THE MASQUERADE BALL OF THE CEOs AND THE MASK OF EXCESSIVE RISK*

Haluk Citci† and Eren Inci‡

16 January 2012

We analyze the effects of CEOs' layoff risk on their risk choice while overseeing a firm. A CEO, whose managerial ability is unknown, is fired if her expected ability is below average. Her risk choice changes the informativeness of output and market's belief about her ability. She can decrease her layoff risk by taking excessive risk and trade off current compensation for layoff risk. The firm may voluntarily or involuntarily allow excessive risk taking even under optimal linear compensation contracts. Above-average CEOs always keep their jobs, but among below-average CEOs, a higher-ability one is more likely to be fired. *JEL* Codes: D82; G32; J33; L21; M12

*We would like to thank session participants at EBES 2011 Conference, Istanbul Economics and Finance Workshop II, CEE Annual Conference on Macroeconomic and Financial Imbalances in National Economies and the World and seminar participants at Istanbul Technical University for their valuable comments. Any remaining errors are our responsibility.

† *Tel.*: 90-216-483-3159; *fax*: 90-216-483-9250. *Address*: Sabanci University - Faculty of Arts and Social Sciences, Orhanli / Tuzla 34956 Istanbul TURKEY. *E-mail address*: halukc@sabanciuniv.edu.

‡ *Tel.*: 90-216-483-9340; *fax*: 90-216-483-9250. *Address*: Sabanci University - Faculty of Arts and Social Sciences, Orhanli / Tuzla 34956 Istanbul TURKEY. *E-mail address*: ereninci@sabanciuniv.edu.

I. Introduction

Excessive risk taking by the CEOs of large financial cooperations is widely believed to have played a great role in the economic and financial crisis of 2008-2009 (Blinder, 2009). Of the executives and commentators surveyed in the financial services sector, 73% consider excessive risk taking to be one of the crucial factors that triggered the crisis (PricewaterhouseCoopers 2008). G-20 leaders announced their commitment in legislating the necessary changes to minimize excessive risk taking. The Basel II framework has been amended to account for motives to take excessive risk. The Dodd-Frank Act prohibited certain compensation arrangements in order to discourage inappropriate risk taking by financial institutions in the US. What motivates managers to take excessive risk? In this paper, we argue that a CEO's career concerns regarding potential termination give her incentive to make the output of her firm as uninformative as possible about her managerial ability. We show that a CEO can achieve this goal by taking excessive risks (*i.e.,* a risk level higher than the socially optimal risk level) while overseeing the firm and that explicit incentives provided by optimal linear compensation contracts cannot prevent CEOs from taking such excessive risks.

We build a principal-agent framework in which a (risk-neutral) firm operates for two periods. We initially assume that there are two types of (risk-neutral) CEOs, high- and low-ability, who are equally likely in the population. All else being equal, the firm produces higher output when managed by a high-ability CEO. No one, including the CEO herself, knows the ability of the CEO. The output of the firm is also influenced by the privately known risk level chosen by the CEO. In the end, the firm may land in one of two possible states (good or bad) and pays the optimal linear compensation contract that allows for any combination of fixed wages and stocks. The novel results of our paper stem from the structure in which CEOs implicitly choose the probability of states by deciding on the risk level. In particular, by her choice of risk, a CEO can increase the probability of the bad state's occurance, as well as the amount of returns in the good state and loss in the bad one.

If the firm believes that the ability of the CEO is below average at the end of the first period, it fires her and hires a new CEO, whose ability is expected to be average in the population. This layoff risk is the source of the CEO's career concerns.[1] By adjusting the risk level, a CEO can overlap possible outputs produced by high- and low-ability CEOs in different state realizations. When the firm observes the overlapped output, it cannot know exactly which ability type in fact produced this output. Because the probability of the bad state increases with risk, if a CEO overlaps the outputs by taking excessive risk, then the firm believes that the overlapped output is more likely to be the bad-state realization of a high-ability CEO than the good-state realization of a low-ability CEO. Consequently, the firm's expectation about the CEO's ability will be higher than average even though each type is *ex ante* equally likely, which means that the CEO is not fired in such an output realization. Moreover, her type is inferred if she turns out to be a high-ability CEO in the good state and thus she is not fired in this state realization, either. By following this strategy, she is fired only if she turns out to be a low-ability CEO in the bad state.[2]

We show that the strategy of overlapping the outputs by taking excessive risk minimizes the probability of being fired when the difference between the two possible abilities is neither too high nor too low. Yet, a risk level that minimizes the probability of being fired is not automatically an equilibrium. It is an equilibrium when the CEO's compensation benefit

1. It is noteworthy that risk-taking decisions interact with layoff risk and compensation incentives in practice. Kempf, Ruenzi, and Thiele (2009) empirically show that layoff risk and compensation effects matter in managers' risk-taking decisions. Chevalier and Ellison (1999) mention that avoiding a possible layoff is the most important career concern. Other papers supporting this hypothesis include Fama (1980), Jensen and Murphy (1990), Berger, Ofek, and Yermack (1997), Bloom and Milkovich (1998), Mehran, Nogler, and Schwartz (1998), Wiseman and Gomez-Mejia (1998), Eckbo and Thorburn (2003), Hong and Kubik (2003), Clarke and Subramanian (2006), Chakraborty, Sheikh, and Subramanian (2007), and Larraza-Kintana *et al.* (2007). The literature also shows that CEO turnover is closely related to the peer performance (Gibbons and Murphy 1990; DeFond and Park 1999; Kaplan and Minton 2011).

2. An interesting property of our model is that, contrary to standard career concerns models *à la* Holmstrom (1982/1999), in which the manager is trapped in taking the expected action (*i.e.*, supplying effort), in our case, it is the firm who is trapped in forming the correct expectation about the probability of the states and the equilibrium risk choice. Thus, as opposed to the common policy debates, making the risk choice more transparent may not solve excessive risk taking.

she derives by taking optimal risk in the first period is dominated in expected payoff by the career benefit she derives by taking excessive risk to minimize her probability of being fired. In such a case, excessive risk taking occurs in equilibrium under the optimal linear compensation contract. This sheds light on the ongoing debate about the (desperate) role of regulation of compensation structures to prevent excessive risk taking.

Policy debates emphasize the CEOs' *responsibility* in the inefficiently high levels of risk taken by large financial corporations. Yet, we show that, in addition to cases in which the firm *involuntarily* allows the CEO to take excessive risk, there are also cases in which it *voluntarily* allows her to take excessive risk. In the former case, the firm involuntarily allows the CEO to take excessive risk because no compensation contract, not even providing the whole return of the project to the CEO, can achieve the optimal risk level. However, in the latter case, although having the CEO take the optimal risk could be profitable for the firm, letting her take excessive risk is even more profitable. This is inefficient from the point of view of society, as the return from excessive risk has negative net present value. Thus, shareholders sometimes share the responsibility of inefficient levels of risk in the firm.

Our results hold even when CEOs are risk averse. We further show that excessive risk taking persists even when there is a continuum of ability types. This case also illustrates an inverse U-shaped relationship between the unobserved ability of the CEO and her layoff risk. Among the below-average CEOs, a higher-ability one is more likely to be fired than a lower-ability one, while above-average CEOs face no layoff risk. Finally, we show that our results are robust to changes in informational assumptions by illustrating the persistence of excessive risk in equilibrium when CEOs privately know their types. Our explanation for excessive risk taking is not limited-liability based, as there is no limited liability for the CEO in the model. That is, in our setting, a CEO does not take higher risks simply because limited liability protects her from downward risks, which is already a well-known explanation in the literature. As a matter of fact, incorporating limited liability to our setting would increase CEOs' risk appetite.

We now explain how our paper relates to prior work. A large body of literature, pioneered by Fama (1980) and Holmstrom (1982/1999), analyzes how career concerns affect the behavior of agents. Holmstrom (1982/1999) finds that, since investing in a project carries the risk of one's type being discovered, a risk-averse manager behaves overly conservatively by not investing in risky projects at all. Holmstrom and Ricart i Costa (1986) elaborate on this idea further and show that conservatism can be fixed if the shareholders can offer a downward rigid wage. Building on Holmstrom's findings, the literature that followed has focused on *managerial conservatism* in a broad sense (see, *e.g.*, Narayanan [1985], Stein [1988], Shleifer and Vishny [1989], Hirshleifer and Thakor [1992], Milgrom and Roberts [1992], Zwiebel [1995], Nohel and Todd [2005], and Malcomson [2011]). Contrary to this literature, we show that managers (even risk-averse ones) have an incentive to take excessive risks if their risk choices influence the probabilities of different states.

Some papers focus on the possibility of signaling of managerial ability. Huberman and Kandel (1993) analyze the reputation concerns of money managers who might possibly over-invest in a risky asset to signal their ability. Huddart (1999) shows that an explicit performance fee may mitigate excessive risk taking of investment advisors who have reputational concerns. Unlike the signaling literature, in our setting the CEO is trying not to flaunt her type but to rather add noise to the market's inferences about it. In that sense, our mechanism is closer to the signal-jamming literature, in which the agent tries to "jam the signal" about her type (Fudenberg and Tirole 1986).

The recent literature on CEO turnover analyzes the impact of performance risk on the firm's ability to infer the unknown ability of its CEO. For example, Bushman, Dai, and Wang (2010) analyze whether firm-specific or systematic risk increases turnover in a setting where risk is exogenous. Instead, we look at the implications of CEO turnover for risk taking when both the risk choice of the CEO and the turnover decision of the firm are endogenous. Hu *et al.* (2011) find a U-shaped relationship between the manager's risk choice and her *prior* relative performance among her peers. We find a similar inverse U-shaped relationship between the CEO's ability and her layoff risk. In our setting, while above-average CEOs face no layoff risk, among below-average ones, lower-ability CEOs have lower layoff risk than do higher-ability ones.

The type of statistical bias that managers try to add into the market's inference about their unknown abilities appears in various ways in the literature. In Milbourn, Shockley, and Thakor (2001), in order to alter the market's assessment about her ability, the manager distorts the probabilities of reputational states that are observed and not observed by overly investigating potential projects. In Scharfstein and Stein (1990), the motivation of the manager is to minimize reputational risk by following the crowd. The closest in spirit to our paper is Hermalin (1993), which shows that a manager can decrease the variance of the posterior estimate of her ability by choosing the riskiest project in terms of variance, as a result of which the principal puts more weight on his prior assessment of the CEO's ability. In our setting, rather than changing the weights on assessments, the CEO changes the posterior assessment itself. Moreover, in Hermalin (1993), the risk-neutral principal is actually indifferent between any risk choices (a risk-neutral manager would also be indifferent). Thus, there is no incentive problem, while in ours, there is a moral hazard in choosing the risk and the firm tries to enhance efficiency with optimal linear contracts.

This paper is the first to show that CEOs (or managers in general) can improve market's expectation about their abilities by taking too high risks, even when they do not know their own abilities. Therefore, CEOs may have strong career-related incentives to take excessive risk. Our structure is novel in that we allow CEOs to choose the probabilities of occurrence of various states, implicitly by choosing the risk level in overseeing the firm. Such a mechanism does not appear in the literature. Contrary to many papers, we show that any linear combination of fixed wage and stocks can do little to prevent excessive risk taking. We distinguish between excessive risk taking against the will of the firm and that with its consent.

The paper is organized as follows. Section II outlines the model. Section III analyzes the case in which the CEOs managerial ability is unknown but the risk level chosen by them is privately known. Section IV extends the two-type analysis of the previous sections to a continuum of types. Section V goes back to the two-type world but extends the model in another dimension by assuming that CEOs privately know their managerial abilities. Section VI concludes. An (online) appendix contains further details and proofs.

II. The Model

We consider a unit mass of risk-neutral CEOs, each of whom may potentially be employed by a risk-neutral firm. CEOs differ in their innate *managerial ability*, which is represented by θ_i, where $i = \{H, L\}$ and $\theta_L < \theta_H$. A CEO with a managerial ability of θ_L (θ_H) is called a *low-ability (high-ability) CEO*. Each type is equally likely in the population, and thus the average ability of a CEO is $\bar{\theta} := (\theta_H + \theta_L)/2$. No one, including the CEO herself, knows the type of a CEO, but the distribution of types in the population is common knowledge. Thus, all parties, including the CEO herself, hold identical prior beliefs over managerial ability. Given her managerial ability, she chooses a privately known risk level while overseeing the firm, $r \in [0, 1]$.[3] There is no borrowing and lending, and neither the firm nor the CEOs discount future payoffs.

The firm operates for two periods, $t = \{1, 2\}$. The output of the firm in any period is determined by both the managerial ability of and the risk choice by its current CEO.[4] If a CEO of managerial ability θ_i chooses a risk level of r_t in period t, then the *realized* output

3. In Holmstrom (1982/1999), Holmstrom and Ricart i Costa (1986), and Hermalin (1993), observability of project riskiness and risk aversion are crucial to the obtained results. In our setting, we do not need to assume that risk choice is unobservable as long as CEOs do not know their abilities because the market correctly predicts this anyway. However, this assumption will be crucial in the asymmetric information case in which CEOs privately know their abilities and each type chooses different risk levels in equilibrium.

4. This specification is consistent with the evidence showing that not only the managerial ability (Chevalier and Ellison 1997; Sirri and Tufano 1998; Del Guercio and Tkac 2002; Falato, Li, Milbourn 2010) but also the managerial style (in our case the risk choice) (Bertrand and Schoar 2003) matter in the firm.

of the firm, $y_t(\theta_i, r_t)$, is

$$(1) \qquad y_t(\theta_i, r_t) = \begin{cases} \theta_i - f(r_t) & \text{with probability } r_t \\ \theta_i + f(r_t) & \text{with probability } 1 - r_t, \end{cases}$$

where $f(r_t)$ is an increasing, concave, and twice-continuously differentiable *risk-return function* with $f(0) \geq 0$. We keep this technological specification fixed throughout the paper. The reservation payoff of a CEO per period is $\underline{u}$, which satisfies $0 < \underline{u} \leq \theta_L$. Thus, the firm may find it profitable to hire a CEO by paying at least her reservation payoff.

The *expected* output of the firm in period t, $E[y_t]$, is[5]

$$(2) \qquad E[y_t(\theta_i, r_t)] = \theta_i + (1 - 2r_t) f(r_t) \quad \forall t = \{1, 2\}, \quad \forall i = \{H, L\}.$$

Given the managerial ability, we interpret this technology as a collection of investment projects with different risk and return pairs resulting in different expected values for each project. In a large number of papers, the choice is between a risky and a riskless project. Our specification is a generalization of this assumption to many projects. With this specification, an increase in risk increases the output in the good state, the loss in the bad state, and the probability of the bad state.[6] Hence, there is an optimal risk level $r_t^* < 1$ that maximizes the expected output of the firm. Although not universal, such a risk specification makes good sense in many real-life situations, especially in those involving risky financial investments.[7] In

5. The assumption that θ_i enters the production technology linearly is quite common in the literature and it is assumed only for simplicity. One can generalize the analysis by having $E[y_t(\theta_i, r_t)] = g(\theta_i) + (1-2r_t)f(r_t)$, and this does not change the qualitative results as long as $g(\theta_i)$ is an increasing function.

6. The specification that the probability of the bad state is increasing in risk level is crucial in our setting. Our results hold as long as the risk-return function is not convex or the state realizations are not asymmetric in favor of the bad state. Moreover, they are independent of the fraction of types in the population as long as the population is not entirely composed of just one type.

7. Our definition combines various attributes of risk that can be seen in the literature. As Sanders and Hambrick (2007) discuss in quite detail, risk is measured differently in different papers. Making larger bets, investing in bets that have more extreme potential outcomes or bets that have higher likelihood of extreme losses are the three major indicators of increased risk taking. Rothschild and Stiglitz (1970) *unambigously* define risk as *mean-preserving spreads* (*i.e.,* moving the probability density from the center to the tails of the distribution while keeping the expected return fixed). However, many real life situations

fact, our specification is very similar to Palomino and Prat's (2003) technology specification in that an investment project is uniquely defined by its expected value and some kind of risk measure r_t which does not need to correspond to variance. We now define what we mean by excessive risk taking.

DEFINITION 1 (EXCESSIVE RISK TAKING) *A CEO who chooses a risk level of r_t is said to be taking excessive risk iff $r_t > r_t^*$.*

In fact, what we find will be even stronger than this. We show that the expected risk-return function, $(1 - 2r_t)f(r_t)$, is negative valued at the equilibrium excessive risk level. That is, the CEO chooses so high a risk level that it is not only higher than the optimal risk level but also results in negative expected return from the contribution of risk to the output. Thus, she chooses a negative NPV project in equilibrium in terms of risk, but her expected managerial ability covers the expected loss due to excessive risk taking, so that the firm may still want to operate.

Contracting between the firm and the CEO is fairly simple. We assume that the firm is not able to offer two-period contracts.[8] Thus, in each period, the firm offers the CEO an individually rational and incentive-compatible compensation contract. We restrict our attention to linear contracts, as they are most frequently observed in practice and well justified in theory (for example, by Holmstrom and Milgrom [1987]). The realized compensation of

involve choosing between projects with different means, as we have in our setting. Using variance as a risk measure in such settings is certainly not ideal, as Hart and Foster (2009) argue in detail. Using second-order stochastic dominance (SOSD) is not ideal, either, because it provides only partial ordering as the dominating distribution must have *at least as high a mean* as the dominated one. Though, our risk measure is *consistent* with SOSD when the latter is able to make comparisons. It is also consistent with Domar and Musgrave's (1944) probability-of-loss-based risk index. Moreover, our risk measure results in the same order with Value at Risk (VaR) and Semivariance measures of risk when the required thresholds for failure in these risk measures are set to zero. Given the technological specifications, our risk measure is *coherent* in the sense that it satisfies monotonicity, subadditivity, positive homogeneity, and translation invariance axioms defined in Artzner *et al.* (1999).

8. This is a standard assumption in career concern models (Gibbons and Murphy 1992; Hermalin 1993; Dasgupta and Prat 2006; Bushman, Dai, and Wang 2010). Hermalin (1993) argues that it is usually infeasible to commit fully to employ the manager at a prespecified compensation in the future. Moreover, if there were signaling considerations, those who have low abilities would be those who demand such contracts. Thus, to signal her type, a high-ability CEO would not want to get such a long-term contract.

the CEO in period t, w_t, is given by

$$(3) \qquad w_t\left(a_t, b_t, y_t\left(\theta_i, r_t\right)\right) = a_t + b_t y_t\left(\theta_i, r_t\right) \quad \forall t = \{1, 2\}, \quad \forall i = \{H, L\},$$

where $a_t \geq 0$ and b_t are compensation parameters. If $b_t = 0$ and $a_t > 0$ in equilibrium, then the contract is a fixed-wage contract, and if $b_t > 0$ and $a_t = 0$, then it provides stock ownership only. All other combinations involve both a fixed wage and stock ownership simultaneously.[9]

Because the CEO's managerial ability is unobserved, the first-period output of the firm is a predictor of her future productivity. Hence, her layoff risk in the second period is influenced by the realized output in the first period, which is influenced by her risk choice. This creates the CEO's career concern in our setting and results in a misalignment between her and the firm's preferences. The CEO maximizes her two-period expected compensation by choosing the risk level in each period, while the firm engages in period-by-period maximization and makes a firing decision in between the two periods, if necessary, upon updating its beliefs based on the first-period output realization.

The sequence of events is as follows. At the beginning of the first period, the firm signs a contract with a CEO that specifies her compensation in this period. Upon employment, the CEO decides how to oversee the firm by choosing a risk level r_1. Then, the first-period output y_1 is realized. The firm pays w_1 to the CEO, updates its beliefs about her managerial ability based on the realized output, and decides whether to fire her. We call a CEO who is hired again in the second period an *old CEO*; and if the firm hires a new CEO in the second period, we call her a *new CEO*. Depending on its firing decision, at the beginning of the second period, the firm signs a new compensation contract with either the old or a

9. For simplicity, we rule out stock-option-based compensation. If we allow for stock options in addition to stocks, the incentives will be even more skewed toward excessive risk taking (see, *e.g.*, Lambert [1986], Ju, Leland, and Senbet [2003], Mehran and Rosenberg [2007], Raviv and Landskroner [2009], Dong, Wang, and Xie [2010]). Moreover, as Murphy (1999) mentions in his well-known review of executive compensation, stock ownership is the most direct way of aligning the preferences of CEOs and shareholders.

new CEO. The CEO chooses a risk level r_2 for the second period. Finally, the second-period output y_2 is realized, the CEO is paid w_2, and the firm is dissolved.

As a benchmark, we first characterize the complete information setting in which both the managerial ability and the risk choice of the CEOs are observable. Obviously, the firm wants to employ a high-ability CEO, and this CEO has no career concern as there is no risk of being fired. As a result, we can obtain the optimal risk level r_t^* from the joint surplus maximization, $\max_{r_t}\{E[y_t(\theta_H, r_t)]\}$, whose first-order condition yields $2f(r_t) = (1-2r_t)f'(r_t)$, from which we can easily see that the optimal risk level satisfies $r_t^* < 1/2$ in any interior solution.[10] The CEO earns just her reservation payoff in expected terms in the optimal compensation contact, which may involve fixed wage and stock ownership in various combinations.

III. Symmetric-Incomplete Information

In the symmetric-incomplete information setting, neither the CEO nor the firm knows the type of the CEO, and only the CEO knows the risk level that she chooses while overseeing the firm. We proceed backwards to solve the model. The next subsection analyzes the second period and shows that the CEO, whether new or old, chooses the optimal risk level in the second period because she no longer has any career concern in this period as the firm will be dissolved after that. It also shows that the firm fires a CEO at the end of the first period if and only if, upon observing the first-period output, it believes that the CEO's ability is less than the average ability in the population. The subsection following the next analyzes the first period and shows that excessive risk taking can be an equilibrium when the difference between the abilities is neither too high nor too low.

10. The second-order condition, $-4f'(r_t) + f''(r_t)(1-2r_t) \leq 0$, holds for all $r_t < 1/2$.

III.A. The Second Period

This subsection derives the firm's optimal firing rule and the risk level that a CEO chooses in the second period. Because the CEO has no career concern in the second period, the problem that the firm faces is a standard moral hazard problem whose solution leaves no surplus to the CEO, who eventually chooses the optimal risk.

The firm maximizes expected output net of expected CEO compensation subject to the individual rationality constraint, which guarantees that the CEO finds it better to sign the compensation contract than to pursue her outside option, and the incentive compatibility constraint, which guarantees that the firm's maximization problem is consistent with the risk choice that results from the CEO maximization problem. The incentive compatibility constraint is given by

$$r_2 \in \arg\max_{\hat{r}_2} E\left[a_2 + b_2\left(\theta + (1 - 2\hat{r}_2)\, f\left(\hat{r}_2\right)\right)\right]. \tag{4}$$

The CEO does not know her ability but rationally expects it to be $\bar{\theta}$ if she is a new CEO. If she is an old CEO, then all terms are conditional on the first-period output realization. Thus, her type is expected to be $\tilde{\theta} := E[\theta_i \mid y_1(\theta, r_1)]$, which is her expected ability given the first-period output $y_1(\theta, r_1)$.

Because the expected compensation is a concave function of r_2 for its positive range, we can comfortably replace the incentive compatibility constraint with its first-order condition. Yet, this first-order condition is exactly the same as the first-order condition of the complete information setting as long as the compensation contract includes some stock ownership (*i.e.*, $b_2 > 0$). Hence, the CEO, whether new or old, chooses the optimal risk level r_2^* in equilibrium.[11] The firm adjusts the compensation parameters such that the individual rationality constraint binds in equilibrium and the CEO gets exactly her reservation payoff,

11. When $b_2 = 0$, the CEO is indifferent between any risk levels, including the optimal one. Thus, the firm wants to offer some stock in equilibrium.

$\underline{u}$, in expected terms. This analysis leads to the following proposition.

PROPOSITION 1 (RISK CHOICE IN THE SECOND PERIOD) *The CEO, whether old or new, chooses the optimal risk level* r_2^* *in the second period.*

CEOs do not have career concerns in the second period because the firm is dissolved at the end of this period. Hence, this proposition predicts that the preferences of CEOs who are closer to end of their careers to be more in line with the preferences of the shareholders. The results in the literature about changes in managers' behavior as their careers evolve are somewhat mixed. Avery and Chevalier (1999) argue that risk taking increases over time as the manager becomes more confident in her abilities. Chevalier and Ellison (1999), Hong, Kubik, and Solomon (2000), and Lamont (2002) provide some evidence for this. Prendergast and Stole (1996) argue the opposite, and Graham (1999) provides evidence in favor of this opposing view.

An obvious but important corollary of the above findings is that if the solution of the firm's maximization problem yields lower profits with $\tilde{\theta}$ than with $\bar{\theta}$, the firm fires the old CEO and hires a new one.[12] This leads to the optimal firing rule.

COROLLARY 1 (OPTIMAL FIRING RULE) *The firm fires the old CEO and hires a new one in the second period iff* $\tilde{\theta} < \bar{\theta}$.

This firing rule is consistent with Hirshleifer and Thakor (1994, 1998), Hermalin and Weisbach (1998, forthcoming), Renée and Ferreira (2007), and Bushman, Dai, and Wang

12. The implicit assumption here is that the reservation payoff of the CEO, $\underline{u}$, remains unchanged despite the fact that beliefs about her type are updated based on the first-period output. In reality, this reservation payoff may adjust (see the arguments in Holmstrom [1982/1999] and Gibbons and Murphy [1992]). Following Bushman, Dai, and Wang (2010), we assume for simplicity that there is downward rigidity in the reservation payoffs because managerial ability is firm specific and valuable only within the organization. Nonetheless, excessive risk taking is possible even when reservation payoffs get updated in response to changes in beliefs about managerial ability. In such a case, a manager's future compensation is still an increasing function of firm's expectation about her ability, and as we show in the text, a manager can increase market's expectation about her ability by taking excessive risk.

(2010), in all of which the CEO is fired if the assessment about her ability is below a particular threshold.

III.B. *The First Period*

This subsection shows the possibility of excessive risk taking in the first period. The optimal firing rule that we derive in the previous subsection says that the firm keeps the old CEO if and only if $\tilde{\theta} \geq \bar{\theta}$. Thus, the CEO has an incentive to influence the market's belief in her ability by her choice of risk. This is in her best interest if the benefit of decreasing her layoff risk is greater than her loss from compensation due to choosing a risk level different from the optimal risk.

We now derive the CEO's probability of being fired at the end of the first period, p. Because there are two types (high and low) and two states (good and bad) in the model, there are four possible state realizations for any given risk level. If the CEO chooses the optimal risk level r_1^*, the firm infers her actual ability upon observing the output, unless by chance outputs coincide at this risk level in any two state realizations.[13] Then, high-ability CEOs are fired with probability zero while low-ability ones are fired with probability one. Given that each type is equally likely in the population, the *ex ante* probability of being fired is 1/2.

Similarly, the firm infers the actual ability of the CEO upon observing the output for any risk level at which the outputs do not overlap for any state realization of the two types. Hence, high-ability CEOs are fired with probability zero while low-ability ones are fired with probability one. Then, once again, the *ex ante* probability of being fired is 1/2. This means that, given any positive amount of stock ownership, r_1^* dominates any such risk level, because the CEO faces the same probability of being fired even when she chooses r_1^* but receives a

13. Because the optimal risk level is less than 1/2, the expectation about the CEO's ability will be below average when outputs coincide at the optimal risk level by chance. Then, the probability of being fired will be higher than 1/2.

higher first-period compensation by doing so.

So, which risk level does a CEO choose in equilibrium? To answer this, we need to consider three cases in terms of the difference between the abilities. The first case is the case in which even the bad-state output of a high-ability CEO is higher than the good-state output of a low-ability CEO for all risk choices and so outputs cannot overlap. This occurs when $\theta_H - f(1) \geq \theta_L + f(1)$ or when the difference between the abilities is high (*i.e.*, $\theta_H - \theta_L \geq 2f(1)$). This is because if this inequality holds, then it should *strictly* hold for all $r_1 \in [0, 1)$ as $f(\cdot)$ is an increasing function. In this case, the firm is able to infer the actual ability of a CEO for all possible output realizations, and thus the probability of being fired is independent of CEO's risk choice and equal to 1/2. Then, again, given any positive amount of stock ownership, r_1^* dominates all other risk choices as it involves the same layoff risk with higher first-period compensation. The following lemma records this result.

LEMMA 1 (CASE 1) *When the difference between the abilities is high (i.e., $\theta_H - \theta_L \geq 2f(1)$), the CEO chooses the optimal risk, $r_1^* < 1/2$, in equilibrium. Her probability of being fired is 1/2.*

In the second case, the difference between the abilities is intermediate (*i.e.*, $2f(1/2) \leq \theta_H - \theta_L < 2f(1)$). Now, by choosing the risk level $\bar{r}_1 = f^{-1}((\theta_H - \theta_L)/2)$, the CEO is able to overlap the bad-state output when she turns out to be a high-ability CEO with the good-state output when she turns out to be a low-ability CEO (*i.e.*, $\theta_H - f(\bar{r}_1) = \theta_L + f(\bar{r}_1)$). If the firm observes this "overlapped" output level, it is not certain about which type could have produced this output. Then, the conditional expectation on the type of the CEO is

$$E[\theta_i \mid y_1] = (1 - \bar{r}_1)\,\theta_L + \bar{r}_1 \theta_H. \tag{5}$$

Because $1/2 \leq f^{-1}((\theta_H - \theta_L)/2)$, we know that $\bar{r}_1 \geq 1/2$; this in turn implies $E[\theta_i \mid y_1] \geq \bar{\theta}$. Therefore, the firm keeps the CEO in the firm when it observes this overlapped output.

Outputs do not coincide in the remaining state realizations, the CEO's type is perfectly inferred, and as a result the high-ability ones are retained while the low-ability ones are fired. Consequently, the probability of being fired is $p = \Pr\{\theta = \theta_L\} \times \Pr\{y_1 = \theta_L - f(\bar{r}_1)\} = \bar{r}_1/2$, which is definitely less than 1/2, the probability of being fired when the CEO chooses a different risk level, and so the outputs do not overlap for any two state realizations.

Choosing $\bar{r}_1$ minimizes the probability of being fired but it is not automatically an equilibrium. By choosing this risk level rather than the optimal risk level, the CEO is minimizing her layoff risk in the second period, but, she is now offered lower compensation in the first period because she did not choose the optimal risk level. For now, we report $\bar{r}_1$ as the risk level that minimizes the layoff risk, but later we derive the conditions under which it becomes an equilibrium.

LEMMA 2 (CASE 2) *When the difference between the abilities is intermediate (i.e.,* $2f(1/2) \leq \theta_H - \theta_L < 2f(1)$*), the risk level that minimizes the probability of being fired is equal to*

$$\bar{r}_1 = f^{-1}\left(\frac{\theta_H - \theta_L}{2}\right) \geq \frac{1}{2}, \tag{6}$$

which is an excessive risk level. In this case, the probability of being fired is $\bar{r}_1/2$*.*

Finally, in the third case, the difference between the abilities is low ($i.e.$, $0 < \theta_H - \theta_L < 2f(1/2)$). Let us first consider the interval $2f(0) < \theta_H - \theta_L < 2f(1/2)$. Following the reasoning we have in Case 2, we obtain (5) once again when the CEO chooses to overlap the outputs. However, this time $E[\theta_i \mid y_1] < \bar{\theta}$ in such a case. She keeps her job only when she turns out to be a high-ability CEO who lands in the good state. Thus, when she overlaps the outputs, her probability of being fired is $p = 1 - \Pr\{\theta = \theta_H\} \times \Pr\{y_1 = \theta_H + f(\bar{r}_1)\} = (1+\bar{r}_1)/2$, which is higher than 1/2, the probability of being fired when she chooses a different risk level, and so the outputs do not overlap for any two state realizations. This suggests that in this case, given any positive amount of stock ownership, r_1^* dominates all other risk choices, including $\bar{r}_1$.[14] In the remaining part of the interval of case 3 (*i.e.*, $0 < \theta_H - \theta_L \leq 2f(0)$) outputs do not match in any way and thus the probability of being fired cannot be any lower than $1/2$, which implies that she chooses the optimal risk level.

Lemma 3 (Case 3) *When the difference between the abilities is low (i.e.,*$0 < \theta_H - \theta_L < 2f(1/2)$*), the CEO chooses the optimal risk,* $r_1^* < 1/2$*, in equilibrium. Her probability of being fired is* $1/2$*.*

In sum, the CEO chooses the optimal risk in equilibrium when the difference between the abilities is high or low, but when the difference between the two abilities is intermediate she may choose a risk level at which the bad-state output of a high-ability CEO coincides with the good-state output of a low-ability CEO. This strategy decreases the output's informativeness about the CEO's ability, which in turn minimizes her layoff risk. Of course, for this to be an equilibrium, it must also be in her best interest to do so, which we focus on next.

In the rest of this subsection, we analyze possible equilibrium risk levels when the difference between the abilities is intermediate. According to Lemma 2, if the CEO chooses $\bar{r}_1$, then her probability of being fired is $\bar{r}_1/2$. If she chooses any other risk level, her probability of being fired is $1/2$. Then, she is better off choosing the optimal risk level r_1^* among all these possible risk levels because her probability of being fired is still $1/2$ but her first-period compensation is higher. This means that r_1^* always dominates all other risk choices, except $\bar{r}_1$, given any positive amount of stock ownership. Thus, the CEO's choice in Case 2 is between $\bar{r}_1$ and r_1^* only.

The firm's maximization problem is the same as in the second period, except now it includes an additional constraint. If the firm wants the CEO to choose the optimal risk level, it must compensate the forgone expected payoff that comes from increased layoff risk

14. If outputs match by chance at the optimal risk level, then the CEO chooses a risk level arbitrarily close to the optimal risk level.

by not choosing $\bar{r}_1$. We call this constraint the *career concern constraint*, which is given by

$$\text{(CC)} \quad E\left[w_1\left(a_1, b_1, y_1\left(\bar{\theta}, r_1^*\right)\right)\right] + \frac{\underline{u}}{2} \geq E\left[w_1\left(a_1, b_1, y_1\left(\bar{\theta}, \bar{r}_1\right)\right)\right] + \frac{(2-\bar{r}_1)\underline{u}}{2} \quad \text{if } r_1 \neq \bar{r}_1.$$

The left-hand side of this constraint is the expected payoff of the CEO if she chooses r_1^* and the right-hand side is that if she chooses $\bar{r}_1$. This constraint is derived as follows. If the CEO chooses $\bar{r}_1$, her probability of keeping her job in the second period, in which she always obtains her reservation payoff $\underline{u}$, is $(2-\bar{r}_1)/2$. Therefore, her second-period expected payoff is $[(2-\bar{r}_1)\underline{u}]/2$ if she chooses $\bar{r}_1$ in the first period. Adding her expected first-period compensation to this term yields the right-hand side of the inequality. If she chooses the optimal risk level r_1^*, the probability of keeping her job in the second period is $1/2$; hence, her expected payoff is $\underline{u}/2$ in the second period. Adding her expected first-period compensation to this term yields the left-hand side of the inequality.

Reorganizing the career concern constraint after employing the linear compensation contract assumption gives

$$\text{(CC')} \qquad (1-2r_1^*)\, f\left(r_1^*\right) - (1-2\bar{r}_1)\, f\left(\bar{r}_1\right) \geq \frac{(1-\bar{r}_1)\,\underline{u}}{2b_1}.$$

This constraint shows that when the career concern is sufficiently strong, it may be stricter than the incentive compatibility constraint; thus, the solution may involve $\bar{r}_1$ chosen by the CEO as a result of the discontinuous jump created by her career concern. The next question is exactly when choosing $\bar{r}_1$ is better than r_1^*, which we proceed to answer now. A necessary condition is that the firm prefers operating when the CEO chooses the excessive risk level $\bar{r}_1$, which holds as long as $\theta_L \geq \underline{u}$, which we have already assumed.

There are two cases to consider in which excessive risk taking occurs in equilibrium. In the first, satisfying the career concern constraint and having the CEO take the optimal risk requires giving her stocks more valuable than the firm's output (*i.e.*, $b_1 > 1$), which the firm cannot afford without incurring a loss. Thus, in such a situation, the firm *involuntarily* allows excessive risk taking in equilibrium. If, therefore,

$$(1 - 2r_1^*)\, f\,(r_1^*) - (1 - 2\bar{r}_1)\, f\,(\bar{r}_1) < \frac{(1 - \bar{r}_1)\, \underline{u}}{2}, \tag{7}$$

then (CC') is satisfied only when $b_1 > 1$, which the firm cannot afford without incurring a loss, and thus we get excessive risk taking in equilibrium. This inequality represents a situation in which the CEO's career benefit of excessive risk taking to hide her type is higher than the expected return from the project. In such a case, the firm cannot compensate the CEO for her career benefit from excessive risk taking, even if it offers the whole first-period return to her. Note that all terms in this inequality are exogenous. Thus, if it holds, then (CC') cannot hold, and excessive risk taking becomes imperative.

In the second case in which there is excessive risk taking in equilibrium, having the CEO take optimal risk is less profitable than letting her take excessive risk. Thus, the firm *voluntarily* allows excessive risk taking.[15] This time, (CC') is satisfied, which requires providing an amount of stock ownership that satisfies $b_1 \geq (\underline{u}(1 - \bar{r}_1))/(2[(1 - 2r_1^*)f(r_1^*) - (1 - 2\bar{r}_1)f(\bar{r}_1)])$. Hence, the lowest possible stock compensation, $b_1 y_1(\bar{\theta}, r_1^*)$, is given by the following amount:

$$\Omega := \frac{\underline{u}\,(1 - \bar{r}_1)\left[\bar{\theta} + (1 - 2r_1^*)\, f\,(r_1^*)\right]}{2\left[(1 - 2r_1^*)\, f\,(r_1^*) - (1 - 2\bar{r}_1)\, f\,(\bar{r}_1)\right]}. \tag{8}$$

Then, if $(1 - 2r_1^*)f(r_1^*) - \Omega < (1 - 2\bar{r}_1)f(\bar{r}_1) - \underline{u}$, the firm voluntarily allows the CEO to take excessive risk. The left-hand side of this inequality is the profit of the firm in case of optimal risk taking and the right-hand side is that in case of excessive risk taking. Reorganizing it

15. In a related vein, Bebchuk and Spamann (2010) also mentions that even after eliminating the excessive risk from the perspective of the common shareholders in banks, there may still remain excessive risk from the perspective of the society because common shareholders are not concerned about preferred shareholders, bondholders, depositors, and tax payers. We get our result for a different reason because we do not have any of these third parties in the model.

yields a condition that looks similar to (7):

$$(1 - 2r_1^*)\, f\,(r_1^*) - (1 - 2\bar{r}_1)\, f\,(\bar{r}_1) < \Omega - \underline{u}. \tag{9}$$

We summarize the above discussion in the following proposition.

Proposition 2 (Excessive risk taking / two-type) *Suppose that the difference between the abilities is intermediate (i.e., $2f(1/2) \leq \theta_H - \theta_L < 2f(1)$). The firm involuntarily allows the CEO to take excessive risk if (7) holds. It voluntarily allows the CEO to take excessive risk if (9) holds.*

The crucial point here is that excessive risk taking is possible even under an optimal compensation contract. If there is excessive risk taking, the optimal contract is given by $b_1 > 0$, and $a_1 = \underline{u} - b_1[\bar{\theta} + f(\bar{r}_1)(1 - 2\bar{r}_1)]$. In the involuntary case, as shown in (7), it is optimal for the CEO to take excessive risk if the expected loss in output that arises from excessive risk taking is less than the career benefit obtained from excessive risk taking. In the voluntary case, as shown in (9), the benefit of decreasing risk to the optimal level is less than the cost of compensating the CEO to let her take the optimal risk.

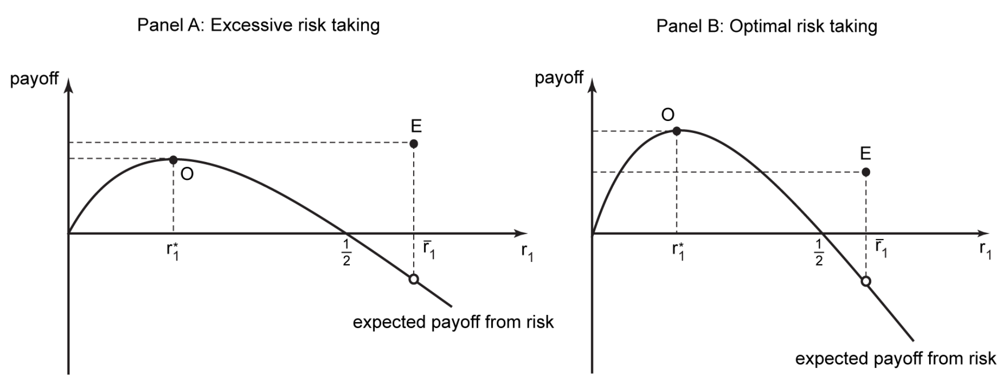

Figure I
Excessive vs. Optimal Risk Taking in Equilibrium

Figure I provides a graphical intuition for excessive risk taking. It shows the expected payoff of the CEO *from risk* for any risk level chosen. Her payoff is increasing up to the optimal risk level r_1^* at point O, and then it is ever decreasing unless her career concern kicks in, which is where her payoff discontinuously jumps up to point E. If this point is above point O, as in Panel A, then the CEO finds it optimal to take excessive risk. This is because the decrease in her first-period compensation due to not taking the optimal risk is less than the career benefit she obtains by minimizing her layoff risk by taking excessive risk. However, if it turns out that point E is below point O, as in Panel B, the CEO takes the optimal risk. It is noteworthy that risk level $\bar{r}_1$ means choosing a negative NPV project in terms of the return from risk. Thus, the risk alone contributes negatively to the firm output in equilibrium, but the return from managerial ability absorbs the loss.[16]

We close this section with some comments on the structure of the model and robustness of the results under different specifications.

The fact that there is just one point jumping up discontinuously as a result of career concern in Figure I is an artifact of our two-type specification. Nevertheless, as we show in Section IV, the same mechanism works when we have a continuum of types, in which case there is a mass of points jumping up and their local maximizer gives us the new $\bar{r}_1$. If it is also the global maximizer (as in Panel A of Figure I), then there is excessive risk taking in equilibrium. Otherwise, the CEO takes the optimal risk (as in Panel B of Figure I).

In our setting, any linear combination of fixed wage and stock ownership is allowed in the optimal compensation contract. At this point, one concern would be if other types of frequently observed compensation contracts work. In Footnote 9, we argued the reasons for which neglecting stock options is harmless. We now explain why bonus contracts may not prevent excessive risk taking in our setting as long as sabotaging output is allowed. A bonus contract pays a fixed sum if the output is above a certain threshold. Suppose, for example, the firm promises to pay a fixed bonus equal to $\underline{u}$ if the CEO obtains $\theta_H + f(r_1^*)$, $\theta_H - f(r_1^*)$,

16. Palomino and Prat (2003) provide a similar figure representing the set of risky portfolios. They mention that the textbook analysis shows only the increasing part of the figure as the decreasing part involves dominated strategies. However, those risk levels are in fact chosen in their analysis as well as ours.

$\theta_L + f(r_1^*)$, or $\theta_L - f(r_1^*)$ and zero otherwise. The CEO is still fired if her assessed ability is below average. Then, she may take excessive risk $\bar{r}_1$, and if she obtains $\theta_H + f(\bar{r}_1)$, she can sabotage the output (perhaps by selling it with too low a price) and make it appear as if she took the optimal risk level r_1^*. Moreover, if one of the parties were risk averse, bonus contracts would not be optimal since they do not involve optimal risk sharing.

Our main results are independent of bilateral risk neutrality. First, unlike the bilateral risk-neutrality case of a standard hidden action problem, the career concern can be so strong that even providing the output of the first period to the CEO may not prevent her from taking excessive risk. Thus, no contract can prevent excessive risk in such a case. The analogy would be a young fund manager who may take excessive risk in managing her own portfolio as a result of her concern that if she does not perform well now she might not receive outside funds in the future. Second, as shown in Appendix A.1, the results remain qualitatively the same even when the CEO is risk averse.

The CEO's possibility of affecting a firing decision with her choice of risk implies behavior consistent with behavioral finance's concept of CEO overconfidence. This literature is based on the hypothesis that many CEOs tend to think that they are better than the average (Malmendier and Tate 2005), and this leads them to be more likely to attribute good outcomes to their managerial ability or style. Hence, the literature argues that overconfident managers overestimate their abilities and underestimate the probability of failure, so their investment decisions are riskier than is ideal. In our setting, each CEO rationally estimates her ability and the probability of failure, but by taking excessive risk, tries to ensure that the market overestimates her ability. Hence, our model provides a rational foundation for CEO overconfidence.

IV. Continuum of Types

The basic insight we get from the two-type analysis of the previous section is that the best strategy for a CEO who does not know her ability is to choose the risk level at which the bad-state output of a high-ability CEO coincides with the good-state output of a low-ability CEO. This strategy decreases the output's informativeness about the CEO's ability and thus maximizes her probability of keeping her position in the second period. Under certain conditions, having outputs coincide may require excessive risk taking in equilibrium. This section extends this line of reasoning to a continuum of CEO types. Our analysis also predicts an inversed U-shaped relationship between unobservable ability and the probability of being fired: while the above-average CEOs do not face any layoff risk, among the below-average CEOs, higher-ability ones are certainly fired while lower-ability ones are fired only with some probability.

The optimal firing rule, derived in Corollary 1, and the optimal second-period compensation contract, which gives the CEO her reservation payoff in the second period, continue to apply in this section. Thus, as in the two-type case, the basic mechanism of the model works as follows. Given that the CEO is paid her reservation payoff in the second period, she trades off the decrease in her layoff risk in the second period by taking excessive risk in the first period for the increase in her expected compensation in the first period by taking optimal risk. There are robust instances in which the former effect dominates the latter in expected payoff, and thus we get excessive risk taking in equilibrium, either by the firm's consent or against its will.

We shall now talk about "the range of abilities" rather than "the difference between the two abilities," as there is now a continuum of abilities rather than just two. In particular, we assume that managerial abilities are uniformly distributed on the interval $[\theta_L, \theta_H]$ with mean $\bar{\theta}$. Just as in the two-type world, it turns out that there are three possible cases to consider in terms of the range of abilities (high, intermediate, and low), and we find excessive risk taking in equilibrium only for the intermediate range of abilities. For brevity, we state only the results for the other two cases in the following lemma, leaving the detailed analysis to Appendix A.2.

Lemma 4 (Cases 1 and 3) *When there is a high (i.e.,* $\theta_H - \theta_L \geq 4f(1)$*) or low (i.e.,* $\theta_H - \theta_L < 2f(1/2)$*) range of abilities in the CEO labor market, the CEO chooses the optimal risk,* $r_1^* < 1/2$*, in equilibrium. Her probability of being fired is* $1/2$*.*

Now, consider Case 2 in which there is an intermediate range of abilities in the CEO labor market (*i.e.*, $2f(1/2) \leq \theta_H - \theta_L < 4f(1)$). This time, we proceed by the guess-and-verify method. We make the *educated* guess that the CEO chooses an $\bar{r}_1$ such that

$$4\bar{r}_1 f(\bar{r}_1) = \theta_H - \theta_L \tag{10}$$

is satisfied. This is the risk level that guarantees that even the worst type is able to overlap her good-state output with the bad-state output of an above-average CEO. The subsequent analysis proceeds as follows. Assuming $\bar{r}_1$ to be the equilibrium risk level, we first derive the probability of being fired. Then, in Appendix A.3, we prove that $\bar{r}_1$ is indeed the risk level that minimizes the probability of being fired. Finally, we show that minimizing the probability of being fired can indeed be an equilibrium under certain conditions.

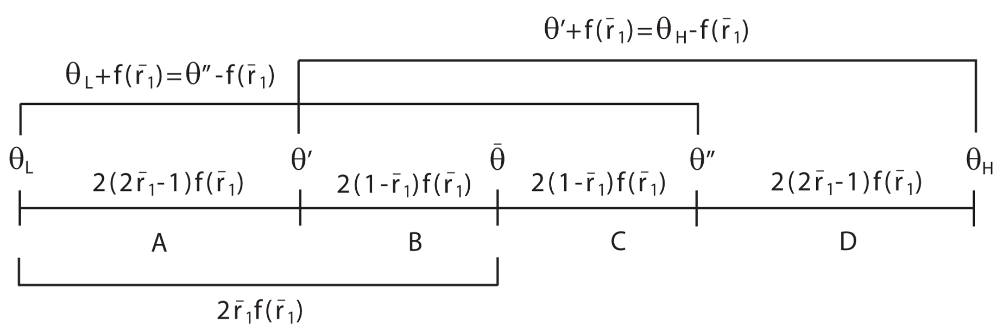

Figure II
The Partition of CEO Types in Case 2

Figure II shows the partition of CEOs on the ability distribution. The partitions are denoted by A, B, C, and D. The ability range of this case guarantees that, given $\bar{r}_1$, there is a θ''-type whose bad-state output coincides with the good-state output of the worst type, θ_L, and the firm's expectation between these two types is exactly $\bar{\theta}$ (that is, $\theta_L + f(\bar{r}_1) = \theta'' - f(\bar{r}_1)$ and $(1-\bar{r}_1)\theta_L + \bar{r}_1\theta'' = \bar{\theta}$). They also guarantee that there is a θ'-type whose good-state output coincides with the bad-state output of the best type, θ_H (that is, $\theta' + f(\bar{r}_1) = \theta_H - f(\bar{r}_1)$). Of course, the expectation between these two types must be higher than $\bar{\theta}$.

Figure II provides the distance between the particular types mentioned in the previous paragraph. Eq. (10) implies that the distance between θ_L and $\bar{\theta}$ and the distance between $\bar{\theta}$ and θ_H are both $2\bar{r}_1 f(\bar{r}_1)$ because $\bar{\theta}$ is the mean of the uniform distribution. Moreover, from the specifications provided in the previous paragraph, one can easily find that the distance between $\bar{\theta}$ and θ'' is $2(1-\bar{r}_1)f(\bar{r}_1)$. Thus, the distance between θ'' and θ_H is $2(2\bar{r}_1 - 1)f(\bar{r}_1)$, which is also the distance between θ_L and θ'. Consequently, the mass in A is equal to the mass in D and the mass in B is equal to the mass in C. Note also that $\bar{r}_1$ is an excessive risk level because it is higher than $1/2$ as a result of the fact that $4\bar{r}_1 f(\bar{r}_1) > 2f(1/2)$ in this case.

We can now derive the probability of being fired in each partition. Because the expectation between θ_L and θ'' is exactly $\bar{\theta}$ at $\bar{r}_1$, the expectation about the ability of a CEO in A must be higher than $\bar{\theta}$ when she obtains the good-state output. Thus, she is rehired in such an output realization. If she obtains the bad-state output, her ability is inferred and she is fired for certain. Thus, the probability of being fired for a CEO in this partition is $\bar{r}_1$. Next, consider a CEO in B. With the given risk level, she is not able to overlap her good-state output with the bad-state output of any existent type and yet her ability is less than $\bar{\theta}$; thus, she is certainly fired in any output realization.

Now consider a CEO in C. Her ability is inferred to be above $\bar{\theta}$ because there is no CEO below $\bar{\theta}$ overlapping her good-state output with the bad-state output of this CEO. Thus, she is rehired for certain. Finally, the bad-state output of a CEO in D coincides with the good-state output of a CEO in A, and thus she is rehired in her bad state. She is rehired for certain in her good state as well, because her output in that state does not coincide with the bad-state output of any existent type above her. Hence, the probability of being fired is zero for a CEO in this partition.

Given the above analysis, the overall probability of being fired is given by $p = \bar{r}_1 \times \Pr\{\theta \in A\} + 1 \times \Pr\{\theta \in B\} + 0 \times \Pr\{\theta \in C\} + 0 \times \Pr\{\theta \in D\}$, or

$$p = \bar{r}_1 \frac{2\,(2\bar{r}_1 - 1)\, f\,(\bar{r}_1)}{4\bar{r}_1 f\,(\bar{r}_1)} + \frac{2\,(1 - \bar{r}_1)\, f\,(\bar{r}_1)}{4\bar{r}_1 f\,(\bar{r}_1)} = \frac{2\bar{r}_1^2 - 2\bar{r}_1 + 1}{2\bar{r}_1}, \tag{11}$$

which is definitely less than 1/2 because $\bar{r}_1 > 1/2$. What remains to be shown is that $\bar{r}_1$ is indeed the risk level that minimizes the probability of being fired, which we prove in Appendix A.3 by comparing the p value in (11) with the ones that stem from other possible risk levels. Thus, we have the following lemma.

LEMMA 5 (CASE 2) *When there is an intermediate range of abilities in the CEO labor market (i.e.,* $2f(1/2) \leq \theta_H - \theta_L < 4f(1)$*), the risk level that minimizes the probability of being fired solves (10), which is an excessive risk level. In this case, the probability of being fired is given by (11).*

In the rest of this section, we look for the equilibrium risk level in Case 2. As in the two-type case, the risk level that solves (10) is not automatically an equilibrium. For that to be an equilibrium, minimizing the probability of being fired must be in the best interest of the CEO. This may be the case when the CEO's compensation benefit by taking optimal risk is dominated in expected payoff by the career benefit she derives by taking excessive risk and hence minimizing her probability of being fired. However, unlike the two-type case in which choosing the excessive risk level $\bar{r}_1$ is the only serious alternative against the optimal risk level, here the CEO may potentially choose a risk level different from the one minimizing the probability of being fired in equilibrium.

As shown in Appendix A.3, choosing a risk level higher than $\bar{r}_1$, satisfying $f(r_1) \in (f(\bar{r}_1), 2\bar{r}_1 f(\bar{r}_1))$, results in a higher probability of being fired than that with $\bar{r}_1$. At the same time, because $\bar{r}_1$ is closer to r_1^*, the first-period compensation is going to be higher with any amount of stock-based compensation. Thus, choosing $\bar{r}_1$ still dominates in expected payoff choosing any risk level satisfying $f(r_1) \in (f(\bar{r}_1), 2\bar{r}_1 f(\bar{r}_1))$. However, if the CEO chooses a risk level satisfying $f(r_1) \in (\bar{r}_1 f(\bar{r}_1), f(\bar{r}_1))$, as shown in the appendix, the probability of being fired is still higher than that with $\bar{r}_1$, yet this time the first-period compensation is going to be higher with any amount of stock-based compensation because this risk level is closer to r_1^*. As a result, the CEO may prefer to trade off the increased probability of being fired for higher first-period compensation. Any such risk level chosen in equilibrium still involves excessive risk taking although it does not minimize the layoff risk. Therefore, unlike the two-type case, excessive risk taking is now an interval rather than just one point (*i.e.*, point E of Figure I. The local maximizer of this interval is the most serious candidate against the optimal risk level, and in fact, if it is also the global maximizer it is the equilibrium.

We now turn to the derivation of the optimal contract. The increase in the probability of being rehired in the second period by choosing $\bar{r}_1$ in the first period is now given by

$$(1 - p(\bar{r}_1)) - (1 - p(r_1^*)) = \frac{3\bar{r}_1 - 2\bar{r}_1^2 - 1}{2\bar{r}_1}. \tag{12}$$

Thus, the new career concern constraint with $\bar{r}_1$ is given by

$$\text{(CC)} \quad E\left[a_1 + b_1 y_1\left(\bar{\theta}, r_1\right)\right] - E\left[a_1 + b_1 y_1\left(\bar{\theta}, \bar{r}_1\right)\right] \geq \frac{\left(3\bar{r}_1 - 2\bar{r}_1^2 - 1\right)\underline{u}}{2\bar{r}_1} \quad \text{if } f(r_1) \notin (\bar{r}_1 f(\bar{r}_1), f(\bar{r}_1)),$$

where the left-hand side is the extra compensation that the firm must provide to the CEO for her expected forgone career benefit by choosing an r_1 such that $f(r_1) \notin (\bar{r}_1 f(\bar{r}_1), f(\bar{r}_1))$, which is shown on the right-hand side. Apart from this change in the career concern constraint, the maximization problem of the firm and its solution remain qualitatively the same. Thus, we provide the following proposition without a proof.

PROPOSITION 3 (EXCESSIVE RISK TAKING / CONTINUUM) *Suppose there is an intermediate range of abilities in the CEO labor market (i.e., $2f(1/2) \leq \theta_H - \theta_L < 4f(1)$). The firm involuntarily allows the CEO to take excessive risk if*

$$f\left(r_1^*\right)\left(1-2r_1^*\right) - f\left(\bar{r}_1\right)\left(1-2\bar{r}_1\right) < \frac{\left(3\bar{r}_1 - 2\bar{r}_1^2 - 1\right)\underline{u}}{2\bar{r}_1}. \tag{13}$$

It voluntarily allows the CEO to take excessive risk if

$$f\left(r_1^*\right)\left(1-2r_1^*\right) - f\left(\bar{r}_1\right)\left(1-2\bar{r}_1\right) < \frac{\left(3\bar{r}_1 - 2\bar{r}_1^2 - 1\right)\left[f\left(r_1^*\right)\left(1-2r_1^*\right) + \bar{\theta}\right]\underline{u}}{2\bar{r}_1\left[f\left(r_1^*\right)\left(1-2r_1^*\right) - f\left(\bar{r}_1\right)\left(1-2\bar{r}_1\right)\right]} - \underline{u}. \tag{14}$$

In both cases, the equilibrium risk level r_1 satisfies $f(r_1) \in (\bar{r}_1 f(\bar{r}_1), f(\bar{r}_1)]$, where $\bar{r}_1$ is defined by (10).

Eqs. (13) and (14) are respectively the counterparts of (7) and (9) in the continuum of types case. Note that if these equations hold for $\bar{r}_1$ and if another risk level satisfying $f(r_1) \in (\bar{r}_1 f(\bar{r}_1), f(\bar{r}_1)]$ dominates $\bar{r}_1$, then these conditions hold for that risk level as well. Thus, the proposition applies for any risk level in that interval, not just for $\bar{r}_1$. The intuitions for (13) and (14) are the same as those provided for Proposition 2. Eq. (13) says that the career benefit the CEO derives from excessive risk taking is higher than the compensation benefit she derives in the first period by taking optimal risk, even when she is offered the whole first-period return. Thus, the firm cannot design a linear compensation contract that implements the optimal risk, even if it wants to do so. Eq. (14) gives the condition under which the expected profit of the firm is higher with excessive risk than that with optimal risk. As in the two-type case, one can easily see that none of our results stems from our assumption that the CEO is risk neutral.

So far, we have shown that all results of the two-type case extend to the continuum of types case. This case also provides an important prediction that we do not have in the two-type case. Consider a CEO whose ability is below $\bar{\theta}$ in Figure II. If she is in A, then she is able to overlap her good-state output with the bad-state output of an above-average CEO; thus she is not fired in such a state. However, if she is in B, then she is not able to overlap her good-state output with the bad-state output of any existent type in the ability distribution. As a result, a CEO in B is fired for certain whereas one in A is fired only with probability r_1, which means that, among those who are below average, a worse type is less likely to be fired than a better type. However, those who are in C and D, all of whom are above average, are not fired in any case. Thus, there is an inverse U-shaped relationship between unobserved ability and the probability of being fired.

Proposition 4 (Ability and layoff risk) *There is an inverse U-shaped relationship between the unobserved ability and the probability of being fired.*

The intuition for this result is as follows. By taking risk, a lower-ability CEO can disguise her type more convincingly because her good-state output is not going to be very high anyway. Hence, she has some chance of successfully substituting the return from managerial ability with the return from risk. The firm is skeptical to some extent, but it is not 100% sure if the CEO is below average ability or not. A higher-ability (but still below average) CEO is also able to do the same substitution, but this time the observed output is so high that the firm believes that there is no way that this CEO is above average ability. That is, if the CEO ends up with an unbelievably high output, then the firm is certain that this output is coming from a lucky below-average type who gambled and thus fires her without hesitation.

V. Asymmetric Information

This section relaxes our information assumptions by assuming that CEOs privately know their abilities in the two-type setting. Now that CEOs know their abilities, different types can choose different risk levels in equilibrium. The reservation payoff of a high-ability CEO is now u_H, which satisfies $u_H \leq \theta_H$, and that of a low-ability CEO is u_L, which satisfies $u_L \leq \theta_L$. It is natural to assume that $u_H > u_L$, considering that high-ability CEOs have higher outside options. We also make the following assumption, which rules out the possibility of separation in the second period.

Assumption 1 (No separation) $\theta_H - \theta_L \geq 3(u_H - (u_L/2))$ *and* $\theta_L/\theta_H \geq u_L/u_H$.

The two expressions in this assumption ensure that the firm cannot offer a contract aimed only at the low- and high-ability CEOs, respectively.[17] Hence, knowing that CEOs have no career concerns in the second period, the firm offers a pooling compensation contract that attracts both types. In turn, both CEO types choose the optimal risk level in the equilibrium, and thus no agency problem arises in this period, as in the previous sections. Because a pooling contract is offered in the second period, the optimal firing rule derived in the symmetric-incomplete information model continues to hold in this information setting.

To show the possibility of excessive risk taking in equilibrium, we now turn to the analysis of the first period. Allowing for asymmetric information extensively enlarges the strategy space of the CEOs. In the two-type world, a CEO may choose to overlap the good-state output of a low-ability CEO with the bad-state output of a high-ability CEO. Now that she knows her own ability, she can even overlap good states with good states and bad states with bad states. We first show that the firm's expectation about the CEO's ability is higher

17. Unlike many asymmetric information problems, here the firm may want to attract only the low-ability CEOs because it may eliminate the moral hazard aspect of the problem. That is, the benefit of hiring a low-ability CEO (who is going to choose optimal risk) with a separating contract may outweigh the benefit of hiring a CEO (who will not choose the optimal risk if she is a low-ability one) with a pooling contract.

than $\bar{\theta}$ in each of these cases. Thus, it is not *ex ante* clear which one is the best strategy for the CEO under various conditions.

LEMMA 6 (STRATEGIES) *Suppose that high-ability CEOs choose the optimal risk level r_1^*. Consider a low-ability CEO.*

1. *If she chooses the risk level $\bar{r}_1 \in [1 - r_1^*, 1)$, at which her good-state output overlaps with the bad-state output of a high-ability CEO (i.e., $\theta_L + f(\bar{r}_1) = \bar{y}_1 = \theta_H - f(r_1^*)$), then, upon observing such an output level, the firm's expectation about her ability is greater than or equal to $\bar{\theta}$. Her overall probability of being fired is $\bar{r}_1$.*

2. *If she chooses the risk level r_1', at which her good-state output overlaps with the good-state output of a high-ability CEO, (i.e., $\theta_L + f(r_1') = y_1' = \theta_H + f(r_1^*)$), then, upon observing such an output level, the firm's expectation about her ability is greater than or equal to $\bar{\theta}$. Her overall probability of being fired is r_1'.*

3. *If she chooses the risk level $\hat{r}_1$, at which her bad-state output overlaps with the bad-state output of a high-ability CEO (i.e., $\theta_L - f(\hat{r}_1) = \hat{y}_1 = \theta_H - f(r_1^*)$), then, upon observing such an output level, the firm's expectation about her ability is greater than or equal to $\bar{\theta}$. Her overall probability of being fired is $1 - \hat{r}_1$.*

The proof is in Appendix A.4. It is obvious that a high-ability CEO is rehired for certain in the second period in any of the three cases of this lemma. Thus, she will in fact choose the optimal risk level r_1^* because this maximizes her first-period compensation without affecting her probability of being fired.[18] However, a low-ability CEO's choice is among $\bar{r}_1$, r_1', $\hat{r}_1$, and r_1^*, depending on the case.

18. There is an exceptional measure-zero case in which the good-state output of low-ability CEOs coincides with the bad-state output of high-ability CEOs by chance at the optimal risk level (*i.e.*, $\theta_L + f(r_1^*) = \theta_H - f(r_1^*)$). In this case, as we show in Appendix A.5 (Case 6), a high-ability CEO chooses a risk level arbitrarily close (but not equal) to r_1^*.

The strategies defined in Lemma 6 are not always viable. First, none is viable when $\theta_H - f(r_1^*) \geq \theta_L + f(1)$, because in such a case, a low-ability CEO cannot overlap her output with the output of a high-ability CEO in any state. Second, as the first part of Lemma 6 suggests, $\bar{r}_1$ is not an effective strategy for a low-ability CEO when $\bar{r}_1 \in [0, 1 - r_1^*)$, because it does not decrease her probability of being fired. Third, choosing r_1' is a viable strategy only if $\theta_H + f(r_1^*) < \theta_L + f(1)$; otherwise, there exists no r_1' overlapping the good-state output of a low-ability CEO with that of a high-ability CEO. This requires $\theta_H - \theta_L \in (0, f(1) - f(r_1^*))$. Fourth, choosing $\hat{r}_1$ is a viable strategy only if $\theta_L - f(0) > \theta_H - f(r_1^*)$; otherwise, there exists no $\hat{r}_1$ overlapping the bad-state output of a low-ability CEO with that of a high-ability CEO. This requires $\theta_H - \theta_L \in (0, f(r_1^*) - f(0))$.

The requirements in the above paragraph result in six different cases in terms of the difference between the abilities. For brevity, we consider only Case 2 here, in which excessive risk taking occurs in equilibrium, and leave the analysis of the remaining cases to Appendix A.5. Case 2 is of particular interest because the CEO employs the same strategy of the previous sections in this case, namely overlapping the good-state output of a low-ability CEO with the bad-state output of a high-ability CEO by choosing $\bar{r}_1$. However, excessive risk taking is not limited to this case. In fact, there is another case (Case 4) in which low-ability CEOs may potentially overlap their good-state outputs with those of high-ability CEOs by choosing the excessive risk level r_1'.[19]

In Case 2, $\theta_H - \theta_L \in [f(r_1^*) + f(1 - r_1^*), f(r_1^*) + f(1))$ and thus neither r_1' nor $\hat{r}_1$ is viable. Assume, for the moment, that a high-ability CEO chooses the optimal risk level r_1^*. We know from the first part of Lemma 6 that the firm rehires the low-ability CEO if she chooses $\bar{r}_1$ and overlaps her good-state output with the bad-state output of a high-ability CEO. Hence, the only serious candidate against choosing r_1^* is choosing $\bar{r}_1$ in this case. Now, consider a high-ability CEO. The firm keeps her if it observes the overlapped output. It keeps her

19. In Case 4, low-ability CEOs may even overlap their bad-state outputs with that of high-ability CEOs by choosing the *insufficient* risk level $\hat{r}_1$.

even when she produces a different output, because her ability, which is high, is inferred. Thus, if she chooses the optimal risk level, her probability of being fired is zero regardless of her output, which means that she has no incentive to deviate from this risk level, as it will also maximize her first-period compensation. The firm needs to pay some positive amount of stock ownership to guarantee this, which it certainly does. This discussion results in the following lemma.

LEMMA 7 (CASE 2) *If $\theta_H - \theta_L \in [f(r_1^*) + f(1 - r_1^*),\ f(r_1^*) + f(1))$, then high-ability CEOs choose the optimal risk level r_1^* in equilibrium and their probability of being fired is zero. Low-ability CEOs choose either the optimal risk level r_1^*, in which case their probability of being fired is one, or the excessive risk level $\bar{r}_1$ so that their good-state output coincides with the bad-state output of a high-ability CEO, in which case their probability of being fired is $\bar{r}_1$.*

We now show the possibility of excessive risk taking in equilibrium. The firm hires just one CEO and therefore its contract offer must provide at least u_H if it wants to contract with a high-ability CEO. The first expression in Assumption 1 (*i.e.*, $\theta_H - \theta_L \geq 3(u_H - (u_L/2))$) ensures that attracting both types is better for the firm than attracting only the low-ability CEOs. The second expression in Assumption 1 (*i.e.*, $\theta_L/\theta_H \geq u_L/u_H$) ensures that if both types choose the optimal risk, then any contract that satisfies the individual rationality constraint of a high-ability CEO also satisfies that of a low-ability CEO. In the case of excessive risk taking, a low-ability CEO chooses $\bar{r}_1$, which means that choosing $\bar{r}_1$ makes her better off for all compensation schemes than choosing r_1^*. Consequently, if high-ability CEOs participate in the excessive risk taking case, low-ability CEOs will participate as well.

The career concern constraint of a low-ability CEO guarantees that the extra compensation that she gets by choosing the optimal risk rather than $\bar{r}_1$ is higher than her career benefit by choosing $\bar{r}_1$:

$$\text{(CC)} \quad [a_1 + b_1\theta_L + b_1(1-2r_L)f(r_L)] - [a_1 + b_1\theta_L + b_1(1-2\bar{r}_1)f(\bar{r}_1)] \geq$$
$$(1-\bar{r}_1)\frac{[\theta_L + (1-2r_2^*)f(r_2^*)]u_H}{\theta_H + (1-2r_2^*)f(r_2^*)} \quad \text{if } r_L \neq \bar{r}_1.$$

Here, $1-\bar{r}_1$ that appears on the right-hand side is the probability that the CEO keeps her job and the remaining term is her expected compensation in the second period. Then, if

$$(15) \qquad (1-2r_1^*)f(r_1^*) - (1-2\bar{r}_1)f(\bar{r}_1) < (1-\bar{r}_1)\frac{[\theta_L + (1-2r_2^*)f(r_2^*)]u_H}{\theta_H + (1-2r_2^*)f(r_2^*)}$$

is satisfied, the compensation contract cannot satisfy (CC) for all $a_1 \in \Re^+$ and $b \in [0,1]$, in which case a low-ability CEO chooses $\bar{r}_1$ rather than r_1^*. Because a high-ability CEO chooses r_1^* and a low-ability CEO chooses $\bar{r}_1$, the expected output of the firm is the same for all $b_1 \in (0,1]$, and hence the optimal compensation contract is the one that minimizes the compensation without violating the constraints. The following proposition summarizes our findings.

PROPOSITION 5 (EXCESSIVE RISK TAKING / ASYMMETRIC INFORMATION) *If* $\theta_H - \theta_L \in [f(r_1^*) + f(1-r_1^*), f(r_1^*) + f(1))$ *and (15) holds, then, in the pooling equilibrium, low-ability CEOs choose the excessive risk level* $\bar{r}_1$ *while high-ability CEOs choose the optimal risk level* r_1^*.

VI. CONCLUSION

The risk choice of a CEO who is concerned with her career may differ from the risk choice that maximizes the shareholders' return or society's social return. The question is in what way the risk choice will be distorted. The managerial conservatism literature suggests that a top manager is likely to be less entrepreneurial and take too little risk because she would like to oversee the firm with the least amount of problems and the minimum risk of obtaining bad states; thus this literature advises shareholders to design compensation contracts that encourage the manager to take higher risk.

In this paper, we provide an alternative. We analyze how CEOs layoff risk affects their risk choice in the firm under optimal contracts and optimal firing decisions. We allow for any linear combination of fixed-wage and stock compensation and show that there are market structures in which explicit incentives are not helpful in preventing CEOs from taking excessive risk. Because a CEO is replaced by a new CEO if her expected ability is below average, in trying to limit her layoff risk, she chooses a risk level that minimizes the informativeness of the first-period output about her unknown ability. In our setting, this can be achieved with excessive risk taking when the range of managerial abilities is neither too high nor too low. The CEO increases the risk until the good-state output when she turns out to be a low-ability type coincides with the bad-state output when she turns out to be a high-ability type.

Our results stem from the novel mechanism in which CEOs can influence the probabilities of states by choosing their risk level. In particular, taking higher risk makes the bad state more likely. Hence, while the firm foresees that the CEO will take excessive risk, once it observes the overlapped output level, it has to statistically infer that this is more likely to be the output of a high-ability CEO who is in the bad state than the output of a low-ability CEO who is in the good state, as the bad state is more likely with excessive risk taking. Although the firm is not fooled by the actions of the CEO, its expectation about the CEO's ability is that it is above average, despite the fact that each type is equally likely in the population.

Whether there is excessive or too little risk in the market is obviously a sector-specific question. A president of a university may opt for a quiet life while a surgeon may push for surgery even though it is not entirely necessary. We believe that the banking industry, or the financial sector in general, is an example of excessive risk taking. The structure of financial markets are so complicated that shareholders cannot entirely and precisely evaluate whether the observed return is due to the CEO's ability or to pure luck. Using financial derivatives, CEOs can simply gamble on anything and possibly get rewarded for taking massive risks for short-term revenues. For one reason or another, there is a mismatch between the preferences of shareholders and CEOs, and we believe that there always will be.

Faculty of Arts and Social Sciences, Sabanci University

References

Artzner, Philippe, Freddy Delbaen, Jean-Marc Eber, and David Heath, "Coherent Measures of Risk," *Mathematical Finance*, 9 (1999): 203-228.

Avery, Christopher N., and Judith A. Chevalier, "Herding over the Career," *Economics Letters*, 53 (1999), 327-333.

Bebchuk, Lucian A., and Holger Spamann, "Regulating Bankers' Pay," *Georgetown Law Journal*, 98 (2010), 247-287.

Berger, Philip G., Eli Ofek, and David L. Yermack, "Managerial Entrenchment and Capital Structure Decisions," *Journal of Finance*, 52 (1997), 1411-38.

Bertrand, Marianne, and Antoinette Schoar, "The managing with style: The Effect of Managers on Corporate Policy," *Quarterly Journal of Economics*, 118 (2003), 1169-1208.

Blinder, Alan, "Crazy Compensation and the Crisis," *The Wall Street Journal*, May 28, 2009, p. A15.

Bloom, Matthew C., and George T. Milkovich, "Relationships among Risk, Incentive Pay and Organizational Performance," *Academy of Management Journal*, 43 (1998), 283–297.

Bushman, Robert, Zhonglan Dai, and Xue Wang, "Risk and CEO Turnover," *Journal of Financial Economics*, 96 (2010), 381-398.

Chakraborty, Atreya, Shahbaz Sheikh, and Narayanan Subramanian, "Termination Risk and Managerial Risk Taking," *Journal of Corporate Finance*, 13 (2007), 170-188.

Chevalier, Judith, and Glenn Ellison, "Risk Taking by Mutual Funds as a Response to Incentives," *Journal of Political Economy*, 105 (1997), 1167-1200.

— —, "Career Concerns of Mutual Fund Managers," *Quarterly Journal of Economics*, 114 (1999), 389-432.

Clarke, Jonathan, and Ajay Subramanian, "Dynamic Forecasting Behavior by Analysts: Theory and evidence," *Journal of Financial Economics*, 80 (2006), 81-113.

Dasgupta, Amil, and Andrea Prat, "Financial Equilibrium with Career Concerns," *Theoretical Economics*, 1 (2006), 67-93.

DeFond, Mark L., and Chul W. Park, "The Effect of Competition on CEO Turnover," *Journal of Accounting and Economics*, 27 (1999), 35-36.

Del Guercio, Diane, and Paula Tkac, "The Determinants of the Flow of Funds of Managed Portfolios: Mutual Funds vs. Pension Funds," *Journal of Financial and Quantitative Analysis*, 37 (2002), 523-557.

Dong, Zhiyong, Cong Wang, and Fei Xie, "Do Executive Stock Options Induce Excessive Risk Taking?," *Journal of Banking and Finance*, 34 (2010), 2518-2529.

Eckbo, B. Espen, and Karin S. Thorburn, "Control Benefits and CEO Discipline in Automatic Bankruptcy Auctions," *Journal of Financial Economics*, 69 (2003), 227-258.

Falato, Antonio, Dan Li, and Todd Milbourn, "To Each According to His Ability? CEO pay and the market for CEOs," Working Paper, 2011.

Fama, Eugene F., "Agency Problems and the Theory of the Firm," *Journal of Political Economy*, 88 (1980), 288-307.

Foster, Dean P., and Sergiu Hart, " An Operational Measure of Riskiness," *Journal of Political Economy*, 117 (2009), 785-814.

Fudenberg, Drew, and Jean Tirole, "A 'Signal Jamming' Theory of Predation," *RAND Journal of Economics*, 17 (1986), 366-76.

Gibbons, Robert, and Kevin J. Murphy, "Relative Performance Evaluation for Chief Executive Officers," *Industrial and Labor Relations Review*, 43 (1990), 30-51.

— —, "Optimal Incentive Contracts in the Presence of Career Concerns: Theory and Evidence," *Journal of Political Economy*, 100 (1992), 468-505.

Graham, John R., "Herding among Investment Newsletters: Theory and Evidence," *Journal of Finance*, 54 (1999), 237-268.

Hermalin, Benjamin, "Managerial Preferences Concerning Risky Projects," *Journal of Law, Economics and Organization*, 9 (1993), 127-135.

Hermalin, Benjamin E., and Micheal S. Weisbach, "Endogenously Chosen Boards of Directors and Their Monitoring of the CEO," *American Economic Review*, 88 (1998), 96-118.

— —, "Information Disclosure and Corporate Governance," *Journal of Finance*, forthcoming, 2011.

Hirshleifer, David, and Anjan V. Thakor, "Managerial Conservatism, Project choice, and Debt," *Review of Financial Studies*, 5 (1992), 437-470.

— —, "Managerial Performance, Boards of Directors and Takeover Bidding," *Journal of Corporate Finance*, 1 (1994), 63-90.

— —, "Corporate Control through Board Dismissals and Takeovers," *Journal of Economics and Management Strategy*, 7 (1998), 489-520.

Holmstrom, Bengt, “Managerial Incentive Problems: A Dynamic Perspective,” in *Essays in Economics and Management in Honour of Lars Wahlbeck*, (Helsinki: Swedish School of Economics, 1982) (reprinted in the *Review of Economic Studies*, 66 (1999), 169-182).

Holmstrom, Bengt, and Paul Milgrom, “Aggregation and Linearity in the Provision of Intertemporal Incentives,” *Econometrica*, 55 (1987), 303-28.

Holmstrom, Bengt, and Joan Ricart i Costa, “Managerial Incentives and Capital Management,” *Quarterly Journal of Economics*, 101 (1986), 835-860.

Hong, Harrison, and Jeffrey D. Kubik, “Analyzing the Analysts: Career Concerns and Biased Earning Forecasts,” *Journal of Finance*, 58 (2003), 313-351.

Hong, Harrison, Jeffrey D. Kubik, and Amit Solomon, “Security Analysts’ Career Concerns and Herding of Earnings Forecasts,” *RAND Journal of Economics*, 31 (2000), 121-144.

Hu, Ping, Jayant R. Kale, Marco Pagani, and Ajay Subramanian, “Fund Flows, Performance, Managerial Career Concerns, and Risk Taking,” *Management Science*, 57 (2011), 628-646.

Huberman, Gur, and Shmuel Kandel, “On the Incentives for Money Managers: A Signalling Approach,” *European Economic Review*, 37 (1993), 1065-1081.

Huddart, Steven, “Reputation and Performance Fee Effects on Portfolio Choice by Investment Advisers,” *Journal of Financial Markets*, 2 (1999), 227-271.

Jensen, Michael C., and Kevin J. Murphy, “Performance Pay and Top-Management Incentives,” *Journal of Political Economy*, 98 (1990), 225-64.

Ju, Nengjiu, Hayne E. Leland, and Lemma W. Senbet, “Options, Option Repricing and Severance Packages in Managerial Compensation: Their eEfects on Corporate Risk,” Working Paper, 2003.

Kaplan, Steven N., and Bernadette Minton, “How has CEO Turnover Changed?,” *International Review of Finance*, forthcoming, 2011.

Kempf, Alexander, Stefan Ruenzia, and Tanja Thiele, “Employment Risk, Compensation Incentives, and Managerial Risk Taking: Evidence from the Mutual Fund Industry,” *Journal of Financial Economics*, 92 (2009), 92-108.

Lambert, Richard A., “Executive effort and the Selection of Risky Projects,” *RAND Journal of Economics*, 17 (1986), 77-88.

Lamont, Owen A., “Macroeconomic forecasts and Microeconomic Forecasters,” *Journal of Economic Behavior and Organization*, 48 (2002), 265-280.

Larraza-Kintana, Martin, Robert M. Wiseman, Luis R. Gomez-Mejia, and Theresa M. Welbourne, “Disentangling Compensation and Employment Risks Using the Behavioral Agency Model,” *Strategic Management Journal*, 28 (2007), 1001-1019.

Malcomson, James M., “Do Managers with Limited Liability Take More Risky Decisions? An Information Acquisition Model,” *Journal of Economics and Management Strategy*, 20 (2011), 83-120.

Malmendier, Ulrike, and Geoffrey Tate, “CEO Overconfidence and Corporate Investment,” *Journal of Finance*, 60 (2005), 2661-2700.

Mehran, Hamid, and Joshua Rosenberg, "The Effect of Employee Stock Options on Bank Investment Choice, Borrowing, and Capital," Federal Reserve Bank of New York Staff Reports 305, 2007.

Mehran, Hamid, George E. Nogler, and Kenneth B. Schwartz, "CEO Incentive Plans and Corporate Liquidation Policy," *Journal of Financial Economics*, 50 (1998), 319-349.

Milbourn, Todd T., Richard L. Shockley, and Anjan V. Thakor, "Managerial Career Concerns and Investment in Information," *RAND Journal of Economics*, 32 (2001), 334-351.

Milgrom, Paul, and John Roberts, *Economics, Organization and Management*, ed. (New Jersey NJ: Prentice-Hall, Englewood Cliffs, 1992).

Murphy, Kevin J., "Executive Compensation," in *Handbook of Labor Economics*, Orley Ashenfelter and David Card ed. (Amsterdam, North Holland, 1999).

Narayanan, M. P., "Observability and the Payback Criterion," *Journal of Business*, 58 (1985), 309-323.

Nohel, Tom, and Steven Todd, "Compensation for Managers with Career Concerns: The Role of Stock Options in Optimal Contracts," *Journal of Corporate Finance*, 11 (2005), 229-251.

Palomino, Frederic, and Andrea Prat, "Risk Taking and Optimal Contracts for Money Managers," *RAND Journal of Economics*, 34 (2003), 113-37.

Prendergast, Canice , and Lars Stole, "Impetuous Youngsters and Jaded Old-Timers: Acquiring a Reputation for Learning," *Journal of Political Economy*, 194 (1996), 1105-1134.

PricewaterhouseCoopers, *Reward: A New Paradigm?*, 2008.

Raviv, Alon, and Yoram Landskroner, "The 2007-2009 Financial Crisis and Executive Compensation: Analysis and a Proposal for a Novel Structure," NYU Finance Working Paper No. FIN-09-003, 2009.

Renée, Adams, and Daniel Ferreira, "A Theory of Friendly Boards," *Journal of Finance*, 62 (2007), 217-250.

Rothschild, Michael, and Joseph E. Stiglitz, "Increasing Risk: I. A Definition," *Journal of Economic Theory*, 2 (1970), 225-243.

Sanders, W. M. Gerard, and Donald C. Hambrick, "Swinging for the Fences: The Effects of CEO Stock Options on Company Risk-Taking and Performance," *Academy of Management Journal,* 50 (2007), 1055-1078.

Scharfstein, David S., and Jeremy C. Stein, "Herd Behavior and Investment," *American Economic Review*, 80 (1990), 465-479.

Shleifer, Andrei, and Robert W. Vishny, "Management Entrenchment: The case of Manager-Specific Investments," *Journal of Financial Economics*, 25 (1989), 123-139.

Sirri, Erik R., and Peter Tufano, "Costly Search and Mutual Fund Flows," *Journal of Finance*, 53 (1998), 1589-1622.

Stein, Jeremy C., "Takeover Threats and Managerial Myopia," *Journal of Political Economy*, 96 (1988), 61-80.

Wiseman, Robert M., and Luis R. Gomez-Mejia, "A Behavioral Agency Model of Managerial Risk Taking," *Academy of Management Review*, 23 (1998), 133-153.

Zwiebel, Jeffrey, "Corporate Conservatism and Relative Compensation," *Journal of Political Economy*, 103 (1995), 1-25.

Online Appendix to

THE MASQUERADE BALL OF THE CEOs AND THE MASK OF EXCESSIVE RISK

(not intended for publication)

Haluk Citci and Eren Inci

A. Appendix (not intended for publication)

A.1. Risk-averse CEO

This appendix shows that the possibility of excessive risk taking is not coming from the assumption of bilateral risk neutrality. Assume, for the sake of the argument, that the firm continues to be risk neutral but the CEO becomes risk averse, and we are in Case 2 of the two-type world. Then, the first-period maximization problem of the firm is

$$\text{(A.1)} \qquad \max_{a_1,b_1,r_1} \bar{\theta} + (1-2r_1)\, f(r_1) - \left(a_1 + b_1 y_1\left(\bar{\theta}, r_1\right)\right)$$

$$s.t.$$

$$\text{(IR)} \qquad E\left[u\left(a_1 + b_1 y_1\left(\bar{\theta}, r_1\right)\right)\right] \geq \underline{u}$$

$$\text{(IC)} \qquad r_1 \in \arg\max E\left[u\left(a_1 + b_1 y_1\left(\bar{\theta}, \hat{r}_1\right)\right)\right]$$

$$\text{(CC)} \qquad E\left[u\left(a_1 + b_1 y_1\left(\bar{\theta}, r_1\right)\right)\right] - E\left[u\left(a_1 + b_1 y_1\left(\bar{\theta}, \bar{r}_1\right)\right)\right] \geq \frac{(1-\bar{r}_1)\underline{u}}{2} \quad \text{if } r_1 \neq \bar{r}_1,$$

where the constraints are respectively the individual rationality, incentive compatibility, and career concern constraints; $u(\cdot)$ is a concave utility function; and

$$\text{(A.2)} \quad E\left[u\left(a_1 + b_1 y_1\left(\bar{\theta}, r_1\right)\right)\right] = \frac{1-r}{2}\left[u\left(a_1 + b_1\left(\theta_L + f(r_1)\right)\right) + u\left(a_1 + b_1\left(\theta_H + f(r_1)\right)\right)\right] + \frac{r}{2}\left[u\left(a_1 + b_1\left(\theta_L - f(r_1)\right)\right) + u\left(a_1 + b_1\left(\theta_H - f(r_1)\right)\right)\right].$$

When $b_1 = 0$, then we have $E[u(a_1 + b_1 y_1(\bar{\theta}, r_1))] = u(a_1)$ for all $r \in [0,1]$. Thus, (CC) is not satisfied when $b_1 = 0$. Now, assume that

$$\text{(A.3)} \qquad E[u\left(y\left(\bar{\theta}, \tilde{r}_1\right)\right)] - E[u\left(y\left(\bar{\theta}, \bar{r}_1\right)\right)] < \frac{(1-\bar{r}_1)\underline{u}}{2},$$

where $\tilde{r}_1$ is a risk level that maximizes the CEO's first-period payoff. This inequality means that (CC) cannot be satisfied when $a_1 = 0$ and $b_1 = 1$. Then, it cannot be satisfied for all $b_1 \in [0,1]$, either, because $E[u(a_1 + b_1 y_1(\bar{\theta}, r_1))]$ is a continuous function. Thus, there is excessive risk taking whenever (A.3) is satisfied.

A.2. Proof of Lemma 4

Case 1 ($\theta_H - \theta_L \geq 4f(1)$): Let us consider each possible $r_1 \in [0,1]$ in turn. If $r_1 = 0$, then the output will be $\theta + f(0)$ for certain, in which case the firm infers the ability of the CEO from the output. Thus, she is fired only when it turns out that her ability is less than $\bar{\theta}$, which implies a probability of being fired of 1/2. Similarly, if $r_1 = 1$, then the output will be $\theta - f(1)$ for certain, and once again the firm perfectly infers the ability, which implies a probability of being fired of 1/2. If the CEO chooses $r_1 \in (0,1)$, then there must be two (interior) types θ' and θ'' such that the good-state output realization of the θ'-type coincides with the bad-state output realization of the θ''-type (*i.e.*, $\theta' + f(r_1) = \theta'' - f(r_1)$), and the firm's expectation about the CEO's ability after observing this output is $\bar{\theta}$ (*i.e.*, $(1-r_1)\,\theta' + r_1\theta'' = \bar{\theta}$). Then, one can easily find that $\theta' = \bar{\theta} - 2r_1 f(r_1)$ and $\theta'' = \bar{\theta} + 2(1-r_1)f(r_1)$ for any risk choice $r_1 \in (0,1)$.

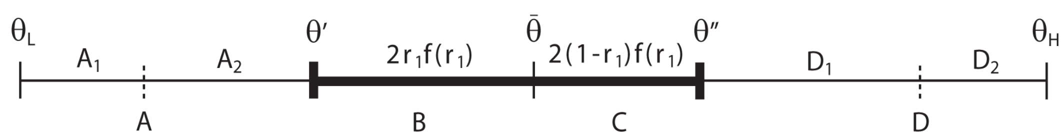

FIGURE III
The Partition of CEO Types in Case 1

Figure III shows the partition of CEOs on the ability distribution when the risk level is $r_1 \in (0,1)$. Because $\theta_H - \theta_L \geq 4f(1)$, one can easily show that $\bar{\theta} - 2f(r_1) > \theta_L$, which implies that the mass of CEOs in A is larger than that in C. Thus, there is a subpartition of $A = A_1 \cup A_2$ such that the mass in A_2 is equal to the mass in C and A_1 is the residual partition. Similarly, one can show that $\bar{\theta} + 2f(r_1) < \theta_H$ for all $r_1 \in (0, 1)$ and thus the mass of CEOs in D is higher than that in B. Hence, there is a subpartition of $D = D_1 \cup D_2$ such that the mass in D_1 is equal to the mass in B and D_2 is the residual partition.

Given r_1, the good-state output realization of a CEO in A_1 coincides with the bad-state output realization of a below-average CEO, which means that the expectation about her ability is always lower than $\bar{\theta}$ regardless of the state. However, given r_1, the bad-state output realization of a CEO in D_2 does not coincide with the good-state output realization of any below-average CEO. Thus, the expectation about her ability is always higher than $\bar{\theta}$ regardless of the state.

We can now derive the probability of being fired in each partition. If the CEO is in A_1, then she is fired for certain because the firm knows that her ability is below average for any output realization. On the other hand, if the CEO is in A_2, her good-state output coincides with the bad-state output of a CEO in C. If she is in the bad state, the firm infers that her ability is less than $\bar{\theta}$ and fires her. If she is in the good state, the firm still fires her, because the expectation of θ' matching with θ'' is $\bar{\theta}$ and thus the expectation of any $\theta \in A_2$ matching with any $\theta \in C$ must be lower than $\bar{\theta}$.

If the CEO is in B, her good-state output coincides with the bad-state output of a CEO in D_1 and the expectation about her ability is higher than $\bar{\theta}$, which means that she is not fired in case of the good state. However, she is fired if she is in the bad state, as the firm infers that her ability must be less than $\bar{\theta}$. Thus, her probability of being fired is r_1 if she is in this partition. Next, consider a CEO in C. Her bad-state output coincides with that of a CEO in A_2 and thus her expected ability is less than $\bar{\theta}$, which means that she is fired in such a state. Otherwise, she is in the good state in which case her ability is inferred to be higher than $\bar{\theta}$ and thus the firm keeps her for certain. Thus, the probability of being fired in this partition is also r_1.

Now consider a CEO in D_1. The bad-state output realization of a CEO in this partition coincides with the good-state output realization of one in B, and the expectation about her ability is higher than $\bar{\theta}$, which means that she is not fired in such a state. She is not fired even when she is in the good state because the firm infers that her ability is higher than $\bar{\theta}$. Thus, a CEO in D_1 is rehired for certain. Finally, a CEO in D_2 is also rehired for certain, because, as previously argued, her ability is perfectly inferred.

Given the above analysis, the overall probability of being fired is given by $p = 1 \times \Pr\{\theta \in A\} + r_1 \times \Pr\{\theta \in B\} + r_1 \times \Pr\{\theta \in C\} + 0 \times \Pr\{\theta \in D\}$, or

$$p = \frac{\bar{\theta} - \theta_L - 2r_1 f(r_1)}{\theta_H - \theta_L} + r_1 \frac{2r_1 f(r_1)}{\theta_H - \theta_L} + r_1 \frac{2(1-r_1) f(r_1)}{\theta_H - \theta_L} = \frac{1}{2}. \tag{A.4}$$

Because the CEO faces the same probability of being fired for any choice of r_1, she chooses the optimal risk level r_1^* as it is the best choice in terms of her first-period compensation, as long as she is given some stock ownership. Thus, she chooses the optimal risk in equilibrium.

Case 3 ($\theta_H - \theta_L < 2f(1/2)$): If $\theta_H - \theta_L \leq 2f(0)$, outputs do not match in any case and thus the probability of being fired cannot be lower than 1/2. Now, consider the remaining part of the interval (*i.e.*, $2f(0) < \theta_H - \theta_L < 2f(1/2)$). This time, one can easily obtain (11) once again by going through exactly the same calculations as in Case 2. Yet, this time, $\theta_H - \theta_L = 4\bar{r}_1 f(\bar{r}_1) < 2f(1/2)$ which implies $\bar{r}_1 < 1/2$. Therefore, we have $p = 1/2$, which means that the minimum probability of firing for any risk choice is not less than the one with the optimal risk level r_1^*. But, then, because r_1^* is both the risk level that maximizes the first-period compensation and the risk level that minimizes the probability of being fired, it is the equilibrium risk level.

A.3. The Probability of Being Fired with a Continuum of Types

This appendix shows that the guessed risk level $\bar{r}_1$ of Case 2 of the continuum of types is indeed the risk level that minimizes the probability of being fired. We prove this claim in two steps. In the first step, we show that the probability of being fired is higher for any $r_1 \in [0, \bar{r}_1)$ than the one with $\bar{r}_1$. In the second step, we prove the same thing for any $(\bar{r}_1, 1]$.

Step 1: Show that $p(r_1) > p(\bar{r}_1)$ for all $r_1 \in [0, \bar{r}_1)$.

Subcase *i*: Suppose that the risk level satisfies $f(r_1) = \bar{r}_1 f(\bar{r}_1) < f(\bar{r}_1)$. Then, there must be a θ'-type and a θ''-type such that the good-state output of the θ'-type coincides with the bad-state output of the θ''-type, and thus $\theta' = \theta_L + 2(1 - r_1)f(r_1)$ and $\theta'' = \theta' + 2f(r_1)$, and the expectation about them is $(1 - r_1)\theta' + r_1\theta'' = \bar{\theta}$.

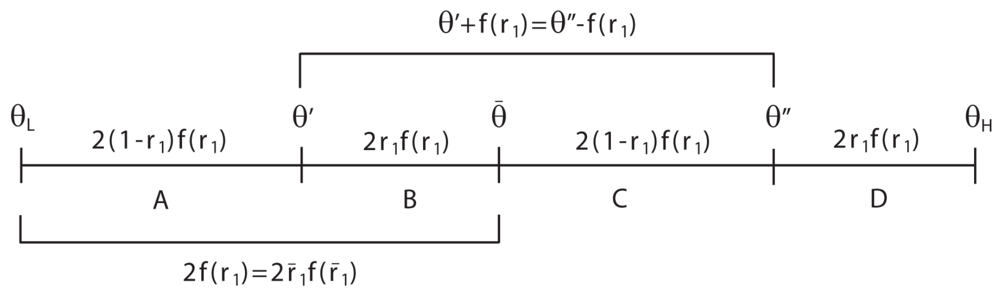

FIGURE IV
The Partition of CEO Types in Case 2 (Subcase *i* of Step 1)

Figure IV portrays the subcase. Note that the mass in A is equal to the mass in C, and the mass in B is equal to the mass in D. It is easy to see that the probability of being fired is one in A, r_1 in B and C, and zero in D. Therefore, the overall probability of being fired is

$$p(r_1) = \frac{2(1 - r_1) f(r_1)}{4f(r_1)} + \frac{2r_1 f(r_1)}{4f(r_1)} = \frac{1}{2}, \tag{A.5}$$

which is obviously higher than $p(\bar{r}_1)$.

Subcase *ii*: Suppose that the risk level satisfies $f(r_1) < \bar{r}_1 f(\bar{r}_1) < f(\bar{r}_1)$. Then, there must be a θ'-type and a θ''-type such that the good-state output of the θ'-type coincides with the bad-state output of the θ''-type, and thus $\theta' = \theta_L + 2\bar{r}_1 f(\bar{r}_1) - 2r_1 f(r_1)$ and $\theta'' = \theta' + 2f(r_1)$, and the expectation about them is $(1 - r_1)\theta' + r_1\theta'' = \bar{\theta}$.

Figure V portrays the subcase. Unlike Subcase *i*, in this subcase, the mass in A is larger

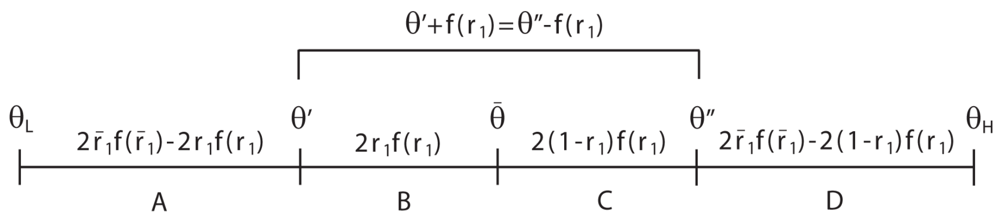

FIGURE V
The Partition of CEO Types in Case 2 (Subcase *ii* of Step 1)

than the mass in C because $2\bar{r}_1 f(\bar{r}_1) > 2f(r_1)$ and hence $2\bar{r}_1 f(\bar{r}_1) - 2r_1 f(r_1) > 2(1-r_1)f(r_1)$. For the same reason, $2\bar{r}_1 f(\bar{r}_1) - 2(1-r_1)f(r_1) > 2r_1 f(r_1)$ and hence the mass in D is larger than the mass in B. It is easy to see that the probability of being fired is one in A, r_1 in B and C, and zero in D. Therefore, the overall probability of being fired is

$$\frac{2\bar{r}_1 f(\bar{r}_1) - 2r_1 f(r_1)}{4\bar{r}_1 f(\bar{r}_1)} + r_1 \frac{2f(r_1)}{4\bar{r}_1 f(\bar{r}_1)} = \frac{1}{2}, \tag{A.6}$$

which is obviously higher than $p(\bar{r}_1)$.

Subcase *iii*: Suppose that the risk level satisfies $\bar{r}_1 f(\bar{r}_1) < f(r_1) < f(\bar{r}_1)$. Then, there must be a θ'-type, a θ''-type, a θ'''-type, and a θ''''-type such that $\theta' = \bar{\theta} - 2r_1 f(r_1)$, $\theta'' = \theta_H - 2f(r_1)$, $\theta''' = \theta_L + 2f(r_1)$, and $\theta'''' = \bar{\theta} + 2(1-r_1)f(r_1)$, as shown in Figure VI. Then, the mass in A is equal to the mass in E, the mass in C is equal to the mass in D, and the mass in B is equal to the mass in F. Then, it is easy to see that the probability of being fired is one in A and C, r_1 in B and E, and zero in D and F.

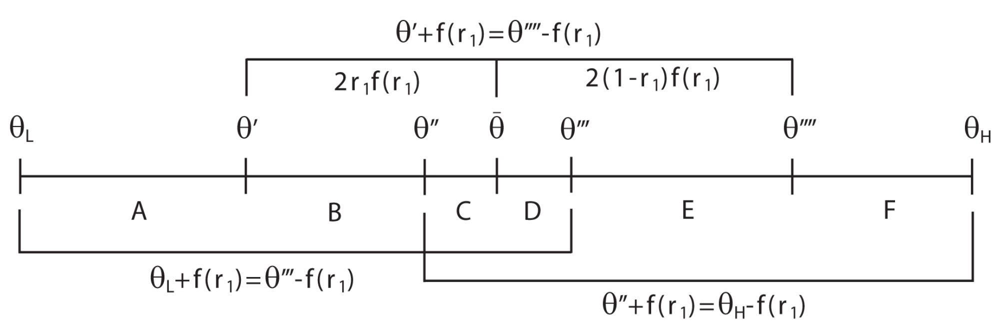

FIGURE VI
The Partition of CEO Types in Case 2 (Subcase *iii* of Step 1)

The probabilities of being fired in each partition are given by

$$\Pr\{\theta \in A\} = \Pr\{\theta \in E\} = \frac{2\bar{r}_1 f(\bar{r}_1) - 2r_1 f(r_1)}{4\bar{r}_1 f(\bar{r}_1)} \tag{A.7}$$

$$\Pr\{\theta \in B\} = \Pr\{\theta \in F\} = \frac{2\bar{r}_1 f(\bar{r}_1) - 2(1-r_1) f(r_1)}{4\bar{r}_1 f(\bar{r}_1)} \tag{A.8}$$

$$\Pr\{\theta \in C\} = \Pr\{\theta \in D\} = \frac{2f(r_1) - 2\bar{r}_1 f(\bar{r}_1)}{4\bar{r}_1 f(\bar{r}_1)}. \tag{A.9}$$

Therefore, the overall probability of being fired is

$$\begin{aligned} p(r_1) &= \frac{2\bar{r}_1 f(\bar{r}_1) - 2r_1 f(r_1)}{4\bar{r}_1 f(\bar{r}_1)} + r_1 \frac{2\bar{r}_1 f(\bar{r}_1) - 2(1-r_1) f(r_1)}{4\bar{r}_1 f(\bar{r}_1)} \\ &\quad + \frac{2f(r_1) - 2\bar{r}_1 f(\bar{r}_1)}{4\bar{r}_1 f(\bar{r}_1)} + r_1 \frac{2\bar{r}_1 f(\bar{r}_1) - 2r_1 f(r_1)}{4\bar{r}_1 f(\bar{r}_1)} \\ &= \frac{4r_1\bar{r}_1 f(\bar{r}_1) - 2(2r_1 - 1) f(r_1)}{4\bar{r}_1 f(\bar{r}_1)}. \end{aligned} \tag{A.10}$$

The CEO follows this strategy if she can decrease her probability of being fired to a level less than 1/2 by doing so:

$$\frac{4r_1\bar{r}_1 f(\bar{r}_1) - 2(2r_1 - 1) f(r_1)}{4\bar{r}_1 f(\bar{r}_1)} < \frac{1}{2} \tag{A.11}$$

or $2r_1\bar{r}_1 f(\bar{r}_1) - (2r_1-1)f(r_1) < \bar{r}_1 f(\bar{r}_1)$. This is satisfied when $(2r_1-1)[\bar{r}_1 f(\bar{r}_1) - f(r_1)] < 0$, or $r_1 > 1/2$. Now, we need to show that this probability of being fired is higher than $p(\bar{r}_1)$. It is so if

$$\frac{4r_1\bar{r}_1 f(\bar{r}_1) - 2(2r_1 - 1) f(r_1)}{4\bar{r}_1 f(\bar{r}_1)} > \frac{2\bar{r}_1^2 - 2\bar{r}_1 + 1}{2\bar{r}_1}, \tag{A.12}$$

or $(4r_1\bar{r}_1 - 4\bar{r}_1^2 + 4\bar{r}_1 - 2)f(\bar{r}_1) > 2(2r_1-1)f(r_1)$. Because $f(\bar{r}_1) > f(r_1)$, this holds when $4r_1\bar{r}_1 - 4\bar{r}_1^2 + 4\bar{r}_1 - 2 > 2(2r_1 - 1)$, or $(1-\bar{r}_1)(\bar{r}_1 - r_1) > 0$, which is always satisfied. Thus, $p(r_1) > p(\bar{r}_1)$. This completes the proof of the claim that that $p(r_1) > p(\bar{r}_1)$ for all $r_1 \in [0, \bar{r}_1)$.

Step 2: Show that $p(r_1) > p(\bar{r}_1)$ for all $r_1 \in (\bar{r}_1, 1]$.

Subcase *i*: Suppose that the risk level satisfies $f(\bar{r}_1) < f(r_1) = (\theta_H - \theta_L)/2$. Then, the good-state output of the worst type coincides with the bad-state output of the best type, $\theta_L + f(r_1) = \theta_H - f(r_1)$. This means that $\theta + f(r_1) > \theta_H - f(r_1)$ for all $\theta \in (\theta_L, \bar{\theta})$. Thus, the firm infers the ability of the CEO when it observes any output level different from the overlapped output level, and thus fires the CEO with probability 1/2. If it observes the overlapped output level, it fires the CEO with probability $r_1/2$. Thus, the overall probability of being fired is approximately 1/2 and definitely higher than $p(\bar{r}_1)$.

Subcase *ii*: Suppose that the risk level satisfies $f(\bar{r}_1) < (\theta_H - \theta_L)/2 < f(r_1)$. Then, we have $\theta + f(r_1) > \theta_H - f(r_1)$ for all $\theta \in [\theta_L, \bar{\theta})$, which means that the output realization perfectly signals the ability of a CEO. Thus, the overall probability of being fired is 1/2, which is higher than $p(\bar{r}_1)$.

Subcase *iii*: Suppose that the risk level satisfies $f(\bar{r}_1) < f(r_1) < (\theta_H - \theta_L)/2$. Then, there must be a θ'-type and a θ''-type such that $\theta' = \theta_H - 2f(r_1)$ and $\theta'' = \theta_L + 2f(r_1)$, as shown in Figure VII. The good-state output of the θ_L-type (θ'-type) coincides with the bad-state output of the θ''-type (θ_H-type). However, the expectation for both pairs is above $\bar{\theta}$. It is easy to see that the probability of being fired is r_1 in A, one in B, and zero in C and D.

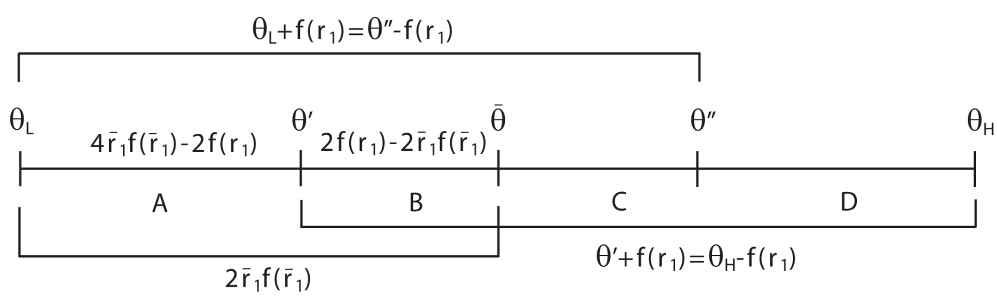

FIGURE VII
The Partition of CEO Types in Case 2 (Subcase *iii* of Step 2)

The overall probability of being fired is given by

$$p(r_1) = r_1 \frac{4\bar{r}_1 f(\bar{r}_1) - 2f(r_1)}{4\bar{r}_1 f(\bar{r}_1)} + \frac{2f(r_1) - 2\bar{r}_1 f(\bar{r}_1)}{4\bar{r}_1 f(\bar{r}_1)} = \frac{2(2r_1 - 1)\bar{r}_1 f(\bar{r}_1) + 2(1 - r_1) f(r_1)}{4\bar{r}_1 f(\bar{r}_1)}. \tag{A.13}$$

Remember that

$$p(\bar{r}_1) = \frac{2(2\bar{r}_1 - 1)\bar{r}_1 f(\bar{r}_1) + 2(1 - \bar{r}_1) f(\bar{r}_1)}{4\bar{r}_1 f(\bar{r}_1)}. \tag{A.14}$$

Thus, $p(r_1) > p(\bar{r}_1)$ if $2(2r_1-1)\bar{r}_1 f(\bar{r}_1) + 2(1-r_1)f(r_1) > 2(2\bar{r}_1-1)\bar{r}_1 f(\bar{r}_1) + 2(1-\bar{r}_1)f(\bar{r}_1)$, which boils down to $4(r_1 - \bar{r}_1)\bar{r}_1 f(\bar{r}_1) + 2(1-r_1)f(r_1) - 2(1-\bar{r}_1)f(\bar{r}_1) > 0$. Because $f(\bar{r}_1) < f(r_1)$, let $f(r_1) = f(\bar{r}_1) + \lambda$ where $\lambda \geq 0$. Then, we have $4(r_1 - \bar{r}_1)\bar{r}_1 f(\bar{r}_1) + 2(1-r_1)(f(\bar{r}_1)+\lambda) - 2(1-\bar{r}_1)f(\bar{r}_1) > 0$. This boils down to $2(r_1-\bar{r}_1)(2\bar{r}_1-1)f(\bar{r}_1) + 2\lambda(1-r_1) > 0$, which always holds because $\bar{r}_1 > 1/2$. Hence, $p(r_1) > p(\bar{r}_1)$. This completes the proof of the claim that $p(r_1) > p(\bar{r}_1)$ for all $r_1 \in (\bar{r}_1, 1]$.

A.4. Proof of Lemma 6

1. When the firm observes $\bar{y}_1$, its expectation about the ability of the CEO will be

$$E[\theta_i \mid \bar{y}_1] = \left(\frac{1-\bar{r}_1}{r_1^* + 1 - \bar{r}_1}\right)\theta_L + \left(\frac{r_1^*}{r_1^* + 1 - \bar{r}_1}\right)\theta_H. \tag{A.15}$$

This is greater than or equal to $\bar{\theta} = (1/2)\theta_L + (1/2)\theta_H$ iff $(1-\bar{r}_1)/(r_1^*+1-\bar{r}_1) \leq 1/2$, which holds for all $\bar{r}_1 \in [1-r_1^*, 1)$. Therefore, the firm keeps her in such a state, which happens with probability $1-\bar{r}_1$; otherwise she is fired, which happens with probability $\bar{r}_1$.

2. When the firm observes y_1', its expectation about the ability of the CEO will be

$$E[\theta_i \mid y_1'] = \left(\frac{1-r_1'}{1-r_1^*+1-r_1'}\right)\theta_L + \left(\frac{1-r_1^*}{1-r_1^*+1-r_1'}\right)\theta_H, \tag{A.16}$$

This is greater than or equal to $\bar{\theta} = (1/2)\theta_L + (1/2)\theta_H$ if $(1 - r_1')/(1 - r_1^* + 1 - r_1') \leq 1/2$, which holds when $r_1' \geq r_1^*$. Note that $f(r_1') - f(r_1^*) = \theta_H - \theta_L > 0$ in this case. Hence, $f(r_1') > f(r_1^*)$, which implies that $r_1' > r_1^*$. Therefore, the firm keeps her in such a state, which happens with probability $1 - r_1'$; otherwise she is fired, which happens with probability r_1'.

3. When the firm observes $\hat{y}_1$, its expectation about the ability of the CEO will be

$$E\left[\theta_i \mid \hat{y}_1\right] = \left(\frac{\hat{r}_1}{r_1^* + \hat{r}_1}\right)\theta_L + \left(\frac{r_1^*}{r_1^* + \hat{r}_1}\right)\theta_H, \tag{A.17}$$

This is greater than or equal to $\bar{\theta} = (1/2)\theta_L + (1/2)\theta_H$ if $(\hat{r}_1)/(r_1^* + \hat{r}_1) \leq 1/2$, which holds when $\hat{r}_1 \leq r_1^*$. Note that $f(r_1^*) - f(\hat{r}_1) = \theta_H - \theta_L > 0$ in this case. Hence, $f(r_1^*) > f(\hat{r}_1)$, which implies that $\hat{r}_1 < r_1^*$. Therefore, the firm keeps her in such a state, which happens with probability $\hat{r}_1$; otherwise she is fired, which happens with probability $1 - \hat{r}_1$.

A.5. Cases under Asymmetric Information

As Lemma 6 suggests, in trying to minimize the probability of being fired, the CEO can choose three possible risk levels: $\bar{r}_1$, r_1', and $\hat{r}_1$. The motivation of a low-ability CEO in choosing these risk levels is to disguise her type with her risk choice. If one of these does not work or current compensation dominates career concern, the CEO chooses the optimal risk level r_1^*. We have already derived the following feasibility conditions in the text. First, none of the strategies is viable when $\theta_H - f(r_1^*) \geq \theta_L + f(1)$ because in such a case a low-ability CEO cannot overlap her output with the output of a high-ability CEO in any state. This forms the lower boundary of Case 1 below. Second, as the first part of Lemma 6 suggests, $\bar{r}_1$ is not an effective strategy for a low-ability CEO when $\bar{r}_1 \in (0, 1 - r_1^*)$ because it does not decrease her probability of being fired. This forms the lower boundary of Case 2 below.

Third, choosing r_1' is a viable strategy only if $\theta_H + f(r_1^*) < \theta_L + f(1)$; otherwise there exists no r_1' overlapping the good-state output of a low-ability CEO with that of a high-ability CEO. This requires $\theta_H - \theta_L \in (0, f(1) - f(r_1^*))$. Fourth, choosing $\hat{r}_1$ is a viable strategy only if $\theta_L - f(0) > \theta_H - f(r_1^*)$; otherwise there exists no $\hat{r}_1$ overlapping the bad-state output of a low-ability CEO with that of a high-ability CEO. This requires $\theta_H - \theta_L \in (0, f(r_1^*) - f(0))$. This condition and the previous one overlap, but both of these conditions are under the lower boundary of Case 2, $f(r_1^*) + f(1 - r_1^*)$.[20] Depending on the technology, the maximum of these two conditions form the lower boundary of Case 3 below and the minimum of them forms the upper boundary of Case 4. There is also the exceptional (and measure zero) case in which $\theta_H - f(r_1^*) = \theta_L + f(r_1^*)$ by chance, which is Case 6. Below, we state all these conditions and analyze them one by one.

Case 1 ($\theta_H - \theta_L \in [f(r_1^*) + f(1), \infty)$): In this case, $\theta_H - \theta_L$ is so high that a low-ability CEO is unable to overlap even her good-state output with the bad-state output of a high-ability CEO even when she takes the maximum risk (*i.e.*, $\theta_H - f(r_1^*) \geq \theta_L + f(1)$). This implies that a high-ability CEO's output when she chooses the optimal risk is higher than a low-ability CEO's output for any risk level. This means that the outputs of different types cannot overlap, $y_1(\theta_H, r_1^*) > y_1(\theta_L, r_1)$, for all $r_1 \in [0, 1]$ in all states. Thus, if the high-ability CEO chooses r_1^*, then the firm infers her ability at the end of the period. As a result, neither a high-ability CEO nor a low-ability one has career concerns, which means that they choose the optimal risk r_1^* in the first period since it maximizes their first-period compensation. Note that the firm needs to pay some positive amount of stock ownership to guarantee this.

LEMMA 8 (CASE 1) *If $\theta_H - \theta_L \in [f(r_1^*) + f(1), \infty)$, then both types choose the optimal risk level r_1^* in equilibrium. Outputs do not overlap in any state combination and thus the firm infers the type of the CEO at the end of the first period. It fires the low-ability CEOs and rehires the high-ability ones.*

Case 2 ($\theta_H - \theta_L \in [f(r_1^*) + f(1 - r_1^*), f(r_1^*) + f(1))$): We analyze this case in the text.

20. $f(r_1^*) + f(1 - r_1^*)$ is trivially higher than $f(r_1^*) - f(0)$. The concavity of the risk-return function implies that $f(r_1^*) + f(1 - r_1^*) > f(1)$, and hence $f(r_1^*) + f(1 - r_1^*) > f(1) - f(r_1^*)$ must hold as well.

Case 3 ($\theta_H - \theta_L \in [\max\{f(1) - f(r_1^*), f(r_1^*) - f(0)\}, f(r_1^*) + f(1 - r_1^*))$): The analysis of this case is trivial because none of the risk levels $\bar{r}_1$, r_1', or $\hat{r}_1$ is viable in this case. Hence, the type of a CEO will be inferred anyway and her probability of being fired will be independent of her risk choice. Consequently, she chooses the optimal risk level in order to maximize her first-period compensation. Note that the firm needs to pay some positive amount of stock ownership to guarantee this.

Lemma 9 (Case 3) *If $\theta_H - \theta_L \in [\max\{f(1) - f(r_1^*), f(r_1^*) - f(0)\}, f(r_1^*) + f(1 - r_1^*))$, then both types of CEOs choose the optimal risk level r_1^* in equilibrium. Outputs do not overlap in any state combination and thus the firm infers the type of the CEO at the end of the first period. It fires the low-ability CEOs and rehires the high-ability ones.*

Case 4 ($\theta_H - \theta_L \in [\min\{f(1) - f(r_1^*), f(r_1^*) - f(0)\}, \max\{f(1) - f(r_1^*), f(r_1^*) - f(0)\})$): The analysis of this case is also trivial. If $f(1) + f(0) > 2f(r_1^*)$, then overlapping her good-state output with that of a high-ability CEO is optimal for a low-ability CEO and thus she chooses r_1'. If, however, $f(1) + f(0) < 2f(r_1^*)$, then overlapping her bad-state output with that of a high-ability CEO is optimal for a low-ability CEO, and thus she chooses $\hat{r}_1$. Because a high-ability CEO is always rehired, she does not have career concerns and chooses the optimal risk level to maximize her first-period compensation. These are the risk levels that minimize the probability of being fired, not necessarily the equilibrium values. If minimizing the probability of being fired is not optimal, a low-ability CEO chooses the optimal risk level r_1^*. The risk level r_1' is higher than the optimal risk level r_1^* but the risk-return function may have positive or negative NPV depending on whether the risk level is above or below 1/2.

Lemma 10 (Case 4) *If $\theta_H - \theta_L \in [\min\{f(1) - f(r_1^*), f(r_1^*) - f(0)\}, \max\{f(1) - f(r_1^*), f(r_1^*) - f(0)\})$, then a high-ability CEO chooses the optimal risk level r_1^* in equilibrium and rehired for certain in the second period. A low-ability CEO minimizes her probability of being fired by choosing the excessive risk level r_1' if $f(1) + f(0) > 2f(r_1^*)$ and overlap her good-state* *output realization with the good-state output realization of a high-ability CEO. In this case, she is fired with probability r_1'. She minimizes her probability of being fired by choosing the insufficient risk level $\hat{r}_1$ if $f(1)+f(0) < 2f(r_1^*)$ and overlaps her bad-state output realization with the bad-state output realization of a high-ability CEO. In this case, the firm rehires the high-ability CEO while it fires the low-ability CEO with probability $1-\hat{r}_1$.*

Case 5 ($\theta_H - \theta_L \in (0, \min\{f(1)-f(r_1^*), f(r_1^*)-f(0)\})$): If a low-ability CEO chooses r_1', her probability of being fired is r'; if she chooses $\hat{r}_1$, her probability of being fired is $1-\hat{r}_1$. Hence, the CEO chooses r_1' or $\hat{r}_1$ in order to minimize her probability of being fired. We know that $\theta_L + f(r_1') = \theta_H + f(r_1^*)$ and $\theta_L - f(\hat{r}_1) = \theta_H - f(r_1^*)$. Thus, $r_1' \geq 1-\hat{r}_1$ if and only if

$$f^{-1}\left(\theta_H - \theta_L + f\left(r_1^*\right)\right) + f^{-1}\left(\theta_L - \theta_H + f\left(r_1^*\right)\right) \geq 1. \tag{A.18}$$

However, it might be the case that the CEO still chooses r_1' if her compensation benefit in the first period overweighs her career benefit in expected payoff. However, it turns out that this is not the case. The compensation of a low-ability CEO is weakly higher with $\hat{r}_1$ than with r_1'. If the firm offers a fixed wage, then evidently her compensations under both risk choices are equal. Suppose that the firm offers a positive amount of stock ownership. Combining $\theta_L + f(r_1') = \theta_H + f(r_1^*)$ and $\theta_L - f(\hat{r}_1) = \theta_H - f(r_1^*)$ gives $|f(\hat{r}_1) - f(r_1^*)| = |f(r_1') - f(r_1^*)|$, and because the risk-return function is concave, this implies $|\hat{r}_1 - r_1^*| < |r_1' - r_1^*|$. We know that the expected return is a continuous and concave function in its positive range and that it is maximized at r_1^*. Thus, expected return decreases as we move away from the r_1^*. As a result, the expected return is always higher under $\hat{r}_1$ than it is under r_1'. Hence, if a positive amount of stock ownership is offered, then the current compensation is higher under $\hat{r}_1$ than it is under r_1'. This means that we get *insufficient risk taking* in equilibrium.

LEMMA 11 (CASE 5) *If $\theta_H - \theta_L \in (0, \min\{f(1)-f(r_1^*), f(r_1^*)-f(0)\})$, then a high-ability CEO chooses the optimal risk level r_1^* whereas a low-ability CEO chooses the insufficient risk* *level $\hat{r}_1$ as long as (A.18) is satisfied. In this case, the low-ability CEO overlaps her bad-state output with the bad-state output of a high-ability CEO, and the firm rehires the high-ability CEO while it fires the low-ability CEO with probability $1 - \hat{r}_1$.*

This lemma is a sufficiency condition for the insufficient risk choice $\hat{r}_1$. If (A.18) is not satisfied, whether the CEO chooses $\hat{r}_1$ or r_1' depends on the exact trade-off between layoff risk and current compensation. As a matter of fact, the choice between $\hat{r}_1$ and r_1' is very sensitive to the technology. For example, if the risk-return function is linear, then probability of being fired with r_1' is lower than that with $\hat{r}_1$ and the first-period compensation is same for both risk choices. Hence, r_1' would certainly dominate $\hat{r}_1$.

Case 6 ($\theta_H - \theta_L = 2f(r_1^*)$): This is a knife-edge case in which $\theta_L + f(r_1^*) = \breve{y}_1 = \theta_H - f(r_1^*)$. When the firm observes $\breve{y}_1$, its expectation about the type of the CEO will be $E[\theta_i \mid \breve{y}_1] = r_1^*\theta_H + (1 - r_1^*)\theta_L$, which is less than $\bar{\theta}$ because $r_1^* < 1/2$. Therefore, the firm fires the CEO after such an observation. It fires the CEO even when it observes $y(\theta_L, r_1^*) \neq \breve{y}_1$ in which case her ability is inferred. Thus, she has no career incentive, which means that she chooses r_1^* in the first period as it maximizes her first-period compensation. However, by differentiating her output from the output of a low-ability CEO, a high-ability CEO can decrease her probability of being fired from r_1^* to zero. As a result, the high-ability CEO will choose something arbitrarily close (but not equal to) r_1^*. These are the risk levels that minimize the probability of being fired, not necessarily the equilibrium values. If minimizing the probability of being fired is not optimal, she chooses the optimal risk level r_1^*.

LEMMA 12 (CASE 6) *If $\theta_H - f(r_1^*) = \theta_L + f(r_1^*)$, low-ability CEOs take the optimal risk level r_1^* whereas high-ability CEOs choose a risk level that is arbitrarily close (but not equal to) r_1^*. Outputs do not overlap in any state combination and thus the firm infers the type of the CEO at the end of the first period. It fires the low-ability CEOs and rehires the high-ability ones.*