A New Robust Position Control Algorithm for a Linear Belt-Drive

Aleš Hace
University of Maribor
ales.hace@uni-mb.si

Karel Jezernik
University of Maribor
karel.jezernik@uni-mb.si

Asif Šabanović
Sabanci University
asif@sabanciuniv.edu.tr

Abstract

The paper deals with a linear belt-driven servomechanism. It proposes new position tracking control algorithm that has been designed by Sliding Mode Control theory. The selected sliding manifold was extended by non-rigid modes of the elastic servodrive. However, the proposed control scheme retains simple and practical for implementation. The experiments presented in the paper show that it effectively suppresses vibrations and furthermore extends the closed-loop bandwidth.

1. Introduction

The use of timing belts in drive trains is attractive because of their high speed, high efficiency, long travel lengths and low-cost [9]. On the other hand, they yield higher transmission error since they feature elasticity, compliance and often more friction then screw ball drives [12]. Consequently, belt-drives suffer from lower repeatability and accuracy; moreover, the belt-drives introduce more resonance frequencies, which can cause vibrations if not suppressed.

A conventional control approach often fails if vibrations appear in a servodrive operation [16]. Therefore, an advanced control scheme must be applied in order to achieve accurate position tracking performance. Vibration analysis is a good foundation for improved design method [23]. Although a control signal filtering method may improve performance [4], closed-loop stability is better with the use of state observer [2], [17]. Hori has presented advanced methods that deal with torsional vibrations in elastical servodrives [10]. Disturbance observer [11], [22], acceleration feedback [5], [20] or joint torque feedback [13], [18] may be also applied to suppress vibrations. Although the research of the flexible joint/link control is widely present in the robotics related works, the methods often do not address the load side position control problem, which is a key issue in position tracking of a linear servodrive.

Plant parameter variations, uncertain dynamics, and disturbances, are issues that have to be addressed to guarantee robust system stability and high performance of a linear belt-driven servomechanism. Some authors have involved Sliding Mode Control (SMC) theory in order to improve robustness of an elastic system [6], [14], [15]. Hace [8] and later Šabanovič [19] addressed a linear servomechanism with a timing belt. It has been shown that vibration-free performance can be achieved by introduction of belt-stretch control. In [8] the author has proposed the advanced motion control law with the inner-loop controller for vibration suppression and robust position control scheme in the outer-loop. A 2-stage design has been applied in the development of the vibration-free position controller. The SMC design in the outer loop has involved the position control objective, and a PD-type belt-stretch control scheme in the inner loop has been added to extend closed-loop bandwidth.

In this paper, a new robust position control algorithm for the linear belt-driven servomechanism is presented. The model of a linear beltdrive is briefly described in the 2nd section. The 3rd section presents derivation of the proposed algorithm. Experimental results are shown in the 4th section, which follows with conclusions in the 5th section.

2. Mathematical Model

A typical linear belt-drive is presented by Fig.1. It

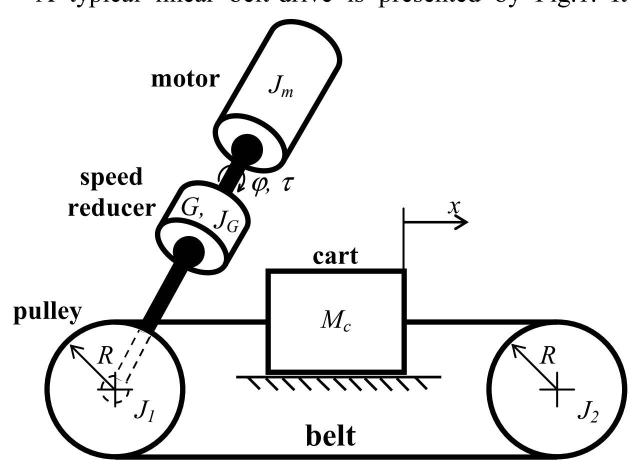


Fig.1. Linear belt-driven servomechanism

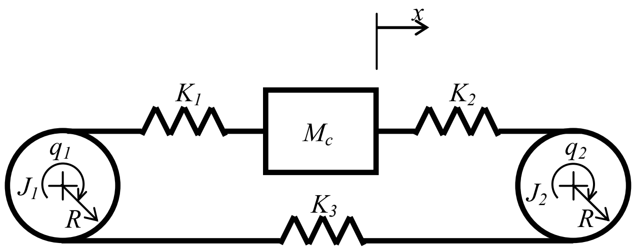


Fig.2. Spring model of the belt drive

consists of a motor, a speed reducer and a belt drive. The belt drive converts rotation of the motor into linear motion of the cart. The cart represents the load side of the system. The belt-drive (Fig. 2) consists of a timing belt and two pulleys: a driving pulley and a driven pulley that stretch the belt. It represents a complex non-linear distributed parameter system. The mathematical model of the belt-driven servomechanism can be obtained using modal analysis. Let assume that the motor can ensure a high-dynamic torque response with a negligible time delay, link between motor shaft and the driving pulley is totally rigid, and no backlash is present in the system.

Furthermore, let linear mass-less spring characterizes elasticity of the belt in the multi-mass system with concentrated parameters. Friction present in the motor and pulley bearings, the speed reducer, the cart guidance is considered as an unknown disturbance. Then we can obtain a sixth order mathematical model (1), where:

J_1, J_2	the inertia moment of the driving and the driven pulley, respectively
J_G, J_m	the inertia moment of the speed reducer and the motor, respectively
M_c	the mass of the cart
G	the speed reducer ratio
R	the radius of the pulleys
K_1, K_2, K_3	the position dependant elasticity coefficients of the belt
q_1, q_2, φ	the angular position of the driving pulley, driven pulley, and the motor
x	the cart position
τ	the torque developed by the motor
τ_{f1}, τ_{f2}	the friction torque which affects the pulleys
f_f	the friction force on the cart

The model (1) is a highly-coupled and nonlinear higher order system with external disturbances which enter at the driving side as well as the load side. However, the pulley inertia is small in comparison with the motor and the load side inertia. Therefore, the model can be simplified and reduced to a two-mass system:

$$\begin{aligned} J\ddot{\varphi} + \tau_f &= \tau - L \cdot Kw \\ M\ddot{x} + f_f &= Kw \\ w &= L\varphi - x \end{aligned} \tag{2}$$

where J denotes inertia of the motor side (approximately equals the motor inertia), M denotes mass of the load side (approximately equals the cart mass), τ_f stands for friction on the motor side, and w denotes belt-stretch, respectively. L denotes a transmission constant ($L = \partial x / \partial \varphi$). We assume constant elasticity coefficient K. The mathematical model (2) can be further rearranged according to a vibration analysis of belt-drives [1]. One can express dynamics of the belt stretch w. If we assume unit transmission constant ($L = 1$) to simplify further algebra, then it yields:

$$\begin{aligned} J\ddot{w} + K_w w &= \tau - \tau_{wf} \\ M\ddot{x} + f_f &= Kw \end{aligned} \tag{3}$$

where $\tau_{wf} = \tau_f - \kappa f_f$, $K_w = K(1+\kappa)$. $\kappa = J/M$ is so called inertia ratio. The block scheme of the linear belt-driven servomechanism design model is presented by Fig.3.

3. Control Design

SMC law can be used if the uncertainties in the model structure are bounded with known bounds [21]. Let $\dot{z} = f(z) + b(z)u + d(t)$. The goal of the control design is to find a control input u that restricts the motion of the system states z to a selected sliding manifold. SMC action with discontinuities on the sliding manifold $\sigma(t,z) = 0$ may enforce sliding mode if the derivative of Lyapunov function candidate $v = \sigma^2/2$ is negative definite. One such solution can be found if the derivative can have form $\dot{v} = -D\sigma^2$, $D > 0$. From condition $\sigma\dot{\sigma} = -D\sigma^2$ one can derive control $u = u_{eq} + D\sigma$ that assures invariant system motion in sliding mode if disturbance $d(t)$ complies to matching conditions [3]. Equivalent control u_{eq} is solution of $\dot{\sigma}\big|_{\sigma=0} = 0$.

$$\begin{aligned} \left(J_1 + G^2(J_G + J_m)\right) \cdot \ddot{q}_1 + \tau_{f1} &= G\tau - R \cdot \left[K_1(x) \cdot \left(Rq_1 - x\right) - K_3 \cdot \left(Rq_2 - Rq_1\right)\right] \\ J_2\ddot{q}_2 + \tau_{f2} &= R \cdot \left[K_2(x) \cdot \left(x - Rq_2\right) - K_3 \cdot \left(Rq_2 - Rq_1\right)\right] \\ M_c\ddot{x} + f_f &= K_1(x) \cdot \left(Rq_1 - x\right) - K_2(x) \cdot \left(x - Rq_2\right) \end{aligned} \tag{1}$$

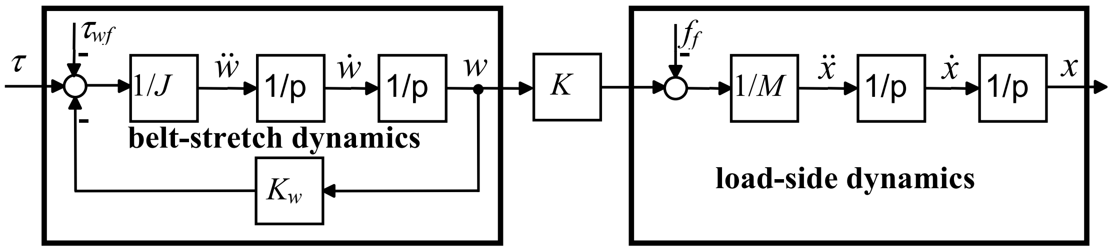


Fig.3. Belt-stretch model block scheme of linear servomechanism

SMC law reduces the system order and decouples from disturbances. However, SMC law has some disadvantages related to well known chattering in the system due to the discontinuous bang-bang control action. This phenomena is undesirable in the control of mechanical systems, since it causes excessive control action leading to increased wear of the actuators and to excitation of the high order unmodeled dynamics. Consequently, the demanded performance can not be achieved, or even worse - mechanical parts of the servo system can be destroyed. This paper follows chattering-free SMC design introduced by Hace [7], which augments the original system with additional system state in order to eliminate discontinuities on control signal. The SMC design has been applied for a rigid mechanical system, the switching function was chosen as

$$\sigma_e = r(t) - \left(\ddot{x} + K_v\dot{x} + K_p x\right) \tag{4}$$

where $r(t)$ and x denote reference signal and actual position, respectively. This paper extends this idea to the non-rigid mechanism model. Let equation (5)

$$\begin{aligned} J\ddot{w} + K_w w &= \tau - \tau_w^{dist} \\ M\ddot{x} &= Kw - f^{dist} \end{aligned} \tag{5}$$

describes nominal model, where τ_w^{dist} and f^{dist} involve unmodeled and uncertain dynamics as friction or parameter uncertainties and variation. In this paper is proposed that the sliding mode manifold is constructed by the following switching function:

$$\sigma = r(t) - \left[\ddot{x} + K_v\dot{x} + K_p x + \gamma\left(\ddot{w} + \alpha\dot{w}\right)\right] \tag{6}$$

The sliding mode manifold now involves also belt elasticity. The portion of belt-stretch dynamics ($\ddot{w} + \alpha\dot{w}$) is added to the definition of the switching function. In the case of "stiff" belt performance ($\ddot{w} = 0$, $\dot{w} = 0$) the switching function (6) can be reduced to (4): $\sigma = \sigma_e$. However, in order to reduce vibrations due to belt compliance and elasticity, the control design parameters K_v, K_p, α, and γ, shall be selected in order to shape asymptotically stable motion dynamics on the sliding manifold.

The sliding manifold is constructed so to allow driving position error to zero at vibration-free operation. The control law can be now derived following the SMC procedure. From condition $\dot{\sigma}(\dot{\tau} = \dot{\tau}_{eq}) = 0$ and by simple algebra application one can find the equivalent control τ_{eq} from (5) and (6):

$$\tau_{eq} = \frac{\beta}{\omega_0^2}(J+M)a^c - J\left(\alpha\dot{w} + (\beta - \omega_0^2)w\right) + \tau^{dist} \tag{7}$$

where $\tau^{dist} = \tau_w^{dist} + \kappa\gamma^{-1}f^{dist}$ denotes disturbance, $a^c = r(t) - \left(K_v\dot{x} + K_p x\right)$, α and $\beta = \gamma^{-1}K/M$ are the design parameters that shape the system motion dynamics when $\sigma = 0$. ω_0 is a natural frequency of the belt-drive, which can be computed from the equation $\omega_0^2 = K(1/J + 1/M)$. The control law can be derived from condition $\dot{\sigma} = -D\sigma$ in order to obtain control signal in the form $u = u_{eq} + D\sigma$. However, the disturbance signal τ^{dist} from (7) is not known in practice. Therefore, the equivalent control signal τ_{eq} is replaced with the estimated value $\hat{\tau}_{eq}$. It yields:

$$\dot{\tau} = \dot{\hat{\tau}}_{eq} + \frac{\beta}{\omega_0^2}(J+M)\cdot D\sigma \tag{8}$$

where the equivalent control signal estimation is determined by (9).

$$\hat{\tau}_{eq} = \frac{\beta}{\omega_0^2}(J+M)a^c - J\left(\alpha\dot{w} + (\beta - \omega_0^2)w\right) \tag{9}$$

The control law (8) has two components. One is representing estimation of the equivalent control. Another is robust controller representing the disturbance estimation and the convergence to the selected sliding mode manifold. Consequently, the system motion projection on the σ-space is governed by (10),

$$\dot{\sigma} + D\sigma = \frac{\omega_0^2}{\beta}\frac{1}{J+M}\dot{\tau}^{dist} \tag{10}$$

which in combination with the equation $\sigma\dot{\sigma} = -D\sigma^2$ proves asymptotically stable reaching phase: convergence to stable origin $\sigma = 0$ can be guaranteed if $\dot{\tau}^{dist} = 0$. Then the derivative of the Lyapunov function $v = \sigma^2/2$ is negative definite, i.e. $\dot{v} = -D\sigma^2$, $D > 0$. In systems with high sampling rate fast convergence rate can be achieved. If disturbance changes slowly ($\dot{\tau}^{dist} \approx 0$), the control law can keep the system states close to the sliding manifold ($\sigma \approx 0$) that can allow for good performance in practice. However, the proposed controller cannot decouple the closed-loop dynamics from the load-side disturbance since the matching condition fails. The system dynamics when $\sigma = 0$ is governed by (11).

$$\dddot{x} + \alpha\ddot{x} + \beta\ddot{x} + \beta K_v \dot{x} + \beta K_p x = r(t) - \frac{\ddot{f}^{dist} + \alpha \dot{f}^{dist}}{M} \quad (11)$$

The selection of the sliding manifold guarantees elimination of steady state error. Furthemore, proper design of the control parameters can desensitize from the load-side disturbance dynamics.

4. Results

The proposed control algorithm is simple for application, no high order signal derivatives are required, and however, the signals of position and velocity of the motor and the cart, respectively, are necessary for implementation. A design method for selecting of the proper control parameters is another significant feature for practical application. It can be figured out from closed-loop dynamics analysis.

Let choose the reference signal $r(t)$ by (12),

$$r(t) = \ddot{x}^r(t) + K_v \dot{x}^r(t) + K_p x^r(t) \quad (12)$$

where $x^r(t)$ is reference position trajectory and $e = x^r(t) - x$ is position error. Then, the closed-loop dynamics can be described by (10) and (13).

$$\ddot{e} + K_v \dot{e} + K_p e = \sigma_e$$
$$\ddot{\sigma}_e + \alpha\dot{\sigma}_e + \beta\sigma_e = \beta\sigma + \left(\ddot{a}^c + \alpha\dot{a}^c\right) + \frac{\ddot{f}^{dist} + \alpha\dot{f}^{dist}}{M} \quad (13)$$

On the sliding manifold ($\sigma = 0$), the position error dynamics is driven by derivatives of the signal a^c and the signal of load-side disturbance f^{dist}, respectively. The first portion could be eliminated by modification of sliding manifold definition, i.e. by inclusion of higher-order derivatives of the reference position trajectory. However, in order to desensitize the system motion from the load-side friction one shall carefully choose the design parameters. From (13) two transfer functions can be considered to determine the disturbance sensivity:

$$F_\sigma(s) = F_{\sigma1}(s)F_{\sigma2}(s) = \frac{1}{s^2 + \alpha s + \beta}\frac{s}{s + D} \quad (14)$$

$$F_e(s) = F_{e1}(s)F_{e2}(s) = \frac{1}{s^2 + K_v s + K_p}\frac{s^2 + \alpha s}{s^2 + \alpha s + \beta} \quad (15)$$

$F_\sigma(s)$ links σ_e with disturbance signal τ^{dist}, and $F_e(s)$ links position error with load-side disturbance. The amplitude responses in frequency domain are depicted by Fig. 4 and 5, respectively, where $\omega_\beta = \sqrt{\beta}$. As shown by Fig.4, performance of the robust controller from (8) is a key issue in desensitizing from the disturbance. It allows for implementation of rapid vibration-free belt response, which in turn can also help to reject load side disturbance (see Fig. 5). Moreover, although high value of gain β can extend robust operational bandwidth, it is always limited in a practical application due to neglected higher order system dynamics and discrete implementation of the control algorithm. Furthermore, in beltdrives a time delay in force transmission always occurs, that can cause unstable belt response if too high operational bandwidth is prescribed.

The experiments were conducted on a low-cost timing belt-servomechanism. A DC-motor was attached to the belt-drive via a gearbox with speed reduction ratio G=29. The maximum travel length of the cart was about 2m. The

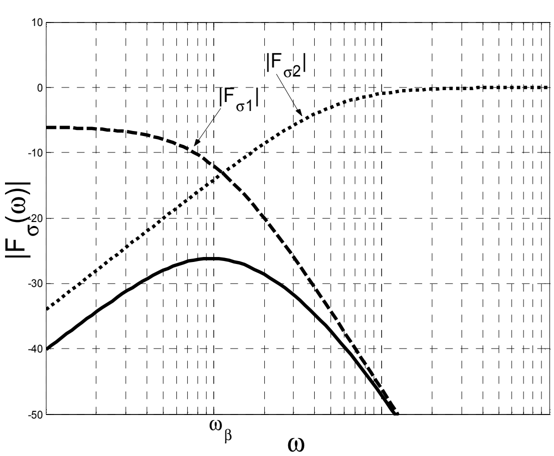


Fig.4. Frequency characteristic of $F_\sigma(s)$

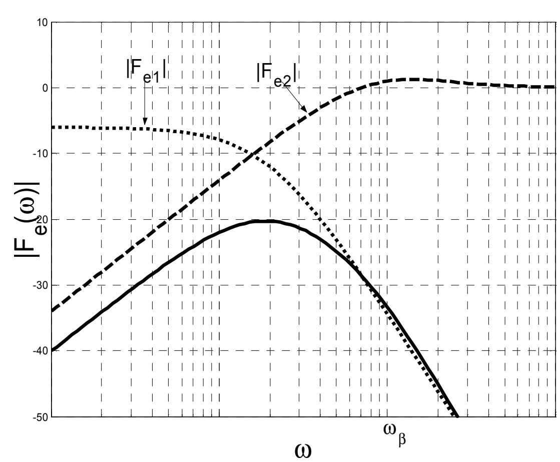
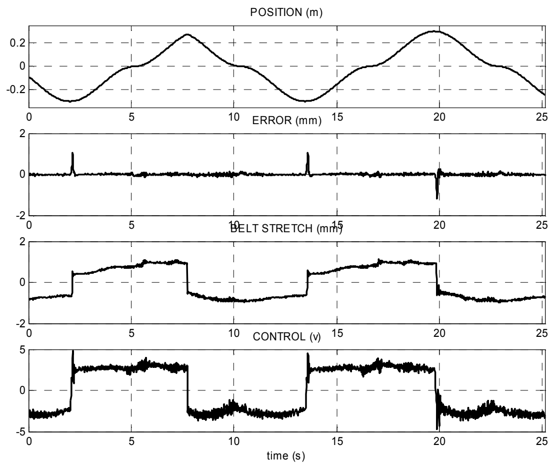
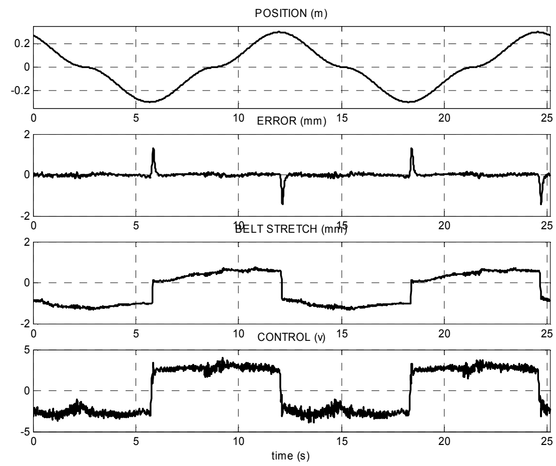


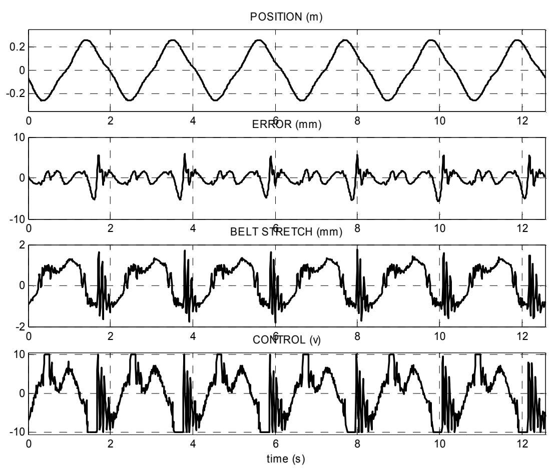
Fig.5. Frequency characteristic of $F_e(s)$



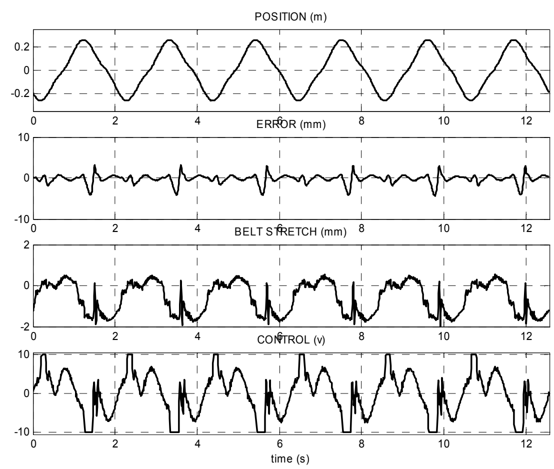
a) control scheme [8]: low speed



b) proposed control scheme: low speed



c) control scheme [8]: high speed



d) proposed control scheme: high speed

Fig.6. Experimental results

position reference trajectory was shaped by $\sin^2$ profile with amplitude of 30cm. The "belt-stretch" has been calculated on the basis of measurement of the cart position and motor angle.

Fig. 6 shows results achieved by the control scheme introduced in [8] (diagrams a) and c)), and by the control scheme proposed in this paper (diagrams b) and d)) at low and high speed, respectively. The error curve shows peaks at velocity zero crossings due to backlash and stiction effect. Large backlash in the servomechanism was demonstrated by the "belt-stretch" curve. Although the backlash phenomena can not be compensated in a moment, the tracking error was compensated with prescribed dynamics and then kept close to zero value. Vibrations were also effectively suppressed. However, if $\gamma = 0$ and retaining the position control parameters K_v, K_p at same value (see (6)) experiments showed unstable response of the system. The proposed control scheme was compared with the controller introduced in [8]. At low speed, the later control scheme performs slightly better in terms of position error. Then we increased the reference speed that caused higher position tracking error. In this case, the proposed control scheme assured better position error tracking performance. It performed more robustly at extended bandwidth.

5. Conclusion

The paper has proposed new control algorithm for position following task of a linear belt-driven servomechanism. It has utilized SMC theory and has extended the switching function definition in order to include also non-rigid modes due to the belt elasticity. The proposed control scheme is practical for the implementation, since it involves only position and velocity signals. Higher order derivatives are not necessary. The proposed control scheme utilizes two sensors: one for motor angle measurement and another for cart position measurement. The control design method suggests easy control parameters tuning procedure. The experiments have shown that the proposed control scheme effectively suppresses vibrations and furthermore extends position closed-loop bandwidth. However, in the future, further consideration shall show if motor angle measurement may be replaced by state observer values, thus eliminating one sensor from the control scheme.

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