Praise Worthy Prize

International Review of Automatic Control (I.RE.A.CO.), Vol. 2, n. 1
January 2009

Sensorless Wave Based Control of Flexible Structures Using Actuator as a Single Platform for Estimation and Control

Islam S. M. Khalil, Asif Sabanovic

Abstract – *In this paper we present a wave based control approach for flexible structures using the actuator as a single platform for measurement and control without picking any measurements from the system. Using the reflected wave to the actuator from the system along with the actuator parameters as the current and velocity to estimate the flexible system's parameters. And using those estimated parameters to achieve robust controllers keeping the system free from any measurements and attached sensors. In this paper all the sensors attached to the system were used just to verify the performance of the controller not to have feed back from the system.*

Keywords: *Mechanical Wave, Torque observer, Position Estimation, Estimation based PID controller*

Nomenclature

x_o	Linear actuator displacement
m_o	Linear actuator mass
f	Input forcing function
n	Number of lumped masses
$fref_l$	Reflected force from mass number l
$fref_{tot}$	Total reflected force wave at the actuator
j_m	Motor Inertia
B_m	Viscous damping coefficient
τ_m	Motor toque
τ_{ref}	Reflected torque wave
j_{mn}	Nominal motor inertia
B_{mn}	Nominal viscous coefficient
k_{tn}	Nominal motor torque constant
Δj_m	Deviation from actual motor inertia
ΔB_m	Deviation from actual viscous damping
Δk_t	Deviation from actual motor torque constant
v	Wave propagation speed
$u(t,x)$	Axial displacement along the flexible system
ρ	Material density
G	Modulus of rigidity
$\hat{\tau}_{ref}$	Estimated reflected torque Wave
i_a	Motor torque
g_{dist}	Cut of frequency of the low pass filter
ω_i	Natural frequencies of the system
m_i	i^{th} Lumped mass
x_i	i^{th} mass position
$\hat{x}_i(t)$	Estimated position of i^{th} mass
K_p	Proportion gain
K_d	Derivative gain
K_i	Integral gain

I. Introduction

Importance of flexible structures is growing everyday due to the crucial need for efficient systems with better load to weight ratio that became an essential parameter in the design of robots and manipulators. On the other hand dealing with flexible system keeps the control problem more complex as the controller has to achieve several tasks as robust motion control, active vibration damping, and estimation of systems parameters as no sensors will be attached to the system for the feed back purpose. They will be just used for the purpose of verification of controller performance. In order to do so we consider the reflected wave from the flexible system as a feed back that carrying all the information about the system including the natural frequencies, stiffness of the joints, damping coefficients and disturbance added to the system. For this purpose all the work will be done in the platform of the actuator including measurements, estimations, analysis of reflected wave and control. In the last few decades researchers used some approaches to perform a robust motion control with active vibration damping. O'Connor introduced a solution for the gantry cranes [1] based on wave approach. Wave based control was presented for the motion control of lumped flexible robots [2]-[3] and [4] keeping into mind that measurement was picked from the system i.e. from the first mass of the lumped system that is not applicable if the system is continuous and infinite number of masses exist. Resonance ratio control [5]-[6] was used in order to suppress the torisional vibrations of two resonant masses. Direct velocity feed back [7] was used for the same purpose. In [8] wave variables were used to for system analysis and control. Input shaping [9] was introduced in order not to excite certain oscillation modes of the flexible systems.

Manuscript received and revised December 2008, accepted January 2009

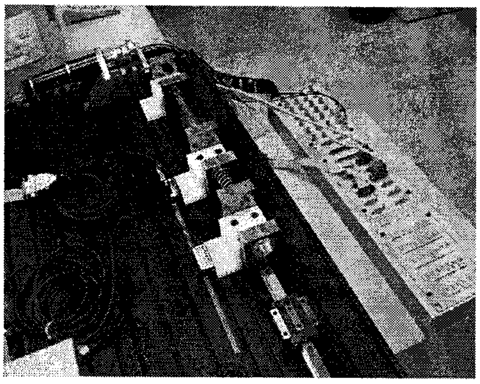

Fig. 1. Experimental setup

Min max control [10], Uniform damping control [11], L_2 and H_∞ optimized control [12]. In [13] wave in a finite length beam was controlled by a piezoelectric actuator. Each of the previous approaches was dealing with the active vibration damping and motion control from different perspective but the single common feature among these approaches is picking some measurements from the system and using this measurement as a normal feed back to the controller using some sensors that are attached somewhere to the system. but the problem arises when we start dealing with the very flexible structures as flexible beams at which the use of the stain gages to measure the deflections is limited by the dynamic range of the patch and constrained for limited range of deflections also the use of visual feed back require very high speed cameras if the response of the beam is required to be fast. This in turns implies that sensors might cause some problems for some systems that motivate us to think about controlling such systems from the actuator platform by using another approach to have feed back from the system rather than using sensors. Using this idea the wave concept appears into the picture by launching an initial wave to the flexible system and waiting for the reflected wave from the boundary condition at the end of the system. Considering the reflected wave as the natural feed back from the system that carries all the information about the system. By analyzing the reflected wave some interesting information could be extracted as the positions and the stiffness matrix along with the assumption that we know some little information about the system as its mass. For this purpose we propose a lumped mass spring system connected to a DC motor via a crank and link connection and the system is interfaced to an 1103 DSPASE controller. Fig. 1. shows the experimental setup at which the experimental work was performed. The paper is organized as follow in section II system is modeled, observer is designed, wave is analyzed and position controller is presented based on the estimated positions of the discrete masses of the system. In section III overall control scheme is presented and Experimental results are discussed and finally conclusions are presented in section V.

II. Problem Formulation

II.1. Modeling and Analysis

II.1.1. Lumped Mass Spring System

Considering a linear lumped mass spring system attached to a linear actuator as shown in Fig. 2. Where F is the forcing function generated by the actuator and lunched to the lumped mass spring system. The dynamic equations of motion describing the system are as follow.

$$m_0\ddot{x}_o + k_1(x_0 - x_1) = F \tag{1}$$

$$m_1\ddot{x}_1 - k_1(x_0 - x_1) + c_2(\dot{x}_1 - \dot{x}_2) + \\ + k_2(x_1 - x_2) = 0 \tag{2}$$

$$m_2\ddot{x}_2 - k_2(x_1 - x_2) - c_2(\dot{x}_1 - \dot{x}_2) + c_3(\dot{x}_2 - \dot{x}_3) + \\ k_3(x_1 - x_2) = 0 \tag{3}$$

$$m_3\ddot{x}_3 - k_3(x_2 - x_3) - c_3(\dot{x}_2 - \dot{x}_3) + c_4(\dot{x}_3 - \dot{x}_4) + \\ k_4(x_3 - x_4) = 0 \tag{4}$$

$$m_4\ddot{x}_4 - c_4(\dot{x}_3 - \dot{x}_4) - k_4(x_3 - x_4) = 0 \tag{5}$$

Rewriting the above equations $k_0(x_0 - x_1)$ can be expressed as follow (eq (6)):

$$k_1(x_0 - x_1) = \sum_{i=1}^{3} m_i\ddot{x}_i + c_4(\dot{x}_3 - \dot{x}_4) + k_4(x_3 - x_4)$$

Where $k_0(x_0 - x_1)$ represents a force wave that is reflected back from the system including all the system parameters and information as shown in equation (6). And from equation (1) $k_0(x_0 - x_1)$ could be expressed as follow:

$$f_{ref_{tot}} = k_0(x_0 - x_1) = F - m_0\ddot{x}_0 \tag{7}$$

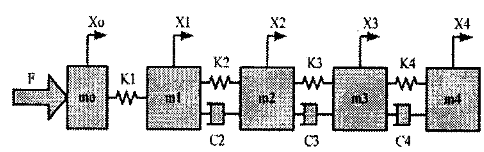

Fig. 2. Lumped mass spring model

Which indicates that the reflected force wave could be detected using the linear actuator parameters along with the knowledge forcing function. This by its turns implies that all the system parameters have a contribution on the value of the reflected wave or the actuator parameters. Generally equation (6) could be expressed as follow:

$$K_l(x_{l-1} - x_l) = \sum_{i=l}^{n} m_i\ddot{x}_i = \sum_{i=l}^{n} f_{ref_l} \tag{8}$$

Where l is the index of the mass at which the force wave is reflected back. Since the goal of this work is to use the actuator as a single platform for measurement and estimation the we use $l = 1$ to observe the total reflected wave from the system:

$$f_{ref_{tot}} = f_{ref_l} + f_{ref_2} + \dots + f_{ref_n} \quad (9)$$

Which indicates that the reflected force wave is composed of a force wave component from each mass of the lumped flexible system that can be detected from the platform of the actuator.

II.1.2. Motor Dynamics

The dynamics of the actuator attached to the system shown in Fig. 2 is as follow:

$$j_m\ddot{\theta}_m + B_m\dot{\theta} = \tau_m - \tau_{ref} \quad (10)$$

$$j_l\ddot{\theta}_l + B_l\dot{\theta}_l - \tau_{ref} = 0 \quad (11)$$

It turns out that attaching a rotational actuator to the linear lumped system the reflected force will be mapped into torque by a linear transformation:

$$\tau_{ref} = R(f_{ref_{tot}}) \quad (12)$$

Where R is a linear transformation between the reflected force wave and the torque. Equivalently we can say that looking at the linear lumped mass spring system the wave propagating through the system is considered as a force wave on the other hand when we consider that motion is launched using a rotational actuator the reflected force will be transformed into reflected torque wave.

II.2. Reflected Torque Estimation

In order to keep the actuator as to keep the system free from measurement torque observer [6] is used in order to estimate the reflected torque. From equation (10):

$$(j_{mn} + \Delta j_m)\ddot{\theta}_m + (B_{mn} + \Delta B_m) + \tau_{ref} = i_a(k_{tn} + \Delta k_t) \quad (13)$$

$$j_m = j_{mn} + \Delta j_m \quad (14)$$

$$B_m = B_{mn} + \Delta B_m \quad (15)$$

$$k_m = k_{mn} + \Delta j_m \quad (16)$$

Rearranging equation (13):

$$j_{mn}\ddot{\theta} + B_{mn}\dot{\theta} + \tau_{dist} = i_a k_{tn} \quad (17)$$

$$\tau_{dist} = \tau_{ref} + \Delta j_m\ddot{\theta} + \Delta B_m\dot{\theta}_m + i_a\Delta k_t \quad (18)$$

Assuming that reflected torque is larger than disturbance due to the model parameters uncertainties $\tau_{ref} >> \Delta j_m\ddot{\theta} + \Delta B_m\dot{\theta}_m + i_a\Delta k_t$.

Equation (17) could be written as follow:

$$j_{mn}\ddot{\theta} + B_{mn}\dot{\theta} + \tau_{ref} = i_a k_{tn} \quad (19)$$

And the structure of the torque observer [5] is shown in Fig. 3 using a low pass filter with cut of frequency of g_{dist} in order to compensate the effect of the direct differentiation.

$$\hat{\tau}_{ref} = \frac{g_{dist}}{s+g_{dist}}\tau_{ref} \quad (20)$$

The difference between reflected and estimated torque wave is illustrated in Fig. 4.

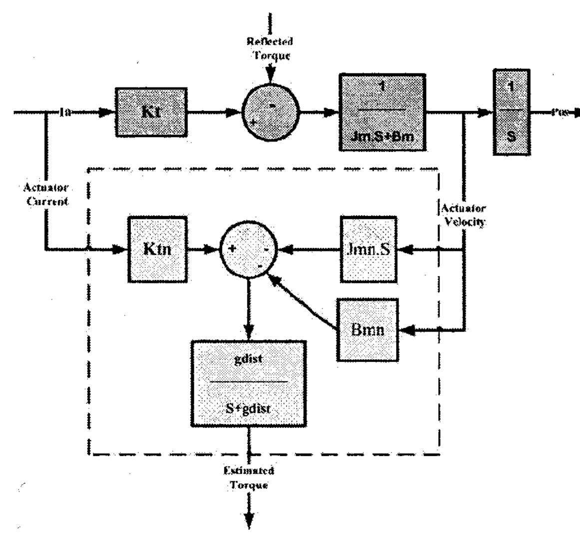

Fig. 3. Reflected torque observer

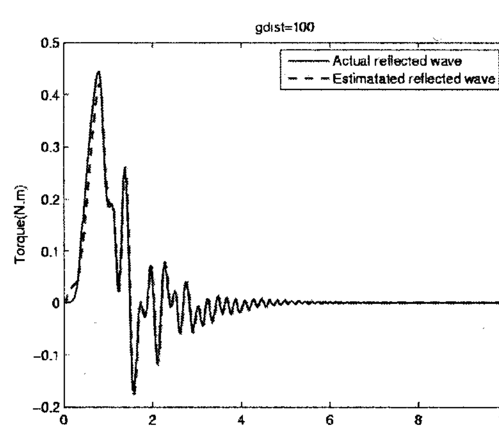

Fig. 4. Reflected and estimated torque

II.3. Theoretical Wave Propagation

Equations (6), (8) are describing a force wave that is reflected back to the mass m_l.

But it does not show the nature of propagation through the entire lumped system that can be show using the wave equation as follow:

$$\frac{\partial^2 u(t,x)}{\partial t^2} - v^2 \frac{\partial^2 u(t,x)}{\partial x^2} = f(t,x) \quad (21)$$

Which represent a one dimensional wave equation propagating through the system [14]:

$$v = \sqrt{\frac{G}{\rho}} \quad (22)$$

The solution of equation (21) can be used to show the behavior of the propagating wave along the system:

$$u(x,t) = \frac{1}{2}[f(x+vt) + f(x-vt)] + S + R \quad (23)$$

where:

$$S = \frac{1}{2v}\int_{x-vt}^{x+vt} g(s)ds \quad (24)$$

$$R = \frac{1}{2v}\int_0^t \int_{x-v(t-\tau)}^{x+v(t-\tau)} f(s,\tau)dsdt \quad (25)$$

$$g(x) = \frac{\partial u(0,x)}{\partial t} \quad (26)$$

Where $g(s)$ and $f(s,\tau)$ are the initial velocity of the wave and the forcing function.

And the solution show that the wave will be splinted into two portions $f(x-vt)$propagating to the right and $f(x+vt)$ propagating to the left as shown in Fig. 5.

Where $T1 < T2 < T3 < T4 < T5$.

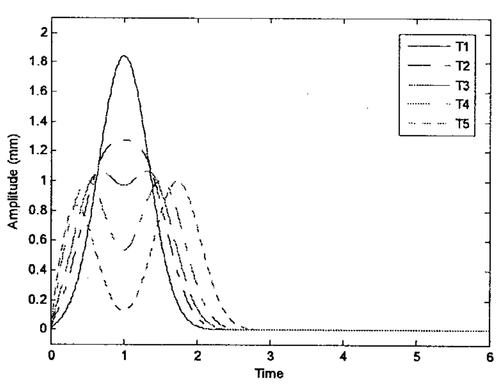

Fig. 5. Wave splits into two equal opposite portions

II.4. Reflected Wave Factorization

Considering the number of lumped masses as $n = 3$ and as shown in Fig. 6. Such system has three modes of oscillation. Considering the rigid mode we can conclude the following from equation (8):

$$f_{ref_{tot}} = (m_1 + m_2 + m_3)\ddot{x} \quad (27)$$

And in general if we assumed that the system in its rigid mode we can express the motion of the system by the following expression:

$$x(t) = \frac{1}{\sum_{i=1}^{n} m_i}\int_0^t \int_0^t f_{ref_{tot}}(\tau)\, d\tau d\tau \quad (28)$$

Where n is the number of lumped masses and $f_{ref_{tot}}$ could be obtained from the reflected estimated torque wave. Recalling equation (12) we conclude:

$$\hat{x}(t) = \frac{1}{\sum_{i=1}^{n} m_i}\int_0^t \int_0^t \frac{\hat{\tau}_{ref}(\tau)}{r} d\tau d\tau \quad (29)$$

And the reflected estimated torque $\hat{\tau}_{ref}$ by its turn could be obtained from the actuator parameters $\dot{\theta}_m$and i_a as shown in equation (19).

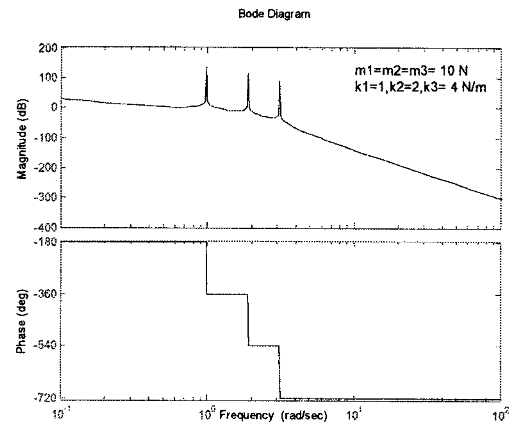

Fig. 6. Frequency response of the lumped system

It turns out that the position of the rigid lumped mass spring system could be estimated by equation (29) using nothing but the actuator parameters without picking any measurement from the system. Where $x(t)$ and $\hat{x}(t)$ are the actual and estimated positions of the lumped rigid spring mass system. Fig. 7 shows the response of estimated and actual position of the lumped rigid masses to an impulse response input. In order to generalize the idea of using the actuator parameters to estimate the position of the lumped system the other flexible modes should be taken into consideration and equation (29) will not be valid for the position estimation of the flexible lumped system's masses. By factorizing the estimated reflected torque $\hat{\tau}_{ref}(\tau)$ to its fundamental components (29) could be rewritten as follow:

$$\hat{x}_i(t) = \frac{1}{m_i}\int_0^t \int_0^t \frac{\hat{\tau}_{ref_i}(\tau)}{r} d\tau d\tau \quad (30)$$

In order to investigate the validity of (30) non rigid oscillation mode should be excited. Fig. 6 shows the frequency response of the lumped system and the following relations shows the frequencies at which different modes could be obtained.

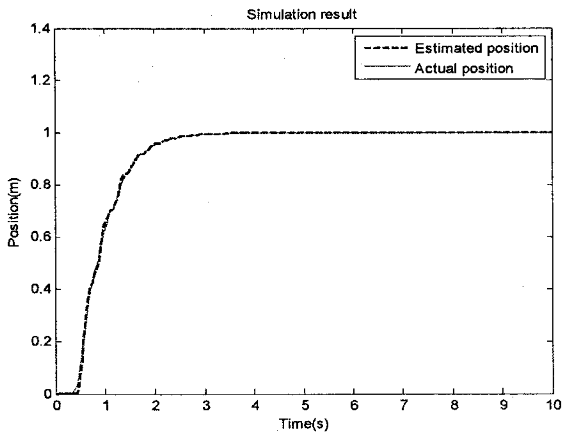

Fig. 7. Estimated and actual lumped masses position

$$\omega_i = a_i \sqrt{\frac{K}{m}} \tag{31}$$

Where $a_1 = 0.44504, a_2 = 1.2471, \quad a_3 = 1.8025$ and ω_i is ith natural frequency of the flexible system [15].

Fig. 8. shows the response of the last mass to a sinusoidal input used to excite other flexible modes of the system in order to clarify that each mass has its own contribution on the reflected wave that should be factorized according to the system's natural frequencies and equation (30) is used to estimate the position according to the estimated factorized reflected wave.

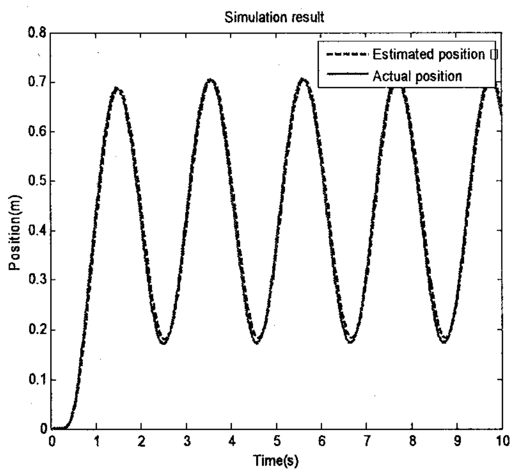

Fig. 8. Estimated and actual last mass position

II.5. Convergence Experimental Results

Figs. 9 show experimentally the difference between the actual and estimated last mass response for arbitrary inputs and in each the estimated position has the same global behavior with some delay due to the nature of estimation that depends on the reflected wave from the system.

II.6. Position and Vibration Control

From the simulations along with the experimental results we can conclude that the position of the lumped masses could be estimated with a Sensorless manner by just picking the measurement from the actuator.

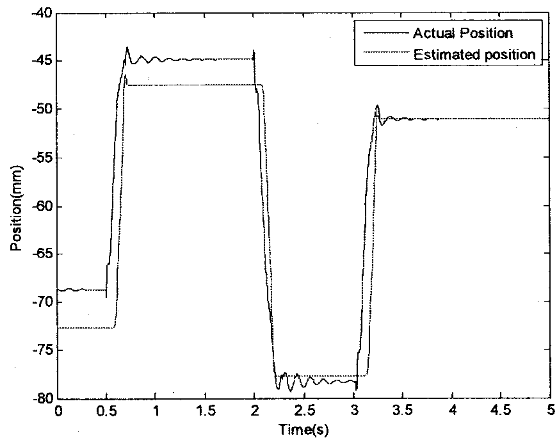

Fig. 9(a)

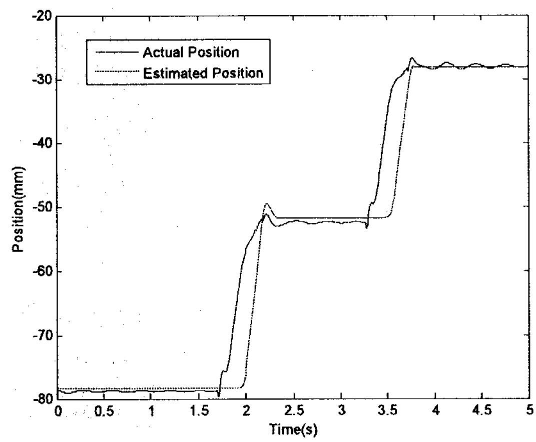

Fig. 9(b)

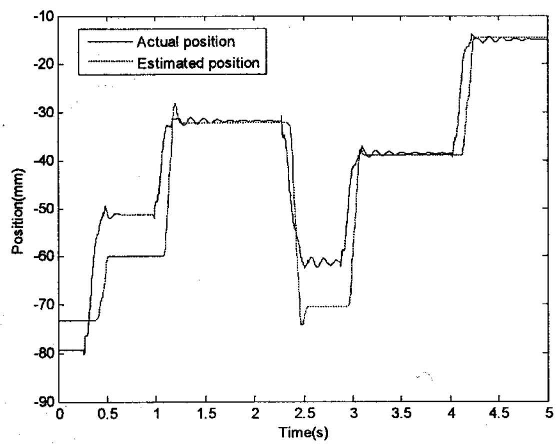

Fig. 9(c)

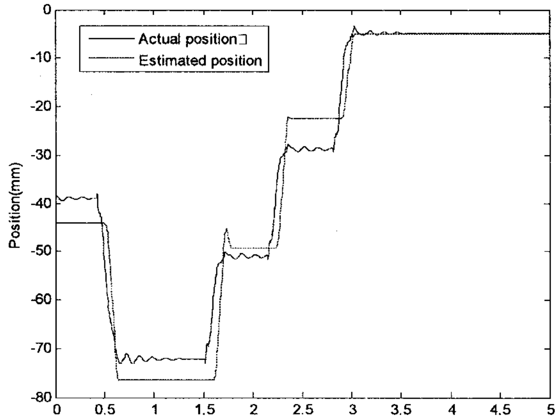

Fig. 9(d)

Figs. 9. Estimated and actual last mass position (a), (b), (c) and (d) convergence experiments for different arbitrary references

It turns out that we can use that estimate instead of attaching sensors to the masses of the lumped system and using the difference between a reference input and the estimate of any of the lumped masses position an error signal is obtained for the control purpose and a *PID* controller could be used and its parameters K_p, K_i and K_D could also be tuned to obtain different responses of the system.

$$e(t) = x_{ref}(t) - \hat{x}_i(t) \tag{32}$$

$$u(t) = k_p e(t) + k_i \int_0^t e(t)dt + k_d \frac{de}{dt} \tag{33}$$

III. Overall Control And Experiments

The overall system consists of four subsystems controller, actuator, flexible lumped system and a chain of estimators as shown in Fig.10. The controller starts with forcing an impulse to the system in other words the controller launches an initial wave to the system that is free from any sensors. And since the system has a finite length the force wave will be reflected back when it reaches the boundary condition at the end of the system. The reflected force or torque wave will be observed from the actuator platform using the actuators current and velocity. Estimated wave is factorized and position is estimated by assuming that the masses are known. Using the estimated position we can design a controller that is using an estimate of the measurement instead of the measurement itself. Equivalently we can say that controller needs to have a feed back from the system that is considered as a reflected wave that could be obtained by sending an impulse at the very beginning of the control task and followed a chain of estimators to develop an estimate of the position of each mass of the system. Experiments were performed on the setup shown in Fig.1. The Rainshow linear encoders are used for the purpose of verification not for feed back as the controller is considering the feed back is appearing naturally from the reflected wave from the system. Dc motor with the following specifications was used as the platform of estimation and control $j_m = 64.2gcm^2, l = 1.68\,mh, k_m = 105\,{}^{mNm}/_{A}, k_b = 90.9\,{}^{rpm}/_{v}$.

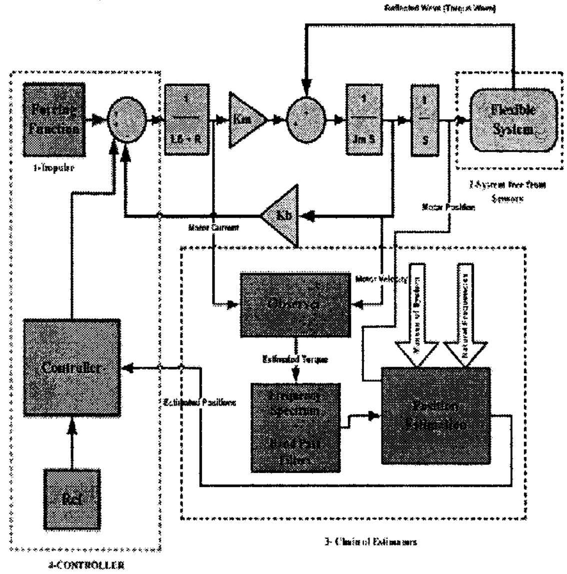

Fig. 10. Overall control scheme

For the reflected torque wave observer the $g_{dist} = 100$. And the experiments were performed for two sets of lumped mass loads. Fig. 11. shows the closed loop response of the last mass to a step input when a $1kg$ load is attached to both last and middle mass. Increasing the load to $2kgs$ at the same masses should be followed by modifying the estimator input and the closed loop response is shown in Fig. 12. With larger overshoot and slower response. The same controller parameters are used in both experiments. Moreover the controller parameters could be tuned to have a family of closed loop responses or to satisfy certain performance requirements.

IV. Conclusion

Actuator can be used as a single platform for estimation and Control if the reflected wave from the system is considered as a natural feed back from the system and a control scheme could be developed without picking any measurement from the system by closing the loop using the estimated position of any of the lumped masses of the system. But this approach requires the exact knowledge of the masses and requires updating the input of the position estimator beside the offline impulse natural frequency test in order to detect the natural frequencies of the system that will be used to design the low, high and band pass filters to factorize the total reflected wave into components corresponding to each lumped mass of the flexible system. Finally a *PID* controller is used experimentally by closing the loop with the estimated position instead of the actual position. A steady state error might be found in the final response due to the uncertainty of the exact value of the masses and the cumulative error of the estimators.

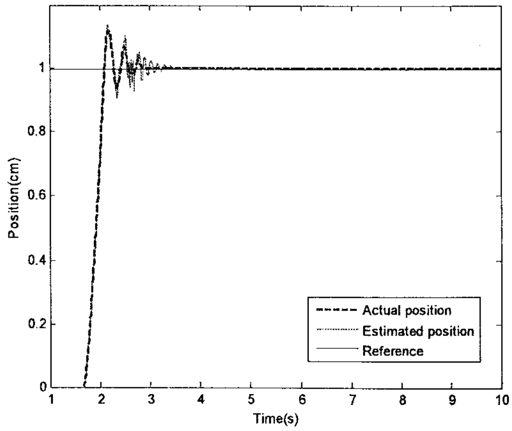

Fig. 11. Response of the last mass and its estimate to a step input-(k_p=2, k_i=0, k_D=1)

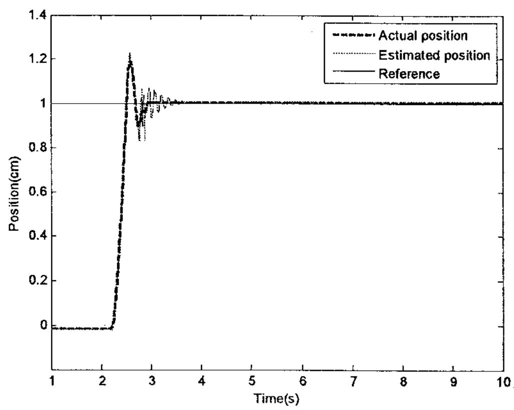

Fig. 12. Response of the last mass and its estimate to a step input- (k_p=2, k_i=0, k_D=1)

Acknowledgements

The authors gratefully acknowledge the financial assistance provided by SanTez project number 00183.STZ.2007-2.

References

[1] W.J.O'Connor,"A gantry crane problem solved," *ASME J. Dyn. Syst., Meas., Control*, vol.125, no.4, pp. 569-576, Dec. 2003.

[2] W.J.O'Connor and D.Lang,"Position Control of Flexible robot arms using mechanical waves,"*ASME J. Dyn.Syst., Meas., Control*, Vol.120, no.3, pp.334-339, Sep. 1998.

[3] W.J.O'Connor, "Wave-Echo Control of lumped flexible systems," *J. Sound Vibrat.*, vol. 298, no. 4-5 pp. 1001-1018, Dec. 2006.

[4] W.J.O'Connor *"Wave-Based Analysis and Control of Lumped-Modeled Flexible Robots,"* IEEE Trans on Robotics, Vol, 23, no, 2, April, 2007.

[5] K. Yuki, T. Murakami, and K. Ohnishi,"*Vibration Control of 2 mass resonant system by resonance ratio control,*" in Proc. Int. Conf. IEEE Industrial Electronic Society (IECON'93), Nov. 1993, Vol. 3, pp. 2009-2014.

[6] Seiichiro Katsura, Kouhei Ohnishi,"*Force Servoing by Flexible Manipulator Based on Resonance Ration Control,*" IEEE Trans on Industrial Electronic, Vol.54, NO.1, February 2007.

[7] Balas, M.J., 1978."*Feedback Control of Flexible System,*" IEEE Trans. Autom.Control, AC-23, pp.673- 679.

[8] G.Nieneyer, and J.Slotiv,"*Using Wave variable for System analysis and Robust Control,*" IEEE ICRA Albuquerque, NM, April 1997.

[9] Z. Mohammed, and M.O. Tokhi, "*Vibration control of a single-link flexible manipulator using command shaping techniques*", Pros. Inst. Mech. Eng., part I,J Systems and Control Engineering, 216, 2002, pp. 191-210.

[10] Singh, T., 2002,"Minimax design of Robust Controllers for Flexible Systems," J. *Guid. Control Dyn.* 25(5), pp. 868-875.

[11] Silverberg, L., 1986,"Uniform Damping Control of Spacecraft," *J. Guid. Control Dyn.* 9, pp. 221-227.

[12] Safonov, M. G., Chiang, R. Y., and Flashner, H., 1991,"*Hι* Robust Control for a Large Space Structure," *J. Guid. Control*, 14, pp. 513-519.

[13] Pines, D. J., and Von Flotow, A.H., 1990,"Active Control of Bending Wave propagation at Acoustic frequencies," *J. Sound Vib.*, 142,pp. 391-412.

[14] Walter. A., Strauss, *Partial Differential Equations* (Wiley, 1993)

[15] Singiresu, S.Rad, *Mechanical Vibrations* (Pearson Prentice Hall, 2003)

Authors' information

Islam S.M. Khalil received the B.S. degree in mechanical engineering from Helwan University, Cairo, Egypt in 2006; He is currently a Master candidate in Mechatronics department, Faculty of engineering and natural science, Sabanci University, Turkey.

His research interests are in the area of modeling and control of dynamical systems, motion and vibration control and Mechatronics.

Asif Sabanovic (M'85-SM'03), He received B.S '70, M.S. '75, and Dr Sci. '79 degrees in Electrical Engineering all from University of Sarajevo, Bosnia and Herzegovina. He is with Sabanci University, Istanbul, Turkey. Previously he had been with University of Sarajevo; Visiting Professor at Caltech, USA, Keio University, Japan and Yamaguchi University, Japan Head of CAD/CAM and Robotics Department at Tubitak - MAM, Turkey.

His fields of interest include power electronics, sliding mode control, motion control and Mechatronics.