A Natural Limit on the Observable Periods of Anomalous X-ray Pulsars and Soft Gamma-ray Repeaters

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Abstract. We investigate the dependence of the evolution of neutron stars with fallback disks on the strength of the magnetic dipole field of the star. Using the same model as employed by Ertan et al. (2009), we obtain model curves for different dipole fields showing that the neutron stars with magnetic dipole fields greater than $\sim 10^{13}$ G on the surface of the star are not likely to become anomalous X-ray pulsars (AXPs) and soft gamma-ray repeaters (SGRs). Other sources with conventional dipole fields evolve into the AXP phase if their disk can penetrate the light cylinder. The upper limits to the observed periods of AXP and SGRs could be understood if the disk becomes inactive below a low temperature around 100 K. We summarize our present and earlier results indicated by the evolutionary model curves of these sources with an emphasis on the importance of the minimum disk temperature and the X-ray irradiation in the long-term evolution of AXPs and SGRs with fallback disks.

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INTRODUCTION

Magnetic dipole field of a young neutron star, rotating in vacuum or with a fallback disk, plays an important role in the rotational evolution of the star. Efficiencies of both magnetic torques and disk torques depend on the strength of the dipole field, $B_0$, on the surface of the star. For a neutron star in vacuum, the only external torque acting on the star is the magnetic dipole torque and $B_0$ is estimated from the period, $P$, and period derivative, $\dot{P}$, of the star ($B_0 \propto \sqrt{P \dot{P}}$). When there is an active fallback disk around the neutron star, the torque acting on the star is usually much more efficient than the magnetic dipole torque for a given $B_0$, and the value of $B_0$ inferred from the observed $\dot{P}$ could be much less than that estimated using magnetic dipole torques. The disk torque depends on the mass-flow rate $\dot{M}$ at the inner disk, $P$ and $B_0$. In addition, the efficiency of the disk torque depends on the fastness parameter $\omega_* = \Omega_* / \Omega_K$ which is the ratio of the star’s angular velocity to the Keplerian angular velocity at the inner disk.

In the magnetar model [10], anomalous X-ray pulsars (AXPs) and soft gamma-ray repeaters (SGRs) [8] are assumed to spin down by magnetic dipole torques. With this assumption, $B_0$ inferred from $P$ and $\dot{P}$ measurements are found to be greater than $10^{14}$ G. This does not explain the period clustering (2 - 12 s), and there are some
difficulties in explaining X-ray, optical and IR properties of AXP/SGRs in the persistent and enhancement phases of these sources. Super-Eddington soft gamma-ray bursts of AXP/SGRs indeed require strong magnetic fields. We proposed in a series of earlier papers on fallback disk model that the magnetar fields powering the bursts could be in higher multipole components of the magnetic field which are localized close to the surface of the star rather than in the large scale dipole field. The upper limit on the period derivative of the recently discovered SGR 0418+5729 gives $B_0 < 10^{13}$ G [9], and strongly support the idea that these bursts do not require magnetar dipole fields. It was shown that the observed properties of this source ($L_x$, $P$ and $\dot{P}$) can be reached by a neutron star with a fallback disk if $B_0 \sim 10^{12}$ G [2].

In the frame of the fallback disk model [1, 3], AXP/SGRs are powered by accretion from a fallback disk and spin down by the disk torques. The results of recent work based on this model require $B_0 \sim 10^{12} - 10^{13}$ G to explain the optical and IR observations of the sources [5]. In our earlier work [6], we showed that $P$, $\dot{P}$ and X-ray luminosities of AXP/SGRs could be accounted for by the evolution of neutron stars interacting with fallback disks consistently with the statistical properties of these sources provided that the disk becomes inactive at a low critical temperature $T_p \sim 100$ K. This requires that the fallback disks of AXP/SGRs be viscously active at present within radii greater than about $10^{12}$ cm depending on the X-ray irradiation strength, as proposed by Erkan et al. (2007) to explain the broad band optical and IR spectrum of AXP 4U 0142+61. In Section 2, based on the calculations by Erkan et al. (2009), we explain the effect of the field strength, X-ray irradiation and the critical temperature on the evolution of AXPs and SGRs through illustrative model curves with different $B_0$ values. Our results and conclusions are summarized in Section 3.

WHY IS THERE AN UPPER LIMIT TO THE OBSERVED PERIODS OF AXP AND SGRS?

The periods of all AXP/SGRs are clustered in 2 - 12 s range. Why don’t we observe AXP/SGRs with periods above this range? This could be explained by the effect of the disk passivization below a minimum disk temperature $T_p$ on the evolution of the period and the X-ray luminosity, $L_x$, of these sources [6].

The details of the code developed to study the evolution of the neutron stars with fallback disks are described in Erkan and Erkut (2008) and Erkan et al. (2009). In the present work, using this code we calculate the luminosity, period and period derivative curves of a neutron star with a given initial period ($P_0 = 200$ ms) and a particular disk mass for different magnetic fields. We perform the calculation for $T_p = 0$, and $T_p = 100$ K. Our results here can be extended to lower $P_0$ and lower disk masses provided that the inner disk can penetrate the light cylinder at some early phase of evolution. A detailed analysis investigating the effect of the initial disk mass and the initial period on the evolution of the sources can be found in Erkan et al. (2009).

The evolution of the $L_x$ and $P$ curves with $T_p = 100$ K are seen in Fig. 1. In this model, the outer disk radius $r_{out}$ is determined by the current temperature profile of the disk taking $r_{out} = r(T = 100$ K). The outer edge of the active disk propagates to smaller radii with decreasing $L_x$ leaving a passive outermost disk region with $r > r_{out}$. Towards
FIGURE 1. Luminosity, period and period-derivative evolution of model sources for $T_p = 100$ K, with different $B_0$ values. In the middle panel, $B_0$ values are given in units of $10^{12}$ G. It is seen that the model sources with $B_0 = 5 \times 10^{12}$ G and $B_0 = 1 \times 10^{13}$ G do not evolve into the AXP/SGR phase. The $B_0$ values given here show the strength of the field on the equator of the neutron star. For the other illustrative sources, unless they are very close, it is not likely to observe periods higher than the AXP periods, since the X-ray luminosities have already decreased below observable levels when the periods start to increase beyond the observed AXP period range.

At the end of the AXP phase, propagation speed of $r_{\text{out}}$ increases as it comes closer to the inner disk, and thereby leads to a sharp decline in X-ray luminosity. It is seen in Fig. 1 that all model sources have similar $L_x$ evolution, since all of them have the same initial disk mass and all these disks can penetrate the light cylinder from the beginning of the evolution for chosen $P_0$ and $B_0$ values in the model. When the inner disk radius $r_{\text{in}}$ is less than the radius of the light cylinder $r_{\text{LC}}$, we assume that all the matter arriving at the inner disk is accreted onto the neutron star. Actually, it is likely that a fraction of the inner disk matter could be propelled from the system while the remaining fraction could accrete onto the star. Therefore, the X-ray luminosities presented here could be taken as upper limits to actual luminosities. A similar evolution with lower accretion efficiency can be obtained by increasing the disk mass.
For \( T_p = 100 \, \text{K} \), the \( L_x \) cutoff puts an upper limit to the observable periods of AXP/SGRs. For the sources with weaker magnetic dipole fields, \( r_{in} \) comes out of the light cylinder earlier than it does for the sources with stronger dipole fields. After this point, \( P \) remains almost constant because of decreasing efficiency of the disk torque [2]. As seen in Fig. 1, the model sources with higher \( B_0 \) values tend to evolve beyond the observed period range of AXP/SGRs. These sources with such long periods are not likely to be observable due to very low \( L_x \) in this late phase of evolution. The results here are obtained by using a particular disk mass. Detailed work on the evolutionary curves tracing the possible initial disk mass and the initial period ranges [6] also shows that a minimum disk temperature of \( \sim 100 \, \text{K} \) gives reasonable results in explaining the properties of AXP/SGRs. In Fig. 1, we also give evolution curves of model sources with \( B_0 = 5 \times 10^{12} \, \text{G} \) and \( B_0 = 1 \times 10^{13} \, \text{G} \) which cannot become AXP/SGR. It is seen in Fig. 1, that these two illustrative model sources do not enter the AXP/SGR phase defined by the observed range of \( L_x \), \( P \) and \( \dot{P} \) values. We obtain similar results for other possible disk masses and conclude that neutron stars with \( B_0 > 10^{13} \, \text{G} \) are not likely to become AXP/SGRs.

For comparison, we repeat the calculations taking \( T_p = 0 \) (Fig. 2), meaning that the disk remains always active at all radii. In this model, \( L_x \) curves do not show a cutoff phase, since the entire disk remains viscously active and mass transfer is not cut off. In this case, the model sources evolve approaching rotational equilibrium, and in the subsequent phases remain close to equilibrium. Comparing \( L_x \) and \( P \) evolutions seen in Fig. 2, we see that the predictions of this model are not consistent with the observations. In the late phases of evolution, the periods of the model sources increase above the range of AXP/SGR periods, while their X-ray luminosities remain at observable levels. From the results of this model, considering also the fact that older AXP/SGRs should be more abundant compared to younger AXP/SGRs, we would expect to see many more AXP/SGRs in the luminosity range of these sources with periods above the range of the observed periods, which is in contradiction with the observations. The absence of still bright sources at longer periods requires that the disk must turn off. It is only natural that a fallback disk will indeed turn off as it cools below some \( T_p \), and the ionization fraction becomes too small to drive the magneto-rotational generation of viscosity for an active disk [6, 2].

There is some uncertainty in the actual value of minimum temperature \( T_p \) of the active disk, since there is a degeneracy between the irradiation efficiency \( C \) and \( T_p \). The irradiation parameter \( C \) is restricted by the IR and X-ray observations of AXP/SGRs to a range \( \sim 1.0^{-4} - 7 \times 10^{-4} \) [4]. In the present work, we use the same irradiation efficiency (\( C = 10^{-4} \)) as that employed in Ertan et al. (2009) and in our earlier works. If \( C \) is increased to its maximum value in this range, to obtain a similar evolution to that seen in Fig. 1 it is sufficient to decrease \( T_p \) to \( \sim 50 \, \text{K} \). The values of \( T_p \) greater than about 300 \, \text{K} are not likely, since either the mass-flow rate remains too low for the inner disk to penetrate the light cylinder or the lifetime of the disk becomes too short to explain the AXP/SGRs (see [6] for details). These low critical disk temperatures indicate that the fallback disk of an AXP with an X-ray luminosity of \( \sim 10^{35} \, \text{erg s}^{-1} \) is active within a radius of the order of \( 10^{12} \, \text{cm} \).

As seen in the bottom panel of Fig. 1, the \( \dot{P} \) values of the model sources remain in the \( \dot{P} \) range of AXP/SGRs when their inner disks are inside the light cylinder. When possible
ranges of $B_0$ and the disk mass that can produce AXP/SGRs are traced, we see that $\dot{P}$ of the sources approaching the rotational equilibrium decrease to about $\sim 10^{-13} \text{ s s}^{-1}$. These $\dot{P}$ values indicated by the model are also in good agreement with the observations.

**CONCLUSIONS**

Our present and earlier results [6] show that young neutron stars with fallback disks and conventional magnetic dipole fields evolve into the $L_x$, $P$ and $\dot{P}$ range of AXP/SGRs for a large range of disk masses. The upper limits to the observed periods of AXP/SGRs are accounted for by turn-off of the fallback disks at a temperature $T_p \sim 100 \text{ K}$. This result could be tested by the observations of AXP/SGRs in the near and mid IR bands, and seems to be consistent with SPITZER observations of AXP 4U 0142+61 [11, 5].

From the observed X-ray luminosities of AXP/SGRs, we expect that the fallback disk of a typical AXP with $L_x \sim 10^{35} \text{ erg s}^{-1}$ be active within a radius around or larger than

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**FIGURE 2.** Luminosity, period and period-derivative evolution of model sources for $T_p = 0$, with different $B_0$ values. For comparison we also present a model curve with $T_p = 100 \text{ K}$. In the middle panel, $B_0$ values are given in units of $10^{12} \text{ G}$ (See the text for explanations).
\sim 10^{12} \text{ cm depending on the efficiency of X-ray irradiation.}

We also show that the neutron stars with magnetic dipole fields higher than about $10^{13}$ G on the surface of the star are not likely to become AXP/SGRs. For this class of model sources it is not possible to reach $P$, $\dot{P}$ and $L_{X}$ properties of AXP/SGRs at the same epoch. In addition, periods of these sources pass through the AXP/SGR periods in a relatively short time, while period derivatives remain above the AXP/SGR period-derivative range (Fig.1).

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