Tombuş, O., Ertek, G., Atay, C., Kökten, G., and Tombuş, A.C. (2010) "A New Network Flow Model for Determining theAssortment of Roll Types in Packaging Industry." IEEE International Conference on Systems, Man, Cybernetics. October 10-13, 2010, Istanbul, Turkey.

A New Network Flow Model for Determining the Assortment of Roll Types in Packaging Industry

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Abstract— **This paper reports work motivated by a real world assortment problem in packaging industry. A novel network flow model has been developed to solve the problem of selecting the optimal set of roll types for use in production. The model can incorporate fixed costs that depend on the number of elements in the assortment as well as the selected roll types. While the trade-off between inventory cost and cost of waste is resolved optimally through the model, graphical understanding of the trade-off can bring insights into the decision making process. This graphical analysis has been demonstrated on a computational example.**

Keywords— ***assortment problem, cutting stock, packaging industry, shortest path problem, dynamic programming***

I. INTRODUCTION

Inventory holding and generation of paper waste are two major cost contributors in packaging industry, where papers are cut in various widths and lengths from paper rolls with varying widths and lengths. Companies typically hold high variety of roll types (identified based on their widths) in inventory for the purpose of decreasing the wasted scrap paper. A job (identified by its width and length) is cut from the roll with the minimum width, that is wider than the width of the given job. However, this results in higher inventory holding cost, accompanied with additional burden in operational planning. Thus, there is a clear trade-off between the cost of waste and the cost of inventory holding while deciding on the *assortment of roll types* to keep in inventory, and analytical modeling is required for optimal decision making.

This paper contains two contributions: 1) It extends an existing *network flow model* [1] (pages 11-12) for the aforementioned assortment problem for enabling the incorporation of new types of cost structures while preserving the network flow structure of the model; 2) It demonstrates how the solutions of the developed model can be used by decision makers for deciding on the assortment of roll types.

One of the most important issues in optimization modeling is the complexity of the solution algorithm used to solve the problem to optimality. Network flow problems are a class of integer programming optimization problems with a special structure. Integer programming models are *NP-hard*, meaning that they grow increasingly and non-polynomially more complex in size when their basic components, the number of variables and constraints, grow linearly. Thus, it becomes computationally infeasible to solve such problems to optimality as the problem instances grow. However, network flow problems are polynomially solvable, meaning that the time to solve such problems to optimality grows as a polynomial function of the number of variables and constraints.

The paper is organized as follows: The one-dimensional cutting stock problem is described in Section II. Related literature is summarized in Section III. The problem is modeled as a shortest path problem in Section IV and is solved through dynamic programming. Finally, a computational example is provided in Section V to demonstrate how the model solutions can be used for exploring the trade-off between various decisions and deciding on the assortment of roll types.

II. THE PLANNING PROBLEM

The research described in this paper is motivated by one of the largest packaging companies in Turkey, producing cardboard packaging for glassware products. Paper rolls are cut by guillotine machines from the rolls with the minimum possible width that satisfy the dimensions required by the job. Customer orders and thus the jobs to be cut in the plant show high variability, due to a multitude of factors such as product variety, seasonality, packaging characteristics. Rolls with different widths are kept in inventory and are used in production with the objective of minimizing waste paper. The growing number of roll types used in production have resulted in an increase in the amount of inventory and subsequently operational costs, including the costs of inventory holding, warehousing space and material handling. Furthermore, there exists a limited space for the inventory of paper rolls, which requires the resolution of the dilemma of having to incur increased paper waste while trying to reduce the number of roll types.

The decision problem is selection of the most appropriate assortment of roll types, from among a set of possible roll types. The objective is the minimization of total cost, which is composed of costs of inventory holding and paper waste, when the number of roll types to be used is fixed and is given as a constraint. The sets, parameters, decision variables, and the network modeling of the problem are given in Section IV.

III. RELATED LITERATURE

A. Cutting Stock Problem

In this paper, our main aim is to determine the best assortment policy for roll types where the inventory cost can be decreased without a significant increase in terms of the waste paper amount. The first stream of research investigated the cutting stock problem in roll paper cutting industry and was pioneered by the work of Gilmore and Gomory [2]. The problem considered in [2], as well as its succeeding literature, allow for multiple cuts from the same roll, yielding a combinatorial optimization problem. [3] notes that “the objective function, ... should minimize the percentage of waste rather than the amount of it” in the linear programming models for this problem. Restricting the problem by limiting the minimum and maximum number of sheets per pattern, as [4] did, did not yield efficient results to cutting stock problem due to its sequential approach. In summary, the NP-hard nature of the classic cutting stock problem does not allow for polynomial solution times when modeled as a linear program [5].

The cutting stock problem is polynomially solvable only if a single sheet is cut from the rolls (one-dimensional cutting stock problem) and thus the combinatorial nature of the problem is eliminated. Under this assumption, the problem can be treated as a network flow problem and can be solved through a dynamic programming approach, as described in [1].

As a related study with a different focus, [6] discusses a variation of the one-dimensional cutting stock problem where the control of defects is a major concern.

B. Case Studies in Packaging Industry

In this paper, decision making is based on two conflicting objectives, which are cost of inventory holding and the cost of cutting loss. As mentioned by [7], cutting a roll into its final dimensions is done according to a pattern which is determined by customer orders. This paper aims to achieve a balance between these two costs through a network flow model and a corresponding dynamic programming model. The only work that was encountered in the literature is [3], where the customer service impact of inventory is incorporated into the analysis through a simulation model.

IV. NETWORK MODELING OF THE PROBLEM

Similar to [1], the problem is formulated as a shortest path problem by constructing a directed network $\mathcal{G}(\mathcal{N}$.. Let denote the number of possible roll types and where denote the number of selected roll types. Figure 1 demonstrates the network constructed for a problem instance with I : and R :. While [1] constructs a single-dimensional network with $(I +$ nodes, we construct a two-dimensional network with $2 + \sum_{r}^{I}$ nodes and $\sum_{r=R}^{I}$ arcs. This added dimension for the set of roll types allows the incorporation of more complicated setup costs into the problem.

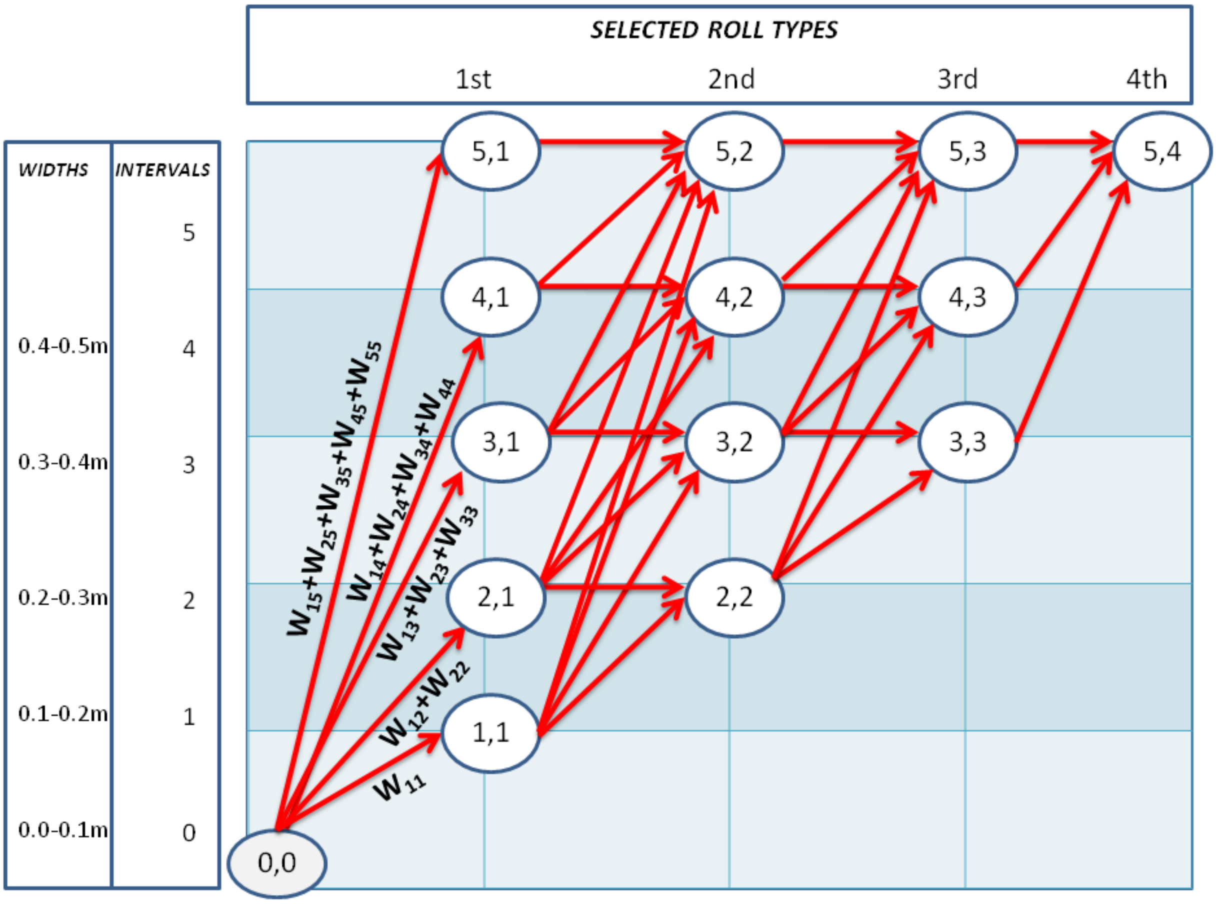

Figure 1. Demonstration of the network model, for a problem instance with five intervals and three roll types to be selected.

In the network model, the indices of the intervals (widths of the roll types) are placed along the axis and the indices of the selected roll types are placed along the axis. Having a positive flow through node (means that the th selected roll type (at stage of the network) corresponds to interval . In other words, it has a length that is equal to r the upper bound of interval . represents the target number of roll types to be selected. The condition R should be satisfied, since the roll types can be selected only from among those that are available. When R, the solution is trivial: All the candidate roll types, corresponding to each of the available intervals, are selected.

Source and sink nodes are included in the problem formulation, with a unit flow emanating from the source and terminating at the sink. The source node represents the initial stage in the solving of the problem (where no roll type is yet selected) and the sink node represents the last stage (where all the selected roll types are determined, and the assortment is completed).

Set of arcs consists of three subsets: Those that emanate from the source, those that terminate at the sink, and those between transient nodes.

The cost of traversing an arc a is defined as $C(a)$ and is calculated based on W_{ir}, the waste generated if jobs in interval i are cut from selected roll type r. The calculations are detailed in this section.

The lengths of rolls are assumed infinite and the thickness of rolls are considered as identical for each roll type.

The mathematical notation and the shortest path model is given next:

A. Sets

$\mathcal{I}$: Set of intervals, $i = 1, 2, 3, \ldots . I$

$\mathcal{R}$: Set of candidate roll types, $r = 0, 1, 2, \ldots R, R + 1$

$\mathcal{J}$: Set of all jobs to be cut from the selected roll types, $j = 1, 2, 3, \ldots . J$

$\mathcal{J}_i$: Set of jobs that fall in interval i

B. Nodes

$\mathcal{N}$: Set of nodes in the network representation

$\mathcal{N} = \dot{n} \cup \bar{\mathcal{N}} \cup \ddot{n}$

$\dot{n} = (0,0)$: source node

$\ddot{n} = (I, R + 1)$: sink node

$\bar{\mathcal{N}}$: set of transient nodes

$\bar{\mathcal{N}} = \{(i, r): 1 \leq r \leq R, r \leq i \leq I\}$:

$(i, r) \in \bar{\mathcal{N}}$: node that represents r^{th} selected roll type having length $\Gamma(i)$

C. Arcs

$\mathcal{A}$: Set of arcs in the network representation

$\mathcal{A} = \dot{\mathcal{A}} \cup \bar{\mathcal{A}} \cup \ddot{\mathcal{A}}$

$\dot{\mathcal{A}}$: set of arcs that emanate from the source node (and terminate at transient nodes)

$\ddot{\mathcal{A}}$: set of arcs that (emanate from transient nodes and) terminate at the sink node

$\bar{\mathcal{A}}$: set of arcs that emanate from transient nodes and terminate at the transient nodes

$\dot{\mathcal{A}} = \{((0,0), (i(r), 1))\}$

$\bar{\mathcal{A}} = \{((i_1, r), (i_2, r + 1)): i_1 + 1 \leq i_2 \leq I, 1 \leq r \leq R\}$

$\ddot{\mathcal{A}} = \{((i, R), (I, R + 1)): R \leq i \leq I\}$

D. Decision Variables

$$X(a) = \begin{cases} 1 & \text{if arc } a \text{ has positive flow through it} \\ 0 & o/w \end{cases}$$

$$X((i,j),(k,j+1)) = \begin{cases} 1 & \text{if contidition } Cond \text{ is satisfied} \\ 0 & o/w \end{cases}$$

where

condition C_c: "jobs in intervals $(i+$ to are assigned to the $(j+$st selected roll type"

E. Lookup Tables

$i(r)$: the index of the interval corresponding to the length of the r^{th} selected roll type

I : Upper bound value of interval

F. Parameters and Waste & Cost Calculations

V_r : width of r^{th} selected roll type

v_j : width of job j

l_j : length of job j

W_{ir} : waste generated if jobs in interval i are cut from selected roll type r

$$W_{ir} = \sum_{j \in J_i} (V_r - v_j) l_j$$

K_{ir}: fixed cost (setup and fixed inventory holding) of selecting the r^{th} roll type

K_R: overall setup cost of having only s different rolltype at the inventory

$C(a)$: cost of traversing arc $a = (n_1, n_2)$, which is the total waste produced by traversing that arc.

$C((i,j),(k,j+1))$: total waste if the jobs within interval $(i+1)$ to k are cut from the roll which corresponds to the $(j+1)^{st}$ interval

$$C((i,j),(k,j+1)) = K_{ik} + \sum_{r=i+1}^{k} W_{r,k}$$

G. Discussion

The model in [1] can handle only as the fixed cost component. However, in most real world situations, besides the real or perceived cost of adding roll type , there is also a cost associated with adding yet another roll type. Since the fixed cost of adding the th roll type to the assortment is not necessarily same as adding the th roll type (where s), the fixed cost has as an index.

Finally, another type of fixed cost can be incorporated, thanks to the two-dimensional network flow representation of the problem that takes into account the number of selected roll types. K_R represents the overall setup cost of having exactly R different roll types. This fixed cost can be added to the optimal value of the objective function, and will enable a more accurate calculation of the objective function value. Even though it will not affect the optimal solution, it will affect the objective function value, and will be important in comparing alternative scenarios (problem instances) with varying values of R.

H. Minimum Cost Flow Model

// Minimize total cost

min $C_{total} = K_R + \sum_{a \in \mathcal{A}} C(a)X(a)$

s.t.

// Flow out of the source node

$\sum_n X(\dot{n}, n) = 1$

// Flow into the sink node

$\sum_n X(n, \ddot{n}) = 1$

// Flow balance

$\sum_{n_1} X(n_1, n_2) = \sum_{n_2} X(n_2, n_3), \quad \forall n_2 \in \bar{\mathcal{N}}$

// Nonnegative decision variables

$X(n_1, n_2) \geq 0$

V. COMPUTATIONAL EXAMPLE

In this section we demonstrate how the model solutions can be used for exploring the trade-off between various decisions and deciding on the assortment of roll types. For this, a hypothetical computational example was constructed to mimic a confidential dataset from the real-world. The dynamic programming model was coded in MATLAB to solve the shortest path model for multiple values of (number of selected roll types) and the results were plotted.

A. Problem Instance

The problem instance consists of 207 jobs, ranging from 590 to 1390 millimeters in width and from 600 to 1386 meters in length. The set of candidate roll types were assumed to consist of 10 elements, with the widths being uniformly distributed. Since the main goal of this section is to demonstrate how computational results can be used in decision making, fixed costs were assumed to be zero to simplify computations.

B. Waste vs. Number of Selected Roll Types

"How does the generated waste change depending on the number of selected roll types?"

Figure 2 answers this question graphically for the constructed example. Since the fixed costs were all assumed to be zero, the total cost C_t monotonically decreases as the number of selected roll types increases. From the figure, a decision maker can quantitatively observe the trade-off between and C_t.

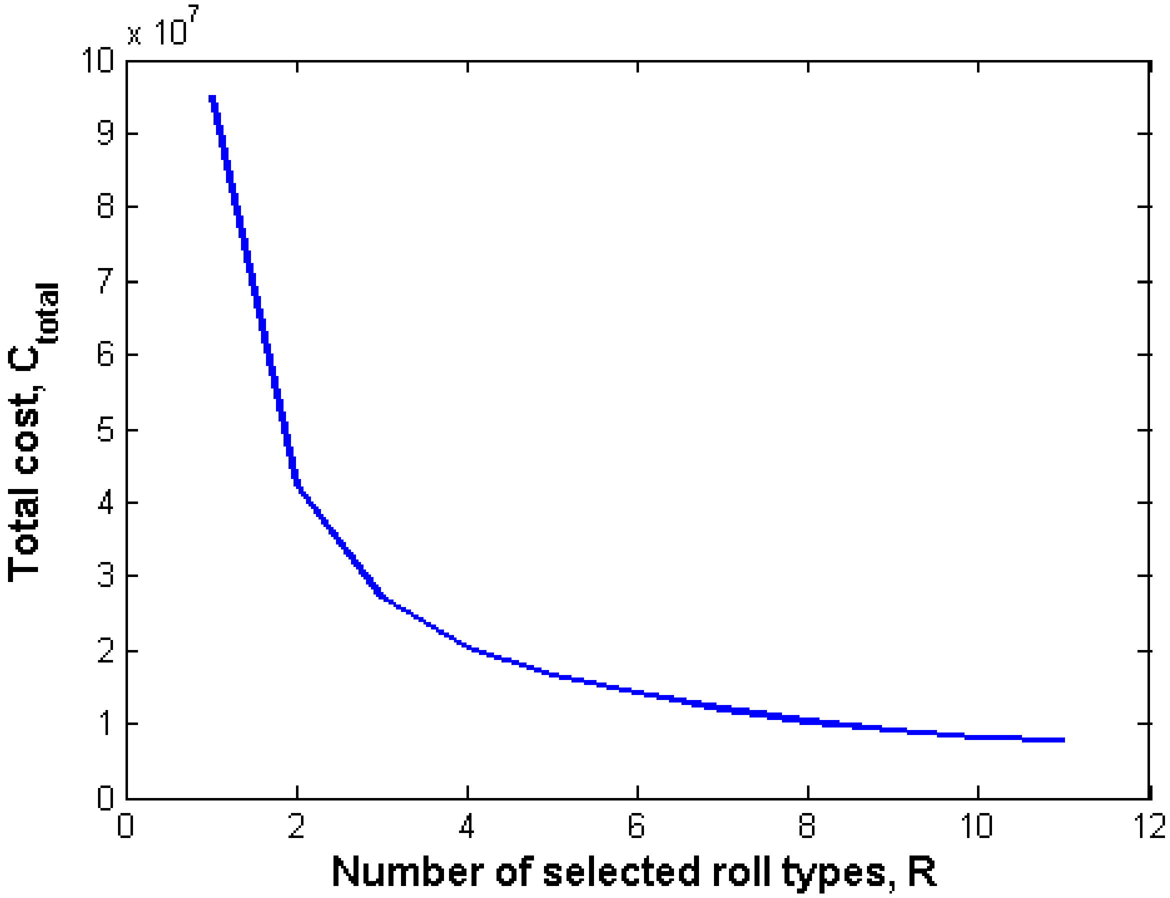

Figure 2. Total waste (m²) for the computational example, depending on the number of selected roll types.

C. Optimal Selection of Roll Types for Different Number of Selected Roll Types

"How does the assortment change depending on the number of selected roll types?"

Figure 3 answers this question. This second type of analysis involves the observation of the specific roll types that are selected, for different values of R. Each path in the figure corresponds to a specific value of R, and the circles on the paths correspond to the widths of the selected rolls. This type of graphical analysis shows which roll types consistently appear in the assortment. For example, in Figure 3, certain roll widths appear again and again after a certain value of R.

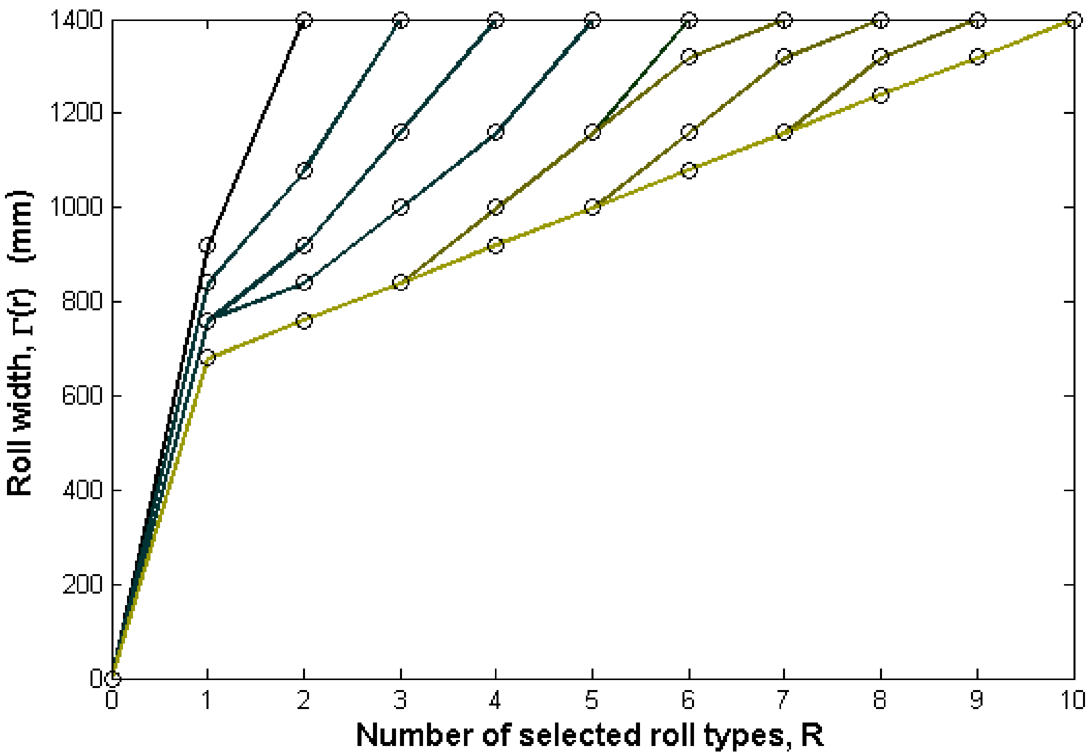

Figure 3. Optimal roll widths for the computational example, depending on the number of selected roll types.

VI. CONCLUSION AND FUTURE WORK

This paper reported work on a one-dimensional cutting stock problem in cardboard packaging. A novel network flow model has been developed to solve the problem and setup costs that depend on the number of elements in the assortment can be incorporated through the new model. Graphical demonstration of the trade-off between inventory cost and cost of waste has been demonstrated on a computational example.

ACKNOWLEDGMENT

The authors thank Sabancı University alumni Halide Ezgi Özbağ for bringing the problem into the attention of the authors.

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