

# Estimation Based Control of Flexible Systems-Sensorless Wave Based Technique

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**Abstract**—This paper presents an algorithm for parameters and positions estimation of lumped flexible systems. As soon as the parameters and the positions are estimated they can be used to design virtual sensors that can be moved along the system to estimate the position of any lumped mass keeping the system free from any attached sensors. The virtual sensors are nothing but a chain of estimators that are connected at the end of each other, starting with two actuator's measurements and ending up with system parameters and all the system lumped positions. An estimation Based PID controller is presented based on the feedback of the virtual sensor's estimates instead of the actual measurement.

## I. INTRODUCTION

Feedback control relies on the measurements picked using some sensors attached to the system or estimations provided by especially designed observers. In other words, control system design relies on some actual sensor's measurement and some other virtual sensor's estimations. Motion control of a system with lumped masses requires attaching a sensor to the point of interest. Therefore, position control of such systems requires number of sensors equal to the number of generalized coordinates that determine the position of the system or moving a single sensor to the mass required to be controlled. In this paper a parameter and position estimation algorithms is introduced and conventional PID controller is replaced with an Estimation based one that can be used for both motion and vibration control of lumped flexible systems. In doing so actuator is used to launch fourier synthesized control inputs and to receive reflected mechanical waves that can be estimated from its current and velocity. Surprisingly enough, this reflected waves contains not only the system load information but it also provides a complete picture about the damping coefficients, joint stiffness, parameter disturbances and externally applied forces. the algorithm introduced by this paper shows that its possible to achieve a motion and vibration control of any of the lumped masses of the flexible system without attaching any sensors to the system. In [1], [2] the reflected mechanical waves were considered as disturbance on the actuator that can be estimated by an observer designed in [3]. Robust motion control is achieved when this disturbance is rejected in [4] by turning the system into acceleration control if the inertia and motor constants are assumed to be unity [5]. In this paper the disturbance is considered as

reflected mechanical wave from the system and instead of rejecting these waves by the control input it is used to extract the system parameter information and to estimate the position of the lumped masses of the flexible system. Oconar [6], [7] pointed out that actuator can be used to launch mechanical waves to the system and to absorb the reflected waves keeping the system free from residual vibrations but requires the measurement of the first lumped mass and assuming that system is free from external applied forces. Vibration control can be achieved by a variety of approaches [8], point-to-point vibration control is an affective method to set the position of the last mass at certain position insuring that the system is free from any potential or kinetic energy [9], that implies vibrationless motion control, but the position of the masses have to be measured. Multi-switch Bang-Bang control [10], command shaping [11] and laplace domain synthesis [12] are very efficient vibration control techniques that requires some measurements from the system. This paper presents an algorithm that is based on mechanical waves analysis in order to estimate and control the position and vibration of any of the system lumped masses. The paper is organized as follows, in section 2 the mechanical waves are estimated using actuator parameters and parameters information is obtained, rigid and flexible motion of the lumped flexible system are estimated in section 3 then estimation based PID controller is presented. Finally section 4 includes the experimental results and the conclusions.

## II. PARAMETER ESTIMATION

### A. Modal Analysis of Lumped Flexible System

For the inertial lumped flexible system shown in Fig. 1 with  $n$  number of generalized coordinates the equation of motion

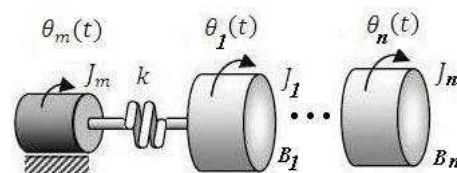


Fig. 1. Lumped flexible inertial system

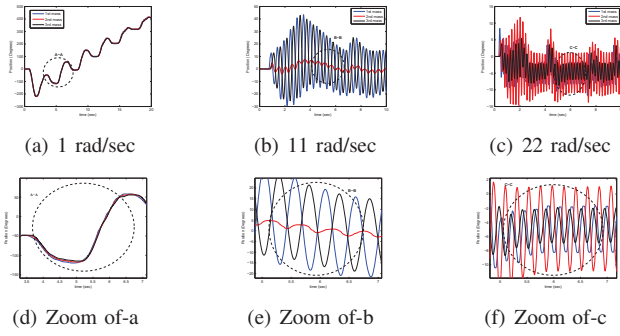


Fig. 2. Modal matrix experimental interpretation

is

$$\mathbf{J} \ddot{\underline{\theta}} + \mathbf{B} \dot{\underline{\theta}} + \mathbf{K} \underline{\theta} = \underline{\mathbf{T}} \quad (1)$$

where  $\mathbf{J}$ ,  $\mathbf{B}$  and  $\mathbf{K}$  are the inertia, damping and stiffness matrices, respectively.  $\underline{\theta}$  and  $\underline{\mathbf{T}}$  are the generalized coordinates and torque inputs vectors, assuming zero damping and equal inertial masses for a three degree of freedom flexible system. Taking laplace transform of eq.2 and putting it in the following linear system form

$$\mathbf{A} \underline{\theta} = \underline{\mathbf{T}} \quad (2)$$

solving for the eigenvalues of the homogenous version of eq.2 and finding the corresponding eigenvectors we get the following modal matrix

$$\mathbf{M} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -2 \\ 1 & -1 & 1 \end{bmatrix} \quad (3)$$

Figure.2 shows the experimental interpretation of the modal matrix, where the first eigenvector represents the rigid body motion of the system as all system masses have constant amplitudes with respect to each other and in phase. The second eigenvector indicates that the second mass is not moving while the first and third are oscillating with the same amplitude but out of phase. According to the third eigenvector the first and third masses have the same amplitude and in phase while the third mass has twice the amplitude and out of phase. In any event, the point here is to indicate that if the forcing function is filtered or fourier synthesized so that the input has zero energy at the system resonances all the lumped masses will be moving with the same amplitude with respect to each other. Moreover, the number of generalized coordinates used to uniquely describe the system will be reduced from  $n$  to a single generalized coordinate. The equation of motion for the first lumped mass is

$$J_m \ddot{\theta}_m + B(\dot{\theta}_m - \dot{\theta}_1) + k(\theta_m - \theta_1) = \tau_1 = i_a k_t \quad (4)$$

making the following definition

$$\tau_{\text{ref}} \triangleq B(\dot{\theta}_m - \dot{\theta}_1) + k(\theta_m - \theta_1)$$

where  $\tau_{\text{ref}}$  is the reflected torque wave on the actuator that can be estimated using the actuators current and velocity,  $i_a$  and  $k_t$  are the motor current and torque constant.

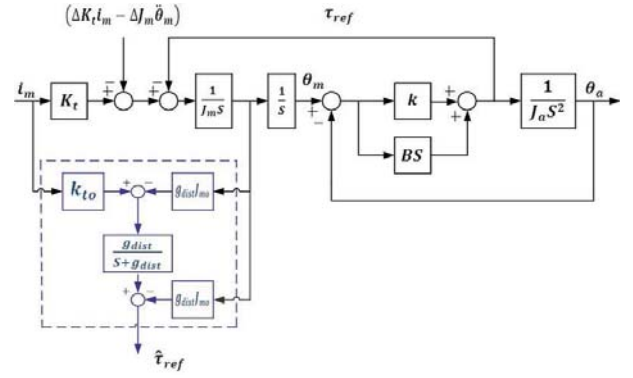


Fig. 3. Torque observer using actuator parameters

## B. Reflected Torque Estimation

Considering the parameters variation Eq.(4) becomes

$$(J_{m_o} + \Delta J_m) \ddot{\theta}_m + \tau_{\text{ref}} = i_a (k_{t_o} + \Delta k_t) \quad (5)$$

where  $J_{m_o}$  and  $k_{t_o}$  are the nominal inertia and torque constant, while  $\Delta J_m$  and  $\Delta k_t$  are the variation from these nominal values. rearranging terms

$$J_m \ddot{\theta}_m - i_a k_{t_o} = -\tau_{\text{ref}} + \Delta k_t i_a - \Delta \ddot{\theta}_m J_m \quad (6)$$

and assuming that the reflected torque wave is much larger than the parameter variation disturbance  $\tau_{\text{ref}} \gg \Delta k_t i_a + \Delta J_m \ddot{\theta}_m$ . Fig.3 shows the block diagram implementation of the reflected torque estimation process through a low pass filter with a cutoff frequency  $g_{\text{dist}}$  used to control the convergence speed of the estimated variable to the actual one along with reducing the level of noise amplification due to the differentiation process.

## C. Rigid Body Motion Estimation

As the reflected wave has been estimated using the actuator parameters it can be written as

$$\hat{\tau}_{\text{ref}} \triangleq B(\dot{\theta}_m - \dot{\theta}_1) + k(\theta_m - \theta_1)$$

assuming that the input forcing function doesn't contain any energy at the system resonance frequencies. This can be achieved by filtering and/or fourier synthesizing the control input. In this special case the motion of the system can be described by a single generalized coordinate as all the masses are moving with the same amplitude with respect to each other. The position estimate is given as

$$\hat{\theta}(t) = \frac{1}{\sum_{i=1}^n J_i} \int_0^t \int_0^{\tau} \hat{\tau}_{\text{ref}} d\tau d\tau \quad (7)$$

and for linear flexible system

$$\hat{x}(t) = \frac{1}{\sum_{i=1}^n m_i} \int_0^t \int_0^{\tau} \hat{\tau}_{\text{ref}} d\tau d\tau \quad (8)$$

where  $\hat{\theta}(t)$  is the rigid body motion estimate.

#### D. Parameters Estimation

The estimate of the reflected torque can be rewritten using the estimate of the rigid body position of the system as follows

$$\widehat{\tau}_{\text{ref}} \triangleq B(\dot{\theta}_m - \widehat{\dot{\theta}}) + k(\theta_m - \widehat{\theta}) \quad (9)$$

making the following definitions

$$\begin{aligned} \underline{\xi} &\triangleq (\theta_m - \widehat{\theta}) \\ \underline{\eta} &\triangleq (\dot{\theta}_m - \widehat{\dot{\theta}}) \end{aligned} \quad (10)$$

where  $\underline{\xi}$  is a vector of data points representing the difference between the actuator and estimated systems positions,  $\underline{\eta}$  is the time derivative of these data points.  $B$  and  $k$  are the damping coefficient and the joint stiffness, assuming that these parameters are uniform along the flexible lumped system, rewriting Eq.9

$$\widehat{\tau}_{\text{ref}} = k \underline{\xi} + B \underline{\eta} \quad (11)$$

putting the previous equation in the following matrix form

$$\begin{bmatrix} \underline{\xi} & \underline{\eta} \end{bmatrix} \begin{bmatrix} K \\ B \end{bmatrix} = \widehat{\tau}_{\text{ref}} \quad (12)$$

where  $\widehat{\tau}_{\text{ref}}$  is a vector of reflected torque wave data points, making the following definition

$$\mathbf{A} \triangleq \begin{bmatrix} \underline{\xi} & \underline{\eta} \end{bmatrix}$$

Equation 12 represents an over determined system, where the number of equations are more than the number of unknowns. Therefore, the optimum parameters can be determined as follows

$$\begin{bmatrix} \widehat{K} \\ \widehat{B} \end{bmatrix} = \mathbf{A}^\dagger \widehat{\tau}_{\text{ref}} \quad (13)$$

where  $\mathbf{A}^\dagger$  is the pseudo inverse of  $\mathbf{A}$ ,  $\widehat{k}$  and  $\widehat{B}$  are the stiffness and damping estimates

### III. FLEXIBLE MOTION ESTIMATION

#### A. Recursive estimation equations

As the control input may contain some energy at the resonance frequencies of the system which will excite any of the system's flexible modes as it was shown in Fig.2. In this case Eq.7 will no longer be valid. Therefore, we need to describe the motion of the lumped masses through the entire frequency range of the system regardless to the frequency content of the forcing function. The equation of motion describing the system with the estimated parameters is

$$\mathbf{J} \ddot{\underline{\theta}} + \widehat{\mathbf{B}} \dot{\underline{\theta}} + \widehat{\mathbf{K}} \underline{\theta} = \mathbf{T} \quad (14)$$

recalling (9) and rearranging the terms, we get the following first order differential equation

$$\begin{aligned} \widehat{B}\dot{\theta}_1 + \widehat{k}\theta_1 &= \alpha \\ \alpha &\triangleq \widehat{B}\dot{\theta}_m + \widehat{k}\theta_m - \widehat{\tau}_{\text{ref}} \end{aligned} \quad (15)$$

solving the previous differential equation we get

$$\widehat{\theta}_1(t) = e^{-\frac{\widehat{B}}{k}t} \int_0^t \beta e^{\frac{\widehat{B}}{k}\tau} d\tau + e^{-\frac{\widehat{B}}{k}t} c_1 \quad (16)$$

where  $\widehat{\theta}_1(t)$  is the estimate of the first inertial mass position, regardless to the frequency of the forcing function. For the first equation of motion we have

$$\begin{aligned} \widehat{B}\widehat{\dot{\theta}}_2 + \widehat{k}\theta_2 &= \gamma \\ \gamma &\triangleq J_1\widehat{\dot{\theta}}_1 - \widehat{B}(\dot{\theta}_o - \widehat{\dot{\theta}}_1) - \widehat{k}(\theta_o - \theta_1) + \widehat{B}\widehat{\dot{\theta}}_1 + \widehat{k}\theta_1 \\ \zeta &\triangleq \frac{\gamma}{\widehat{B}} \end{aligned} \quad (17)$$

solving (17) we get

$$\widehat{\theta}_2(t) = e^{-\frac{\widehat{B}}{k}t} \int_0^t \zeta e^{\frac{\widehat{B}}{k}\tau} d\tau + e^{-\frac{\widehat{B}}{k}t} c_2 \quad (18)$$

where  $\widehat{\theta}_2(t)$  is the estimate of the second lumped inertial mass. And the general position estimate of any of the lumped masses of the system is

$$\widehat{\theta}_i(t) = e^{-\frac{\widehat{k}}{\widehat{B}}t} \int_0^t \Omega e^{\frac{\widehat{k}}{\widehat{B}}\tau} d\tau + e^{-\frac{\widehat{k}}{\widehat{B}}t} c_i \quad (19)$$

where

$$\begin{aligned} \Omega &\triangleq \frac{\Psi}{\widehat{B}} \\ \Psi &\triangleq g(J_{i-1}, \widehat{\theta}_{i-1}, \widehat{\dot{\theta}}_{i-1}, \widehat{\ddot{\theta}}_{i-1}, \widehat{k}, \widehat{B}) \end{aligned}$$

#### B. Estimation Based-PID Controller

The position of any of the lumped masses can be estimated by the previous process, the estimate can be used as a virtual feed back instead of the actual measurement taken using any attached sensor. The error is no longer defined as the difference between some desired reference and an actual one, it became the difference between the reference and the estimate of the actual position

$$\widehat{e}(t) = \theta_{\text{ref}}(t) - \widehat{\theta}_i(t) \quad (20)$$

the control law of the Estimation based PID control is

$$u(t) = k_p \widehat{e}(t) + k_i \int_0^t \widehat{e}(t) dt + k_d \frac{d\widehat{e}(t)}{dt} \quad (21)$$

#### C. Summary of the Estimation Based Control Process

The steps of the entire process are:

- 1) Fourier synthesize the input such that it contains zero energy at the system resonances.
- 2) Reflected torque estimation using actuator parameters.
- 3) Rigid body motion estimation using (7) or (8).
- 4) Estimate the uniform system parameters using (13).
- 5) Use the recursive formula (19) to determine the estimate of the  $i^{\text{th}}$  mass required to be controlled.
- 6) Feeding back the position estimate of the  $i^{\text{th}}$  mass to the controller to accomplish the motion and vibration control assignment.

Figure.4 shows the entire estimation and control process which is based on two measurements from the actuator. and then a chain of estimators are designed. The previous estimation algorithm is based on extracting the systems parameters information when the system is oscillating in the low frequency range rigidly, then using these parameters in the general position estimator that can estimate the position of any lumped mass regardless to the frequency content of the control input.

#### IV. EXPERIMENTAL RESULTS

The implementation of the pervious algorithm is performed on a three degree of freedom flexible system with parameters summarized in Table.I.

TABLE I  
EXPERIMENTAL PARAMETERS

Parameter	Value	Parameter	Value
$J_1$	5152.99 gcm <sup>2</sup>	$g_{dist}$	100 rad/sec
$J_2$	5152.99 gcm <sup>2</sup>	$g_{lpf}$	100 rad/sec
$J_3$	6192.707 gcm <sup>2</sup>	$f_{init}$	1 rad/sec
$J_m$	209 gcm <sup>2</sup>	$k_{act}$	1.627 KN/m
$k_b$	235 rpm/v	$k_t$	40.6 mNm/A

where  $g_{lpf}$  is the velocity low pass filter cut off frequency,  $f_{init}$  is the forcing function's frequency that keeps the system in its rigid mode,  $k_{act}$  is the theoretical spring constant along the flexible system that is known before hand by the following calculation

$$k_{act} = \frac{Gd}{8c^3n} = \frac{70 \times 10^9 \times 2}{8 \times (\frac{8}{2})^3 \times 21} = 1.627 \text{ kN/m} \quad (22)$$

where  $G$  is the modulus of rigidity,  $c$  is the spring ratio,  $d$  and  $n$  are the coil diameter and the effective number of turns.

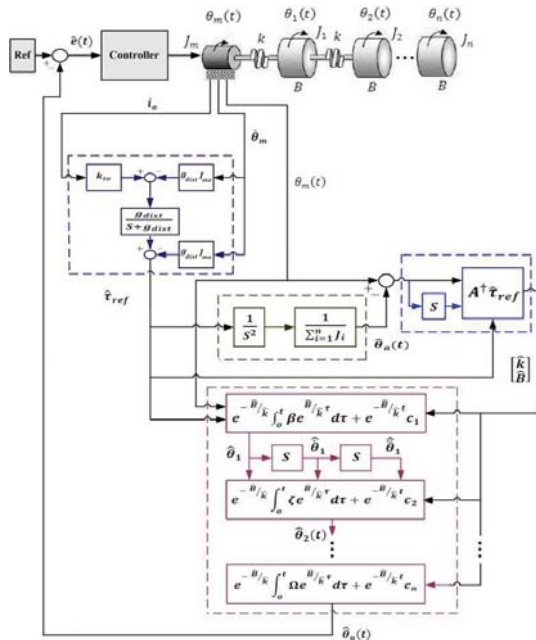


Fig. 4. Estimation Based-Control of Flexible system

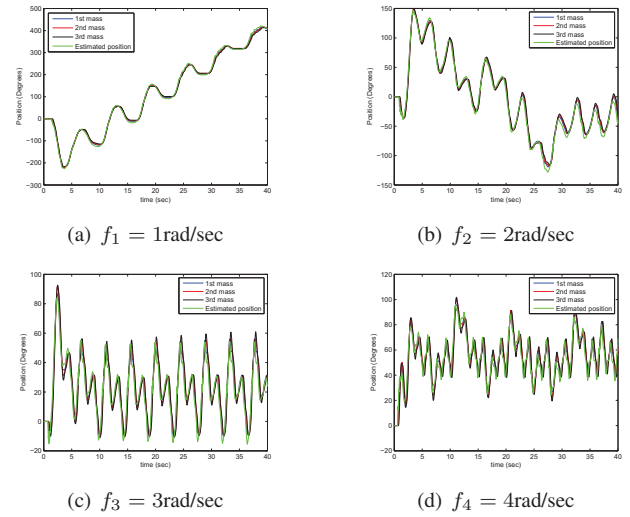


Fig. 5. Rigid body motion estimation

#### A. Rigid Body Motion Estimation-Experimental Results

System parameters are extracted when the system is in its rigid mode. The idea behind doing this process is to drop the unknown lumped masses positions to reduce the number of unknown from  $n$  to only single unknown which is the position if the entire rigid system that can be determined using (7) after estimating the reflected torque wave. In doing so, the input forcing function is filtered so that its energy content at the system's resonance frequencies is zero. Fig. 5 shows the motion of the three masses and estimates of their positions for different frequencies. It turns out that below 4 rad/sec the estimation of the rigid motion is identical to the actual systems position. In other words Eq.7 is valid in the low frequency range below 4 rad/sec. Therefore, the parameter estimation experiment have to be performed in this frequency range.

#### B. Uniform Parameters Estimation-Experimental Result

The parameter estimation experiment requires the estimate of the reflected torque along with the difference between the actuator position and the rigid body position estimate data point vector and its derivative. Using Eq.13 the system parameters are determined and Table.II summarizes the obtained experimental results for both the joint stiffness and the damping coefficient that are assumed to be uniform along the flexible system. Using the obtained average estimated parameters, the reflected torque wave is reconstructed and compared with the estimated one as shown in Fig.6. The magnified plot of Figure.6-a is too noisy because of the direct differentiation effect that doesn't represent a problem as it is just used to compare the reflected wave with the reconstructed wave using the optimum estimated uniform parameters.

$$\hat{k}_{avg} = \frac{\sum_{i=1}^n k_i}{n} = \frac{\sum_{i=1}^5 k_i}{5} = 1.566 \text{ kN/m} \quad (23)$$

$$\hat{B}_{avg} = \frac{\sum_{i=1}^n B_i}{n} = \frac{\sum_{i=1}^5 B_i}{5} = 0.0882 \text{ Nsec/m} \quad (24)$$



comparing the average estimated stiffness with the theoretical value that is known before hand through Eq.23 we conclude that the difference is less than 5 percent and these parameters can be used for further estimation steps.

### C. Position Estimation

The position estimation process is performed using Eq.19, where the position estimate of the  $i^{th}$  mass requires the determination of all the previous masses positions estimates. Figure.7-a shows the flexible system oscillation when an arbitrary forcing function contains some energy at the system's resonances. While other figures show the difference between the actual position and the estimated one using Eq.19. The estimated position seems to be identical to the actual mass position that encourage us to use the estimate as a feedback to the controller instead of the actual measurement.

### D. Sensorless Position Control-Experimental Results

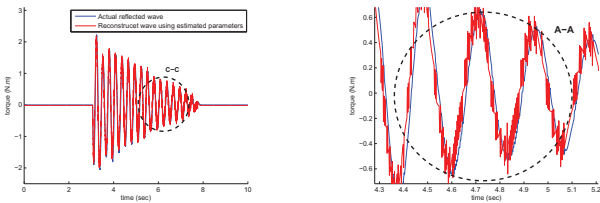
Figure 4 shows the entire sensorless estimation and control process, where no measurements were taken from the system. Just the actuator parameters are measured and used by the previous chain of estimators to detect the position of any lumped mass in the system.

1) *Set-point tracking experiment-1st mass:* Using the positions estimates of the lumped inertial masses as a feedback instead of the actual measurement makes it possible to switch the estimates and feeding them back easily to the controller. Figure.8 shows the sensorless control process of the first mass, where its position estimate is fed back to the controller. The magnified plot shows 0.1 degrees steady state error in the final response, the other figures shows the response of the other two masses. The objective here is to control the position of any particular mass along with active vibration damping of the other masses keeping the system free from any kinetic and potential to minimize any residual vibrations.

2) *Set-point tracking experiment-2nd mass:* In order to control the position of the second mass with minimum residual vibration of the system the estimate of the second mass has to

TABLE II  
PARAMETERS ESTIMATION RESULTS

Par	1st Exp	2nd	3rd	4th	5th
$k$ KN/m	1.579	1.533	1.645	1.511	1.562
$\hat{B}$ Nsec/m	0.088	0.087	0.088	0.089	0.089



(a) reflected and reconstructed torque using the estimated parameters

(b) Magnified plot of a

Fig. 6. Uniform system's parameters estimation

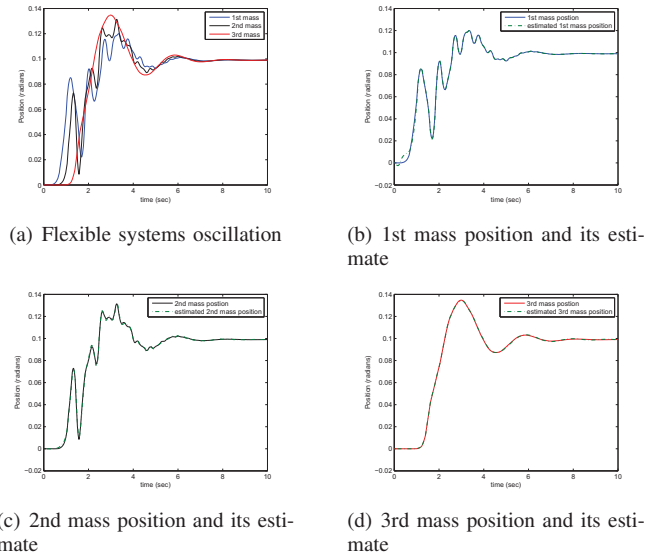


Fig. 7. Flexible motion estimation

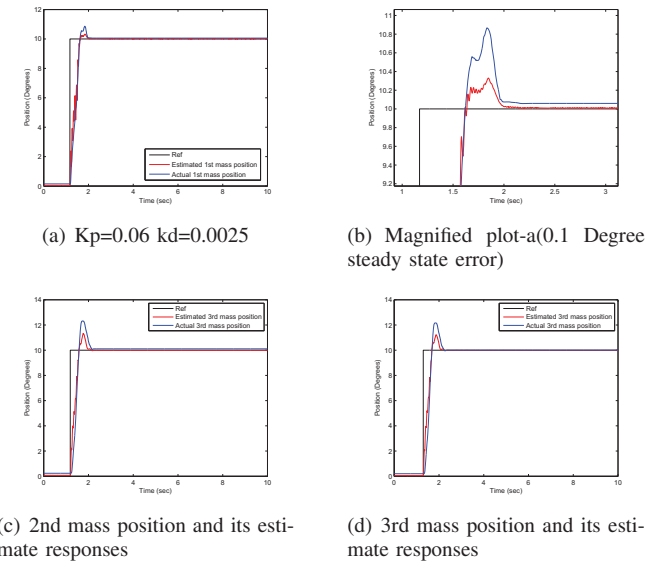


Fig. 8. Sensorless control of the 1st mass-Based on the first mass position estimate feed back

be switched to the controller instead of the first mass. Figure.9-a shows the response of the second mass when its estimate was fed back to the controller, its magnified plot shows 0.15 degrees steady state error in the final response.

3) *Arbitrary trajectory tracking experiment:* Figure.10 shows the third mass trying to track two different arbitrary references. In doing so the estimate of the third mass position was fed back to the controller. Indeed, for a time varying trajectory tracking feed back control is not enough and a feed forward control input have to be added to the control law. Surprisingly enough that this feed forward control input can be obtained without taking any measurement from the system as system dynamics and parameters have been estimated through the proposed algorithm.

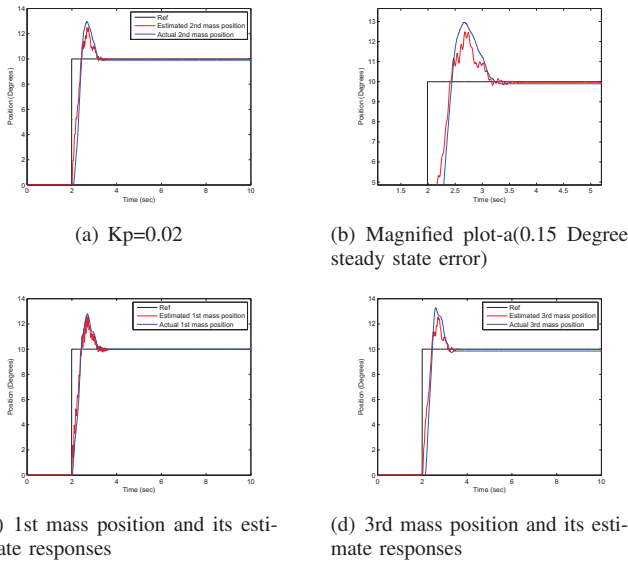


Fig. 9. Sensorless control of the 2nd mass-Based on the second mass position estimate feed back

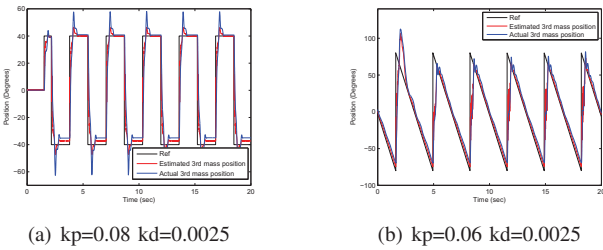


Fig. 10. Sensorless control of the 3rd mass-Based on the third mass position estimate feed back

## V. CONCLUSION

Uniform parameters and dynamics of any lumped flexible system can be obtained from the reflected mechanical waves on the actuator. By extracting the information of the system parameters from the low frequency region, where the number of generalized coordinates describing the systems position is dropped from  $n$  to one generalized coordinate. This simple procedure allows the estimation of the damping coefficient and the joint stiffness, that is similar to an optimization problem. The obtained estimates are nothing but the optimum parameters that minimizes the norm square of errors. Using this information along with the actuator parameters the flexible motion of the system can be estimated by a chain of estimator or by some recursive computations. The experiments show promising results but the main draw back of the proposed algorithm is the steady state error in final response when the estimate of the position is used instead of the actual one the controller brings the estimate to the desired reference but if there was any error between the estimate and the actual position the control action will not be able to bring the actual position back to the desired reference keeping the system with a steady state error in the final response. The amount of steady state error depends on the accuracy of the estimation process

that is not a single estimation but a chain of estimations working on the output of each other. However, reducing the steady state error relays on more accurate estimates. The steady state error is not only a result of the inaccurate estimation but also due to the dependence of one of the estimators on an optimization process, where the estimated parameters are obtained by solving an over determined system. On one hand the proposed algorithm didn't prove how to get rid of the steady state error at the final response but on the other hand it provides a sensorless parameter estimation procedure and flexible motion estimation algorithm. Surprisingly enough that taking the derivatives of the position estimates provides all the system dynamics that can be used to estimate the external torques and disturbances due to the system interaction with the environment that is not included in this article.

## ACKNOWLEDGMENT

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