

Optimal Motion Control and Vibration Suppression of Flexible Systems with Inaccessible Outputs

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Abstract—This work addresses the optimal control problem of dynamical systems with inaccessible outputs. A case in which dynamical system outputs cannot be measured or inaccessible. This contradicts with the nature of the optimal controllers which can be considered without any loss of generality as state feedback control laws for systems with linear dynamics. Therefore, this work attempts to estimate dynamical system states through a novel state observer that does not require injecting the dynamical system outputs onto the observer structure during its design. A linear quadratic optimal control law is then realized based on the estimated states which allows controlling motion along with active vibration suppression of this class of dynamical systems with inaccessible outputs. Validity of the proposed control framework is evaluated experimentally.

Keywords—Action-reaction state observer, motion control, reaction force observer, vibration suppression.

I. INTRODUCTION

The optimal linear quadratic control law of the form $u^*(t) = -R^{-1}B^TK\hat{x}(t)$, requires measuring or estimating dynamical system states. Such problem has been solved by using either sensors to measure dynamical system states or by designing state observers. The class of dynamical systems we consider in this work do not have accessible outputs or measurements cannot be made due to some constrains, e.g., Mariaana and Heikki [1] pointed out that there exist at least two major problems that makes it difficult to automate the micromanipulation systems, namely the poor understanding of the interaction phenomena and the difficulty of making measurement at microscale. A question naturally arises: Can we realize an optimal control law of a dynamical system when neither of its outputs are measured or when these dynamical systems are required to be free from any attached sensors to overcome their associated problems or even when it is impractical to make measurement due to limited space constraints? It would be natural, however, to devise observers to estimate dynamical system states that in turn requires having measurements from the dynamical system to be used as basis of the estimation process during the design of the state observer. Unfortunately, the dynamical systems we consider in this work do not have any accessible outputs. Therefore, we consider a novel state observer, with

a Luenberger [2]-[3] like structure except that it does not requires measuring dynamical system states to develop the error variables between the actual and estimated states, it rather utilizes the incident reaction forces that occurs upon any excitation generated at the interface plane between these systems and their actuators. These incident reaction forces are instantaneous and can be conceptually considered as feedback like forces from these dynamical systems.

The interaction forces along dynamical systems were considered as propagating mechanical waves that must be launched with the proper amount and at the right time in order to position a non-collocated mass to a reference position without residual vibrations [4], [5]. However, the wave transformation was utilized in [14]-[17] in order to preserve passivity of a communication network in the presence of time delay. On the other hand, Ohnishi [6]-[10] considered these interaction forces as disturbances that have to be suppressed in the attainment of robust motion control [11].

In this work, estimated reaction forces are used as basis of the state estimation process during the design of the proposed state observer. The optimal motion control and vibration suppression control law is then realized based on the estimated states. This allows performing a motion control assignment along with vibration suppression without taking any measurement from the plant side, reaction forces are rather conceptually considered as natural feedback from these systems which can be used in the design of state observers.

The remainder of this paper is organized as follows. In Section II, the energy content of the dynamical system along with the controller induced energy are used to formulate a performance index, then the optimal control law is derived. In Section III, a state observer is presented which consists of a reaction force observer, disturbance observer and a Luenberger like state observer. This observer is designed and utilized to estimate dynamical system states without injecting any of its outputs onto the observer structure during its design. The estimated states are then used in the realization of the optimal control law which guarantees precise motion control along with vibration suppression at the end of the travel.

Experimental results are included in Section IV, where a multi degrees of freedom flexible system is conceptually considered as a dynamical system with inaccessible outputs. A state observer is designed for this system then the regulation control law is used to regulate its states to the origin or any target position. Eventually, conclusions and final remarks are included in Section V.

II. MOTION CONTROL AND VIBRATION SUPPRESSION

The state space representation of the system we consider can be written as

$$\dot{x} = Ax + Bu, \quad y = Cx \quad (1)$$

where $x \in \mathbb{R}^{n \times 1}$ and $y \in \mathbb{R}^{n \times 1}$ are system state and output vectors. $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times 1}$ and $C \in \mathbb{R}^{n \times n}$ are system matrix, input and output distribution vectors, respectively. $u \in \mathbb{R}^{1 \times 1}$ is a single input to the dynamical system.

In order to perform vibrationless motion control, the following performance index is used

$$J(x(t), u(t), t) = \Upsilon + \frac{1}{2} \int_{T_0}^{T_f} (x^t Q x(t) + u(t)^t R u(t)) dt \quad (2)$$

$$\Upsilon \triangleq x^t(t_f) H x(t_f)$$

where, R is a symmetric positive definite matrix, i.e., $R^t = R$, $R > 0$, while Q is at least symmetric semi-definite matrix, i.e., $Q^t = Q$, $Q \geq 0$. Q and R will be selected such that the first and second terms of the performance index integrand represent the energy content of the system and the energy induced by the control input, respectively.

The previous performance index can be rewritten using the estimated states which will be discussed in the next section. Therefore, (2) can be written as

$$J(\hat{x}(t), u(t), t) = \Upsilon + \frac{1}{2} \int_{T_0}^{T_f} (\hat{x}^t Q \hat{x}(t) + u(t)^t R u(t)) dt \quad (3)$$

$$\Upsilon \triangleq \hat{x}^t(t_f) H \hat{x}(t_f)$$

consequently the Hamiltonian can be written as follows

$$\mathcal{H}(\hat{x}(t), u(t), \hat{p}(t), t) \triangleq \Gamma + \hat{p}^t(t) [A \hat{x} + Bu(t)] \quad (4)$$

$$\Gamma \triangleq g(\hat{x}(t), u(t), t) = \frac{1}{2} (\hat{x}^T Q \hat{x} + u^t(t) R u(t))$$

$\hat{x}(t)$ is a vector of the estimated states while $\hat{p}(t)$ is the corresponding vector of system co-states. Differentiating the Hamiltonian with respect to states, co-states and control, the necessary conditions for the plant sensorless optimal control can be represented as follows

$$\dot{\hat{x}}^*(t) = \frac{\partial \mathcal{H}(\hat{x}(t), u(t), \hat{p}(t), t)}{\partial \hat{p}} \quad (5)$$

$$\dot{\hat{p}}^*(t) = - \frac{\partial \mathcal{H}(\hat{x}(t), u(t), \hat{p}(t), t)}{\partial \hat{x}} \quad (6)$$

$$\frac{\partial \mathcal{H}(\hat{x}(t), u(t), \hat{p}(t), t)}{\partial u} = 0 \quad (7)$$

the following matrix differential equation can be obtained using the previous necessary conditions

$$\begin{bmatrix} \dot{\hat{x}}^*(t) \\ \dot{\hat{p}}^*(t) \end{bmatrix} = \begin{bmatrix} A & -BR^{-1}B^t \\ -Q & -A \end{bmatrix} \begin{bmatrix} \hat{x}^*(t) \\ \hat{p}^*(t) \end{bmatrix} \quad (8)$$

solving the previous matrix differential equation for estimated states and co-states we obtain

$$\begin{bmatrix} \hat{x}^*(t_f) \\ \hat{p}^*(t_f) \end{bmatrix} = \begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{bmatrix} \begin{bmatrix} \hat{x}^*(t) \\ \hat{p}^*(t) \end{bmatrix} \quad (9)$$

Where Ψ is the state transition matrix. Using the following boundary condition

$$\hat{p}(t_f) \triangleq H \hat{x}(t_f) \quad (10)$$

combining (9) and (10), we obtain

$$\hat{p}(t) = (H \Psi_{12} - \Psi_{22})^{-1} (\Psi_{21} - H \Psi_{11}) \hat{x}(t) \quad (11)$$

taking partial derivative of Hamiltonian with respect to the control input

$$u^*(t) = -R^{-1} B^t \hat{p}(t) \quad (12)$$

using (11) in (12) we obtain

$$u^*(t) = -K \hat{x}(t) \quad (13)$$

where

$$K = R^{-1} B^t (H \Psi_{12} - \Psi_{22})^{-1} (\Psi_{21} - H \Psi_{11}) \quad (14)$$

III. ACTION REACTION STATE OBSERVER

It can be shown that the state space representation (1) can be written as follow

$$\dot{x}_a = A_a x_a + B_a u + B_{react} f_{react}(x, \dot{x}) \quad (15)$$

$$\dot{x}_p = A_p x_p + B_p f_{react}(x, \dot{x}) \quad (16)$$

$$x = [x_a \quad | \quad x_p]$$

(15) and (16) are the state space representations of the actuator and the dynamical system required to be controlled in the absence of its outputs or states (x_p). The subscript (a) and (p) denote the single input and the plant, respectively. $f_{react}(x, \dot{x})$ is the reaction force between the dynamical system (16) and its actuator (15). $f_{react}(x, \dot{x})$ is written as an implicit function of the entire dynamical system (1) states (x) since the reaction force is a function of the actuator and the plant states. Therefore, we propose a state observer for the plant (16) of the following form

$$\dot{\hat{x}} = A \hat{x} + Bu + M (\widehat{f_{react}}(x_a, \dot{x}_a) - \widehat{f_{react}}(\hat{x}, \dot{\hat{x}})) \quad (17)$$

where $\widehat{f_{react}}(x_a, \dot{x}_a)$ is the incident reaction force which can be observed using the actuator variables, $\widehat{f_{react}}(\hat{x}, \dot{\hat{x}})$ is the computed reaction force using the estimated states and the reaction force mathematical model which is assumed to be known a priori. M is the state observer vector gain.

The estimated reaction force can be observed using the following reaction force observer

$$\widehat{f}_{reac}(x_a, \dot{x}_a) = \frac{g_{reac}}{s + g_{reac}} [g_{reac} \widehat{\Delta m}_a \dot{x}_a + i_a \widehat{\Delta k}_f + \widehat{d}] - g_{reac} \widehat{\Delta m}_a \dot{x}_a \quad (18)$$

where $g_{reac} \in \mathbb{R}^+$ is the positive reaction force observer gain. i_a is reference current input. m_a and k_f are the actuator mass and force constant. $\widehat{\Delta m}_a$ and $\widehat{\Delta k}_f$ are the identified actuator parameter deviations. A procedure to determine $\widehat{\Delta m}_a$ and $\widehat{\Delta k}_f$ can be found in [18]-[19] through an off-line experiment. \widehat{d} is the observed disturbance force which can be determined through the following observer [10]

$$\begin{aligned} \widehat{d} &= \frac{g_{dist}}{s + g_{dist}} [g_{dist} m_{an} \dot{x}_a + i_a k_{fn}] - g_{dist} m_{an} \dot{x}_a \\ &= \frac{g_{dist}}{s + g_{dist}} [i_a k_{fn} - s m_{an} \dot{x}_a] = \frac{g_{dist}}{s + g_{dist}} d \end{aligned} \quad (19)$$

where $g_{dist} \in \mathbb{R}^+$ is the positive disturbance observer gain. m_{an} and k_{fn} are the nominal actuator mass and nominal force constant, respectively. Equations (17), (18) and (19) represent a state observer for dynamical systems with inaccessible outputs where measurements have only to be taken from the actuator side, whereas plant states (x_p) are not measured at all. The estimated states obtained through (17) can then be used in the optimal control law (13). Figure 1 illustrates the architecture of the control system, where the disturbance observer (DOB), reaction force observer (RFOB) and the action reaction state observer (ARSO) are used to estimate the plant states (x_p) from measurement taken from the single actuator attached to the system. Convergence stability of the observer (17) along with the necessary and sufficient conditions of the observability of the dynamical system states with inaccessible outputs are shown in [12]. However, it was shown that the action reaction state observer for the dynamical system of form (1) can be designed if the system matrix (A) has distinct eigenvalues. Therefore, this condition has to be checked before designing a state observer of the form (17). In Fig.1, the function $f(\cdot)$ depends on the model of plant in contact with the actuator. However, a mass spring model can be used to model the reaction force between the actuator and a dynamical system without any loss of generality. Figure 1 indicates that the control system does not depend on any measurement from the plant (\dot{x}_p), the incident reaction force is rather considered as a natural feedback from the dynamical plant on the actuator, then estimated using a reaction force observer as shown in Fig.1.

IV. EXPERIMENTAL RESULTS

In order to verify the validity of the control system, experiments were conducted on a single input multiple outputs flexible dynamical system as depicted in Fig.2. The dynamical plant consists of three degrees of freedom, this plant is conceptually considered as a dynamical plant with inaccessible states in order to examine the validity of the proposed control system. However, encoders are attached to each degree of

freedom in order to compare the estimated states with the actual ones. The three degrees of freedom plant is attached to a single input as shown in Fig.2 from which position or velocity measurement is taken along with the current reference input. The experimental parameters are included in Table.I

The force observer and the disturbance observer gains were set to 628 rad/s, whereas the action reaction state observer gain vector is selected such that observer (17) becomes twice faster than the control system. However, before selecting the proper gain of the state observer (17), the observability condition of this class of dynamical systems has to be checked. It was shown in [13] that the eigenvalues of the system matrix (A) have to be distinct in order to design state observers of the form (17). It can be easily shown that the system matrix for the dynamical system depicted in Fig.2 under the assumption that contacts between the lumped masses and their slides are smooth enough that its behavior can be accurately governed with a linear model in the neighborhood of a given operating point.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -a & -b & a & b & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ d & e & -2d & -2e & d & e & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & f & g & -2f & -2g & f & g \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & h & r & -h & -r \end{bmatrix}$$

$$a \triangleq \frac{k}{m_a}, \quad b \triangleq \frac{c}{m_a}, \quad d \triangleq \frac{k}{m_1}, \quad e \triangleq \frac{c}{m_1}$$

$$f \triangleq \frac{k}{m_2}, \quad g \triangleq \frac{c}{m_2}, \quad h \triangleq \frac{k}{m_3}, \quad r \triangleq \frac{c}{m_3}$$

where, c and k are the viscous damping coefficient and spring stiffness, respectively, that can be identified. Therefore, using these identified parameters along with the given ones from Table.I, eigenvalues of the dynamical system turned out to be distinct. The following observer vector gain was utilized through the whole experiment.

$$M = [0.3 \quad 0.1 \quad 0.3 \quad 0.3 \quad 0.1 \quad 0.2 \quad 3 \quad 3]'$$

An identity regulation matrix (R) was used in the performance index (2), the diagonal entries of the matrix (Q) were selected such that the first term of the performance index integrand represents both potential and kinetic energy of the plant. Therefore, (2) represent the energy trapped in the system along with energy induced by the controller. The optimal state feedback vector gains (K)

$$K = [10.03 \quad 2.12 \quad 6.75 \quad 1.43 \quad 6.55 \quad 1.43 \quad 6.45 \quad 1.42]'$$

The experimental results of the regulation control law (13) are depicted in Fig.3, the phase portrait shown in Fig.3-a illustrates the behavior of the first non-collocated mass while

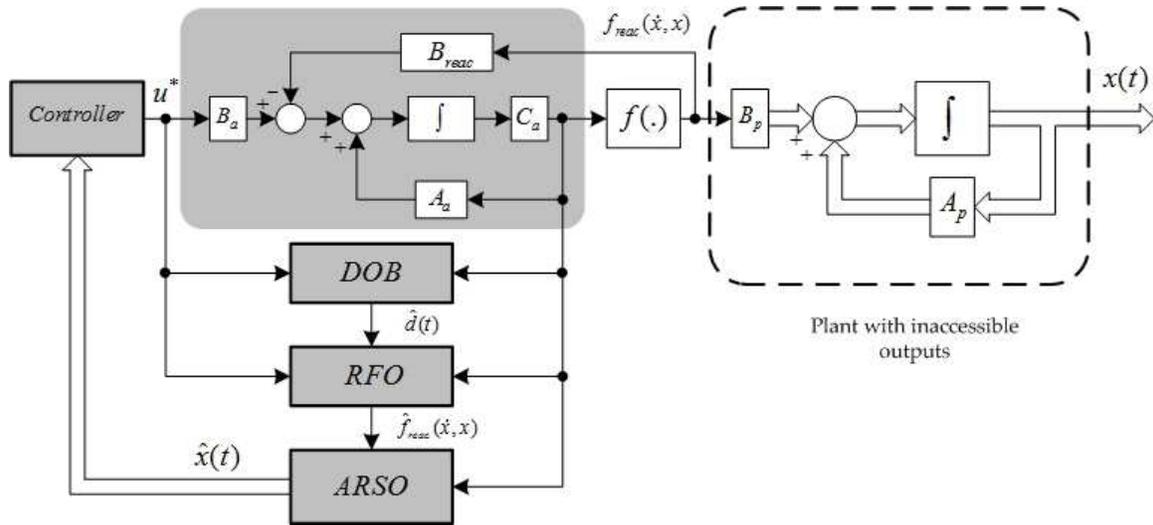


Fig. 1. Control system of a dynamical system with inaccessible outputs.

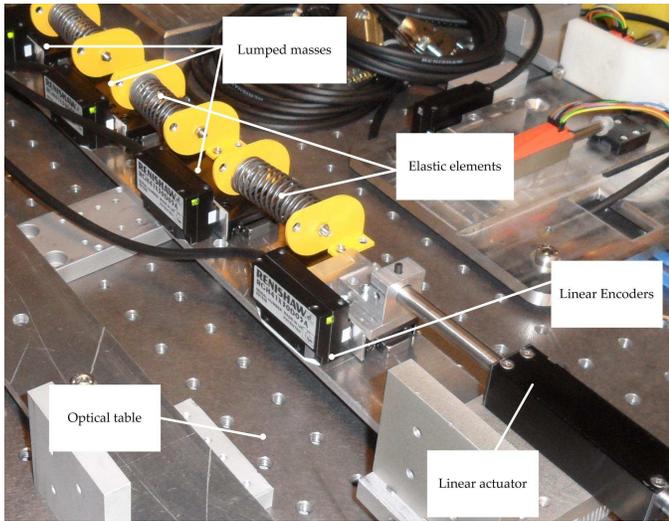


Fig. 2. Experimental setup.

Fig.3-b illustrates the second mass phase portrait to the optimal control law (13) which is used to regulate the second non-collocated mass to pre-specified reference. The phase portraits show that the second non-collocated mass is positioned with minimum residual vibration. Similarly, the phase portrait for the second and first non-collocated masses are illustrated in Fig.4 for different target position reference. The previous phase portraits indicate that the even in the absence of the plant outputs, an optimal control law can be realized.

In Fig.5 and Fig.6 the optimal regulating control law regulates the system to the origin with minimum residual vibration of the non-collocated masses.

V. CONCLUSION

Optimal motion control and vibration suppression of systems with inaccessible outputs can be achieved from measurements taken from their actuators rather than having multiple

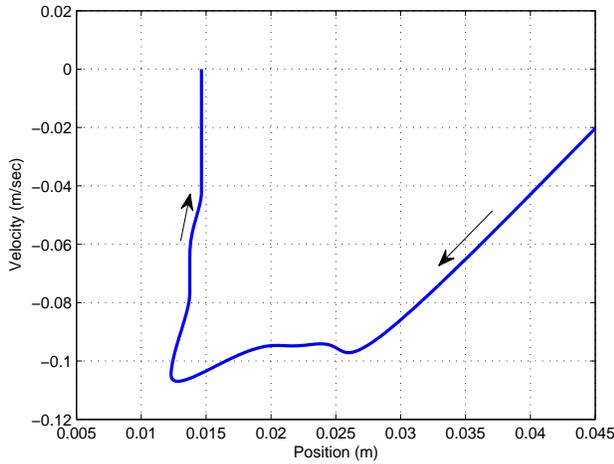
sensors attached to their structure. The reaction forces are conceptually considered as feedback like forces which can be used as replacement for system measurements which might be unavailable or can not be measured. Reaction force at the point of interface between the flexible dynamical system and its single input is estimated using a reaction force observer. The estimated reaction forces is then injected onto the state observer rather than the flexible system outputs in order to guarantee convergence of the estimated states to the actual ones. The estimated states are used instead of the actual ones in the realization of the optimal motion and vibration suppression control law.

Experimental results demonstrated the validity of the proposed controller by regulating the system either to the origin or any target position with minimum residual vibration even in the complete absence of the dynamical plant states. Therefore, the proposed controller can be used for a class of dynamical systems at which measurement cannot be made.

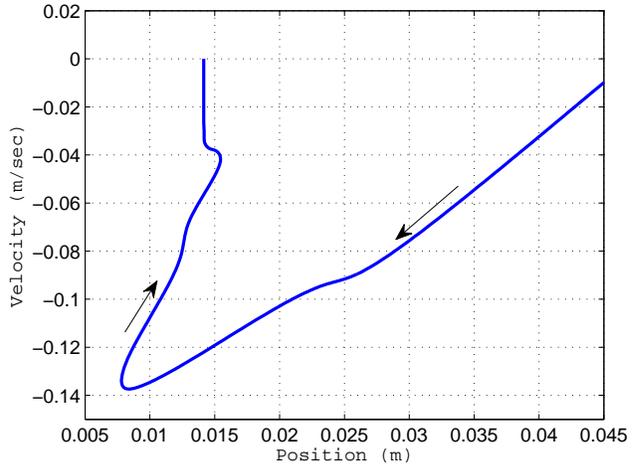
The proposed observer has three degrees of freedom, since it depends on a reaction force observer, disturbance observer and a Luenberger like state observer. Therefore, (g_{reac}) , (g_{dist}) and (M) have to be selected such that the overall observer is at least twice faster than the control system. Experimentally, this was achieved by tuning the disturbance observer and reaction force observer gains first. Then, the vector gain vector (M) is

TABLE I
EXPERIMENTAL PARAMETERS

Actuator force constant	k_{fn}	6.43	N/A
Actuator Nominal mass	m_{an}	0.059	kg
Lumped masses	$m_{1,2,3}$	0.019	kg
Force observer gain	g_{reac}	628	rad/s
Disturbance observer gain	g_{dist}	628	rad/s
Sampling time	T_s	1	ms

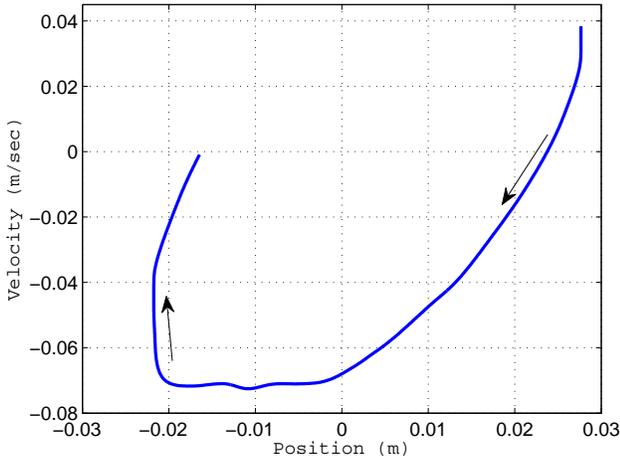


(a) 1st non-collocated mass phase portrait

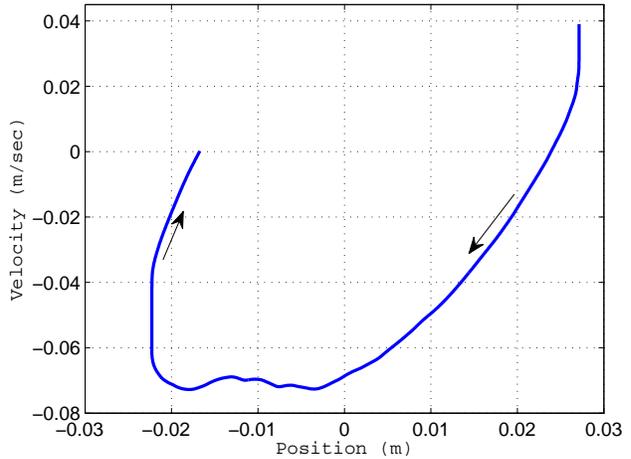


(b) 2nd non-collocated mass phase portrait

Fig. 3. Experimental results of optimal states regulation of a dynamical system with 3-dof (Optimal regulation of the second non-collocated mass to a reference position).



(a) 1st non-collocated mass phase portrait



(b) 2nd non-collocated mass phase portrait

Fig. 4. Experimental results of optimal states regulation of a dynamical system with 3-dof (Optimal regulation of the second non-collocated mass to a reference position).

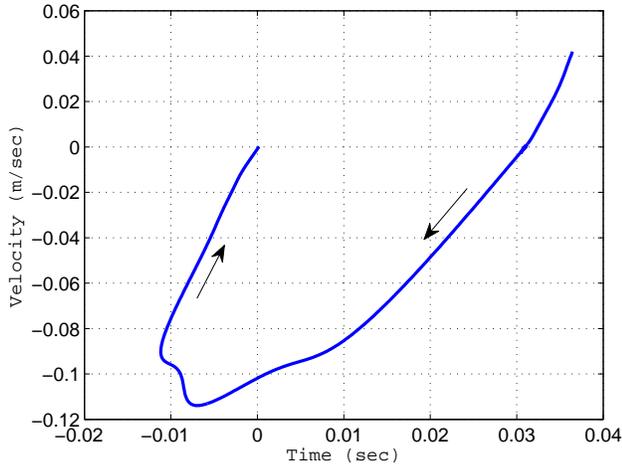
selected upon the required performance of the state observer.

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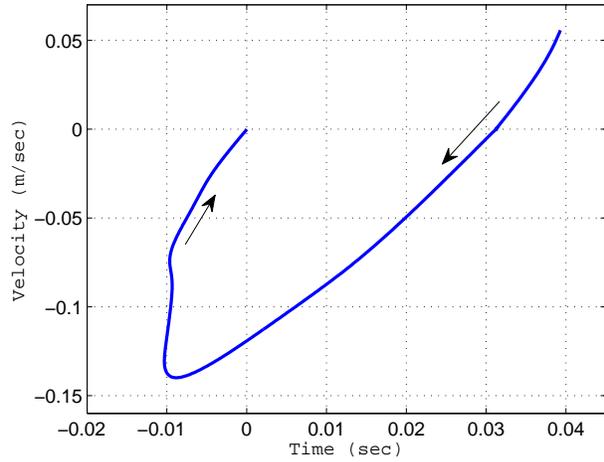
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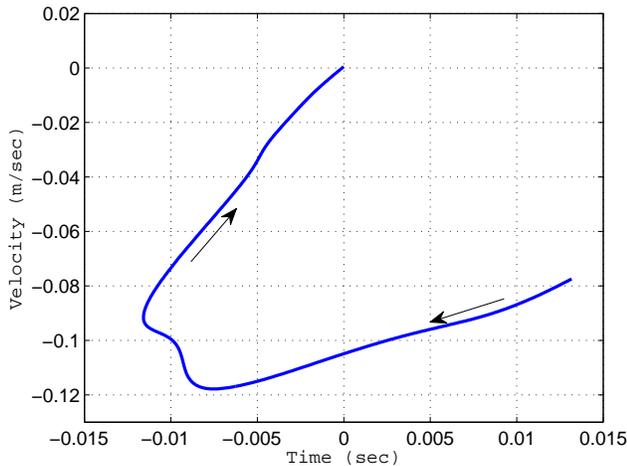


(a) 1st non-collocated mass phase portrait

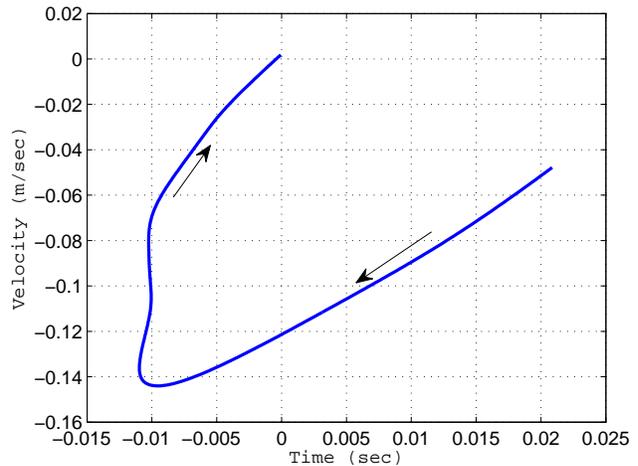


(b) 2nd non-collocated mass phase portrait

Fig. 5. Experimental results of optimal states regulation of a dynamical system with 3-*dof* (Optimal regulation of the second non-collocated mass to the origin).



(a) 1st non-collocated mass phase portrait



(b) 2nd non-collocated mass phase portrait

Fig. 6. Experimental results of optimal states regulation of a dynamical system with 3-*dof* (Optimal regulation of the second non-collocated mass to the origin).

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