

# FREE PARKING FOR ALL IN SHOPPING MALLS\*

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## Abstract

We show why a shopping mall prefers to provide parking for free and embed the parking costs in the prices of the goods. This holds if the mall has monopoly power or prices competitively; if there is parking validation or a trade-off between shopping and parking spaces. It is also the second-best social optimum. Generally, the equilibrium lot size is too small, yielding a rationale for minimum parking requirements. In urban malls, parking fees may be positive because individuals can use the lot without intending to shop, and lots may become too large because of the trade-off between shopping and parking spaces.

**Keywords:** land use; lot size; parking fee; parking requirements; shopping mall

**JEL Classification:** D42; H21; L91; R41; R52

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# 1 Introduction

The average Joe does not think much about parking, but economists should. Other than money and credit cards, parking is probably the most important intermediate good in the modern economy. Needless to say, it consumes a vast quantity of natural resources. The total amount of land devoted to parking in the US would cover several New England states (Jakle and Sculle, 2004, pp. 1-2). And the price put on this commodity is very low. The US Department of Transportation (1990) found that parking was free for 99% of car trips. Shopping malls are one of the largest contributors to the stock of parking spaces. There are over 100,000 shopping malls in the US. A typical shopping mall has 4-6 parking spaces per 1,000 sq. ft. of gross leasable area, suggesting that the average mall has more space allocated to parking than stores (International Council of Shopping Centers and Urban Land Institute, 2003; henceforth, ICSC and ULI, 2003). More interestingly, in the same survey, 94% of the malls reported that they charged no fee for parking and 4% did not respond to this question. Thus, we can only be sure that 2% of these parking spaces have any fee whatsoever. Shoup (2005) estimates the cost of a parking space in the US to be at least \$125 a month. He then asks, “if parking costs so much, why is it usually free?”

Based on these figures, the current literature finds parking too cheap and its quantity too high, especially given the negative externalities from congestion and the air pollution produced by parking. This research focuses mainly on urban areas where these negative externalities are most severe. There has also been increasing attention to *minimum parking requirements* which affect land use by forcing property developers to allocate certain amounts of land for parking (van Ommeren and Wentink, forthcoming). Many towns and cities continue to impose these requirements even though no one knows where they come from or what they are based on (Shoup, 1999, 2005, 2006). Is underpricing an issue for all forms of parking? Are minimum parking requirements unnecessary or irrational as put forward by some economists and urban planners, or are they justified for some land uses? Our

paper finds that parking is priced properly in shopping malls, that both society and the shopping mall want parking to be free, and furthermore that society generally wants to require minimum parking lot sizes.

Our contribution to the literature begins by noting that there are three parties in the parking problem, one of which previous research has not analyzed. These parties are the customer, the primary demander of parking; the parking lot, the supplier; and the store, which acts as a secondary demander for parking. Because stores rely on parking for business, they are vitally concerned with how much parking is available and its price. Most of the literature on economics of parking focuses on the customers and the negative externalities they impose on each other by parking. Some papers—notably Arnott (2006) and Arnott and Rowse (2009b)—focus on the incentives of the parking lot provider; but no paper combines all three parties.

This paper is a first analysis of the shopping mall parking problem. What makes shopping mall parking an interesting research question is that the parking provider and the stores are combined into one economic entity. This simplifies the analysis of the optimal price and quantity of parking. Shopping malls have two methods to charge customers for parking. They can raise either the price of the good or the parking fee to absorb the costs of parking. What we find is that both the shopping mall and society want the price of the good to absorb the entire cost of parking. Moreover, a social planner always wants the mall to provide more than the profit-maximizing amount of parking. This justifies the common implementation of minimum parking requirements. To our knowledge, ours is the first attempt to explain the foundations of minimum parking requirements.<sup>1</sup>

The key to our analysis is recognizing that sometimes shoppers do not find what they want. We do not argue that this is the normal outcome, merely that it occurs. For example,

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<sup>1</sup>Arnott (2006) discusses potential effects of minimum (and maximum) parking requirements and van Ommeren and Wentink (forthcoming) quantify the welfare loss caused by them, but they do not focus on explaining their rationale.

one of the authors is still searching for a footstool high enough for his son. The other author's wife went to seven different malls before buying a dress to wear to a wedding. When this outcome is possible, charging a parking fee to risk-averse customers is like charging them for losing a lottery. Thus, both the mall and society prefer to have the cost of parking embedded in the price of the goods. In fact, the mall would like to fully insure the marginal customer, who is indifferent between visiting and not visiting the mall, but this would require negative parking fees and is not implementable. This result holds as long as customers are risk averse, but the degree of risk aversion does not matter. Thus, it holds no matter how small the good is relative to the customer's wealth. It is also quite robust. It holds if the shopping mall has monopoly power or prices competitively. It holds even if we allow the shopping mall to provide parking vouchers or if it faces a trade-off between the space devoted to shopping and to parking.

We derive these results in a model where a monopolist shopping mall sells one good to homogeneous customers. We treat the shopping mall as a monopolist because we want to avoid competitive explanations for free parking. A standard result is that prices decrease in the face of competition; thus if we find parking is free for a monopolist then we expect that it will be free for a shopping mall in a competitive environment. We verify this in a reduced-form competitive variation of our model. Furthermore, notice that the shopping mall's goal is not to insure the customer. Like the principal-agent model with moral hazard, it simply finds that insuring the customer maximizes its profits. In addition, although in our model we have homogeneous customers and one good, we are not arguing that every customer faces this uncertainty. Rather, what is important for our argument is that the marginal customer does, or, if there are heterogeneous goods, then the customer faces this uncertainty with regard to some of the goods he would like to purchase.

ICSC and ULI (2003) reported that shopping malls that charge for parking are mostly located in large urban areas. In keeping with the survey findings, our results change when we look at an urban mall. We define a shopping mall as *suburban* if the only reason to use

the parking lot of the mall is to shop at the mall; otherwise it is *urban*. In suburban malls, parking is an intermediary for transactions in the mall. It does not interfere with traffic and there is no significant land allocation trade-off between lot size and shopping space. So, the parking fee can be embedded into the price of the good. In urban malls, however, parking is more like a commodity and thus should be priced independently, which means that the mall begins behaving like a parking garage. In maximizing profit, the urban mall has to balance providing insurance to shoppers with trying to extract surplus from non-shoppers as well. This results in positive parking fees if the latter motive dominates.

The second crucial difference of the urban mall is the land allocation trade-off that prevents the mall from expanding the parking lot without shrinking the size of the store space. In such environments, it is no longer clear that society wants a larger parking lot than the shopping mall. This may explain why some large cities and smaller towns in the US, such as San Francisco, Seattle, San Antonio, Portland, Oregon, Cambridge (Massachusetts), Redmond (Washington), Queen Cree (Arizona), and Concord (North Carolina) have begun also regulating the maximum parking lot size. The UK also started imposing maximum parking requirements throughout the country, most likely because land allocation trade-off is more intense in an island country.

A small but rapidly growing literature in economics deals with parking. One group of papers focuses on the price of parking, while a relatively smaller group looks at land use and parking requirements. To our knowledge, no paper in the literature analyzes the economics of shopping mall parking. The whole literature is shaped by Vickrey's (1954) idea of pricing parking at its social opportunity cost, just like any other commodity. Prominent books on urban transportation, such as Arnott, Rave, and Schob (2005) and Small and Verhoef (2007), provides significant coverage of underpricing of parking in urban areas that results largely from cruising for parking. Perhaps the most influential work in both parking pricing and land use is Shoup (2005), which underlines the high cost of free parking in all of its modes. While it is undoubtable that parking has high social costs, we argue that it may be better

for society to reflect these costs in the price of goods.

Early work focuses on how congestion externalities influence parking fees. Glazer and Niskanen (1992) point out that hourly parking fees may increase congestion by causing shorter parking durations. Building on Vickrey's bottleneck model (1969), Arnott, de Palma, and Lindsey (1991) analyze the optimal temporal-spatial dispersion of parking fees and derive the prices that eliminate queuing and induce drivers to park at the most distant parking spaces first. Anderson and de Palma (2004) build on similar arguments in a linear-city model and find that the optimal price can be attained if parking is priced in a monopolistically competitive fashion. Anderson and de Palma (2007) show that the same result holds even when they endogenize land use. Arnott and Rowse (1999) consider a circular city and find that the optimal fee is equal to the externality imposed, but because there are multiple equilibria, the optimal parking fee may not work since traffic may end up at the bad equilibrium.

A recent surge of work elaborates on cruising for parking. Arnott and Inci (2006) find the optimal fees and quantity of on-street parking to eliminate cruising for parking, and Arnott and Inci (2010) analyze the transient dynamics of downtown parking and traffic in a similar model. Arnott and Rowse (2009b) consider the optimal on-street parking capacity when parking garages compete with on-street parking. Arnott and Rowse (2009a) further extend this model to allow for heterogeneity in parking duration and value of time, and analyze the imposition of parking time limits. Building on Calthrop (2001), Calthrop and Proost (2006) analyze the optimal parking fee when both on- and off-street parking are available. They find that the on-street parking fee should be set at the marginal cost of off-street parking. Arnott and Rowse (2009b) include in this equalization that the prices should be adjusted by the cruising necessary to find on-street parking. On the empirical side, van Ommeren, Wentink, and Dekkers (2011) estimate the cost of cruising in Amsterdam, and using a nation-wide dataset, van Ommeren, Wentink, and Rietveld (forthcoming) provide comprehensive descriptive information on cruising for parking and its determinants in the Netherlands.

Arbatskaya, Mukhopadhaya, and Rasmusen (2009) provide another rationale for why parking lots in shopping malls are so large and why this may be socially desirable. The basic insight is that if the demand for parking will be higher than supply by a small amount, the welfare loss from this situation is not limited to a few drivers being unable to find a parking space. In such a situation, all drivers will preemptively arrive early to avoid being shut out of the parking lot, and as a result welfare will significantly decrease. This effect is similar to Vickrey (1969), in which drivers arrive early to the bottleneck to avoid the delay.

There is relatively little analysis of the optimal lot size for off-street parking. Arnott (2006) derives the capacity chosen by parking garages and considers the potential effects of minimum and maximum parking requirements. In practice, determining the minimal lot size is at best an ad-hoc practice. Shoup (1999, 2005) rightfully criticizes minimum parking requirements because no one knows the criteria behind them or whether they are appropriate for a particular development. Shoup (1999) reports from Willson's (1996) survey of 144 planning directors: two of the most frequently used methods in setting the parking requirements are surveying nearby cities and consulting the handbooks of Institute of Transportation Engineers. This leads him to conclude that minimum parking requirements distort land use and to recommend eliminating them for all land uses. We show that they are well justified and could be structurally based for shopping mall lots.

The paper is organized as follows. Section 2 presents the base model, derives the equilibrium and socially optimal parking fees followed by a discussion of parking validation and a competitive variation of the model. Section 3 modifies the base model to analyze the lot size in equilibrium as well as the social optimum. Section 4 discusses two urban complications: the possibility of free riding on parking spaces and land allocation trade-off between lot size and shopping space. Section 5 concludes. An appendix contains a proof and a discussion of two alternative specifications of the model.

## 2 The Parking Fee

We start off by describing the base model, which we shall extend in various ways in the next sections. A risk-neutral monopolist shopping mall sells one good, which has no cost, at a non-negative price of  $P$ . The only way to reach the mall is by car, and thus the mall has to provide parking spaces for incoming customers, which costs  $c > 0$  per unit of parking. The parking fee is denoted by  $t$ . Both  $P$  and  $t$  are determined by the mall and both of them are common knowledge.<sup>2</sup>

There are also strictly risk-averse customers whose utility function is represented by  $u(\cdot)$  with  $u' > 0$  and  $u'' < 0$ . All customers have the same initial wealth of  $w > 0$  and the reservation value of not visiting the mall of  $r > 0$ . The reservation value represents the savings in fuel and time from not shopping plus the value of the activity that customers can engage in instead of shopping. Each customer either purchases one or zero units of the good. The value of the good to a customer is  $v$ , which has the common knowledge distribution  $F(v)$  with support  $[0, \bar{v}]$  and density  $f(v) > 0$ . We assume that  $F(v)$  has the standard monotone hazard rate property or that  $f(v)/(1 - F(v))$  is increasing. We also assume that  $\bar{v}$  is high enough to cover the mall's costs. In particular, if the mall provides parking only for the highest type, then it makes a strictly positive profit on that type. This is sufficient for the mall to exist.

The critical innovation in our model is that when customers go shopping they do not always find the good that they want. Sometimes a customer searches all day and, in the end, leaves empty-handed. We represent this by saying that the probability that the good sold at the shopping mall is the customer's desired good is  $\rho \in (0, 1)$ . Formally, this means that with probability  $\rho$  the customer realizes that the good has value  $v$  and purchases it if  $v \geq P$ , and with probability  $1 - \rho$  the good has a value of zero and the customer does not

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<sup>2</sup>In reality, the mall generally does not own any of the stores in the mall and does not directly sell the goods itself. However, the stores almost certainly have a strong input into the parking fee; thus treating the mall owner and the stores as separate entities is an unnecessary complication for our purposes.



purchase it.

## 2.1 Equilibrium

Having described the economic environment, we are now in a position to calculate the equilibrium parking fee offered by the mall. Consider first the customer's problem. If he does not go to the shopping mall, he gets his reservation utility  $u(w + r)$  with certainty. There are, however, two possibilities if he goes to the shopping mall. He gets  $v - P$  if it turns out that the good sold at the shopping mall is his desired good, but he also has to pay a parking fee of  $t$ . As a result, his utility in this case is  $u(w + v - P - t)$ , which is realized with probability  $\rho$ . On the other hand, he gets zero if it turns out that the good sold at the shopping mall is not his desired good, and he still has to pay the parking fee. As a result, his utility in this case is  $u(w - t)$ , which is realized with probability  $1 - \rho$ . Consequently, the expected utility of visiting the shopping mall,  $E(u|P, t)$ , is

$$E(u|P, t) = \rho u(w + v - P - t) + (1 - \rho) u(w - t). \quad (1)$$

A customer visits the shopping mall if his expected utility of doing so is at least as high as his reservation utility:  $E(u|P, t) \geq u(w + r)$ , which defines the (unique) value of the good to the *marginal customer* who is indifferent between visiting and not visiting the shopping mall,  $\tilde{v}(P, t)$ :

$$\tilde{v}(P, t) \equiv u^{-1} \left( \frac{u(w + r) - (1 - \rho) u(w - t)}{\rho} \right) - w + P + t. \quad (2)$$

Note, for future reference, that<sup>3</sup>

$$\tilde{v}_P = 1 \quad \text{and} \quad \tilde{v}_t = \frac{(1 - \rho) u'(w - t)}{\rho u'(w + \tilde{v} - P - t)} + 1 \geq \frac{1}{\rho}. \quad (3)$$

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<sup>3</sup>Here and throughout the paper, we write  $x_y$  for  $\partial x / \partial y$ .

Consider now the mall's problem. A customer who visits the shopping mall buys a good with probability  $\rho$ , yielding an expected payoff of  $\rho P$  to the mall. In addition, it collects  $t$  from each customer who visits the shopping mall, but it costs  $c$  to provide a parking space for each of them. Thus, the payoff of the mall per customer is  $\rho P + t - c$ . Given that there are  $1 - F(\tilde{v}(P, t))$  such customers visiting the shopping mall, the mall's objective is to maximize

$$\Pi(P, t) = [1 - F(\tilde{v})](\rho P + t - c) \quad (4)$$

subject to the rationality constraint  $\rho P + t - c \geq 0$ , which ensures that it is optimal for the mall to operate the business, and the non-negativity constraint  $P \geq 0$ , which ensures the non-negativity of the price of the good. At this point, we do not require  $t$  to be non-negative because it might be optimal for the mall to subsidize parking. In fact, this will turn out to be the case.

The objective function of the mall is concave. Ignoring the rationality and non-negativity constraints for the moment, the first-order conditions of the problem are

$$\Pi_P = -f(\tilde{v}) \tilde{v}_P (\rho P + t - c) + (1 - F(\tilde{v})) \rho \quad (5)$$

$$\Pi_t = -f(\tilde{v}) \tilde{v}_t (\rho P + t - c) + (1 - F(\tilde{v})). \quad (6)$$

If both first-order conditions are zero, then we can eliminate the terms related to the distribution of customers' valuations by combining the two first-order conditions. Then, the solution of the problem is characterized only by the properties of the marginal customer, to whom the mall would like to provide no surplus. This is summarized in the following relation.

$$\frac{\tilde{v}_P}{\rho} = \tilde{v}_t. \quad (7)$$

Notice that  $\tilde{v}_P/\rho = 1/\rho$ , which is the lower bound for  $\tilde{v}_t$ . To reach this lower bound, one can see from the equation for  $\tilde{v}_t$  (equation 3) that  $P = \tilde{v}$ , or that the price of the good is equal

to the valuation of the marginal customer. Putting this condition into equation 2 yields  $t^* = -r$ .

It is optimal for the mall to *fully insure* the marginal customer for the risk that he is taking by searching for the good at the mall. This solution requires the mall to subsidize parking even when the customer does not buy the good, which makes the implementation of such pricing practically impossible. Otherwise, anyone who shows up at the shopping mall can claim that the good is not his desired good and enjoy the parking subsidy. This means that we have to look for a constrained solution in which the parking fee cannot be negative. This requires adding yet another non-negativity constraint to the maximization problem,  $t \geq 0$ .

The characterization of the constrained solution is easy and entails the free provision of parking,  $t^* = 0$ . We know that the monopolist mall provides the marginal customer nothing more than his reservation utility, which implies that whatever the marginal customer gets in equilibrium must have a certainty equivalent of  $u(w + r)$ . However, the mall cannot fully insure him this time because full insurance requires subsidizing parking, which is ruled out by the inequality constraint  $t \geq 0$ . Nonetheless, it can still employ a pricing scheme that gives the customer an expected utility of  $u(w + r)$ . As a result, the equilibrium price of the good can be derived from

$$P^* = \frac{1 - F(\tilde{v})}{f(\tilde{v})} + \frac{c}{\rho}. \quad (8)$$

The left-hand side of this expression is strictly increasing in  $P$  and the right-hand side is weakly decreasing; thus there exists a unique  $P^*$ . The first term on the right-hand side is the standard monopoly mark-up in this type of a model. The second term guarantees that revenue is higher than costs. This discussion leads to our first important result.

**Proposition 1 (Equilibrium parking fee)** *Free provision of parking is the unique equilibrium.*

Notice that this result depends only on customers being risk averse; the degree of risk aversion does not matter. Thus, it holds even when the good is small relative to the customer's wealth level. Moreover, risk-neutral customers with a binding time constraint behave as if they were risk averse (see, for example, Drewianka (2008) and the references therein). It is also noteworthy that, since there are no administrative or operational costs related to collecting parking fees in the model, parking is not free because the mall finds these costs important and thus bundles them in the price of the good. Finally, notice that only the marginal customers have to have  $\rho < 1$ . If there are some customers who are certain to get what they want, they are indifferent between paying parking fees and having the fee embedded in the price of the good. The marginal customers will cause the mall to set the parking fee at zero, even in very large malls.

A graphical analysis of the solution is given in Figure 1. The payoffs to customers when they buy the good are shown on the  $y$ -axis and when they do not on the  $x$ -axis. The indifference curve of the marginal customer in equilibrium is represented by  $I_1$ , which gives him a utility level of  $u(w + r)$ . The indifference curves of all other customers who visit the shopping mall are lined up in the shaded area. We also show two iso-profit lines of the mall, one passing through the full insurance point and another through the equilibrium point, which are denoted by  $\Pi_1$  and  $\Pi'_1$ , respectively. Note that  $\Pi_1$  is associated with a higher profit level for the mall than  $\Pi'_1$  since it is much closer to the origin, and thus it leaves lower payoffs to the customer in case of "full insurance." The 45°-line, or the *certainty line*, gives the set of all offers on which the customer gets the same payoff in both states of the world (*i.e.*, when he ends up buying the good and when he does not).

The key to the unconstrained solution is that the mall is risk neutral whereas customers are risk averse, which means that it is optimal for the mall to offer a price for the good and a parking fee pair such that the marginal customer ends up at point  $A$  in Figure 1, where he gets the same payoff in both states. In this solution, the marginal customer is indifferent between visiting the shopping mall and not, and all other customers in the shaded area earn

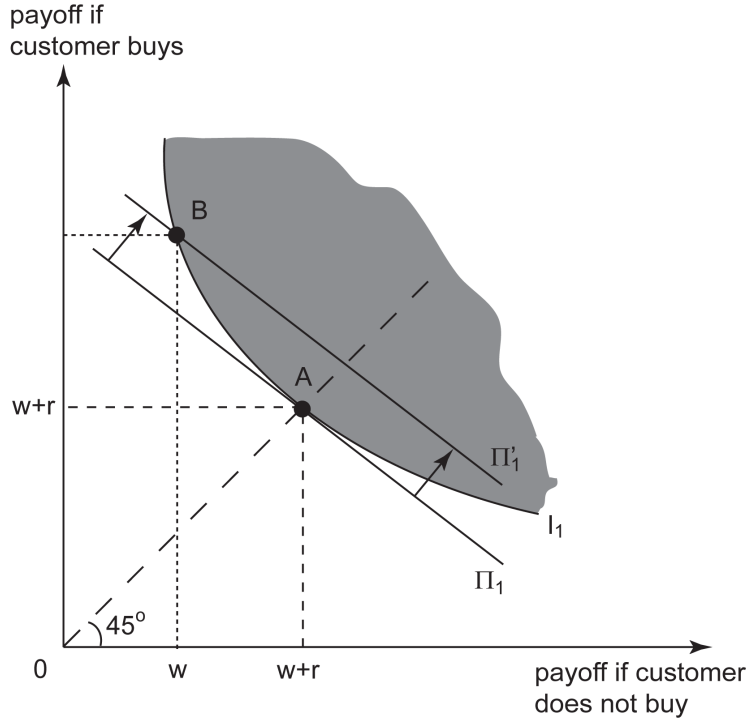


Figure 1: Free Provision of Parking

rent. The profit of the mall is represented by the iso-profit line  $\Pi_1$  in this unconstrained solution. As explained before, this solution is not implementable in practice because it requires subsidizing parking whether the customer buys a good or not.

In the constrained solution, we impose the restriction that the parking fee cannot be negative. In any such solution, the maximum attainable payoff of the customer when he does not buy the good is  $w$ . Among all these solutions, the best one in terms of profits is point  $B$ , which gives the marginal customer the maximum attainable payoff when he does not buy the good, and the corresponding payoff represented by the  $y$ -axis of point  $B$  when he buys it. In this constrained solution, as in the unconstrained solution, the marginal customer gets an expected utility of  $u(w+r)$  since he is still on the same indifference curve. Yet, the profit of the mall is now lower since it moves from the iso-profit line  $\Pi_1$  to the iso-profit line  $\Pi'_1$ .

Some comments on the robustness of our model specifications are in order. First, our

results hold for any degree of risk aversion. So, as previously mentioned, the results hold no matter how small the good is relative to the customer's wealth. Second, for ease of explication, we call  $\rho$  the customer's probability of finding his desired good; however, one can interpret  $\rho$  more broadly as the probability of purchase. In that case, even when the customer is certain to find his desired good at the mall (*e.g.*, an exact brand of an LCD TV), he might not purchase it right away, and thus his probability of purchase is less than one. Third, allowing for some number of customers with  $\rho = 1$  does not alter our results as long as the probability of purchase in the aggregate is less than one from the perspective of the mall. That is, the mall looks at the marginal customer who will almost always have  $\rho < 1$ . Fourth, the implications of the model are the same if customers buy a bundle of goods, some of which are sure to be found in the mall (*e.g.*, toothpaste or bread), as long as at least one good is not found with a probability of one. In all of these cases, in trying to maximize its profits, the mall has incentive to "provide insurance" to the customers for the risk they face by searching for the good at the mall.

## 2.2 Welfare

Since we have found that the mall prefers providing parking for free, the natural next question is whether a social planner agrees. We assume that the social planner maximizes total welfare,  $W(P, t)$ , defined as the sum of customers' net utility,  $U(P, t)$ , and the mall's profit defined in equation 4. Therefore,

$$W(P, t) = U(P, t) + \Pi(P, t), \quad (9)$$

where  $U(P, t)$  is the integration (over the valuations of customers) of the maximum of customer's expected utility from visiting the mall and his reservation utility:

$$U(P, t) = \int_0^{\bar{v}} \max [E(u|P, t), u(w + r)] dF(v), \quad (10)$$

where  $E(u|P, t)$  is given in equation 1.

The derivative of total welfare with respect to  $t$  is

$$W_t = \int_{\bar{v}}^{\bar{v}} E_t dF(v) + \Pi_t. \quad (11)$$

It is easy to find the sign of this derivative. As we have shown above,  $\Pi_t$  is negative for all  $P$  and  $t \geq 0$ . Moreover, customers do not like fees since  $E_t = -(1 - \rho) u'(w - t) - \rho u'(w + v - P - t) < 0$ . Thus,  $W_t < 0$  and the socially optimal parking fee is also equal to zero.

This is, of course, the social optimum in a second-best sense. By removing the constraint  $t \geq 0$ , we can find the first-best parking fee and show that it is less than  $-r$ . The derivative of total welfare with respect to  $P$  is

$$W_P = \int_{\bar{v}}^{\bar{v}} E_P dF(v) + \Pi_P, \quad (12)$$

where  $E_P = -\rho u'(w + v - P - t) < 0$ . This means that, in the unconstrained equilibrium (which requires  $\Pi_P = \Pi_t = 0$ ), both  $W_P$  and  $W_t$  are negative, meaning that both the price of the good and the parking fee of the unconstrained solution ( $t^* = -r$ ) are too high. Therefore, the first-best social optimum requires *subsidizing free parking*, but for the reasons explained in the mall's problem, such a solution is not implementable. This leads to the second-best optimum in which parking is free. We record the results of this discussion in the following proposition.

**Proposition 2 (Social optimum)** *Free provision of parking is socially optimal in a second-best sense, but the price of the good that the mall charges is too high because of its monopoly power.*

One may notice that this welfare analysis does not include the social costs of parking.

Shoup (2005) argues that the high cost of parking is due to social costs that stem from congestion and pollution externalities. One may be tempted to think that such social costs may encourage the social planner to charge positive parking fees. After all, it is the shopping (which always results in parking) that causes the social costs, not the purchasing. However, it turns out that this is not the case. When society has two ways of covering the costs of parking, it would rather have the price of the good absorb the costs (both private and social) rather than impose a direct parking fee.

To facilitate this idea, let the social costs of demand be  $SC(Q)$ , where  $Q = 1 - F(\tilde{v})$  and modify the welfare function as follows:

$$W(P, t) = U(P, t) + \Pi(P, t) - SC(1 - F(\tilde{v})). \quad (13)$$

The first-order conditions of welfare maximization are

$$W_P = \int_{\tilde{v}}^{\bar{v}} E_P dF(v) - f(\tilde{v}) \tilde{v}_P (\rho P + t - c - SC') + (1 - F(\tilde{v})) \rho \quad (14)$$

$$W_t = \int_{\tilde{v}}^{\bar{v}} E_t dF(v) - f(\tilde{v}) \tilde{v}_t (\rho P + t - c - SC') + 1 - F(\tilde{v}). \quad (15)$$

Both conditions are satisfied when

$$\left( \frac{\int_{\tilde{v}}^{\bar{v}} E_t dF(v) + 1 - F(\tilde{v})}{\int_{\tilde{v}}^{\bar{v}} \frac{E_P}{\rho} dF(v) + 1 - F(\tilde{v})} \right) \frac{\tilde{v}_P}{\rho} = \tilde{v}_t. \quad (16)$$

This expression differs from the one in equation 7 only by the term in the large parenthesis, which can be shown to be smaller than one. This implies that  $\tilde{v}_P/\rho > \tilde{v}_t$ , and since  $\tilde{v}_P = 1$ , we have  $1/\rho > \tilde{v}_t$ . On the other hand, we also know that  $\tilde{v}_t \geq 1/\rho$ . Thus, we have a contradiction, and the socially optimal parking fee is its lower bound, zero.

From equation 16 one can see that, as long as the cost of parking is a constant marginal cost, free parking will be socially optimal in a more general environment. For example,



if there were heterogeneous customers who each wanted only one of a set of goods, then welfare could be expressed as the sum of welfare for each good, and the results above would immediately generalize. The most difficult case would be if customers were homogeneous but wanted to buy multiple individually priced goods. This would give rise to interesting cross subsidization issues—which good’s price should bear the brunt of the cost of parking? However, we believe that the socially optimal parking fee would still be zero. From these two cases, we can proceed to general demand curves. We also expect that these results would continue to hold for more general cost functions.

### 2.3 Parking validation

The reader might be disturbed by the inequity of free parking in our model. After all, everyone who comes to the shopping mall raises the costs of the mall. Why do some get off without paying for it? Perhaps if the mall used parking vouchers, this would give the mall an incentive to charge non-buyers a fee. However, it turns out that parking vouchers do not change the mall’s incentives. It may now either subsidize buyers for parking or not charge them at all, but non-buyers will still park for free. This means that free provision of parking is one of the equilibria even when validation is allowed. More importantly, if there were transaction costs associated with collecting the parking fees in the model, free provision of parking would be the only equilibrium. As a matter of fact, ICSC and ULI (2003) report that 86 percent of the shopping malls do not have a parking validation program.

In a parking validation system, customers are allowed to validate the receipt of the good that they buy when they are leaving the parking lot. This allows the mall to charge a different parking fee to the customers who buy a good than those who do not. Let the parking fee when a customer buys a good be  $t_b$  and that when he does not be  $t_{nb}$ . Now, the

marginal customer's valuation is given by<sup>4</sup>

$$\tilde{v}(P, t_b, t_{nb}) \equiv u^{-1} \left( \frac{u(w+r) - (1-\rho)u(w-t_{nb})}{\rho} \right) - w + P + t_b. \quad (17)$$

Note that  $\tilde{v}_P = \tilde{v}_{t_b} = 1$ , and  $\tilde{v}_{t_{nb}} = ((1-\rho)/\rho)u'(w-t_{nb})/u'(w+\tilde{v}-P-t) \geq (1-\rho)/\rho$ .

The mall's maximization problem when we allow for validation is

$$\max_{P, t_b, t_{nb}} \{(1 - F(\tilde{v}))(\rho P + \rho t_b + (1 - \rho)t_{nb} - c)\}. \quad (18)$$

By setting the first-order conditions with respect to  $P$  and  $t_{nb}$  to zero, we get  $((1-\rho)/\rho)\tilde{v}_P = \tilde{v}_{t_{nb}}$ . Again, the right-hand side is the lower bound for  $\tilde{v}_{t_{nb}}$  and  $t_{nb}^* = -r$ , but as before this solution is not implementable and thus  $t_{nb}^* = 0$ . Customers would not validate their parking if  $t_b > t_{nb}$ , so we can conclude that  $t_b^* \leq 0$ , or, in words, the mall prefers either to subsidize buyers' parking or to provide them parking for free.

The price of the good and the parking fee for buyers are not uniquely determined this time:

$$P^* + t_b^* = \frac{1 - F(\tilde{v})}{f(\tilde{v})} + \frac{c}{\rho}. \quad (19)$$

However, the equilibrium payoffs will always be as in the base model. All solutions are represented by point  $B$  in Figure 1. The mall may want to subsidize buyers' parking, but if it does so, this subsidy will increase the price of the good in a one-to-one ratio. That is, it may appear to be helping out the buyers, but it is indeed just transferring a fee between the two prices. The following proposition summarizes our findings.

**Proposition 3 (Parking validation)** *Free provision of parking is an equilibrium even when*

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<sup>4</sup>In this section and in the rest of the paper, we uniformly use the same notation of the base model:  $\tilde{v}$  for valuation of the marginal customer,  $\Pi$  for profit of the mall,  $U$  for sum of customers' net utility, and  $W$  for total welfare. We are not going to distinguish notation in different models even when, for example, the valuation of the marginal customer is given by a different expression than the one in the base model. We shall, however, use superscripts or decorations whenever we think that keeping the same notation can be confusing.

*parking validation is allowed. The mall can either charge no parking fees at all or provide free parking to those who do not validate their tickets while subsidizing the rest upon validation.*

Notice that one solution of parking validation is  $t_b^* = t_{nb}^* = 0$ , the solution we found in the base model. More interestingly, if there are any transaction costs associated with collecting the parking fees such as administrative or operational costs, this solution becomes the only solution. This explains why validation programs are so infrequent in shopping malls.

## 2.4 Competitive pricing

The reader may also be concerned that the results depend on the mall being a monopolist. Perhaps if it were not able to extract supranormal profits with the price of the good, it would be optimal to try to extract some surplus through a parking fee. We can partially respond to this concern by assuming that the price of the good is determined competitively (*i.e.*, it is not a function of demand). To do this, we can assume that the shopping mall has more than one store that offers the same good, and that the stores are Bertrand competitors. This, one should notice, is actually closer to a proper model of a shopping mall—where the mall does not own the stores.<sup>5</sup>

Assume that the mall requires a profit of  $\pi > 0$  per customer. Optimizing over  $\pi$  would be the same as optimizing over  $\rho P + t - c$ , so we do not want to optimize over this value. Instead, we assume that it is fixed, perhaps as a result of competition with other shopping malls in the area. The outside option of visiting other shopping malls could, in a reduced form, be part of what determines  $r$ —the reservation value of not visiting the given mall. The exact size of  $\pi$  is not important for our analysis as long as it is strictly positive.

Formally, the mall can determine the parking fee,  $t$ , and charge the stores a per-customer

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<sup>5</sup>Another natural way of modeling competition is to introduce spatial competition between shopping malls as in Anderson and de Palma (2004), Arnott (2006), and Arnott and Rowse (2009). In such a model, however, the shopping mall is a *local* monopolist, and this is closer to our base model.

fee,  $\chi = \pi + c - t$ . The stores then compete by choosing their prices, resulting in an equilibrium price of the good of  $P = \chi/\rho$  due to Bertrand competition. The mall, then, maximizes its profit given by

$$\Pi(\pi, t) = (1 - F(\tilde{v})) \pi, \quad (20)$$

where  $\tilde{v}$  is the same as in equation 2.

The constraints for this model are  $P = \chi/\rho$  and  $t \geq 0$ . After simplifying the analysis by replacing  $P$  by  $\chi/\rho$ , we get

$$\Pi_t = f(\tilde{v}) \pi \left( \frac{\tilde{v}_P}{\rho} - \tilde{v}_t \right). \quad (21)$$

This expression is strictly negative, implying that the equilibrium parking fee is zero. To see this, notice that if  $\tilde{v} > P$  then  $\tilde{v}_P/\rho < \tilde{v}_t$ , and  $\tilde{v} > P$  if  $t \geq 0$ . Thus, the equilibrium  $t$  is zero. The following proposition records this result.

**Proposition 4 (Competitive pricing)** *Free provision of parking is an equilibrium even when the price of the good is set competitively.*

### 3 Parking Lot Size

We now extend the base model to analyze the equilibrium size of a parking lot. In the base model, the equilibrium size is just enough to meet total demand. In order to make parking lot size a meaningful choice variable, demand must vary, which would then allow the mall to choose between having a very large lot that is rarely used or a very small one that is usually full. The ICSC and ULI (2003) survey strongly supports that demand varies. It inquired if and when the lots are at capacity, and 43% responded that it never was. Of those who reported that the lot was sometimes full, the times this occurred were on weekends, holidays, days before forecasted snow, back-to-school sales, and the Christmas shopping season.

Motivated by the survey findings, we assume that the total possible demand,  $M$ , is a

random variable with distribution  $G(M)$  and density  $g(M)$  for  $M \in [\underline{M}, \bar{M}]$  where  $\underline{M} > 0$ . Therefore, the base model is just a special case in which  $M = 1$ . To assure (local) concavity of the mall's profit function and uniqueness of the equilibrium, in addition to the monotone hazard rate property of the distribution of  $v$ , we impose that the hazard rate of the distribution of  $M$ ,  $g(M)/(1 - G(M))$ , is higher than  $1/M$  for all  $M \in [\underline{M}, \bar{M}]$ .

We assume that the price of the good, the parking fee, and the lot size have to be determined before  $M$  is realized and thus cannot depend on  $M$ . This assumption is clearly very reasonable for both the parking fee and the lot size. As for the price of the good, while much of the variation in the ICSC and ULI (2003) survey is predictable, most of the time either price does not vary or it varies inversely with  $M$  (such as discounts during the high demand season of Christmas), which clearly signals that the price of the good is somewhat orthogonal to  $M$ , that is, determined largely by something other than  $M$ . Hence, we believe that our assumption is reasonable for the price of the good, too.

The parking lot size is denoted by  $l$ . Customers know both their valuations of the good and total possible demand. If the demand is higher than the lot capacity (*i.e.*,  $M(1 - F(\tilde{v})) > l$ ) and yet customers' valuation of the good is sufficiently high (*i.e.*,  $v \geq \tilde{v}$ ), then we assume that they decide to go to the shopping mall with the appropriate probability so that exactly a mass  $l$  of customers visits the shopping mall with the aim to purchase the good.

Assuming that customers randomize before visiting the shopping mall is a simplifying assumption. This keeps  $\tilde{v}$  given in equation 2 unchanged. In Appendix A.2, we work out two other reasonable alternatives. In the first, all customers with  $v \geq \tilde{v}$  decide to go to the shopping mall, but some will have to leave without shopping because they cannot find a parking space. In the second, among all customers with  $v \geq \tilde{v}$  only those who have higher valuation of the good decide to go to the shopping mall. Under both of these alternative specifications, the key results of this section are qualitatively the same.

### 3.1 Equilibrium

Given our assumptions, there are two possibilities: either all individuals who demand the good can shop at the mall or the parking lot saturates and some customers do not visit the mall. Therefore, the effective demand for the shopping mall is  $\min [M (1 - F(\tilde{v})), l]$ , and consequently the expected demand,  $D(P, t, l)$ , is

$$D(P, t, l) = \int_{\underline{M}}^{\tilde{M}} \min [M (1 - F(\tilde{v})), l] dG(M) \quad (22)$$

$$= (1 - F(\tilde{v})) \int_{\underline{M}}^{\tilde{M}} M dG(M) + l \int_{\tilde{M}}^{\bar{M}} dG(M) \quad (23)$$

$$= G(\tilde{M}) (1 - F(\tilde{v})) E[M | M \leq \tilde{M}] + (1 - G(\tilde{M})) l, \quad (24)$$

where  $\tilde{M}(1 - F(\tilde{v})) = l$ , and  $E[M | M \leq \tilde{M}]$  represents the expected value of  $M$  for  $M \leq \tilde{M}$ . Notice that if  $G(\tilde{M}) = 1$ , then this demand curve is the same as in Section 2. We provide  $D(P, t, l)$  in three different forms, the first of which is the raw form. It is easier to obtain some derivatives of concern by using equation 23 and more intuitive to provide some expressions in the form of equation 24. For future reference, note that  $D_P = -f(\tilde{v}) G(\tilde{M}) E[M | M \leq \tilde{M}] < 0$ ,  $D_t = -f(\tilde{v}) \tilde{v} G(\tilde{M}) E[M | M \leq \tilde{M}] < 0$ , and  $D_l = 1 - G(\tilde{M}) > 0$ .<sup>6</sup>

The profit function of the mall is

$$\Pi(P, t, l) = D(P, t, l) (\rho P + t) - lc. \quad (25)$$

Here, we subtract  $lc$  from the revenue because the cost of a parking lot is determined by the size of the lot, not by how much of it is used. Notice that if there were no uncertainty then

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<sup>6</sup>Although  $D_l$  looks like simply the derivative of the last term of  $D(P, t, l)$ , we need some algebraic manipulation to get this result. By using the derivation rule under integral sign in equation 23, we get  $D_l = \tilde{M}_l \tilde{M} (1 - F(\tilde{v})) g(\tilde{M}) + 1 - G(\tilde{M}) - g(\tilde{M}) \tilde{M}_l l$ . Substituting for  $\tilde{M}_l = 1/(1 - F(\tilde{v}))$  and  $l = \tilde{M}(1 - F(\tilde{v}))$  yields  $D_l = 1 - G(\tilde{M})$ . Similarly, the terms coming from the derivative of  $\tilde{M}$  in  $D_P$  and  $D_t$  disappear because  $\tilde{M}$  is the upper bound in the first integral of equation 23 and lower bound in the second.

$D(P, t, l) = l$  and this equation would be identical to the profit expression in the base model. The profit maximization problem is still subject to similar rationality and non-negativity constraints of the base model. It should be clear that  $l = \bar{M}(1 - F(\tilde{v}))$  only if  $c = 0$ , because if  $l = \bar{M}(1 - F(\tilde{v}))$  then there is no benefit of increasing the parking lot size. It should also be clear that  $l > \underline{M}(1 - F(\tilde{v}))$  because otherwise  $D(P, t, l) = l$  and there is no cost to raising the fees.

Setting  $\Pi_P = \Pi_t = 0$  yields  $\tilde{v}_P/\rho = \tilde{v}_t$  which means  $t^* = -r$ . Thus, once again, we get  $t^* = 0$ . Given this, there is no problem in setting  $\Pi_l = 0$ . The first-order condition with respect to  $l$  represents the marginal benefit of an additional parking space minus the marginal cost of it. The marginal benefit is the probability that the parking space is used times the revenue from that space if it is used. The probability is the probability  $M > \tilde{M}$ , or  $1 - G(\tilde{M})$ . This yields the equilibrium lot size:

$$l^* = G^{-1} \left( 1 - \frac{c}{\rho P} \right) (1 - F(\tilde{v})). \quad (26)$$

Notice that another way of writing this condition is that  $G(\tilde{M}) = 1 - c/(\rho P)$ . From this we can see that  $\rho P \geq c$ , and one can show that the profit of the mall is positive when this is true. Finally, the equilibrium price of the good is determined by the first-order condition with respect to  $P$ :

$$P^* = \frac{1 - F(\tilde{v})}{f(\tilde{v})} \left( 1 + \frac{(1 - G(\tilde{M})) \tilde{M}}{G(\tilde{M}) E[M|M \leq \tilde{M}]} \right), \quad (27)$$

and the equilibrium  $(P^*, t^*, l^*)$  is unique.<sup>7</sup> The following proposition summarizes the results of this section.

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<sup>7</sup>When there is no demand uncertainty, this price expression boils down to equation 8 of the base model. First, recognize that  $E[M|M \leq \tilde{M}] = \tilde{M}$  with no demand uncertainty. Then, using  $G(\tilde{M}) = 1 - c/(\rho P)$  in equation 27 results in  $P^* = [(1 - F(\tilde{v}))(\rho P^*)]/[f(\tilde{v})(\rho P^* - c)]$ . After simple manipulation we arrive at equation 8.

**Proposition 5 (Lot size model - equilibrium)** *If the demand to go to the shopping mall varies, then*

(i) *the equilibrium price of the good satisfies equation 27,*

(ii) *the equilibrium parking fee is zero,*

(iii) *the equilibrium lot size is given by equation 26.*

### 3.2 Welfare

Let us now turn to welfare analysis. Our main goal here is to figure out the criteria behind imposing minimum parking requirements. Minimum parking requirements exist all over the world. They specify the minimum amount of parking that must be provided by any land use.

As in the base model, we define welfare,  $W(P, t, l)$ , as the sum of customers' net utility,  $U(P, t, l)$ , and the mall's profit,  $\Pi(P, t, l)$ . The latter is defined in equation 25 and the former is given by

$$U(P, t, l) = \int_{\underline{M}}^{\bar{M}} M \int_0^{\bar{v}} \max[\alpha E(u|P, t) + (1 - \alpha) u(w + r), u(w + r)] dF(v) dG(M), \quad (28)$$

where  $E(u|P, t)$  is the expected utility given in equation 1, and  $\alpha$  is the probability of visiting the shopping mall, which we calculate next.

According to our assumptions, customers whose valuations are higher than  $\tilde{v}$  decide to go to the shopping mall with the appropriate probability so that exactly a mass  $l$  of them try to purchase the good. This probability is 1 when there is sufficient number of parking spaces for all those with  $v \geq \tilde{v}$ . Otherwise, when  $M > \tilde{M}$ , there will be only  $l$  parking spaces but  $M(1 - F(\tilde{v})) > l$  individuals who demand them. Therefore, the probability of visiting the shopping mall should be  $l / (M(1 - F(\tilde{v})))$  in this case. Both cases can be summarized



with the following probability measure.

$$\alpha = \frac{\min [M (1 - F (\tilde{v})), l]}{M (1 - F (\tilde{v}))}. \quad (29)$$

The utility expression given in equation 28 deals with all possibilities at once. In the cases in which the total possible demand is low (*i.e.*,  $M \leq \tilde{M}$ ),  $\alpha = 1$  and those whose valuations of the good are higher than  $\tilde{v}$  visit the shopping mall and obtain  $E(u|P, t)$  in expected terms while those whose valuations are less than  $\tilde{v}$  do not visit the shopping mall and obtain their outside option  $u(w + r)$ . In the cases in which the total possible demand is high (*i.e.*,  $M > \tilde{M}$ ), those whose valuations are less than  $\tilde{v}$  still do not visit the shopping mall and obtain their outside option. However, this time, there are not enough parking spaces to fulfill the demand of those with  $v > \tilde{v}$ , and so each individual will go to the shopping mall with probability  $\alpha = l / (M (1 - F (\tilde{v})))$ . When this happens, by the Law of Large Numbers, we expect to see exactly  $l$  individuals showing up in the parking lot.

We know that  $\Pi_l(P, t, l) = 0$  at the mall's profit-maximizing choice of lot size. Thus, the derivative of the welfare function with respect to the lot size at the profit-maximizing lot size, denoted by  $W_l|_{\Pi_l=0}$ , is simply  $U_l(P, t, l)$ :

$$\begin{aligned} W_l|_{\Pi_l=0} = U_l &= \int_{\tilde{M}}^{\bar{M}} M \int_{\tilde{v}}^{\bar{v}} \frac{E(u|P, t) - u(w + r)}{M (1 - F (\tilde{v}))} dF(v) dG(M) \\ &= \left(1 - G(\tilde{M})\right) \int_{\tilde{v}}^{\bar{v}} \frac{E(u|P, t) - u(w + r)}{1 - F(\tilde{v})} dF(v) > 0. \end{aligned} \quad (30)$$

Note that the outer integral in the first line of the expression is over  $[\tilde{M}, \bar{M}]$  because if the size of the parking lot is non-binding then increasing the lot size has no impact on welfare. The second line is easier to interpret. To understand it, note that customers are hurt only if the size of the parking lot is binding, which occurs with probability  $1 - G(\tilde{M})$ . A customer with value  $v$  loses  $E(u|P, t) - u(w + r)$  by not being able to shop, and since only the customers

with values in  $[\tilde{v}, \bar{v}]$  suffer this loss, the expected loss to one of these customers is the integral term in the expression.

Equation 30 is of interest because it says that, given any price of the good and the parking fee chosen by the mall, the *full* social optimum requires a larger parking lot size than the market equilibrium. However, this requires price controls in addition to controlling the lot size. If the social planner cannot control the price of the good, then it might well happen that once it imposes a larger lot size, the mall increases its price, which certainly diminishes or perhaps reverses the welfare-improving effects of increased lot size. However, as shown in Appendix A.1, this does not happen, because one of the implications of a concave objective function is that  $\partial P/\partial l$  is negative. The reason for this is intuitive, if slightly opaque. When  $l$  is increased, the potential pool of customers is increased. This means that decreasing the price by a small amount results in a larger increase in demand, and therefore the profit-maximizing price is lower. Thus, the mall prefers decreasing its price of the good in response to an increase in the lot size. This means that increasing the lot size improves welfare even when the social planner cannot impose price controls. This leads to the following important result.

**Proposition 6 (Foundations of minimum parking requirements)** *If the demand to the shopping mall varies, the lot size it chooses is smaller than the socially optimal lot size, no matter whether or not the social planner controls the price of the good.*

This is why local authorities may want to impose lower bounds on parking lot sizes. To our knowledge, this proposition is the first theoretical attempt to explore the micro-foundations of minimum parking requirements. The intuition behind the result is straightforward. The social planner cares about the loss of utility of those who would like to purchase the good but cannot. The mall cares only about the effect of this on its profits. Thus, the

social planner always wants a larger lot size. The equation for the optimal lot size,  $l^o$ , is<sup>8</sup>

$$l^o = G^{-1} \left( 1 - \frac{c}{\rho P} + \frac{U_l(P, t, l^o)}{\rho P} \right) (1 - F(\tilde{v})), \quad (31)$$

which clearly shows that the social planner wants to increase the lot size to take into consideration the loss of utility of the customers. The social planner can decentralize this solution with appropriate taxes and subsidies.

We should note that if there were a social cost to parking,<sup>9</sup> it could change this result, which may potentially result in society wanting to impose a maximum parking lot size. However, as Arbatskaya, Mukhopadhaya, and Rasmusen (2009) point out, this cost should be subdivided into two costs, a flow cost and a cruising (or queuing) cost. The former is a cost of traffic congestion as individuals are going to the mall; the latter is the social cost imposed by them searching for a parking space when they cannot find one in the lot and by having them come early to be sure to get a parking space. The social cost of flow should be increasing in  $l$ , with a form like  $SC^f(\min[M(1 - F(\tilde{v})], l))$  but the social cost of cruising should be decreasing in  $l$ , with a form like  $SC^c(\max[M(1 - F(\tilde{v})) - l, 0])$ . Thus, whether the net marginal cost of  $l$  is decreasing or increasing must be decided by balancing out these two impacts.<sup>10</sup>

## 4 The Urban Mall

While the models in Sections 2 and 3 explain the situation for most shopping malls, they do not capture two critical issues that malls may face in an urban area. We address these issues here. First, in an urban area, individuals may want to use the mall's parking lot for other

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<sup>8</sup>For simplicity, in this characterization we assume society controls both the price and the lot size.

<sup>9</sup>Parking in typical suburban malls in the US does not interfere with traffic very much.

<sup>10</sup>This argument implicitly considers the parking shortage model that we work out in Appendix A.2.1, where anyone who wants to shop goes to the mall and parks if he can find space.

purposes—to go to a park, restaurant, or store that is not in the shopping mall. Second, the mall has a fixed bound on the amount of space their property can take, so it has to decide what share of that space to devote to parking. We think that this is more of an issue for urban malls, as land is quite expensive in urban areas whereas it is negligible for suburban malls that typically allocate very large asphalted lands for parking, which do not alter the store size decisions.

In order to facilitate comparison with the base model, we analyze these two features one at a time. Section 4.1 shows that if individuals want to use the parking lot for other purposes, then the mall may want to charge positive parking fees. Section 4.2 shows that if the mall has a trade-off between parking and shopping spaces, parking is still provided for free but society may want to impose minimum or maximum parking requirements depending on the trade-off.

## 4.1 Positive parking fees

One question that has been left unanswered is why we observe positive parking fees in urban areas. The base model considers a shopping mall that is primarily car dependent, like most shopping malls in the US. In an urban mall, however, there may be individuals who have no intention of shopping at the mall parking in the lot. They want to go somewhere in the urban area that does not have its own parking lot and use the shopping mall's lot because they cannot find more convenient parking elsewhere.

In this variation of model, customers will have two decisions to make: whether to go to the urban area and whether to shop at the shopping mall. We continue to denote their reservation value of not coming to the urban area with  $r$ . But now, individuals get an additional payoff of  $n > 0$  if they go to the urban area (but not to the mall). Therefore, their reservation payoff becomes  $r + n$ . In order to achieve this value, they must park in the mall's parking lot and pay the parking fee  $t$ . We assume that  $n > c$  so that it can be profitable

to provide parking to non-shoppers. For simplicity, we further assume that if an individual goes to the urban area, he can either shop at the mall or do some alternative activity, but he cannot do both.

We start off by showing that  $t > n$  cannot happen in any equilibrium. In such a case, all those who park in the lot would be shoppers because  $u(w + r + n - t) < u(w + r)$ . But then, we immediately see that the mall would like to decrease the parking fee as much as possible due to its “insurance” motive in maximizing its profit. This means that the mall does not want a parking fee higher than  $n$ . Thus, in any equilibrium, it must be that  $t \leq n$  in which case all individuals go to the urban area because  $u(w + r + n - t) \geq u(w + r)$ .

An individual shops at the mall if  $E(u|P, t) \geq u(w + r + n - t)$ . The valuation of the marginal customer in this case is

$$\tilde{v}(P, t) \equiv u^{-1} \left( \frac{u(w + r + n - t) - (1 - \rho)u(w - t)}{\rho} \right) - w + P + t. \quad (32)$$

For future reference, note that  $\tilde{v}_P = 1$  and  $\tilde{v}_t = [(1 - \rho)u'(w - t) - u'(w + r + n - t)] / [\rho u'(w + \tilde{v} - P - t)] + 1$ . It is no longer possible for us to find a lower bound for  $\tilde{v}_t$ , but we will show that the parking fee should be set at  $n$  if  $\tilde{v}_t \leq 1/\rho$ .

Since  $t \leq n$ , the shopping mall always has a parking demand of 1. Thus, the mall earns  $t$  per customer and incurs a cost of  $c$  per each no matter what. Some of these individuals using the lot shop at the mall and each of those leaves a revenue of  $\rho P$ . This means that the profit of the mall,  $\Pi(P, t)$ , is

$$\Pi(P, t) = (1 - F(\tilde{v}))\rho P + t - c. \quad (33)$$

The first-order conditions of the mall's profit maximization problem are

$$\Pi_P = -f(\tilde{v}) \tilde{v}_P \rho P + \rho(1 - F(\tilde{v})) \quad (34)$$

$$\Pi_t = -f(\tilde{v}) \tilde{v}_t \rho P + 1. \quad (35)$$

Then, the equilibrium price of the good is determined by

$$P^* = \frac{1 - F(\tilde{v})}{f(\tilde{v})}. \quad (36)$$

Plugging this into the derivative with respect to  $t$ , we find that  $\Pi_t = -(1 - F(\tilde{v})) \rho \tilde{v}_t + 1$ . Since  $1 - F(\tilde{v}) \leq 1$ , we know that  $\Pi_t \geq -\rho \tilde{v}_t + 1$ , and the left-hand side of this expression is positive if  $\tilde{v}_t \leq 1/\rho$ . This requires that  $(1 - \rho)(u'(w - t) - u'(w + \tilde{v} - P - t)) \leq u'(w + r + n - t)$ . Since  $u'' < 0$ , we know that  $u'(w - t) > u'(w + r + n - t) > u'(w + \tilde{v} - P - t) > 0$ , and if individuals are sufficiently risk averse then it may be that  $u'(w - t) - u'(w + \tilde{v} - P - t) > u'(w + r + n - t)$ . However, as  $\rho$  gets larger this condition is always satisfied, and we find that  $t^* = n$ . This should be considered as the normal case for our analysis. After all,  $\rho$  is the probability that customers find what they want at the mall. If it is too low, then obviously the mall is doing something wrong. Achieving  $\rho = 1$  may be infeasible but it should be assumed to be close. This gives us the following result.

**Proposition 7 (Positive parking fees)** *If the probability of customers being able to purchase the good is high enough, then  $t^* = n$ .*

Therefore, positive parking fees can happen in equilibrium if individuals are able to free ride on the mall's parking spaces. There is indeed a range of  $\rho$  values such that  $0 < t^* \leq n$ . Here, we focus only on  $t^* = n$  since it is sufficient to show the underlying reason for a positive parking fee in equilibrium. To understand this result intuitively, first notice that if  $\tilde{v}_t \leq \tilde{v}_P/\rho = 1/\rho$  then the mall is driving more customers away by increasing the price of the

good than it is by increasing the parking fee. How is this done? When the mall increases the parking fee, it also makes the outside option less desirable, in essence giving these individuals more of an incentive to shop at the mall.

A clearer way to understand positive parking fees is to look at the three groups of individuals who park in the lot. The first group is the “winners” who shopped at the mall and found what they wanted. This group is completely indifferent over all parking fees since the mall’s pricing strategy guarantees that a higher parking fee is compensated with a lower price of the good. The second group is the “losers” who shopped and did not find what they wanted. They do not want to pay for parking since they are leaving empty-handed, and because of its insurance motive, the mall does not want to charge them, either. This group has a mass of  $(1 - \rho)(1 - F(\tilde{v}))$ . Finally, unlike in suburban malls, there are “non-shoppers” as a third group who only want to park. They have a mass of  $F(\tilde{v})$ . The only way to get any revenue out of non-shoppers is to charge a parking fee. As  $\rho$  gets large enough, the incentive not to charge losers is outweighed by the incentive to charge non-shoppers.

One might notice that  $P$  and  $t$  are never affected by the cost of providing a parking space even if the probability of customers being able to purchase the good is not high enough. This is because, as long as the mall decides to provide parking to all individuals, the cost of providing parking is essentially fixed, and like all fixed costs it has no effect on optimized values. One may wonder why we assume that the mall can provide parking for all who want it. A more complex model where  $n$  has a distribution would be necessary to analyze this issue in depth, and this is beyond the scope of this paper.

## 4.2 Lot share: from too little to too much

In the base model, the mall is making four decisions. It chooses the price for the good, the parking fee, the parking lot size, and the size of the shopping area. We have simplified the analysis by ignoring the last of these choices, but clearly the benefit of a larger store is

that the probability that a customer finds the good that he wants increases. In an urban area, increasing the size of the stores may require decreasing the size of the parking lot. We investigate this trade-off here.

We assume that the shopping mall occupies a fixed amount of land and that the mall is in a position to decide how to allocate this land between parking and shopping spaces. Notice that in this model there must be vertical and horizontal limits on the size of the property. The horizontal limits may be imposed because the cost of acquiring more land is prohibitive. The vertical limit may be imposed by increasing costs as the building gets taller or by government regulations.

We shall employ the simplest characterization of the trade-off between space allocated to parking and shopping. We assume that the shopping mall occupies a unit of land. Let the share of land allocated to parking be  $s$ , and therefore the share of land allocated to shops is  $1 - s$ , where  $s \in [0, 1]$ . We assume that increasing  $s$  decreases “good variety” offered, which thus will decrease the probability that a customer finds the good that he wants. We assume that  $\rho(s) \in (0, 1)$  satisfies  $0 < \rho(0) < 1$ ,  $-\rho(s)/s < \rho'(s) < 0$ ,  $\rho''(s) < 0$ . That is, the probability of finding the desired good at the shopping mall is a monotonically decreasing and strictly concave function of  $s$ . The assumption that  $\rho(0) > 0$  along with the assumption that  $\bar{v}$  is high enough guarantees that the solution is interior, and  $-\rho(s)/s < \rho'(s)$  guarantees that the first-order conditions are well behaved.

The marginal customer is now defined by

$$\tilde{v}(P, t, s) \equiv u^{-1} \left( \frac{u(w+r) - (1-\rho(s))u(w-t)}{\rho(s)} \right) - w + P + t. \quad (37)$$

The maximization problem of the mall is

$$\max_{P,t,s} \{ \min [1 - F(\tilde{v}), s] (\rho(s)P + t) - cs \}. \quad (38)$$



Notice that in any solution,  $s = 1 - F(\tilde{v})$ , just like in the base model. If  $s < 1 - F(\tilde{v})$ , then one can increase  $P$  or  $t$  without changing the quantity sold; and if  $s > 1 - F(\tilde{v})$ , then one can decrease  $s$  without changing the quantity sold. Given this constraint, the maximization problem can be rewritten as

$$\max_{P,t} \{(1 - F(\tilde{v}))(\rho(1 - F(\tilde{v}))P + t - c)\}. \quad (39)$$

The first-order conditions of this problem are:

$$\Pi_P = -f(\tilde{v})\tilde{v}_P [(\rho + (1 - F(\tilde{v}))\rho')P + t - c] + (1 - F(\tilde{v}))\rho \quad (40)$$

$$\Pi_t = -f(\tilde{v})\tilde{v}_t [(\rho + (1 - F(\tilde{v}))\rho')P + t - c] + (1 - F(\tilde{v})), \quad (41)$$

and like before in order for both of them to be equal to zero we need  $\tilde{v}_P/\rho = \tilde{v}_t$ . The difference between the analysis in this section and in our base model is that the supply of parking space is no longer perfectly elastic. The mall trades off increased lot size with decreased good variety. This is the reason for the appearance of  $\rho'(s)$  in the equations above. If one assumes that  $\rho'(s) = 0$  then the first-order conditions are identical to those in the base model (equations 5 and 6). This is equivalent to increasing the lots size until the impact of  $s$  on  $\rho(s)$  is negligible.

Given the equilibrium condition  $s = 1 - F(\tilde{v})$ , the formulas for  $\tilde{v}_P$  and  $\tilde{v}_t$  are more complicated than before:

$$\tilde{v}_P = \frac{\rho u'(w + \tilde{v} - P - t)}{-\rho' f(\tilde{v}) [u(w + \tilde{v} - P - t) - u(w - t)] + \rho u'(w + \tilde{v} - P - t)} > 0 \quad (42)$$

$$\tilde{v}_t = \frac{(1 - \rho) u'(w - t) + \rho u'(w + \tilde{v} - P - t)}{-\rho' f(\tilde{v}) [u(w + \tilde{v} - P - t) - u(w - t)] + \rho u'(w + \tilde{v} - P - t)} > 0. \quad (43)$$

One can still show that in order for  $\tilde{v}_P/\rho = \tilde{v}_t$ ,  $t^*$  must be equal to  $-r$ , thus  $t^* = 0$  is the implementable solution.

**Proposition 8 (Land allocation trade-off)** *Free provision of parking is an equilibrium even when the mall faces a land allocation trade-off.*

Finding the equilibrium value of  $P$  is straightforward. We will not spend space analyzing it, other than to say that it exists and is unique. Instead, we immediately turn to welfare analysis. As before, welfare is defined as the sum of customers' utility and the mall's profit. The mall's profit is given by equation 39 and the customers' utility is

$$U(P, t, s) = \int_{\underline{v}}^{\bar{v}} \max [E(u|P, t, s), u(w + r)] dF(v), \quad (44)$$

where

$$E(u|P, t, s) = \rho(s) u(w + v - P - t) + (1 - \rho(s)) u(w - t). \quad (45)$$

The welfare maximizer does not have to satisfy the constraint  $s = 1 - F(\tilde{v})$ , other than respecting the fact that if  $s < 1 - F(\tilde{v})$  then  $P$  will be adjusted to satisfy this constraint and if  $s > 1 - F(\tilde{v})$  then the firm will treat the share of land devoted to parking as a fixed cost. This means that what we are interested in is  $W_s$  around the optimal level of  $s$  and  $P$ , at which point  $\Pi_s = 0$ . Thus, the derivative of the welfare function with respect to  $s$  at the profit-maximizing share of land allocated to parking, denoted by  $W_s|_{\Pi_s=0}$ , simply equals  $U_s(P, t, s)$ :

$$\begin{aligned} W_s|_{\Pi_s=0} &= U_s = \int_{\tilde{v}}^{\bar{v}} E_s dF(v) \\ &= \int_{\tilde{v}}^{\bar{v}} [\rho'(s) (u(w + v - P - t) - u(w - t))] dF(v) < 0. \end{aligned} \quad (46)$$

This equation says that, in general, society wants the parking lot to be smaller. However, this is confounded by the fact that no matter what the relationship between  $s$  and  $1 - F(\tilde{v})$ ,  $\partial P / \partial s \leq 0$  and as before  $W_P|_{\Pi_P=0} = U_P = -\rho(s) \int_{\tilde{v}}^{\bar{v}} u'(w + v - P - t) dF(v) < 0$ . Thus, if society imposes a smaller parking lot, the mall will increase the price of the good and this will

decrease welfare. Therefore, society may not be able to improve welfare by imposing a smaller parking lot without also controlling the price of the good. If, however,  $\partial P/\partial s$  is sufficiently small, then the welfare gain due to decreased parking lot size outweighs the welfare loss due to the increase in the price of the good. Even if  $P$  is set by competitive considerations, it will probably still be that  $\partial P/\partial s < 0$ , because the marginal cost of shopping area is almost surely higher than the marginal cost of a parking lot. Thus, increasing the parking lot size decreases the marginal cost and thus the price of the good. This gives the key result of this section.

**Proposition 9 (Maximum and minimum parking requirements)** *Society may want a larger or smaller parking lot than the shopping mall. If  $\partial P/\partial s$  is small, then it always wants a smaller parking lot.*

This explains the maximum parking requirements that have been imposed by some cities around the world. Notice, however, that these regulations are justified when there is a trade-off between land devoted to parking and shopping. If this trade-off is negligible (as in suburban malls) or there is a way to change regulations to remove this trade-off, then we are back in the model of Section 3.

## 5 Concluding Remarks

We have now taken a first step toward understanding what stores want the price and quantity of parking to be. In the case of the shopping mall—where the parking provider and the store are united—we find that shopping malls want parking to be free. We further find that society also wants parking to be free, and it wants the shopping mall to provide more than the profit-maximizing amount of parking. The main analysis ignored the congestion externality of parking. We feel that it is best to first understand how the shopping mall would operate

independent of these concerns before taking them into consideration. But, when we consider the social cost of parking in a reduced-form manner, we find that society still wants parking to be free, though it may not want to impose minimum parking requirements.

The main message of this paper is not that parking fees should be zero but that the cost of parking should be absorbed in the price of the good. It is not that parking fees are bad but that raising the price of the good is better than raising the parking fee. Free provision of parking is a surprisingly robust result. It holds if the mall provides vouchers, prices in a competitive manner, and even if it has a trade-off between space for shopping and parking. This result changes when individuals want to use the mall's parking lot for other purposes, or if they use it purely as a parking garage. In this case, the mall will recognize this and generally want to raise revenue from these individuals. This explains the observation of positive parking fees in urban malls.

In a model where customers buy heterogeneous goods or more than one unit, the mall could use a parking fee to increase its profits. Many malls that have parking fees do seem to use this form of second-degree price discrimination—having a different parking fee depending on how much you buy, which store you shop at (like movie theaters), etc. Another form of second-degree price discrimination is to charge different parking fees to short-time parkers than long-time parkers. In the face of these incentives, it is surprising that a vast majority of malls do not have parking fees, and our paper provides a rationale for this practice.

We have provided one coherent theory of shopping mall parking. Obviously, other market forces may strengthen our results. For example, transaction costs associated with parking may prevent positive parking fees being charged. However, this explanation is lacking in several dimensions. First, with increased automation, transaction costs should be decreasing, and thus we should see more malls charging parking fees—but we do not. Second, most of these transaction costs would be based on the hourly wages of employees. Hence, we should see more parking fees in areas with large turnover. In contrast, many urban malls have no

parking fees even when they have high traffic. Third, most US towns and cities have minimum lot-size requirements. If parking was being systematically under-priced in malls, then surely governments should be decreasing the supply of parking; instead they force shopping malls to have larger than privately optimal lots. Our model can explain this phenomenon, while transaction-cost theory cannot.

In a competitive environment, the mall may use free parking as a “loss leader” to attract customers. There are two reasons that we do not think this is as fundamental as our approach. First, this argument relies almost completely on the competitive nature of the shopping mall, suggesting that less competitive malls should be more likely to charge parking fees. The empirical evidence is exactly the reverse: in general it is urban malls that charge parking fees, where the competition between malls is fiercest. Our model explains this phenomenon, while the loss-leader argument does not. Second, in our model free parking is socially optimal, while a loss leader is at best welfare neutral.

Another alternative approach is to analyze the problem as a two-sided market (Rochet and Tirole, 2003) in which the shopping mall is a platform that has to attract both stores and customers. In equilibrium, it may well happen that the mall embeds the costs of parking in the rents as a fixed cost and this would not change the pricing decisions of stores, which means that they cannot pass on the costs of parking to the customers. We did not take this approach in order to simplify our exposition. The simplest version of a two-sided market model would have both the store and the shopping mall be monopolists, but this would lead to the double markup problem from the vertical integration literature. In our model, we assume the efficient solution to this problem, merging the mall and the store. A more complex model would require store-specific demand curves and a careful analysis of the structure of competition between the stores. While this is an excellent topic for future research it is too complicated to be pursued here.

This paper is one of the first to justify the standard practice of imposing minimum park-

ing requirements on shopping malls. In some popular press and planning circles, minimum parking requirements are deemed to be the worst planning rules. We, on the other hand, neither want to dismiss them altogether nor do we endorse blindly supporting and mechanically applying them to any land use. Our results are specific to shopping mall parking and they show that there is a sound basis for minimum parking requirements in this context. However, if there is a significant social cost (due to traffic congestion) from parking, towns and cities may want to impose maximum parking rather than minimum requirements. They may also want to do this if stores face a binding constraint on the amount of land that their property can utilize, because then they may have to sacrifice shopping area to increase the parking area.

We do not claim that our results apply to all forms of parking, and we believe that towns and cities should avoid one-size-fit-all policies. Different parking fees and requirements must be imposed for different land uses and in urban versus suburban areas. We believe that more theoretical work on the foundations of parking policies, and empirical work quantifying the effects of those policies, is required. Given the surprising robustness of free parking, the natural next question is how society should price parking in urban areas. Does society want the external costs of congestion to be reflected in the price of goods rather than the parking fee? Under what conditions will it want to charge for parking? Could it possibly want to use hourly parking fees to increase business for urban stores despite the fact that this will increase congestion? How do the results change in the presence of public transportation and modal choice? These questions are left to future research.

# A Appendix

## A.1 Proof of $dP/dl < 0$

The equilibrium price is given by equation 27. Modifying this expression gives

$$\Phi = [f(\tilde{v})P - (1 - F(\tilde{v}))]G(\tilde{M})E[M|M \leq \tilde{M}] - (1 - F(\tilde{v}))\left(1 - G(\tilde{M})\right)\tilde{M} = 0. \quad (\text{A.1})$$

By the Implicit Function Theorem, we have

$$\frac{dP}{dl} = -\frac{\partial\Phi/\partial l}{\partial\Phi/\partial P}. \quad (\text{A.2})$$

Let us first derive  $\partial\Phi/\partial l$ :

$$\begin{aligned} \frac{d\Phi}{dl} &= (f(\tilde{v})P - (1 - F(\tilde{v})))\tilde{M}g(\tilde{M})\tilde{M}_l - (1 - F(\tilde{v}))\left(\tilde{M}_l - \tilde{M}_lG(\tilde{M}) - \tilde{M}g(\tilde{M})\tilde{M}_l\right) \\ &= \tilde{M}_l\left((f(\tilde{v})P - (1 - F(\tilde{v})))\tilde{M}g(\tilde{M}) - (1 - F(\tilde{v}))\left(1 - G(\tilde{M}) - \tilde{M}g(\tilde{M})\right)\right) \\ &= \tilde{M}_l\left(f(\tilde{v})P\tilde{M}g(\tilde{M}) - (1 - F(\tilde{v}))\left(1 - G(\tilde{M})\right)\right). \end{aligned} \quad (\text{A.3})$$

Substituting for  $P$  from equation 27 yields

$$\begin{aligned} \frac{d\Phi}{dl} &= \tilde{M}_l(1 - F(\tilde{v}))\left(\tilde{M}g(\tilde{M}) + \frac{(1 - G(\tilde{M}))\tilde{M}^2g(\tilde{M})}{G(\tilde{M})E[M|M \leq \tilde{M}]} - (1 - G(\tilde{M}))\right) \\ &= \tilde{M}_l(1 - F(\tilde{v}))\left(\tilde{M}g(\tilde{M}) + \frac{(1 - G(\tilde{M}))\left[\tilde{M}^2g(\tilde{M}) - G(\tilde{M})E[M|M \leq \tilde{M}]\right]}{G(\tilde{M})E[M|M \leq \tilde{M}]}\right), \end{aligned} \quad (\text{A.4})$$

where  $\tilde{M}_l = 1/(1 - F(\tilde{v})) > 0$ . Remember that we assume  $g(M) > 0$  and  $1 < Mg(M)$

for all  $M \in [\underline{M}, \bar{M}]$ , which implies that  $G(\tilde{M}) < \tilde{M}g(\tilde{M})$  because  $G(\tilde{M}) \leq 1$ . Multiplying the left-hand side by  $E[M|M \leq \tilde{M}]$  and the right-hand side by  $\tilde{M}$  does not change this inequality since  $0 < E[M|M \leq \tilde{M}] < \tilde{M}$ . Hence,  $g(\tilde{M})\tilde{M}^2 - G(\tilde{M})E[M|M \leq \tilde{M}] > 0$ . Consequently we get

$$\frac{d\Phi}{dl} > 0. \quad (\text{A.5})$$

Now, consider  $\partial\Phi/\partial P$ :

$$\begin{aligned} \frac{d\Phi}{dP} &= [f'(\tilde{v})P + 2f(\tilde{v})]G(\tilde{M})E[M|M \leq \tilde{M}] + [f(\tilde{v})P - (1 - F(\tilde{v}))]\tilde{M}g(\tilde{M})\tilde{M}_P \\ &\quad - \left[ \left( -f(\tilde{v})(1 - G(\tilde{M})) - g(\tilde{M})\tilde{M}_P(1 - F(\tilde{v})) \right) \tilde{M} + (1 - F(\tilde{v}))(1 - G(\tilde{M}))\tilde{M}_P \right] \\ &= [f'(\tilde{v})P + 2f(\tilde{v})]G(\tilde{M})E[M|M \leq \tilde{M}] \\ &\quad + f(\tilde{v})P\tilde{M}g(\tilde{M})\tilde{M}_P + \tilde{M}f(\tilde{v})(1 - G(\tilde{M})) - (1 - F(\tilde{v}))(1 - G(\tilde{M}))\tilde{M}_P, \end{aligned} \quad (\text{A.6})$$

where  $\tilde{M}_P = (\tilde{M}f(\tilde{v}))/(1 - F(\tilde{v}))$ . Hence,

$$\begin{aligned} \frac{d\Phi}{dP} &= [f'(\tilde{v})P + 2f(\tilde{v})]G(\tilde{M})E[M|M \leq \tilde{M}] + f(\tilde{v})P\tilde{M}^2g(\tilde{M})\frac{f(\tilde{v})}{1 - F(\tilde{v})} \\ &= f'(\tilde{v})PG(\tilde{M})E[M|M \leq \tilde{M}] + 2f(\tilde{v})G(\tilde{M})E[M|M \leq \tilde{M}] \\ &\quad + f(\tilde{v})P\tilde{M}^2g(\tilde{M})\frac{f(\tilde{v})}{1 - F(\tilde{v})} \\ &= P \left[ G(\tilde{M})E[M|M \leq \tilde{M}]f'(\tilde{v}) + g(\tilde{M})\tilde{M}^2\frac{f^2(\tilde{v})}{1 - F(\tilde{v})} \right] \\ &\quad + 2f(\tilde{v})G(\tilde{M})E[M|M \leq \tilde{M}] > 0. \end{aligned} \quad (\text{A.7})$$

From the monotone hazard rate property we know that

$$f'(\tilde{v}) + \frac{f^2(\tilde{v})}{1 - F(\tilde{v})} > 0. \quad (\text{A.8})$$



Multiplying the first term of this expression with  $G(\tilde{M})E[M|M \leq \tilde{M}]$  and the second term with  $g(\tilde{M})\tilde{M}^2$  does not change the inequality because  $\tilde{M}^2g(\tilde{M}) - G(\tilde{M})E[M|M \leq \tilde{M}] > 0$ . Then, we conclude that

$$\frac{d\Phi}{dP} > 0. \tag{A.9}$$

Finally, equations A.5 and A.9 imply that  $dP/dl < 0$ , which means that if the lot size is increased the SMO will decrease its price.

## A.2 Alternative Models of Parking-lot Size

In this appendix, we show that the key result of Section 3 holds under alternative rationing rules. The key result we are interested in is that society still wants a larger parking lot than the shopping mall in a full social optimum. In our base model, we assume that if the parking lot is too small then all customers randomize before they go to the shopping mall so that only an  $l$  mass of customers shows up. One may think that assuming that customers know the total potential mass of demand is too strong of an assumption. This would give rise to the model in Appendix A.2.1. In this model, all customers go to the mall and some do not find a parking space. Alternatively, one may be interested in the socially efficient mechanism of only having the highest-value customers shop. This is the model in Appendix A.2.2. In both of these cases, we show that our key result holds.

### A.2.1 Parking shortages

If everyone always goes to the mall and only some find a parking space, this is similar to reducing  $\rho$ . Now, the customer is able to buy the good only if he goes to the shopping mall and if there is an available parking space. Given  $M$ , the probability of parking is  $\alpha = \min [M (1 - F(\tilde{v})), l] / (M (1 - F(\tilde{v})))$ .

Note that

$$\alpha_l = \begin{cases} \frac{1}{M(1-F(\tilde{v}))} > 0 & \text{if } l < M(1-F(\tilde{v})) \\ 1 & \text{if } l \geq M(1-F(\tilde{v})) \end{cases}, \quad (\text{A.10})$$

and

$$\alpha_P = \begin{cases} \frac{lf(\tilde{v})}{M(1-F(\tilde{v}))^2} > 0 & \text{if } l < M(1-F(\tilde{v})) \\ 0 & \text{if } l \geq M(1-F(\tilde{v})) \end{cases}. \quad (\text{A.11})$$

Now,  $\tilde{v}$  is directly affected by  $l$  and  $M$ :

$$\tilde{v} = u^{-1} \left( \frac{u(w+r) - (1-\rho\alpha)u(w-t)}{\rho\alpha} \right) - w + P + t, \quad (\text{A.12})$$

and  $\tilde{v}_l = (\alpha_l/\rho\alpha^2)[u(w-t) - u(w+r)]/u'(w+\tilde{v}-P-t)$ , which is strictly negative when  $\alpha_l > 0$ .

Given that  $t^* = 0$ , the profit of the mall is

$$\Pi(P, t, l) = \int_{\underline{M}}^{\bar{M}} \min[M(1-F(\tilde{v})), l] \alpha dG(M) \rho P - lc. \quad (\text{A.13})$$

The first-order conditions of mall's profit maximization are

$$\begin{aligned} \Pi_P = & -f(\tilde{v}) \tilde{v}_P \rho P \int_{\underline{M}}^{\bar{M}} M \alpha dG(M) + \rho P \int_{\underline{M}}^{\bar{M}} \min[M(1-F(\tilde{v})), l] \alpha_P dG(M) \\ & + \rho \int_{\underline{M}}^{\bar{M}} \min[M(1-F(\tilde{v})), l] \alpha dG(M) \end{aligned} \quad (\text{A.14})$$

$$\begin{aligned} \Pi_l = & -f(\tilde{v}) \tilde{v}_l \rho P \int_{\underline{M}}^{\bar{M}} M dG(M) \alpha + \rho P \int_{\underline{M}}^{\bar{M}} \min[M(1-F(\tilde{v})), l] dG(M) \alpha_l \\ & + \rho P \int_{\underline{M}}^{\bar{M}} \alpha dG(M) - c. \end{aligned} \quad (\text{A.15})$$

Equation A.15 has two new positive terms, resulting in a larger optimal parking lot. The welfare expression will be changed by this. Like before, we only need to analyze the net utility,  $U(P, t, l)$ , which is given by

$$U(P, t, l) = \int_{\underline{M}}^{\bar{M}} M \int_0^{\bar{v}} \max[\rho\alpha u(w + v - P) + (1 - \rho\alpha)u(w), u(w + r)] dF(v) dG(M). \quad (\text{A.16})$$

The derivative of welfare with respect to  $l$  at the profit-maximizing lot size is

$$W_l|_{\Pi_l=0} = U_l = \int_{\bar{M}}^{\bar{M}} M \int_{\bar{v}}^{\bar{v}} (u(w + v - P) - u(w)) \rho\alpha_l dF(v) dG(M) > 0. \quad (\text{A.17})$$

So, we still conclude that a social planner prefers a larger parking lot than the mall in the full social optimum.

### A.2.2 Asymmetric sorting

In another alternative model, instead of everyone going to the parking lot, only the highest-value customers go to purchase the good when the demand is too high, or the critical value for the mall,  $v^{as}$ , is

$$v^{as} = \begin{cases} F^{-1}\left(1 - \frac{l}{M}\right) & \text{if } M(1 - F(\tilde{v})) > l \\ \tilde{v} & \text{if } M(1 - F(\tilde{v})) \leq l \end{cases}, \quad (\text{A.18})$$

where  $\tilde{v}$  is given by equation 2. Note that  $v_l^{as} = -(1/M)(F^{-1})' < 0$  if  $v^{as} > \tilde{v}$ .

The profit of the mall is

$$\Pi(P, t, l) = \int_{\underline{M}}^{\bar{M}} M(1 - F(v^{as})) dG(M) (\rho P + t) - lc. \quad (\text{A.19})$$

Again, after letting  $t^* = 0$ , the first-order conditions of mall's profit-maximization are

$$\Pi_P(P, t, l) = - \int_{\underline{M}}^{\bar{M}} M f(\tilde{v}) dG(M) \rho P + \rho \int_{\underline{M}}^{\bar{M}} M (1 - F(v^{as})) dG(M) = 0 \quad (\text{A.20})$$

$$\Pi_l(P, t, l) = \left(1 - G(\tilde{M})\right) \rho P - c = 0, \quad (\text{A.21})$$

where again  $\tilde{M}(1 - F(\tilde{v})) = l$ .  $\Pi_l(P, t, l)$  is the same as it is in the original model and thus we get the same value for  $l^*$ . However, like before, the net utility is different. In this case, it is more convenient to break the customers into two groups, one of which is able to purchase at the given level of  $M$  and  $P$ :

$$U(P, t, l) = \int_{\underline{M}}^{\bar{M}} M \left[ \int_{v^{as}}^{\tilde{v}} [\rho u(w + v - P) + (1 - \rho) u(w)] dF(v) \right] dG(M) \\ + \int_{\underline{M}}^{\bar{M}} M \left[ \int_0^{v^{as}} u(w + r) dF(v) \right] dG(M). \quad (\text{A.22})$$

The critical derivative is the derivative of welfare with respect to  $l$  at the profit-maximizing lot size:

$$W_l|_{\Pi_l=0} = U_l = \int_{\underline{M}}^{\bar{M}} M [(\rho u(w + v^{as} - P) + (1 - \rho) u(w) - u(w + r)) (-v_l^{as}) f(v^{as})] dG(M) > 0. \quad (\text{A.23})$$

So, once again, we get the same result; the social planner desires a larger parking lot than the mall in the full social optimum.

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