Identifying parameters of a broaching design using non-linear optimisation

Ertunga C. Özelkan*

Systems Engineering and Engineering Management, and Center for Lean Logistics and Engineered Systems, The University of North Carolina at Charlotte, Charlotte, NC 28223, USA
E-mail: ecozelka@uncc.edu

Özkan Öztürk

Department of Mechanical Engineering and Center for Precision Metrology, The University of North Carolina at Charlotte, Charlotte, NC 28223, USA
E-mail: oozturk@uncc.edu

Erhan Budak

Faculty of Engineering and Natural Sciences, Sabanci University, Orhanli, Tuzla, Istanbul, 34956 Turkey
E-mail: ebudak@sabanciuniv.edu

Abstract: Broaching is one of the most recognised machining processes that can yield high productivity and high quality when applied properly. One big disadvantage of broaching is that all process parameters, except cutting speed, are built into the broaching tools. Therefore, it is not possible to modify the cutting conditions during the process once the tool is manufactured. Optimal design of broaching tools has a significant impact to increase the productivity and to obtain high quality products. In this paper, an optimisation model for broaching design is presented. The model results in a non-linear non-convex optimisation problem. Analysis of the model structure indicates that the model can be decomposed into smaller problems. The model is applied to a turbine disc broaching problem which is considered as one of the most complex broaching operations.

Keywords: broaching; machining optimisation; turbine disc broaching.


Biographical notes: Ertunga C. Özelkan is an Assistant Professor of Engineering Management and Associate Director of the Center for Lean Logistics and Engineered Systems at the University of North Carolina at Charlotte. He received his PhD in Systems and Industrial Engineering from the University of Arizona. His research is on systems optimisation, decision analysis, supply chain management and lean production planning.

Özkan Öztürk is a PhD candidate at the Mechanical Engineering Department at University of North Carolina at Charlotte. His research interests are analysis and modelling of machine tools, precision instrument design and metrology.

Erhan Budak is an Associate Professor of Engineering at the Faculty of Engineering and Natural Sciences at the Sabanci University. He received his PhD in Mechanical Engineering from the University of British Columbia. Some of his research interests are high productivity machining, analysis and modelling of machine tools, and design for manufacturing.
1 Introduction

The broaching process is commonly used in the industry for the machining of variety of external and internal features such as keyways, noncircular holes, and fir-tree slots on turbine discs (see e.g., Monday, 1960; Kokmeyer, 1984). Broaching can offer very high productivity and part quality when the conditions are selected properly. It has several advantages over other machining processes. For example roughing and finishing of a complex form on a part can be completed in one stroke of the machine, which would require many passes with another process such as milling. However, achieving high quality and high productivity for the part needs a well-designed process.

Figure 1 Broaching of fir-tree forms on a turbine disc and a broaching tool with 12 different tool segments

Figure 1 show broaching of a typical turbine disc on a horizontal broaching machine and 12 tool segments (also referred as sections) that form the profile. The tooth rise-per-tooth determines the chip thickness, which is different for each section. In general, there is more than one tooth in-cut depending on the pitch of the cutter (also referred as the broach). Different profiles, rise-per-tooth and pitch results in variations in the total chip area, and thus cutting forces in a broaching cycle. The changes in the total chip area cause fluctuations in the load applied on the part and the fixture, and the part and tool deflections, cutting tooth stress and causing uneven wear of cutting teeth.

In broaching, all process parameters except the cutting speed are predefined during the design of the cutting tool. Therefore, it is not possible to modify cutting conditions after cutters are manufactured, unlike for other machining processes where depth-of-cut or feed-rate can be changed easily. This makes tool design the single most important aspect of broaching.

In this paper, the constraints of the process are discussed and a mathematical programming model is presented with applications to machining of fir-tree forms on turbine discs, which is regarded as one of the most difficult broaching operations due to its complex geometry, very tight tolerances and difficult-to-machine work material.

The rest of the paper is organised as follows: next, we will provide a literature review, and proceed by introducing the broaching objectives and the constraints, and present a mathematical programming model. Then, we will provide numerical computations based on industry data. The final section includes a summary of major findings and conclusions.

2 Literature review

Although widely used in industry, there is very limited literature on broaching. The book by Monday (1960) presents the technology of broaching machines, processes and tools in a detailed manner. Although this is a relatively old reference, most of the material in the book still applies to the current broaching operations. Collection of the works edited by Kokmeyer (1984) has several different broaching applications in the industry demonstrating the effectiveness of the process. Terry et al. (1992) present a knowledge-based system approach that can be used in design of broaching tools. Gilornini and Felder (1984) analyse the cutting forces on a single broaching section and compare them with the forces in tapping and slotting. Sutherland et al. (1997) demonstrate the application of a mechanistic force model to gear machining. In one of the recent works, Sajeev et al. (2000) present finite element analysis (FEA) results for the effects of burnishing in broaching. The last section of a broach set usually burnishes the surface to improve surface finish and surface integrity. The analysis done by Sajeev et al. (2000) provides a good insight on the mechanics of the broaching process. Taricco (1995) presents the tool wear affects on the surface integrity of the broached slots, which increases the risk of high tensile stresses on the surface. Budak (2001) illustrates that monitoring of the results are very helpful for identification of the possible improvements on the broaching tool design. More recently, Ozturk and Budak (2003) and Ozturk (2003) used proper cutting models and FEA to model the broaching process. In their study, the cutting conditions are changed by enumeration until a constraint is met to improve the process. An extension of Ozturk and Budak (2003) and Ozturk (2003) can be found in Kokturk and Budak (2004) as well.
The current paper extends the previous research on broaching by first introducing a mathematical programming framework. Unlike the enumeration or pure trial and error approaches for identification of optimal broaching design, (see e.g., Ozturk and Budak, 2003; Kokturk and Budak, 2004), we provide a systematic way to identify the model parameters using non-linear optimisation techniques. In addition, we show that the original broaching problem can be decomposed into simpler optimisation problems under mild assumptions. As far as we know, broaching design has not been analysed in previous studies as we did herein.

3 Model

Before describing the model, we would like to define the notation below.

Let \( i = 1, \ldots, N_s \) denote the index for each section of the broaching tool to be designed with \( N_s \) number of sections.

Then the following variables can be identified as the main decision variables:

\[
\begin{align*}
t_i & \quad \text{chip thickness (mm), } i = 1, \ldots, N_s \\
p_i & \quad \text{pitch of the section (mm), } i = 1, \ldots, N_s
\end{align*}
\]

Chip thickness is the most critical design decision. It has considerable effect on cutting forces directly related to power consumption and tooth stresses. The pitch of the tool has an impact on the total tool length and it also affects the total broaching force.

A simple example is shown in Figure 2, where the objective is to cut a trapezoidal shape as shown on the left side of the figure. Let’s assume that we would like to design a broach to make this cut as shown on the right side of the figure. This broach will have two sections and two teeth in each section. The first section will cut the rectangular shape marked as Section 1, and the remaining area will be cut by the teeth in Section 2. In this simplistic example, the design objective would be the identification of the chip thickness \( t_1 \) and \( t_2 \), and the pitch of each section \( p_1 \) and \( p_2 \). As also shown in this figure, we assume that the chip thickness and the pitch are constant within each section.

The following is a list of other parameters:

\[
\begin{align*}
A_{1i} & \quad \text{taper angle of the tooth (degrees)} \\
A_{2i} & \quad \text{rake angle (degrees)} \\
AMP & \quad \text{available machine power (Watts - Nm/sec)} \\
b_i & \quad \text{chip width (mm)} \\
B_i & \quad \text{width of the tooth (mm)} \\
c_1, c_2, c_3, c_4 & \quad \text{pitch related constants} \\
K_{tc}, K_{te}, K_{fc}, K_{fe} & \quad \text{cutting constants (units of MPa, N/mm, MPa, N/mm, respectively)} \\
L_{ram} & \quad \text{ram length (mm)} \\
m & \quad \text{is the number of teeth in-cut} \\
n_i & \quad \text{number of teeth in a section} \\
PS & \quad \text{permissible stress (MPa)} \\
SF & \quad \text{safety factor} \\
T_i & \quad \text{top length of the tooth (mm)} \\
V & \quad \text{cutting speed (m/sec)} \\
w & \quad \text{thickness of the part to be cut (mm)}
\end{align*}
\]

The reader may also refer to Figure 6 in the Appendix for an illustration of the general tooth geometry and the related parameters.

Figure 2  Broach design example

3.1 Objective function

Since higher productivity and lower cost are the objectives, it is desired to increase the material removal rate (MRR), which is computed as volume removed per unit time.
3.2 Power

The available machine power (AMP) capacity of every machine is limited. As the cutting forces increase, the power consumption also increases. This constraint ensures that the power consumed due to all tangential cutting forces created by the teeth in-cut are less than the AMP.

\[ [(K_{it} + K_{tt})^2 + (K_{it} + K_{tt})^2]^{0.5} \]

\[ b_1.3c_{i1}^{0.374}c_2^{0.082}c_4^{0.356}p_i^{-0.064}b_i^{-1.09} \]

\[ T_i^{0.072}A_i^{0.088} - PS/\text{SF} \leq 0 \] (4)

Chip space

\[ t_i b_i - (0.331)p_i^{1.8929}(1-c_d)^{0.816} \]

\[ (c_{i1})^{1.14}(c_{i2})^{0.026}(c_{i3})^{-0.0891}A_{i2}^{0.0388} \leq 0 \] (5)

Chip thickness

\[ 0.012 \leq t_i \leq 0.065 \] (6)

Non-negativity

\[ p_i \geq 0 \] (7)

For subsequent discussions, we will refer to this formulation as the original broaching problem (or OBP for short). Note that OBP is a non-linear optimisation problem that has 2Ns decision variables, a non-linear objective function, 3Ns non-linear constraints, \( N_s + 1 \) linear constraints, and \( N_s \) bound constraints on the decision variables. As it is illustrated in the numerical computations section, it can be verified that this problem has a non-convex solution space due to constraints (4) and (5). While in general non-convex problems can be difficult to solve, as it will be shown below, a close examination of this problem indicates that under some mild assumptions, this problem can be decomposable into \( N_s \) simpler optimisation problems in which we identify the optimal pitch and the chip thickness for each section separately.

Assumption A: Let \( C_i \) \( i = 1, \ldots, N_s \) denote the cut area for each section of the broaching tool. It is assumed that the cut area is known.

This assumption is reasonable since each broaching tool would be tailor-made for making a specific cut which the engineer or the designer is familiar with.

Lemma 1: If Assumption A holds, the broaching problem given in equations (1)-(7) is equivalent to the following modified problem with a minimisation objective.

Minimise \[ z = \sum_{i=1}^{N_s} \left( \frac{C_i}{B_i t_i} - 1 \right)p_i \] (1′)

Subject to Constraints (2) – (7)

Proof: The proof uses the fact that \( C_i = n_i B_i t_i \). Substituting \( n_i = \frac{C_i}{B_i t_i} \) into (1) and recognising that \( b_i, B_i, w, V \) and \( N_s \) are constant parameters yields (1′). □.

Lemma 2: The broaching optimisation problem with objective function (1′) can be decomposed into \( N_s \) two dimensional optimisation problems as follows:
Minimise \( z_i = \left( \frac{C_i}{B_i l_i} - 1 \right) p_i \)

Subject to

\[ \text{Constraints (2), (4) - (7)} \]

(1'')

**Proof:** The proof requires recognising first that the objective function (1'') and the left hand side of constraint (3) are the same. Since the minimisation problem will identify the minimal possible total length imposed by the sections, comparing the right hand side to the \( L_{\text{ram}} \) (ram length) as in (3) is merely for feasibility reasons. Thus constraint (3) may be removed from the formulation, and a feasibility check can be made after the optimisation problem is solved. Since constraint (3) is the only constraint tying all sections together, removal of it makes the broaching optimisation problem decomposable \( \square \).

For subsequent discussions, we will refer to the decomposed broaching problem as DBP for short. One of the questions of interest is what happens when the solution of DBP violates the ram length constraint (3). In other words, can there be a situation when the DBP is infeasible and the OBP is feasible. As we state next in the following lemma this situation cannot occur.

**Lemma 3:** Infeasibility of DBP implies infeasibility of the OBP.

**Proof:** Let \( t_i^* \) and \( p_i^* \) denote the optimal solution obtained by solving DBP, and similarly \( t'_i \) and \( p'_i \) denote the optimal solution obtained by solving OBP. Assume by contradiction that DBP has an infeasible solution such that

\[ \sum_{i=1}^{N} \left( \frac{C_i}{B_i l_i} - 1 \right) p_i > L_{\text{ram}}, \quad \text{and OBP has a feasible solution} \]

such that

\[ \sum_{i=1}^{N} \left( \frac{C_i}{B_i l_i} - 1 \right) p'_i \leq L_{\text{ram}}. \]

Based on this,

\[ \sum_{i=1}^{N} \left( \frac{C_i}{B_i l'_i} - 1 \right) p'_i < \sum_{i=1}^{N} \left( \frac{C_i}{B_i l_i} - 1 \right) p_i^* \]

should hold, and furthermore, there should exist some \( i \) for which

\[ \left( \frac{C_i}{B_i l'_i} - 1 \right) p'_i < \left( \frac{C_i}{B_i l_i} - 1 \right) p_i^* \]

holds. Note that this inequality is not possible, since by definition of optimality

\[ \left( \frac{C_i}{B_i l'_i} - 1 \right) p'_i \]

should be the minimum possible value by solving DBP, which completes the proof \( \square \).

Based on Lemma 3, we conclude that if the solution of DBP is not feasible, then the design might need to consider a different machine with a longer ram length. An alternative solution could be to investigate changing other broach parameters, which might be a more economically feasible option.

It can be shown that DBP is still non-linear in the objective function and non-convex both in the objective function and constraints. On the other hand it is a simpler problem to solve than OBP due to the following simplifying observation described in the Lemma 4 below.

**Lemma 4:** Let \( p_i = z_i^k \left( \frac{C_i}{B_i l_i} - 1 \right) \) denote the objective function line (OFL) for any selected \( z_i = z_i^k \in \mathbb{R}^+ \) based on the objective function (1''). Then OFL originates from (0, 0) and has a nonnegative slope as \( t_i \) increases. Furthermore, the slope of OFL is proportional and positively correlated to \( z_i^k \).

**Proof:** It is easy to see that OFL originates from the origin by simply setting \( t_i = 0 \). Next, we evaluate the slope of OFL as

\[ \frac{dp_i}{dt_i} = \frac{z_i^k C_i B_i}{(C_i - B_i t_i)^2}, \]

and observe that \( \frac{dp_i}{dt_i} \geq 0 \) since \( z_i^k, C_i, B_i > 0 \). Consequently, any increase (decrease) in \( z_i^k \) will increase (decrease) the slope of OFL proportionally \( \square \).

The result of Lemma 4 is interesting since it basically indicates that the solution to the DBP can be found by setting \( z_i^k = 0 \) first, and then by increasing \( z_i^k \) to make a sweep of OFL counter clockwise until the feasible region is reached. The first point(s) where the feasible region is reached would give the optimal solution \((t_i^*, p_i^*)\) and \( z_i^* \).

### 4 Solution approach

The DBP can be solved using several different non-linear optimisation approaches [see e.g., Bazaraa et al. (1993) and Bunday (1984) for a review of various traditional techniques, and for recent application examples of genetic and artificial intelligence techniques, see e.g., Jin et al. (2008), and Thangavel et al. (2007)]. Due to the non-convexity of the feasible region, application of non-linear optimisation algorithms would only guarantee a locally optimal solution. Typical approach to deal with this type of optimisation scenarios is to use a multi-start approach in which multiple runs of the optimisation are carried out, and then the best solution is selected among the identified local optimal solutions. Here, we applied two techniques to solve the DBP as follows:

1. **graphical solution**
2. **multi-start complex method** (Box, 1965)

#### 4.1 Graphical solution

This approach leverages the two-dimensional nature of DBP, and deploys a five-step approach to identify the optimal solution as follows:

**Step 1** start with \( i = 1 \)

**Step 2** graph DBP for section \( i \) to identify the feasible space

**Step 3** graph OFL to identify the constraints identifying optimal point and binding constraints visually
Identifying parameters of a broaching design using non-linear optimisation

249

Step 4 solve the constraints from Step 3 simultaneously to identify the coordinates of the optimal point and the corresponding objective function value

Step 5 if \( i = N_s \), stop, else set \( i = i + 1 \) and go to Step 1.

Note that due to non-linearity, identification of the intersection point of constraints in Step 4 might not be trivial, but it can be done through numerical techniques.

4.2 Multi-start complex method

While the graphical approach is visual, it does have disadvantages due to some manual steps. Here, we also applied the complex method to identify the optimal solutions automatically. The main steps of the method can be sketched as follows:

Step 1 start with \( i = 1 \)

Step 2 generate a population of feasible points in the solution space

Step 3 identify best and worst points, and the centroid of all points (excluding the worst point)

Step 4 use a reflection step to identify a new point, moving towards the improvement direction from the centroid of the population

Step 5 if a better feasible point is found, replace the worst point with the new point.

Step 6 go to Step 8 if the population and the corresponding functional values converge

Step 7 if a better point is not found or implicit constraints are violated, contract the population towards the centroid and go back to Step 3

Step 8 if \( i = N_s \), stop, else set \( i = i + 1 \) and go to Step 1.

The reader can refer to e.g., Bunday (1984) for a more detailed and precise description of the algorithm. For our purposes, the algorithm is coded in Fortran 77 with a multi-start option to do multiple runs, and to select the best solution resulting from all runs.

5 Numerical computations

In this section, an application of the proposed model is presented for industry data. The data, which is for a 12 section broach design \( (N_s = 12) \), is obtained from Pratt & Whitney Canada, a leading manufacturer of jet engines. For each section, optimal chip thickness and the pitch \( (t^*_i, p^*_i) \) were obtained applying both the graphical and the complex methods.

Figure 3 illustrates the graphical approach for Section 1. As seen from this figure, non-convexity is caused by constraints (4) and (5). In Steps 2 and 3 of the graphical approach, the feasible space and the optimal point is identified. As it can be seen from Figure 3, the optimal solution is determined by constraints (2) and (4). In Step 4, we solved the expressions for constraints (2) and (4) simultaneously to identify the coordinates of the optimal point. Here, we used the ‘solver’ functionality of MS Excel to identify the solution as \( t^*_1 = 0.032 \text{ mm} \) and \( p^*_1 = 2.243 \text{ mm} \) with an objective function value of \( z^*_1 = 121.8 \).

Figure 3  Optimal parameters for Section 1 using the graphical approach

Repeating the graphical approach for all 12 sections, the optimal parameters are identified as given in Figure 4.

Figure 4  Optimal parameters for all sections using the graphical approach

Next, the multi-start complex method was applied to identify the optimal parameters for each section. The population (also called the complex) has been formed using four feasible points in the solution space, which are randomly identified to start the algorithm. Five replications of the optimisation runs were performed for each section, and the best solutions from these five runs were selected. In general, the complex method identified the same optimal solutions for each section as in the graphical technique as shown in Figure 4.

Table 1  Section 5 optimal parameters and objective function values for each optimisation run using the multi-start complex method

<table>
<thead>
<tr>
<th>Run no.</th>
<th>( t^*_5 )</th>
<th>( p^*_5 )</th>
<th>( z^*_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0241311</td>
<td>0.6241489</td>
<td>34.3266</td>
</tr>
<tr>
<td>2</td>
<td>0.0241308</td>
<td>0.6241384</td>
<td>34.3265</td>
</tr>
<tr>
<td>3</td>
<td>0.0252295</td>
<td>0.9643447</td>
<td>50.6855</td>
</tr>
<tr>
<td>4</td>
<td>0.0241309</td>
<td>0.6241395</td>
<td>34.3263</td>
</tr>
<tr>
<td>5</td>
<td>0.0240712</td>
<td>0.6233219</td>
<td>34.3680</td>
</tr>
</tbody>
</table>
For most sections, the five replications of optimisation runs converged to the same optimal point. Sections 5 and 9 had one run each, which slightly deviated from the optimal point. To illustrate how the methodology works, Section 5 results are shown in Table 1. As seen in this table, all runs converged to the optimal point except for run 3. Based on the objective function values, we select the run 4 results, which is the lowest of all runs.

Figure 5  Section 1 optimality search path and corresponding objective function values for each optimisation run using the multi-start complex method (a) search path for optimum point (b) objective function and number of function evaluations (see online version for colour)

As described earlier, each optimisation run is initialised with a random population of four feasible points. Thus, the path to optimality and the number of required function evaluations might differ between the runs. This is illustrated in Figures 5(a) and 5(b) for Section 1. As indicated with circles in Figure 5 (b), optimisation runs 1 through 5 identified the same optimal point (\(t^*_1 = 0.032\) mm, \(p^*_1 = 2.243\) mm, \(z^*_1 = 121.8\)) with different number of function evaluations of 80, 95, 59, 64 and 98, respectively.

6 Summary and conclusions

In this study, we provided a mathematical programming formulation for the broaching design problem. The problem yields a non-linear and non-convex optimisation problem. Analysis of the problem structure based on mild assumptions indicate that the original problem (which aims to identify all the design parameters simultaneously) can be decomposed into smaller (thus simpler) optimisation problems where parameters for each section are identified independently. The decomposed problem can be solved using a graphical approach or using multi-start non-linear optimisation algorithms such as the complex method, as we did herein. The multi-start complex method is shown to converge to the optimal solution consistently as verified by the graphical solution. We believe that the proposed model and the methodology can help the engineers in future broach designs.

Acknowledgements

The authors would like to thank Pratt & Whitney Canada for their support.

References


Appendix: Derivation of model formulation

Objective function

The MMR can be calculated as:

\[ MRR = \frac{\text{volume removed}}{\text{process time}} \]

The volume removed per one tooth is

\[ wt_i b_i \]

so the total volume removed is

\[ V_{\text{total}} = w \sum_{i=1}^{N_i} t_i b_i n_i \]

The process time can be calculated as

\[ \text{time} = \frac{\text{distance}}{\text{velocity}} = \frac{w + \sum_{i=1}^{N_i} (n_i - 1) p_i}{V} \]

So the objective function is obtained as:

\[ \text{Max } MRR = \frac{w \sum_{i=1}^{N_i} t_i b_i n_i}{w + \sum_{i=1}^{N_i} (n_i - 1) p_i} \]

Power

To calculate the power, first of all, tangential cutting forces created by teeth in-cut have to be calculated as:

\[ F_{\text{total},i} = mF_{ti} \]

where \( m \) is the number of teeth in-cut and \( F_{ti} \) is the tangential cutting force in section \( i \), computed as \( F_{ti} = K_{\mu} t_i b_i + K_{\tau} b_i \).

So the power requirement can be stated as:

\[ F_{\text{total},i} V \leq AMP \]

\[ m(K_{\mu} t_i b_i + K_{\tau} b_i)V \leq AMP \]

\[ m(K_{\mu} t_i + K_{\tau}) \leq \frac{AMP}{Vb_i} \]

Since \( m \leq \frac{w}{p_i} + 1 \), the following would be a sufficient condition for satisfying this constraint:

\[ \left( \frac{w}{p_i} + 1 \right)(K_{\mu} t_i + K_{\tau}) \leq \frac{AMP}{Vb_i} \]

Ram length

Since we assume that each section has a constant pitch, \((n_i - 1)p_i\) yields the length of each section. Adding all sections together yields the following:

\[ \sum_{i=1}^{N_i} (n_i - 1)p_i \leq L_{\text{sum}} \]

Tooth stress

The resultant cutting force on one tooth \( F_i \) can be obtained as

\[ F_i = \sqrt{F_{ti}^2 + F_{fi}^2} \]

where again \( F_{ti} \) is the tangential cutting force as described earlier and \( F_{fi} = K_{\mu} t_i b_i + K_{\tau} b_i \) is the feed force in section \( i \).

Using the cutting force, the tooth stress can be formulated as follows:

\[ S_i = F_i 1.3H_i^{0.374} B_i^{1.097} R_i^{0.072} A_j^{0.088} R_k^{0.082} l_i^{0.356} \]

which includes several parameters related to the tooth geometry as shown in Figure 6. These parameters can be defined as follows: \( H_i = c_1 p_i \): height of the tooth, \( R_{hi} = c_2 p_i \): gullet radius, \( R_{2i} = c_3 p_i \): pre-gullet radius and \( l_i = c_4 p_i \): land length.

Figure 6   General tooth geometry

In order to avoid tooth breakage the following should hold:

\[ S_i \leq PS / SF \]
where $PS$ denotes permissible stress and $SF$ is the safety factor. By making the substitutions and rearranging yields:

$$
\left( \frac{1}{K_{i}t_{i}+K_{ic}} \right)^{0.5} h_{i} 1.3k_{i}^{0.374}e_{2i}^{-0.082}c_{4i}^{-0.356}p_{i}^{-0.064}B_{i}^{-1.09}T_{i}^{0.072}A_{i}^{0.088} - PS / SF \leq 0
$$

**Chip space**

The cut chip volume can be calculated as

$$
V_{pt,i} = wt_{i}b_{i},
$$

and the gullet space as

$$
V_{Gullet,i} = 0.9456w(p_{i} - l_{i})^{0.816}H_{i}^{1.14}R_{i}^{0.026}R_{2i}^{-0.089}A_{2i}^{0.0388},
$$

where $H_{i}$, $R_{1i}$, $R_{2i}$, and $l_{i}$ are defined as before. In order to prevent breakage, the following needs to hold:

$$
\frac{V_{pt,i}}{V_{Gullet,i}} \leq r, \text{ where } r = 0.35.
$$

By making the substitutions and rearranging the expression yields:

$$
t_{i}h_{i} - (0.331)p_{i}^{1.8929}(1 - c_{3i})^{0.816}c_{4i}^{1.14}c_{2i}^{0.026}c_{3i}^{-0.089}A_{2i}^{0.0388} \leq 0
$$

**Chip thickness**

The following are empirical requirements for chip thickness for preventing rubbing and chipping (see Ozturk and Budak, 2003).

$$
0.012 \leq t_{i} \text{ for rubbing}
$$

$$
t_{i} \leq 0.065 \text{ for chipping}
$$