile of interns and hospitals is as follows:

$$R_{i_1}: \; h_1, h_2, h_3, \emptyset; \; R_{i_2}: \; h_3, h_2, h_1, \emptyset; \; R_{i_3}: \; h_3, h_2, h_1, \emptyset.$$
$$R_{h_1}: \; i_2, i_1, i_3, \emptyset; \; R_{h_2}: \; i_1, i_2, i_3, \emptyset; \; R_{h_3}: \; i_3, i_2, i_1, \emptyset.$$

It is easy to verify that BM outcome at the above problem instance is as follows:

$$BM(R, q) = \begin{pmatrix} i_1 & i_2 & i_3 \\ h_1 & h_2 & h_3 \end{pmatrix}$$

Now, let us consider a false preference of hospital h_1: $R'_{h_1}: i_2, i_3, \emptyset, i_1$. Note that keeping position empty is preferred to intern i_1 with respect to R'_{h_1}. As R'_{h_1} does not represent the true preference of hospital h_1, it does not contradict our starting supposition that hospitals always prefer larger group of interns. Then, BM outcome under the false preference profile $R' = (R'_{h_1}, R_{-h_1})$ is given below:

$$BM(R', q) = \begin{pmatrix} i_1 & i_2 & i_3 \\ h_2 & h_1 & h_3 \end{pmatrix}$$

As we can see from above outcomes, hospital h_1 is better off through matching with a more preferred intern while having the same number of filled positions, showing that it is weakly manipulable.

□

Proof of Theorem 3. We first show that ψ^I is not strongly manipulable. Assume for a contradiction that there exist a problem instance (R, q), hospital h, and R'_h such that $q_h \geq |\psi^I_h(R'_h, R_{-h}, q)| > |\psi^I_h(R, q)|$. This implies that the number of interns applying to hospital h in the course of ψ^I at the true preference profile is lower than its capacity due to our supposition that any hospital prefers larger group of interns.

For ease of notation, we write $R' = (R'_h, R_{-h})$. Let S^k_h and $\tilde{S}^k_h$ be the set of interns applying to hospital h in the k^{th} step of ψ^I at problems (R, q) and (R', q), respectively. Then, we have two cases to consider.

Case 1. If hospital h does not reject any intern at R' in the course of ψ^I, then $\psi^I_h(R',q) = \psi^I_h(R,q)$. This comes from the fact that the number of applying interns to hospital h at R is lower than its capacity q_h. This case, hence, contradicts our starting supposition.

Case 2. Let us assume that hospital h rejects some applying interns in the course of ψ^I at the false preference profile R'. Let $\tilde{k}$ be the first step in which hospital h rejects an applying intern. Then, it implies that $S^k_h = \tilde{S}^k_h$ for all $k \leq \tilde{k}$. Let i be a rejected intern in step $\tilde{k}$. He might cause another one to be rejected by applying to other hospitals after getting rejected by hospital h. Note that this is not the case under the true preference profile, that is, as the number of applying interns to hospital h is lower than its capacity and hospital h prefers any larger group of interns, it never rejects an intern at the true preference profile. While this rejection causes one position to be unfilled by step $\tilde{k}$, it might lead another intern to apply hospital h, which is not the case at the true preference profile. Hence, hospital h can increase its number of filled positions by misreporting its preference only if any rejected intern results in more than one rejected interns from other hospitals who apply to hospital h after rejection chains initiated by the former. This, however, is impossible as, in the course of ψ^I, any rejection chain initiated by an intern might lead at most one other intern to apply to hospital h. As this is true for all other rejected interns by hospital h as well, we have $|\psi^I_h(R',q)| \leq |\psi^I_h(R,q)|$, contradiction our starting supposition. Therefore, ψ^I is not strongly manipulable.

Now, let ψ be any stable mechanism other than ψ^I. Assume that there exist a problem (R,q), hospital h, and false preference R'_h such that $|\psi_h(R,q)| < |\psi_h(R',q)|$.[7] By Corollary 1, $|\psi_h(R,q)| = |\psi^I_h(R,q)|$. On the other hand, by our above proof, $|\psi^I_h(R',q)| \leq |\psi^I_h(R,q)|$. As $\psi_{h'}(R',q) R_{h'} \psi^I_{h'}(R',q)$ for any $h' \in H \setminus h$,[8] we have $|\psi_{h'}(R',q)| \geq |\psi^I_{h'}(R',q)|$ for any $h' \in H \setminus h$. This along with $|\psi_h(R',q)| > |\psi_h(R,q)| = |\psi^I_h(R,q)| \geq |\psi^I_h(R',q)|$ shows that $\sum_{h \in H} |\psi_h(R',q)| > \sum_{h \in H} |\psi^I_h(R',q)|$. This implies that there exists an intern i who is unassigned at matching $\psi^I(R',q)$ and assigned to some hospital at matching $\psi(R',q)$.

[7] We again write $R' = (R_{-h}, R'_h)$.

[8] As ψ^I is the anonymously dispreferred stable matching to all other stable ones by hospitals.

This in turn implies that $\psi_i(R', q) P_i \psi_i^I(R', q) = \emptyset$, which contradicts the fact that ψ^I is the anonymously preferred stable matching by interns to all other stable outcomes. Hence, all stable mechanisms are immune to strong manipulations.

Now, we need to show that stable mechanisms are weakly manipulable. Let us consider a problem instance consisting of $I = \{i_1, i_2, i_3\}$ and $H = \{h_1, h_2\}$ with $q_{h_1} = 2$ and $q_{h_2} = 1$. The preference profile of interns and hospitals is as follows (the preference of hospital h_1 is responsive):

$$R_{i_1} : h_2, h_1, \emptyset;\ R_{i_2} = R_{i_3} :\ h_1, h_2, \emptyset.$$

$$R_{h_1} : i_1, i_2, i_3, \emptyset;\ R_{h_2} : i_2, i_1, i_3, \emptyset.$$

Then, the unique stable matching μ in the problem is given below:

$$\mu = \begin{pmatrix} i_1 & i_2 & i_3 \\ h_2 & h_1 & h_1 \end{pmatrix}$$

Now, let us consider the responsive false preference: $R'_{h_1} : i_1, i_3, \emptyset, i_2$.[9] Then, the unique stable matching under the false preference profile μ' is given below:

$$\mu' = \begin{pmatrix} i_1 & i_2 & i_3 \\ h_1 & h_2 & h_1 \end{pmatrix}$$

As we can see from the above matchings, hospital h_1 is better off under μ' without increasing its filled positions, showing that every stable rule is weakly manipulable.

□

References

Abdulkadiroglu, A., P. A. Pathak, and A. E. Roth (2005): "The New York City High School Match," *American Economic Review*, 95(2), 364–367.

[9] Similar to the proof of Theorem 1 above, keeping position empty is preferred to intern i_2 with respect to R'_{h_1}. As R'_{h_1} does not represent the true preference of hospital h_1, it does not contradict our starting supposition that hospitals always prefer larger group of interns.

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