Graduated Penalty Scheme*

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October 10, 2008

Abstract

Evaders of any dues such as local council tax, motor vehicle tax, tv license fees etc., if detected, can pay promptly the dues plus any fine or postpone, which usually means a larger fine, and potentially imprisonment if payments are not made in full. Dominant among the likely reasons for this *graduated penalty scheme* are 'default tracking costs' and 'imprisonment costs'. Although in conflict with the state's basic objective of deterring evasion, a graduated penalty scheme may emerge as an optimal balance between the dual objectives of deterrence and settlement delay minimization. Based on a welfare-maximizing objective where the state determines optimal monitoring intensity and time profile of fines, an intuitively plausible condition is derived such that the fine scheme is of the graduated type. ***JEL* Classification Numbers: K42, K14, H71**.

Key Words: Evasion/crime, fine default, graduated penalty.

*We thank an anonymous referee of this journal for helpful suggestions, and Parkash Chander for useful discussions on the topic. Any errors are our own responsibility.

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1 Introduction

Collection of unpaid fines is an important problem in law enforcement. According to the year 2005 figures, total unpaid fines in the United Kingdom was of the order of £383 million, with a collection rate around 50%. While such a low collection rate can be attributed to large costs of tracking down defaulters, one cannot escape the feeling that the lack of credible serious consequences for persistent defaulters is one prime reason. To tackle the problem, a package of measures announced by Lord Chancellor Lord Falconer in 2005 was experimented that produced considerable increases in fine collection rates.[1] Among the measures proposed, the most important one, in our view, is to increase initial fines by up to 50% for offenders who fail to pay up on time and even sanction imprisonment if fines do not succeed, thus declaring persistent defaults to be of the category of a criminal offense.

In this paper we ask, why such a graduated fine system with imprisonment as potential outcome for persisting defaulters could be optimal. We decompose the law enforcement process with regard to routine crimes, say, nonpayment of some dues such as local community/council tax, motor vehicle tax, tv license fees etc., into three components within a two-period model. Residents, privately informed about their income potential and current (first-period) income, first decide on whether to pay the dues. Second, evaders if detected face a fine f, which, along with the original amount owed, they either settle promptly or postpone. Third, if an evader postpones, the fine becomes e. Nonpayment of this fine plus the dues out of the realized second-period income leads to a jail term.

The standard Beckerian argument suggests that both f and e be set at their maximal statutory levels to economize on deterrence costs. However, large (or maximal) fines can induce a significant proportion of evaders to postpone payments, hence, increase default tracking costs. A graduated penalty scheme $f < e$ can emerge as an optimal balance between 'deterrence costs' on the one hand and 'default tracking costs' and 'imprisonment costs' on the other. We show that the target group whose penalty-settlement behavior is intended to be influenced by the graduated penalty scheme is a section of "wealth-unconstrained" residents whose

[1] See the BBC News: "Fine dodgers face new penalties," 3 January, 2005 available at http://news.bbc.co.uk/1/hi/uk/4141475.stm. The pilot projects were tested in Cambridgeshire, Cumbria, Devon and Cornwall, South Yorkshire, Cheshire and Gloucestershire, leading to an increase in fine collection by more than 90%.

income realizations in the first period are not too high; these residents can afford paying a large fine, if imposed in period 1, but may still consider postponing due to high marginal benefits from immediate consumption (with tight budgets) if in comparison there is only a small risk of eventual non-payment and imprisonment due to extremely unfavorable realization of the second-period income. To induce these residents to settle fines promptly (and thus save on default tracking costs and potential imprisonment costs), the government would rather impose a smaller fine in the first period but raise the fine, if settlement is deferred.

Our argument for ascending fines on defaulters is not related to discounting, nor does it stem from differential rates of discount for the residents and the state; we assume zero discounting for all the parties. Rather, the crucial element that generates strategic evasion and default decisions is the information structure about current and future incomes: Residents privately know their current, but not future, ability to pay. Residents with very low current incomes can hardly settle fines, and residents with sufficiently high current incomes would always settle fines. Among the rest, those with reasonably sound future income potentials but neither too low nor too high a current income draw will take a somewhat optimistic view of the future (in terms of ability to settle fines and the dues), accordingly, choose to default at an insignificant risk of eventual imprisonment sentence. On the other hand, those with sufficiently large current incomes will be pessimistic about their future ability to pay if their income potentials are not high, so, may pay promptly if they evade and are detected.

The results and graduated penalties approach in this paper can broadly be related to several themes in the literature. One of these is the literature on non-maximal optimal penalties.[2] In our setup fines may be non-maximal for two reasons – one, imposing too high a fine may involve (expected) loss of revenues because residents either pay in full or pay nothing due to lack of income verification (similar to wealth non-verification in Polinsky, 2006); two, non-maximal fines can save on default tracking costs as well as potential imprisonment costs. Another theme is

[2] Explanations for non-maximal sanctions range from risk aversion of residents (Polinsky and Shavell, 1979), adjudication errors and litigation costs (Rubinfeld and Sappington, 1987), impact of sanctions on probability of conviction (Andreoni, 1991) to marginal deterrence considerations (Shavell, 1992; Wilde, 1992; Mookherjee and Png, 1994), and impossibility of adjusting monitoring as a function of the seriousness of the offense (Mookherjee and Png, 1992).

explaining why more severe penalties are imposed on repeat offenders.[3] A preliminary observation in this literature is that some inadequacy in deterrence must exist to warrant conditioning sanctions on offense history. In our setup defaulting on the fine is a further offense, although, it must be emphasized, not a repeat offense as such, in addition to the basic crime of evasion.

Since tracking costs are ultimately fine collection costs, our graduated fines recommendation appears in contrast with a result by Polinsky and Shavell (1992), namely, that optimal fines are larger in the presence of fine collection costs: introducing fine collection costs increases the social harm of the offense and calls for stronger deterrence, hence, an increase in the magnitude of fines. This logic holds in a static setup and depends crucially on the assumption that all detected offenders pay the fines. In our setup, residents have the option to postpone settling their dues and fines. We also highlight an important law enforcement hurdle – non-observability of residents' incomes or wealth – the aspect addressed by Polinsky (2006), who shows that wealth non-observability may lead the authorities to choose a fine–imprisonment combination that induces high-wealth offenders to pay a large fine and low-wealth offenders to face the risk of imprisonment; see also Kim (2007). In contrast to Polinsky's setup, in our model offenders of *all* income levels prefer fines over imprisonment except that some may not have any choice but to accept the latter due to poor income realizations.

The next section develops the model. The formal analysis is contained in sections 3 and 4, and section 5 concludes. The proofs are contained in an Appendix.

2 The model

We consider an economy with a continuum of residents (alternatively, firms, depending on the context) and the state's law enforcement department. Some residents are culpable who each owes, at the outset, some fixed dues x to the state; x could be local council tax, tv license fees, motor vehicle tax, pollution permit fees etc. We will assume all residents are corruptible, i.e., would evade paying the dues if they could get away with it.

There are two periods. The discount factor is common to all residents and the state and will be assumed to equal one, which is equivalent to a zero real interest

[3] See Polinsky and Rubinfeld (1991), Emons (2003; 2007), and Miceli and Bucci (2007).

rate. The last characteristic of a resident is his *income potential* parameter k which, from the point of view of all residents, is an i.i.d. draw from the distribution function $G([1, \kappa])$ with a corresponding density function, $g(k)$, and observed by the resident before any actual income realization occurs. In each period t, the income y_t of a resident is then drawn independently from the 'Uniform' distribution over the support $[0, k]$.[4] Thus, residents face income uncertainty in each period within the bounds $[0, k]$, and this uncertainty is resolved at the start of each period. The type of a resident (i.e., whether culpable or not), the income potential k and income realizations y_1 and y_2, are all private knowledge.

The state relies on two activities, monitoring and enforcement, to ensure that residents pay the fixed dues x, as well as the fines that will be administered as a function of the period in which full payment is made. The monitoring activity, limited to period 1, is of general nature: to detect evasion by any culpable resident with probability $0 < \mu < 1$, the state must incur a cost $c(\mu)$ that is strictly increasing and strictly convex in μ, with $c(0) = 0$.

Enforcement, we assume, is one of perfect tracking – detected evaders can never escape from the system unpunished if the dues and the fines are not fully settled. A resident who is detected has two options: to settle $x + f$ (f is a fine) immediately or postpone payment until period 2. The amount due in period 2 becomes $x + e$. By the end of period 2 an evader must pay the dues or face a severe sanction that we term *imprisonment*.[5] The fines (f, e) have a common statutory upper bound $\overline{f}$. Any nonpayment requires chasing up by enforcement authorities through letters, reminders, court orders etc., which are costly. Imprisonment is a last resort to ensure that the residents comply with the authority and its imposition is uniform over anyone failing to pay the *full* amount due (original plus accumulated fines).

All residents consume their entire disposable income in period 1, and start in period 2 afresh relying on period 2 income draw. To simplify the analysis, we rule

[4]The income, y_t, can be thought of as the income in excess of the absolute minimum income necessary for survival. If $y_t > 0$, it adds to one's utility over and above the bare survival minimum. It is this extra utility summed over a two-period horizon that a resident tries to maximize.

[5]The authority can try to send defaulters to prison selectively by auditing and proving that a defaulter has acted opportunistically (rather than defaulted due to pure bad income draw). If auditing were costless and perfect, opportunistic defaults could be eliminated. Under imperfect auditing, there is scope for opportunistic defaults and the fine schedule will play a role in affecting settlement incentives. As a working assumption, we rule out the possibility of auditing. On auditing and its implications for fines vs. imprisonment to control wrongdoing, see Levitt (1997).

out private saving and borrowing.[6] Residents are risk averse and have a common and constant per-period (net) income utility function $u(y)$ with $u'(.) > 0,\ u''(.) < 0$.

Our main question concerns the time profile of the fines – should the fines in both periods be maximal, or should one or both be non-maximal? Also, should the government adopt a *graduated penalty* structure: $f < e \leq \overline{f}$?

The sequence of events is as follows.

- *Policy choices and draws of nature*: The state determines f and e. Residents privately learn whether they are culpable or not about the dues x, and their income potential k.

- *Period 1*: Residents privately observe their first-period income y_1. Culpable residents who chose not to pay x, if detected, decide whether to settle $x + f$ or postpone. Evaders who settle receive utility $u(y_1 - x - f)$, and those who postpone receive $u(y_1)$.

- *Period 2*: Residents privately observe their second-period income y_2. Detected evaders who postponed settlement decide whether to settle the dues for the final time or go to jail. Anyone not having enough second period income must go to jail. The second period utility from settling $x + e$ is $u(y_2 - (x + e))$, whereas the corresponding utility of a defaulting resident is $u(y_2) + u_{\text{jail}}$. The evader who settled in period 1 receives in period 2 utility $u(y_2)$.

Note that a resident enjoys the full benefits of his second period income even if he continues to default and thus ultimately ends up in prison.[7] However, we assume that residents would always avoid jail whenever possible:

$$-u_{\text{jail}} > u(x + e) - u(0). \tag{1}$$

[6] It is not unreasonable to assume that if a resident lacks the income to pay the fines then he has no choice but to default. Moreover, there might be imperfections in the capital market making it impossible for residents to borrow against future income potentials. In a dynamic tax-evasion model, Andreoni (1992) allowed similar imperfections where tax-payers could not borrow against future bequest incomes. In our context, for the government office to deal with residents on a one-to-one basis and arrange a payment schedule according to income realization is very costly, even if one ignores the difficulty of verifying incomes. Our assumption that the residents consume their current disposable income is to keep the analysis free of the complexity of consumption-saving decisions and focus mainly on the penalty default decisions.

[7] The enjoyment of period 2 income accrues in the form of expenses spent before going to prison, or possibly as a result of income transfers to family members, friends or relatives.

While the stated condition is specifically for a resident with $y_2 = x + e$, strict concavity of preferences guarantees that all residents with $y_2 \geq x + e$ would avoid jail by paying up the dues and the accumulated fines in period 2. We normalize $u(0) = 0$, which implies $u_{\text{jail}} < 0$.

3 Default decision

Given the fines and the enforcement policy, we determine in this section the optimal behavior of a detected evader – whether to settle or postpone – as a function of his income potential k and period 1 realization y_1.

Since borrowing is ruled out, if $y_1 < x + f$, the resident has no choice but to postpone payment. For residents with period 1 income $y_1 \geq x + f$, the settlement/postponement decision is based on the following utility comparison:

$$\begin{aligned}&\max\{u(y_1 - x - f) + \frac{1}{k}\int_0^k u(z)\,\mathrm{d}z,\\ &u(y_1) + [\frac{(x+e)}{k}u_{\text{jail}} + \frac{1}{k}\int_0^{(x+e)} u(z)\,\mathrm{d}z + \frac{1}{k}\int_{(x+e)}^k u(z-(x+e))\,\mathrm{d}z]\}. \quad (2)\end{aligned}$$

The first term is the utility from settling $x + f$ in period 1, while the second term is the utility if payment is postponed until period 2. The following definition will be useful.

Definition 1. **(i)** *For any fixed* k*,* $y_1(k, f, e)$ *is the period 1 income level equalizing the two expressions in (2).* $y_1(k, f, e)$ *is unique when it exists.*

(ii) *For any fixed* y_1*,* $k(y_1, f, e)$ *is the income potential equalizing the two expressions in (2).* $k(y_1, f, e)$ *is unique when it exists.*

A detected evader with income potential k is indifferent between settling the fine plus the dues in period 1, and postponing until period 2, if his period 1 income is $y_1(k, f, e)$. Similarly, a detected evader with period 1 income y_1 is indifferent between settling and postponing if his income potential is $k(y_1, f, e)$. The following proposition describes the behavior of detected evaders as a function of their first-period incomes and income potentials:

Proposition 1. **(i)** *Fix* k, f *and* e*. A detected evader prefers settling his dues and fine in period 1 if* $y_1 > y_1(k, f, e)$*, and would postpone if* $y_1 < y_1(k, f, e)$*; if* $y_1 = y_1(k, f, e)$ *the evader is indifferent between the two options.*

(ii) *Fix y_1, f and e. A detected evader with period 1 income y_1 prefers settling his dues and fine in period 1 if $k < k(y_1, f, e)$, and would postpone if $k > k(y_1, f, e)$; if $k = k(y_1, f, e)$ the evader is indifferent between the two options.*

Note the implications of the two parts of Proposition 1. Since there is no link between realized income in period 1 and the given income potential in period 2, a detected evader with the same k but smaller y_1 may prefer not to settle in period 1 because of the relatively high marginal utility associated with low incomes. On the other hand, given a first-period income, a larger income potential for period 2 may induce the evader to gamble on nonpayment and take the risk of a jail term (the likelihood of which must remain small). Thus, in terms of first-period incomes, relatively *poor residents* gamble and *better-off* ones settle, which is intuitive. In terms of period 2 income potentials, residents with relatively *sound* potential would gamble. We like to make two remarks about this last observation. *First,* it's not that somebody who can settle today doesn't settle simply because he expects to be rich/wealthy tomorrow.[8] Rather a more apt description would be that, even if one's current income allows him to barely settle the fines today, his income may not be that high so that the resident might be hard-pressed to consume today with the realistic expectation that tomorrow's income would enable him to be solvent.[9] Thinking carefully, such expectations (or behavioral pattern) are not unusual by any means and may even be the defining characteristics of defaulters who go for instant gratification and place a premium on today's consumption in the hope of an improved future. *Second,* as shown in part (i) of Proposition 1, anyone with sufficiently high first-period income would settle irrespective of his income potential; thus, part (ii) of Proposition 1 should not be viewed as an unconditional statement – after all, the critical k depends on y_1 and, as will be shown in Proposition 2, is an increasing function of y_1.

[8] If true, this would have resulted in a counter-factual prediction – if an empirical distribution of the second-period income were to be estimated then one would see that the defaulters are systematically richer than non-defaulters, which is clearly unrealistic.

[9] This is similar in flavor to an interesting observation made by Andreoni (1992), who argues that if an agent expects to receive a generous bequest in the future but cannot borrow against the bequest (because it cannot be used as collateral, just like our future income potential cannot be used as collateral), then such an agent will cheat in the current period by evading taxes and thereby transfer consumption from the future period to the current period even if this means detection and punishment. In our context default on penalty payment can be interpreted as an act of "borrowing" against one's expected future income, as in Andreoni's setup.

The prospect of a jail term can induce settlement in period 1 even if the fines are not of the *graduated* type, in particular for a detected evader with a high period 1 income but a low income potential k. Such a person would prefer to pay up in period 1 even if $f \geq e$ because the marginal utility in period 1 is low and, moreover, a low income potential means a high probability of default if payment is postponed, hence a high risk of ending up in jail. This further indicates that, in principle it is possible to even allow the second period fine, e, to be lower than the first period fine, f, and still have some residents pay up in period 1. Our observations on the efficacy of graduated penalties, implied from a Corollary to Proposition 2 and subsequently Proposition 4 and section 4 analysis, should be viewed in this context.

Proposition 2. (i) *$y_1(k, f, e)$ is increasing in k and f, but decreasing in e, for any $y_1(k, f, e) < k$.*

(ii) *$k(y_1, f, e)$ is increasing in y_1 for any $k(y_1, f, e) < \kappa$.*

Thus, an increase in the first-period fine, f, leads to an increase in $y_1(k, f, e)$, hence, in more evaders postponing payment for any given income realizations in period 1. The opposite holds for the second-period fine, e: an increase in e leads to a decrease in $y_1(k, f, e)$, hence in fewer evaders postponing payment for any given income realizations in period 1. The following corollary obtains:

Corollary. *Raising the first-period fine would make fewer detected evaders pay up in period 1, whereas raising the second-period fine would make more detected evaders pay up in period 1.*

This corollary accords nicely with the graduated penalty scheme recommendation, to raise the penalty for evaders who fail to settle promptly. Given that chasing defaulters costs the authority significant resources and considering the additional cost of feeding persistent defaulters in prison, any government's objective must include keeping the defaulters to a minimal size. Thus, any prudent government is likely to adopt a graduated penalty scheme. However, keeping the first-period fine low generates different types of costs. Besides reducing the government's fine collections, the lower fine would encourage additional residents, who previously might not have evaded, to evade now. So the ultimate decision on f cannot be viewed purely from the point of minimizing the size of defaulters for any *given* proportion of residents who evade. The choice of f (along with e and the monitoring intensity parameter, μ) would *endogenously determine* the size of defaulters. Successful law enforcement policy must seek a balance between prevention of the basic crime, evasion, and keeping defaulters to a manageable proportion. In the next section we turn to these issues.

4 Determination of optimal fines

We now explicitly formulate the social welfare function so that the question of the optimal fine schedule can be properly addressed. The analysis in this part will require several technical assumptions. While some of the assumptions are for reasons of tractability, we will also discuss the underlying economic reasons/motivations.

A proper specification of the welfare function requires specification of the various decisions available to a representative resident. The expected payoff from evasion, when the plan is also to postpone settlement, is:

$$\begin{aligned} U_{Evade}(y_1, k|postpone) &= u(y_1) + \mu X + (1-\mu)\frac{1}{k}\int_0^k u(z)\,\mathrm{d}z \\ &= u(y_1) + \mu[X - \frac{1}{k}\int_0^k u(z)\,\mathrm{d}z] + \frac{1}{k}\int_0^k u(z)\,\mathrm{d}z, \end{aligned} \quad (3)$$

where X is the expected second-period payoff from postponing settlement of $x+f$, given by:

$$X = \frac{(x+e)}{k}u_{\text{jail}} + \frac{1}{k}\int_0^{(x+e)} u(z)\,\mathrm{d}z + \frac{1}{k}\int_{(x+e)}^k u(z-(x+e))\,\mathrm{d}z.$$

The expected payoff from evasion when one also settles on detection, assuming income permits, is

$$U_{Evade}(y_1, k|settle) = \mu u(y_1 - x - f) + (1-\mu)u(y_1) + \frac{1}{k}\int_0^k u(z)\,\mathrm{d}z. \quad (4)$$

Finally, paying up the dues x (rather than evade) yields the payoff:

$$U_{Pay}(y_1, k) = u(y_1 - x) + \frac{1}{k}\int_0^k u(z)\,\mathrm{d}z. \quad (5)$$

To rule out the uninteresting case in which a resident has no choice but to evade, we assume that the income potential parameter k is never below the dues plus the maximal penalty, i.e., $x + \overline{f} < 1$ (recall, the support of k is $[1, \kappa]$). So, if a resident realizes his full income potential then he remains solvent. Note, however, it is possible that actual income of a resident in some period is much below x (or $x + \overline{f}$), as $y_t \in [0, k]$.

Definition 2. *Fix any* $0 < \mu < 1$*, and* f*. Define* y_E *to be such that the payoff from non-evasion in (5) equals the payoff from evasion-and-settlement in (4), when such an income level exists:*

$$u(y_E - x) = \mu u(y_E - x - f) + (1 - \mu)u(y_E).^{10} \tag{6}$$

Similarly, define y_X *as the first-period income such that the payoff from non-evasion in (5) equals the payoff from evasion-and-postponement in (3):*[11]

$$u(y_X - x) = u(y_X) + \mu[X - \frac{1}{k}\int_0^k u(z)\, \mathrm{d}z]. \tag{7}$$

Note that y_E depends only on the first-period penalty, f, and the detection probability, μ, but not on the income potential, k, or the second-period penalty, e. On the other hand, y_X depends on k, μ and e, but not on f. We therefore write y_E as $y_E(f, \mu)$ and y_X as $y_X(k, e, \mu)$.

It is implicit that the cutoff income levels should separate residents into two groups according to income levels – one group pays the dues while the other group evades. Note that the difference between the non-evasion payoff in (5) and the payoff from evade-and-postponement choice in (3) is increasing in y_1. Hence the following result can be formally noted:

Lemma 1. ***(Regularity of evasion: Part I)*** *A resident would prefer to evade-and-postpone over non-evasion if* $y_1 < y_X(k, e, \mu)$*, and prefer non-evasion over evasion-and-postponement if* $y_1 > y_X(k, e, \mu)$*; the resident is indifferent between the two options at* $y_1 = y_X(k, e, \mu)$*.*

One cannot draw a similar conclusion (as in Lemma 1) by comparing the non-evasion payoff in (5) with the evade-and-settlement payoff in (4); that is, a resident who prefers non-evasion over evade-and-settlement is not guaranteed to choose not

[10] If there is no such income in the maximal potential income range $[0, \kappa]$ satisfying (6), then let $y_E = \infty$ if for all incomes payoff from evasion-and-settlement is higher than the non-evasion payoff, and $y_E = 0$ if for all incomes payoff from evasion-and-settlement is lower than the non-evasion payoff.

[11] To avoid the problem of existence of the defined variable y_X, define similar to the previous footnote, $y_X = \infty$ if for all incomes payoff from evasion-and-postponement is higher than the non-evasion payoff, and $y_X = 0$ if for all incomes payoff from evasion-and-postponement is lower than the non-evasion payoff.

to evade at larger incomes. The reason is not difficult to see as, after all, the resident's absolute risk-aversion may be decreasing in income in which case one may even expect greater evasion incentives at higher income levels. For a tractable welfare function specification and further analysis, we will assume that larger income realizations in period one are more likely to induce residents paying their dues:

Assumption 1. ***(Regularity of evasion: Part II)*** *A resident would prefer evasion-and-settlement over non-evasion if $y_1 < y_E(f, \mu)$, and the ranking is reversed if $y_1 > y_E(f, \mu)$.*

We do not go into any formal characterization of preferences that would conform to Assumption 1. We conjecture that the assumption is likely to hold under non-decreasing absolute risk aversion and possibly with some other restrictions.

To address issues of optimal penalties, we need to look at their comparative static effects on the two evasion cutoff incomes y_E and y_X and also the related impact on the settlement/postponement decisions through changes in $y_1(k, f, e)$. With this objective in mind, differentiating both sides of (6) with respect to f and simplifying, we obtain:

$$\frac{\partial y_E}{\partial f} = \frac{-\mu u'(y_E - x - f)}{u'(y_E - x) - \mu u'(y_E - x - f) - (1-\mu)u'(y_E)}.$$

The sign indeterminacy in the denominator term above makes it impossible to unambiguously sign $\frac{\partial y_E}{\partial f}$, for all values of μ and f. However, we claim that for optimally chosen μ and f, it must be that $\frac{\partial y_E}{\partial f} < 0$. (If not, $\frac{\partial y_E}{\partial f} > 0$; notice $\frac{\partial y_E}{\partial f} \neq 0$ because clearly optimal $\mu \neq 0$. But then by lowering the penalty f, the cutoff y_E can be lowered, which in turn would lead to, on average, more residents preferring non-evasion over evasion-and-settlement (by Assumption 1) that can only improve social welfare – a contradiction.) But we have not yet formally defined social welfare, which we cannot without assuming a global monotonicity condition (i.e., for all μ and f values): $\frac{\partial y_E}{\partial f} < 0$. To overcome the circularity problem, we impose the following assumption:

Assumption 2. **(Beckerian-type penalty impact on evasion)** *$y_E(f, \mu)$ is decreasing in the first-period penalty f, i.e., $\frac{\partial y_E(f,\mu)}{\partial f} < 0$.*

Intuitively, Assumption 2 can be justified if under the optimal policies an increase in first-period penalty, f, with everything else the same, induces more people to *not evade* the dues x than opt for evade-and-settlement. Raising f, however, is likely to force more residents (who evaded) to default because their first-period incomes may fall short of the combined total of $x + f$.

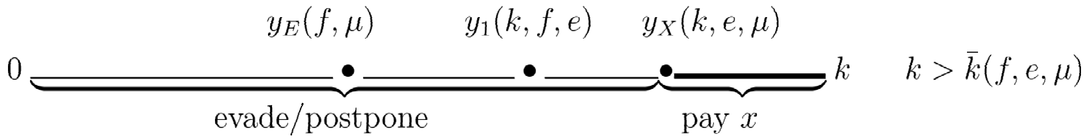
As indicated earlier, Assumption 2 is an intermediate step so that we can formally define the social welfare function. This assumption should, in principle, be verified at the socially optimal configurations of μ and f, but due to lack of explicit solutions we cannot do the verification. Instead, taking this intuitive assumption for granted (for which we have provided some justification above), we derive a qualitative result regarding the optimal fine schedule.

The relative positions of $y_E(f,\mu)$ and $y_X(k,e,\mu)$ determine whether or not a resident with income potential k will ever plan to evade and settle if detected. We note the following properties:

Lemma 2. *$y_X(k,e,\mu)$ is decreasing in e and increasing in k. Moreover, $y_X(k,e,\mu) \geq y_E(f,\mu)$ if and only if $y_1(k,f,e) \geq y_E(f,\mu)$.*

Lemma 2 confirms that the behavior of $y_X(k,e,\mu)$ and $y_1(k,f,e)$ with respect to e and k are qualitatively the same (see Proposition 2). The second statement follows from inspection of the payoff functions in (3), (4) and (5); it is an implication of the requirement that plans of action be time-consistent.[12]

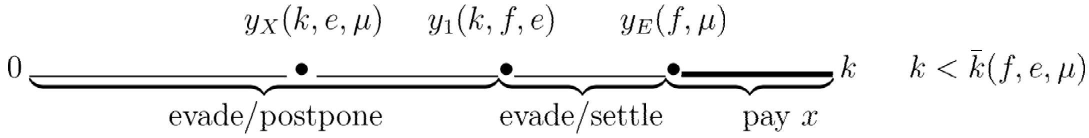
We characterize in Proposition 3 the optimal choice of a type-k resident as a function of his first-period income. The characterization breaks down the income ranges according to evasion/non-evasion and settlement/postponement behavior, which we are going to use below in the specification of the social welfare function. When $y_X(k,e,\mu) \geq y_E(f,\mu)$, the option of evade-and-settle is dominated by non-evasion for $y_1 \geq y_X(k,e,\mu)$, and by evade-and-postpone for $y_1 < y_X(k,e,\mu)$.



When $y_X(k,e,\mu) < y_E(f,\mu)$, the resident will evade if his income is below $y_E(f,\mu)$ and the optimal action upon detection is determined by how his income

[12]If Lemma 2 is violated, there exist income realizations at which it is optimal to evade with a plan to postpone if detected but ex-post deviate from the plan and opt for settlement upon detection. See the proof.

compares with $y_1(k, f, e)$.[13]



Proposition 3. *(i) If $y_X(k, e, \mu) \geq y_E(f, \mu)$, then the type-k resident never chooses the option of evade-and-settlement. The optimal choice is to pay x if $y_1 \geq y_X(k, e, \mu)$, evade and postpone upon detection if $y_1 < y_X(k, e, \mu)$.*

(ii) If $y_X(k, e, \mu) < y_E(f, \mu)$, the resident may choose evade-and-settlement depending on the first-period income. The optimal strategy becomes: pay x if $y_1 \geq y_E(f, \mu)$, evade and settle if $y_1 \in [y_1(k, f, e), y_E(f, \mu))$, and evade and postpone if $y_1 < y_1(k, f, e)$.

An implication of the fact that y_E does not depend on k (by (6)) whereas y_X is increasing in k (by Lemma 2) is that the residents for whom the option evade-and-settlement is strictly dominated (who, therefore, depending on their y_1 would choose either non-evasion or evade-and-postpone) must have large income potentials, at the upper tail of the interval $(1, \kappa]$. The following 'critical income potential' definition is used in lemmas 3 and 4:[14]

Definition 3. *$\bar{k}(f, e, \mu)$ is a critical income potential such that $y_X(k, e, \mu) = y_E(f, \mu)$.*

Lemma 3. *$y_X(k, e, \mu) > y_E(f, \mu)$ for $k > \bar{k}(f, e, \mu)$, and $y_X(k, e, \mu) < y_E(f, \mu)$ for $k < \bar{k}(f, e, \mu)$, given any policy choices e, f and μ.*

Lemma 4. *$\bar{k}(f, e, \mu)$ is increasing in e and decreasing in f.*

The welfare function specification below takes evasion of dues x as the main crime, which the state seeks to optimally deter through fines and costly monitoring

[13]The critical incomes $y_1(k, f, e)$ and $y_X(k, e, \mu)$ differ because the former is conditional on evasion and detection whereas the latter is based on an ex-ante comparison, before the monitoring activity, between the plans to pay dues right away and evade and postpone upon detection. Note that if 'evade and postpone' is the optimal plan of action ex-ante (before monitoring,) then it must also be optimal to postpone ex-post (upon detection.)

[14]The income potential $\bar{k}(f, e, \mu)$ should not be confused with $k(y_1, f, e)$; the latter is a critical income potential we defined in Section 3 to classify detected evaders according to their choices between settlement and postponing.

and enforcement activities. Let us denote tracking costs by T, and imprisonment costs by J, per individual.

Fix a policy (f, e, μ). For a resident with income potential k, let P_k, S_k and XP_k denote the sets of first-period incomes such that the optimal choice is, respectively, not to evade, evade-and-settle, and evade-and-postpone if detected. The sizes of P_k, S_k and XP_k may in general depend on all four variables k, f, e and μ. Note that $S_k = \emptyset$ if $y_X(k, e, \mu) > y_E(f, \mu)$ (by Proposition 3), that is, if $k > \bar{k}(f, e, \mu)$ (by Lemma 3).

The state should maximize the expected dues collection net of the total monitoring and expected enforcement costs.[15] We write the objective function under the assumption that each of the sets P_k and XP_k is non-empty for all $k \in [1, \kappa]$; no matter the income potential k, a resident always has some low y_1 realizations at which he evades (and postpones) and large y_1 realizations close to k at which he pays the dues x.[16] The social welfare function can be written as follows:

$$
\begin{aligned}
W = {} & \\
& \int_1^{\bar{k}} \left[\left\{ \int_{y_E(f,\mu)}^{k} x \cdot \frac{1}{k} \, \mathrm{d}y_1 \right\} + \mu \left\{ \int_{y_1(k,f,e)}^{y_E(f,\mu)} x \cdot \frac{1}{k} \, \mathrm{d}y_1 + \int_0^{y_1(k,f,e)} W_{\text{post}} \cdot \frac{1}{k} \mathrm{d}y_1 \right\} \right] \mathrm{dG}(k) \\
& + \int_{\bar{k}}^{\kappa} \left[\int_{y_X(k,e,\mu)}^{k} x \cdot \frac{1}{k} \, \mathrm{d}y_1 \; + \mu \int_0^{y_X(k,e,\mu)} W_{\text{post}} \cdot \frac{1}{k} \mathrm{d}y_1 \right] \mathrm{dG}(k) - c(\mu), \qquad (8)
\end{aligned}
$$

where

$$
W_{\text{post}} = -T - J \int_0^{x+e} \frac{1}{k} \, \mathrm{d}y_2 + x \int_{x+e}^{k} \frac{1}{k} \, \mathrm{d}y_2 = -T - J \left(\frac{x+e}{k} \right) + x \left(1 - \frac{x+e}{k} \right)
$$

is the state's continuation net expected dues collection from any detected evader who would postpone settlement until period 2.

The first line in (8) captures the residents whose income potentials induce evasion-and-settlement at some middle range of first-period income realizations.

[15] Fines f and e are excluded because they are mere transfers. The qualitative implication of allowing for a state objective that is increasing in fine collections is predictable: the case for maximal fines will become stronger. Excluding the fines is consistent with the utilitarian perspective of maximizing the welfares of all the parties involved – state and the residents.

[16] This is guaranteed if κ is sufficiently large and if the lower bound of the potential income, 1, is sufficiently larger than the dues, x. Allowing for residents who evade at all income realizations or pay dues at all income realizations would substantially complicate the objective function expression without adding/changing any economic insight.

The second line expression captures high potential income residents who are quite optimistic about their second period income to never choose settlement if they evade—they plan to pay $x + e$ in period two, so, choose to postpone. The state maximizes social welfare W by appropriately choosing (μ, f, e) subject to a statutory upper bound $\bar{f}$ to the penalties. Denote the solution by (μ^*, f^*, e^*). We can now state our main result.

Proposition 4. *(i) The optimal detection probability is strictly positive: $\mu^* > 0$.*

(ii) The second-period fine need not be maximal; $e^* < \bar{f}$ is possible.

(iii) The first-period fine f^* is less than the second-period fine e^* if

$$\int_1^{\bar{k}} \left[\frac{x}{k}(\mu^* - 1)\frac{\partial y_E}{\partial f} \right] \mathrm{dG}(k) < \int_1^{\bar{k}} \left[\frac{\mu^*}{k}(x - W_{post})\frac{\partial y_1}{\partial f} \right] \mathrm{dG}(k), \tag{9}$$

evaluated at (μ^*, f^*, e^*).

While not formally noted, it should be obvious that the optimal detection probability $\mu^* < 1$ because 100% detection of evasion is impossible, if not prohibitively costly.

The second-period optimal fine need not be maximal for the simple reason that an increase in this fine induces a revenue loss and a cost increase: Given a fixed population of detected evaders, fewer will be able to settle the accumulated fines plus original dues $x + e$ and more will go to prison, thus, collection of original dues will fall while imprisonment costs will rise. The optimal e balances this negative effect against the standard deterrence-enhancing effect, i.e., the benefits that accrue from the reduction in the measure of evaders.

The first-period fine affects (9) through $\bar{k}$ and the partial derivatives $\partial y_E/\partial f$ and $\partial y_1(k, f, e)/\partial f$. But the intuition for the possibility that $f^* < e^*$ is simple. An increase in f will influence the welfare objective through the behavior of residents whose income potentials are smaller than $\bar{k}$; these residents choose to evade and settle if detected at some intermediate income range within their potentials. The **welfare-enhancing** effect of a rise in f operates through the left-hand side of (9), by reducing y_E at each $k < \bar{k}$, i.e., by increasing the measure of residents who choose not to evade dues in the first place. This term is positive because $\partial y_E/\partial f < 0$ (by Assumption 2) and $\mu^* < 1$. A small μ^* is likely to contribute to the size of this effect. The **welfare-reducing** effect of a rise in f appears in the right-hand side expression in (9). Because $\partial y_1(k, f, e)/\partial f > 0$ (by Proposition 2), a measure of evading residents who previously were planning to settle if detected will now switch to postponing if detected. A large μ^* is likely to contribute to this effect.[17]

Finally, condition (9) shows that, while tracking costs and imprisonment costs are not the sole reasons for the graduated penalty result, these two are clearly the dominant contributing factors: increases in T and J would lower W_{post} and thus increase the right-hand side expression in (9) (with no change in the left-hand side expression).

5 Conclusion

In this paper we study the tradeoffs involved in determination of the optimal fine schedule for detected offenders who have an option to pay soon or postpone. The cost of reducing any of the scheduled fines is the standard deterrence-diluting effect. The expected benefits differ according to whether the fine applies to the early stages following detection or to the final stages where the evader is offered a last chance of settlement before implementation of a more serious sanction such as imprisonment. Reducing the terminal fine will economize on imprisonment costs because ex-post, fewer people will go to jail. On the other hand, reducing the initial fine will induce some detected evaders to switch from postponing payment to early settlement, which economizes on default tracking costs as well as expected imprisonment costs.

We made several simplifying assumptions in the analysis. One of these is that all detected evaders are tracked down. Under imperfect tracking a fraction of evaders can get away without paying the fine, as a result fewer evaders will settle in period 1. Then the case for graduated fines becomes even stronger because tracking costs will increase, hence, adopting a steeper fine schedule is likely to be more cost-effective despite the negative impact on deterrence. We also ruled out private saving to smooth out consumption across the two periods. Allowing residents to save will increase their control over their future ability to pay (or act as a partial insurance against default and imprisonment) and generate an increase in strategic default decisions. This, in turn, strengthens the case for graduated fines. Finally, we ruled out the possibility that real incomes can easily be verified by authorities in the

[17] The optimal detection probability would be small if the monitoring technology is not so effective, displaying rapidly increasing monitoring costs. Note also that $\partial y_1(k, f, e)/\partial f$ depends on μ.

final stage, prior to imprisonment, if dues plus fines are not paid. The case of fully verifiable incomes would call for an income-specific fine schedule and is beyond the scope of this paper.

Appendix

Proof of Proposition 1. (i) The result follows from (2), using $u''(.) < 0$.

(ii) Write the difference between the payoffs from postponing until period 2 and payoffs from paying up in period 1 as:

$$\begin{aligned} D(k, y_1) &= u(y_1) - u(y_1 - x - f) - \frac{1}{k}\int_0^k u(z)\,\mathrm{d}z \\ &+ [\frac{(x+e)}{k}u_{\text{jail}} + \frac{1}{k}\int_0^{(x+e)} u(z)\,\mathrm{d}z + \frac{1}{k}\int_{(x+e)}^k u(z-(x+e))\,\mathrm{d}z] > 0. \end{aligned}$$

Differentiate D partially w.r.t. k to obtain:

$$\begin{aligned} \frac{\partial D}{\partial k} &= \frac{1}{k^2}\int_0^k u(z)\,\mathrm{d}z - \frac{1}{k}u(k) + [-\frac{(x+e)}{k^2}u_{\text{jail}} - \frac{1}{k^2}\int_0^{(x+e)} u(z)\,\mathrm{d}z \\ & -\frac{1}{k^2}\int_{(x+e)}^k u(z-(x+e))\,\mathrm{d}z + \frac{1}{k}u(k-(x+e))] \\ &= \frac{1}{k^2}[\int_{(x+e)}^k u(z)\,\mathrm{d}z - k\{u(k) - u(k-(x+e))\} \\ & -(x+e)u_{\text{jail}} - \int_{(x+e)}^k u(z-(x+e))\,\mathrm{d}z] \\ &= \frac{1}{k^2}[\int_{(x+e)}^k \{u(z) - u(z-(x+e))\}\,\mathrm{d}z - k\{u(k) - u(k-(x+e))\} - (x+e)u_{\text{jail}}] \\ &\underbrace{>}_{\text{by } u''(.)<0} \frac{1}{k^2}[\{k-(x+e)\}\{u(k) - u(k-(x+e))\} \\ & -k\{u(k) - u(k-(x+e))\} - (x+e)u_{\text{jail}}] \\ &= \frac{1}{k^2}[-(x+e)\{u(k) - u(k-(x+e)) + u_{\text{jail}}\}] \\ &\underbrace{>}_{\text{by (1)}} \frac{1}{k^2}[-(x+e)\{[u(k) - u(k-(x+e))] - [u(x+e) - u(0)]\}] \\ &\underbrace{>}_{\text{by } u''(.)<0} \frac{1}{k^2}[-(x+e)\{[u(k) - u(k-(x+e))] - [u(k) - u(k-(x+e))]\}] \\ &= 0. \end{aligned}$$

Since $D = 0$ for $k = k(y_1)$, it follows that $D > 0$ for all $k > k(y_1)$ and $D < 0$ for all $k < k(y_1)$. **Q.E.D.**

Proof of Proposition 2. (i) The result on the impact of k follows from Proposition 1(ii). An increase in k from k_0 to k_1 at income $y_1 = y_1(k_0)$ makes the net payoff $D(k, y_1)$ from postponing payment to period 2 positive, so, a larger income $y_1 > y_1(k_0)$ is necessary to restore $D = 0$ (recall, $\frac{\partial D}{\partial y_1} = u_1'(y_1) - u_1'(y_1 - x - f) < 0$). The impacts of f and e on $y_1(k)$ can be verified in a similar way.

(ii) Since $\frac{\partial D}{\partial y_1} < 0$, an increase in y_1 from y_{10} to some $\hat{y}_1$ with k fixed at $k(y_{10})$ makes the net payoff $D(k, y_1)$ become negative. Thus, a larger k is necessary and sufficient to restore $D = 0$ (because $\frac{\partial D}{\partial k} > 0$; see the argument in Proposition 1 proof). **Q.E.D.**

Proof of Lemma 2. Differentiate both sides of (7) with respect to k and rearrange to obtain:

$$
\begin{aligned}
&\underbrace{[u'(y_X - x) - u'(y_X)]}_{>0} \frac{\partial y_X}{\partial k} \\
= \quad & \mu \left[\frac{-(x+e)}{k^2} u_{\text{jail}} - \frac{1}{k^2} \int_0^{x+e} u(z) \mathrm{d}z - \frac{1}{k^2} \int_{x+e}^{k} u(z - (x+e)) \mathrm{d}z + \frac{1}{k} u(k - (x+e)) \right. \\
& \left. \quad + \frac{1}{k^2} \int_0^k u(z) \mathrm{d}z - \frac{1}{k} u(k) \right] \\
= \quad & \mu \left[\frac{-(x+e)}{k^2} u_{\text{jail}} + \frac{1}{k^2} \int_{x+e}^{k} u(z) \mathrm{d}z - \frac{1}{k^2} \int_{x+e}^{k} u(z - (x+e)) \mathrm{d}z \right. \\
& \left. \quad - \frac{1}{k} \{u(k) - u(k - (x+e))\} \right] \\
= \quad & \mu \left[\frac{1}{k^2} \int_0^{x+e} -u_{\text{jail}} \mathrm{d}z + \frac{1}{k^2} \int_{x+e}^{k} [u(z) - u(z - (x+e))] \mathrm{d}z - \frac{1}{k} \{u(k) - u(k - (x+e))\} \right] \\
\underbrace{\geq}_{\text{by (1)}} \quad & \mu \left[\frac{1}{k^2} \int_0^k [u(z) - u(z - (x+e))] \mathrm{d}z - \frac{1}{k} \{u(k) - u(k - (x+e)\} \right] \\
\geq \quad & \mu \left[\frac{1}{k} \{u(k) - u(k - (x+e)\} - \frac{1}{k} \{u(k) - u(k - (x+e)\} \right] \\
= \quad & 0,
\end{aligned}
$$

implying $\frac{\partial y_X}{\partial k} > 0$.

Similarly, differentiating both sides of (7) with respect to e and rearranging obtain:

$$
\begin{aligned}
& \underbrace{[u'(y_X - x) - u'(y_X)]}_{>0} \frac{\partial y_X}{\partial e} \\
= \quad & \mu \left\{ \frac{1}{k} u_{\text{jail}} + \frac{1}{k} u(x+e) + \frac{1}{k}[-u(x+e-(x+e))] \right\} \\
= \quad & \frac{\mu}{k}[u(x+e) + u_{\text{jail}}] \\
\underbrace{<}_{(by\ (1))} \quad & u(0) = 0,
\end{aligned}
$$

so that $\frac{\partial y_X}{\partial e} < 0$.

The critical income level $y_1(k, f, e)$ equalizes the payoffs in (3) and (4). At $y_1 = y_1(k, f, e)$, the resident, if he evades and is detected, is indifferent between postponing and settling. On the other hand, by definition the income level $y_1 = y_E(f, \mu)$ equalizes the payoffs in (4) and (5). Therefore, if the two critical income levels are identical, i.e., if $y_1(k, f, e) = y_E(f, \mu)$, then at the income level $y_1 = y_1(k, f, e) = y_E(f, \mu)$ the payoffs from all three options (pay, evade/postpone, evade/settle) must be equalized. Clearly, $y_X(k, e, \mu)$ is also equal to $y_1(k, f, e) = y_E(f, \mu)$. Thus, $\{y_1(k, f, e) = y_E(f, \mu)\} \Leftrightarrow \{y_X(k, e, \mu) = y_E(f, \mu)\}$.

To show $\{y_1(k, f, e) > y_E(f, \mu)\} \Leftrightarrow \{y_X(k, e, \mu) > y_E(f, \mu)\}$, consider first the case "$\Rightarrow$" and assume the statement is false. That is, suppose $y_1(k, f, e) > y_E(f, \mu)$ but not $y_X(k, e, \mu) > y_E(f, \mu)$, which implies $y_1(k, f, e) > y_E(f, \mu) \geq y_X(k, e, \mu)$, hence, existence of a range of incomes $[y_X(k, e, \mu), y_E(f, \mu)]$ in which the first best choice is to evade/settle, the second-best choice is not evading, and the worse option is evade/postpone. For any income y_1^s from this range, the resident must therefore prefer to settle rather than postpone if he chooses evasion, which implies $y_1^s > y_1(k, f, e)$ by Proposition 1(i). We have a contradiction.

For the case "$\Leftarrow$," similarly suppose $y_X(k, e, \mu) > y_E(f, \mu)$ but not $y_1(k, f, e) > y_E(f, \mu)$. We must have $y_X(k, e, \mu) > y_E(f, \mu) \geq y_1(k, f, e)$. For any income $y_1^s \in (y_E(f, \mu), y_X(k, e, \mu))$, the resident's ranking of preference, from best to worst, is: evade/postpone, not evading, evade/settle. Therefore, this resident prefers to postpone rather than settlement if he ever evades, implying $y_1^s < y_1(k, f, e)$, a contradiction. **Q.E.D.**

Proof of Proposition 3. (i) Suppose $y_E(f, \mu) \leq y_1 \leq y_X(k, e, \mu)$. The second inequality implies evasion-and-postponement is (weakly) better than non-evasion, whereas by Assumption 1, the first inequality implies that non-evasion is (weakly) better than evasion-and-settlement. Therefore, it must be better overall to evade-and-postpone.

Consider the case $y_1 < y_E(f,\mu) < y_X(k,e,\mu)$. By Proposition 1(i), we know that for a detected evader the difference between the payoffs from postponing payment and settlement (see (2)) is decreasing in y_1. Therefore, since the resident prefers evade-and-postpone over evade-and-settle at $y_1 \in [y_E(f,\mu), y_X(k,e,\mu)]$, his first-best choice for incomes $y_1 < y_E(f,\mu) < y_X(k,e,\mu)$ remains evade-and-postpone.

As for incomes $y_1 \geq y_X(k,e,\mu)$, it is clear that the optimal decision should be to not evade – follows straightforwardly from the definition of $y_X(k,e,\mu)$ and the fact that $u'' < 0$.

(ii) We claim that when $y_X(k,e,\mu) < y_E(f,\mu)$, the relevant critical income for the optimal strategy is $y_1(k,f,e)$, not $y_X(k,e,\mu)$.

We first show that in this case, $y_1(k,f,e) < y_X(k,e,\mu) < y_E(f,\mu)$. To see this, suppose $y_X(k,e,\mu) < y_E(f,\mu)$ and consider a first-period income $y_1 \in [y_X(k,e,\mu), y_E(f,\mu))$. At this income, the resident would rank various options as follows: *evade-and-settle* $\succ$ *not-evade* $\succsim$ *evade-and-postpone*. The first relation ($\succ$) follows from the fact that $y_1 < y_E(f,\mu)$ (by Assumption 1); the second relation ($\succsim$) follows from the definition of $y_X(k,e,\mu)$ and $u'' < 0$. If he evades, upon detection this resident must therefore prefer settlement to postponing, i.e., it must be that $y_1(k,f,e) < y_X(k,e,\mu)$ whenever $y_X(k,e,\mu) < y_E(f,\mu)$.

Applying similar reasoning it is easy to establish that at $y_1 \in [y_1(k,f,e), y_X(k,e,\mu)]$, the resident's optimal strategy is to evade-and-settle. As for the case $y_1 < y_1(k,f,e)$, the optimal strategy is to evade and postpone (by Assumption 1).

Thus, overall, in the case $y_X(k,e,\mu) < y_E(f,\mu)$, the critical income levels at which the optimal strategy switches are $y_1(k,e,f)$ and $y_E(f,\mu)$. **Q.E.D.**

Proof of Lemma 3. The statement $k > \bar{k} \Rightarrow y_X(k,e,\mu) > y_E(f,\mu)$ follows from the fact that $y_X(k,e,\mu)$ is increasing in k while $y_E(f,\mu)$ is independent of k. The same property implies $k < \bar{k} \Rightarrow y_X(k,e,\mu) < y_E(f,\mu)$. **Q.E.D.**

Proof of Lemma 4. We know that $\partial y_X/\partial e < 0$ and $\partial y_X/\partial k > 0$ (by Lemma 2), while $\partial y_E/\partial e = 0$ and $\partial y_E/\partial k = 0$. Therefore, following an increase in e, the income potential that restores the equality $y_X(k,e,\mu) = y_E(f,\mu)$ must be larger; that is, $\bar{k}(f,e,\mu)$ must increase. The proof of the impact of f on $\bar{k}(f,e,\mu)$ follows the same arguments (using also Assumption 2). **Q.E.D.**

Proof of Proposition 4. (i) That the optimal detection probability $\mu^* > 0$ is straightforward: if it were zero, no resident would pay the dues x, which is clearly not welfare-maximizing.

(ii) Differentiate W with respect to e to obtain:

$$\begin{aligned}
\frac{\partial W}{\partial e} &= \left[\int_{y_E(f,\mu)}^{\bar{k}} \frac{x}{\bar{k}}\mathrm{d}y_1 + \mu\left\{\int_{y_1(\bar{k},f,e)}^{y_E(f,\mu)} \frac{x}{\bar{k}}\mathrm{d}y_1 + \int_0^{y_1(\bar{k},f,e)} W_{\text{post}}(\bar{k})\frac{1}{\bar{k}}\mathrm{d}y_1\right\}\right] \mathrm{g}(\bar{k})\frac{\partial \bar{k}}{\partial e} \\
&+ \int_1^{\bar{k}} \left[\mu\left\{\frac{x}{k}\left(-\frac{\partial y_1}{\partial e}\right) + \frac{W_{\text{post}}}{k}\frac{\partial y_1}{\partial e} + \int_0^{y_1(k,f,e)} \frac{\partial W_{\text{post}}}{\partial e}\cdot\frac{1}{k}\mathrm{d}y_1\right\}\right] \mathrm{dG}(k) \\
&- \left[\int_{y_X(\bar{k},e,\mu)}^{\bar{k}} \frac{x}{\bar{k}}\mathrm{d}y_1 + \mu\int_0^{y_X(\bar{k},e,\mu)} W_{\text{post}}(\bar{k})\frac{1}{\bar{k}}\mathrm{d}y_1\right] \mathrm{g}(\bar{k})\frac{\partial \bar{k}}{\partial e} \\
&+ \int_{\bar{k}}^{K} \left[-\frac{x}{k}\frac{\partial y_X}{\partial e} + \mu\left\{\frac{W_{\text{post}}}{k}\frac{\partial y_X}{\partial e} + \int_0^{y_X(k,e,\mu)} \frac{\partial W_{\text{post}}}{\partial e}\cdot\frac{1}{k}\mathrm{d}y_1\right\}\right] \mathrm{dG}(k).
\end{aligned}$$

The second and fourth terms in this expression are of ambiguous signs (since $\partial W_{\text{post}}/\partial e < 0$), whereas the first and the third terms cancel out due to the fact that $y_X(k,e,\mu) = y_E(f,\mu) = y_1(k,f,e)$ at the income potential $k = \bar{k}$. So, the partial derivative $\partial W/\partial e$ can be negative when evaluated at $(\mu^*, f^*, e = \bar{f})$, in which case e^* will be less than the maximal fine $\bar{f}$. On the other hand, if $\partial W/\partial e \geq 0$ at $(\mu^*, f^*, e = \bar{f})$, then $e^* = \bar{f}$.

(iii) Differentiate W with respect to f to obtain:

$$\begin{aligned}
\frac{\partial W}{\partial f} &= \left[\int_{y_E(f,\mu)}^{\bar{k}} \frac{x}{\bar{k}}\mathrm{d}y_1 + \mu\left\{\int_{y_1(\bar{k},f,e)}^{y_E(f,\mu)} \frac{x}{\bar{k}}\mathrm{d}y_1 + \int_0^{y_1(\bar{k},f,e)} W_{\text{post}}\frac{1}{\bar{k}}\mathrm{d}y_1\right\}\right] \mathrm{g}(\bar{k})\frac{\partial \bar{k}}{\partial f} \\
&+ \int_1^{\bar{k}} \left[\frac{x}{k}\left(-\frac{\partial y_E}{\partial f}\right) + \mu\left\{\frac{x}{k}\left[\frac{\partial y_E}{\partial f} - \frac{\partial y_1}{\partial f}\right] + \frac{W_{\text{post}}}{k}\frac{\partial y_1}{\partial f}\right\}\right] \mathrm{dG}(k) \\
&- \left[\int_{y_X(\bar{k},e,\mu)}^{\bar{k}} \frac{x}{\bar{k}}\mathrm{d}y_1 + \mu\int_0^{y_X(\bar{k},e,\mu)} W_{\text{post}}\frac{1}{\bar{k}}\mathrm{d}y_1\right] \mathrm{g}(\bar{k})\frac{\partial \bar{k}}{\partial f}.
\end{aligned}$$

Using the fact that $y_X(k,e,\mu) = y_E(f,\mu) = y_1(k,f,e)$ at $k = \bar{k}$, the first and third-line expressions cancel out and rearranging terms and evaluating at the optimal policy choices (μ^*, f^*, e^*) yields the condition in (9). **Q.E.D.**

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