## Analog Controller for Piezoelectric Actuators

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Analog Controller for Piezoelectric Actuators

Abstract—Today, the digital implementation of the controllers is preferred, mainly from re-programmability point of view. However, depending on the complexity of the plant and DOF to be controlled, digital implementation may require extensive computational power. The necessity for the acquisition of the analog signals on the other hand requires ADC and DAC conversions that compel extra conditions on the system. In this work we are presenting the analog application of the Sliding Mode Control (SMC) like, the well known robust nonlinear controller. As a test bed for the designed system we are using the piezoelectric actuators (PEA), a nonlinear system with hysteresis as major nonlinearity. PEAs are used in many applications to provide sub-micrometer resolution since they theoretically provide unlimited resolution and large bandwidth. Yet, strong hysteretic nonlinear behavior makes PEA control challenging.

Index Terms—Analog Electronics, Disturbance Observer, Motion Control, Piezoelectric Actuator, Sliding Mode Control

I. INTRODUCTION

Many important control problems can be effectively solved using a digital architecture in conjunction with analog-to-digital and/or digital-to-analog conversion. Digital solutions offer two very attractive advantages: (1)-promise to shorten design cycles, and (2)-provide the freedom to reprogram the design in simple ways. This ease-of-change stands in sharp contrast to the great effort required to redesign a typical hard-wired analog implementation.

On the other hand, data conversion requirement is not the only disadvantage of the digital systems; analysis and design methods are more complex for sampled data systems, sampling and resolution can affect the performance, computational delays limit the system bandwidth and degrade accuracy. A less commonly listed disadvantage of digital control is the software development phase which is tedious, error-prone, time consuming, and hence expensive.

For control systems, depending on the complexity of the plant and the degrees of freedom (DOF) to be controlled, digital implementation of an algorithm may be demanding due to the high computational power requirement to run in real time or in the case of small systems an excessive computational power due to the problems of scaling down the digital hardware.

This work aims to develop an analog motion controller for SISO plants of complex nature. As the control algorithm, Sliding Mode Control (SMC) like, the well known robust nonlinear controller is selected as a design framework. Originally designed as a system motion for dynamic systems whose essential open-loop behavior can be sufficiently modeled with ordinary differential equations, Sliding Mode Control (SMC) is one of the effective nonlinear robust control approaches that provide system invariance to uncertainties once the sliding mode motion is enforced in the system [1, 2]. An important aspect of sliding mode is the discontinuous nature of the control action, which switches between two values to move the system motion on so-called “sliding mode” that exist in a manifold and therefore often referred as variable structure control (VSC). The resulting feedback system is called variable structure system (VSS). In this paper we will be using SMC design methodology to arrive to controller that has very small discontinuous component or in some cases no discontinuous component if the exponential convergence to sliding mode manifold is acceptable. In addition we will be trying to make structure as simple as possible in order to allow analog implementation with limited hardware. The aim is in the later stage to try to integrate a whole controller as a part of the high voltage amplifier needed to supply the PZT actuator.

As a test bed for the designed system we selected the position tracking of the piezoelectric actuators (PEA) which are strongly nonlinear plants that occupy important place in micro-actuation world to provide sub-micrometer resolution. Piezoelectric actuators do not suffer from “stick slip” effect mainly caused by the friction between elements of a mechanical system [3]. This property offers an unlimited resolution, and therefore usage in many applications; ultrasonic motors, sports materials [4], aerospace [5], hard disk drives [6], the scanning tunneling microscope (STM) and atomic force microscope (AFM)[7]. Still the achievable resolution in practice can be limited by a number of other factors such as the piezo control amplifier (electronic noise), sensor (resolution, noise and mounting precision) and control electronics (noise and sensitivity to EMI).

In addition to the properties like high bandwidth, stiffness and energy efficiency, they exhibit fundamental hysteresis phenomena [6-14]. Hysteresis yields a rate-independent lag and residual displacement near zero input [15]. The existence of hysteresis limits the performance of the piezoelectric actuator, leads to inaccuracies (up to 10%-15% of the traveling path) and causes undesirable oscillation or even instabilities [16]. Therefore, achieving a high speed and large-range precision positioning of piezo actuators is challenging [17].

The open loop control option is generally shown as a “must” since the position change measurement in the orders of micrometers requires expensive devices. As a results much effort is done to model the hysteresis with invertible functions...
that will be used for compensation purposes [7, 11, 13, 15, 18-20]. In order to design a control scheme that will achieve successful tracking performance without precise dynamic modeling, some fuzzy logic and neural network solutions are presented in the literature. However, due to the limited performance, this research area did not find much popularity [21].

Different nonlinear control strategies are also studied. $H_{∞}$ control [6, 22] offers a good tracking and eliminates the high frequency oscillations for the cost of rounding the corners of the triangular waves and a noticeable delay increasing with frequency. Woronko et al. used SMC to improve machining precisions [23]. Abidi et al. used SMC in conjunction with the disturbance observer for both position and force tracking in piezoelectric actuators [24, 25]. In their work, the lumped parameter model and disturbance observer are used. Bonnail et al. applied SMC on a piezoelectric actuated scanning tunneling microscope to precisely follow the sample surface with the feedback of the tunneling current [26]. Compared to the commercial PI controlled motion, their solution shows a less oscillating tunneling current due to the better tracking of the surface.

The rest of the paper is organized as follows; section II describes the plant and selected model which constitutes a basis for the controller design. The Sliding Mode Controller design together with disturbance observer is presented in Section III. Section IV presents the analog circuit implementation of the algorithm. Section V presents the experimental results while conclusions and areas for future research are presented in Section VI.

II. PLANT: PIEZOELECTRIC ACTUATOR

The piezoelectric actuator consists of an electrical and a mechanical model connected to each other by an appropriate conversion constant $T$ called “electromechanical transformation ratio”. As a result, the dynamics of the piezo-stage can be represented by the following second-order differential equation coupled with hysteresis in the presence of external forces,

$$m_{eff} \ddot{x} + c_{eff} \dot{x} + k_{eff} x = T \cdot [u(t) - h(x, u)] - F_{ext}$$

(1)

where $x$ denotes the displacement of the stage, $m_{eff}$, $c_{eff}$ and $k_{eff}$ denotes the effective mass, effective damping and effective stiffness of the stage respectively, $u$ denotes the input voltage and $h(x, u)$ denotes the unknown, bounded, non-linear hysteresis that has been found to be a function of $x$ and $u$, finally $F_{ext}$ is the external force acting on the stage [13, 27]. Since robust control techniques will be studied, the exact knowledge of the parameters is not so crucial, nominal values can be used.

The model (1) shows that the hysteresis may be perceived as a disturbance. At the same time the lumped disturbance consisting of the external force acting on the system and hysteresis can be estimated, thus allowing the application of the disturbance rejection method in the overall system design.

III. SLIDING MODE CONTROL & DISTURBANCE OBSERVER

In this work, we will consider dynamical systems that can be represented as a class of nonlinear systems, linear in respect with control as described by the following equation

$$\dot{x} = f(x) + B(x) \cdot u + d$$

(2)

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control vector, $f(x) \in \mathbb{R}^n$ is an unknown, continuous and bounded nonlinear function, $B(x) \in \mathbb{R}^{nxm}$ is a known input matrix whose elements are continuous and bounded and $\text{rank}(B(x)) = m$, with $d \in \mathbb{R}^n$ being an unknown, bounded external disturbance. It is assumed that both $f(x) \in \mathbb{R}^n$ and $d \in \mathbb{R}^n$ satisfy the matching conditions and all their components are bounded $\|f_i(x)\|_{yx} \leq M$ and $\|d_i(t)\|_{yx} \leq N$. Fully actuated mechanical systems belong to the class of systems described by (2). Such systems can be interpreted as $m$ interconnected sub-systems

$$\dot{x}_i = h_i(x_i, \dot{x}_i) + b_i(x_i, t) \cdot u_i + g_i(x_i, x_j), \quad h_i(x_i, \dot{x}_i)$$

(3)

and $g_i(x_i, x_j)$ represents Coulomb friction term and the interaction term respectively and they are regarded as disturbance.

The aim is to determine the control input $u = [u_1, \ldots, u_m]^T$ such that the system states $x_1(t), \ldots, x_n(t)$ track the desired trajectories $x_{d_1}(t), \ldots, x_{d_n}(t)$ while control error satisfies selected dynamic requirements.

The controller will be designed in the SMC framework by firstly selecting a suitable sliding manifold that will ensure desired systems dynamics and then selecting control such that the Lyapunov stability conditions are satisfied. Selecting the Lyapunov function candidate in terms of the sliding function is a natural way of guaranteeing the sliding mode existence on the selected manifold and thus having the desired closed loop dynamics. Finally, the necessary control input that will fulfill the requirements of the Lyapunov stability criteria should be selected.

A. Sliding Manifold

For system (2) the natural selection of the sliding manifold is in the following form

$$\sigma = G \cdot e_t = 0,$$

(3)

where tracking error vector is defined as $e_t = [e_1, \ldots, e_n]^T \in \mathbb{R}^n$, $e_i = x_{d_i} - x_i$ and the sliding surface satisfies $\sigma = [\sigma_1, \ldots, \sigma_m]^T \in \mathbb{R}^m$, $G \in \mathbb{R}^{mn}$. The
aim is to drive the system states of the system into the set $S$ defined by $S = \{x : G \cdot e = \sigma(x, x_d) = 0\}$ [28, 29].

For the position tracking problems, the sliding mode manifold can be selected as a combination of the position and velocity error as; $e_t = [e \ e^T] \in \mathbb{R}^2$ with $e = x_d - x$ and $\dot{e} = \dot{x}_d - \dot{x}$.

B. Computation of the Control Action

The continuous control law design, derived from the continuous-time equations, starts with a Lyapunov Function candidate selection;

$$V = \frac{1}{2} \sigma^T \sigma$$

where, $V \in \mathbb{R}$, $V(0) = 0$. The time derivative of the candidate Lyapunov function $\dot{V}$ should be negative definite.

In order to use this condition in selection of the control, we may require that the $\dot{V}$ satisfies some preselected form. Equating the time derivative of this function to a negative definite function like

$$\dot{V} = -\sigma^T \left(D \sigma - \mu \frac{\sigma}{\|\sigma^T \sigma\|}\right)$$

where $D$ is a positive definite symmetric matrix and $\mu > 0$, thus Lyapunov conditions are satisfied. For $\sigma \neq 0$, the control law can be calculated by satisfying the following equation;

$$\sigma + D \sigma + \mu \frac{\sigma}{\|\sigma^T \sigma\|} = 0$$

and the sliding mode conditions are satisfied [30]. The discontinuous term can be selected as small in order to avoid chattering. It was proven [31, 32] that the sliding mode is ensured with continuous control action in the discrete time implementation. Since analog application is targeted, the discontinuous term $\mu \cdot \frac{\sigma}{\|\sigma^T \sigma\|}$ in our application will be omitted, and we will be determining the control action that satisfies the condition $(\sigma + D \sigma) = 0$. This will result is quai sliding mode behavior of the system with an exponential convergence defined by selection of matrix $D$. If that convergence is fast enough then the motion of the system will be in an $\varepsilon$-vicinity of the $\sigma = 0$ for most of the time and the reaching stage will be governed by $\dot{(\sigma + D \sigma)} = 0$. But all further analysis can be easily adopted for the application of expression (6) if the term $D \sigma$ is replaced with $\sigma + \mu \sigma/\|\sigma^T \sigma\|$.

For system (2) with sliding mode manifold (3) the control that satisfies $(\sigma + D \sigma) = 0$ can be determined as

$$u = -(GB)^{-1} (G(f + d - \dot{x}_d) - D \sigma)$$

$$= u_{eq} + (GB)^{-1} D \sigma$$

where $x_d = [x_{d_1}, \ldots, x_{d_n}]$ and $u_{eq}$ is the so-called equivalent control obtained as a solution of the equation $\dot{\sigma} = GB \cdot (u_{eq} - u) = 0$. By substituting (7) into (2) the equations of motion of system (2) in manifold (3) are obtained as $\sigma = G \cdot e_t = 0, \dot{x} = f(x) + B(x) \cdot u_{eq} + d$ and the approach to this solution is governed by equation (6).

To implement this control input, information about the plant dynamics and external disturbances are needed, although they are hard to achieve. Hence, this solution needs the information on the equivalent control thus may be applied for the plants when $u_{eq}$ is known or can be estimated with sufficient accuracy. As proven in [33, 34] with small enough filtering tie constant $\tau = 1/g$, the solution of the differential equation (8) is close to the equivalent control.

$$\tau \dot{z} + z = [u - (GB)^{-1} \cdot \sigma]$$

In order to make controller structure simpler in this paper (8) will be used in order to avoid a direct calculation of the equivalent control from $u_{eq} = -(GB)^{-1} G \cdot (f + d - \dot{x}_d)$; instead use an approximated result $u_{eq} = z$.

$$z = \frac{g}{s + g} \cdot u - K \cdot \frac{g}{s + g} \cdot s \sigma$$

with $K = (GB)^{-1}$. Using the fact that

$$\frac{s}{s + g} = 1 - \frac{g}{s + g}$$

one can avoid direct calculation of the derivative for $\sigma$;

$$z = \frac{g}{s + g} \cdot (u + K \cdot g \cdot \sigma) - K \cdot g \cdot \sigma$$

Equation (8) could be used as it is. However, derivative operation is known to amplify the noise existing in input signals, therefore the calculation in equation (9.3) is realized. Using $u_{eq} = z$ and (7) the applied control $u = z + K D \cdot \sigma$ can be calculated

$$u = \frac{g}{s + g} [u + K \cdot g \cdot \sigma] - K \cdot (g + D) \cdot \sigma$$

Control (10) is suitable for analog implementation since it requires measurement of the sliding mode function and control, yet ideally gives stable motion in manifold $S = \{x : G \cdot e_t = \sigma(x, x_d) = 0\}$. Design parameters $D$ and $C$ should be selected as high as possible in order to achieve fast transients.
C. Approximation Error

The error due to the approximation \( u_{eq} = z \) can be directly calculated using applied control in the following form
\[
\sigma + D\sigma = u_{eq} - z
\] (11)

For small \( \tau = 1/g \), the error in control due to the approximation mainly consists of the high frequency part of the un-approximated control. Using this result, it can be shown that the tracking error due to the control approximation error is bounded. For large \( g \), the estimation error is small, yet as \( g \to \infty \) then \( (u_{eq} - z) \to 0 \); so the approximated system behaves close to the ideal \( \sigma + D\sigma = 0 \).

D. Disturbance Observer

The structure of the observer is based on (1) with the assumption that all the plant parameter uncertainties, nonlinearities and external disturbances can be represented as a lumped disturbance. To represent this, assume that all plant parameters have nominal values denoted with subscript \( N \) and uncertainties shown with \( \Delta \cdot \)
\[
m_{eff} = m_N + \Delta m \quad c_{eff} = c_N + \Delta c
\]
\[
k_{eff} = k_N + \Delta k \quad T = T_N + \Delta T
\] (12)

The displacement of the plant \( x \), and the input to the plant \( u \), are measurable. Hence, the nominal structure of the plant is found as follows
\[
m_N \ddot{x} = T_N \cdot u - F_d
\] (13)
where;
\[
F_d = T_N \cdot h + \Delta T(h + u) + \Delta m \cdot \ddot{x}
\] (14)

is the disturbance on the system.

For simplicity the disturbance observer proposed by Ohnishi et al. can be used [35]. The derivation of the observer is as follows. From (14) the disturbance force is
\[
F_d = T_N \cdot u - m \cdot \ddot{x}
\] (15)

Ohnishi et al. proposes that the observed disturbance force \( \hat{F}_d \) is;
\[
\hat{F}_d = \frac{g}{s + g} \cdot (T_N \cdot u - m \cdot s^2 \ddot{x})
\] (16)
where \( g \) is the constant determining the corner frequency of the first order filter. Since \( x \) and \( u \) are measurable, \( \hat{u} \), the correction to the control output, can then be calculated as;
\[
\hat{u} = \frac{\hat{F}_d}{T_N} = \frac{g}{s + g} \cdot \left( u - \frac{m}{T_N} \cdot s^2 \ddot{x} \right)
\] (17)

To avoid calculation of the second derivative for the position \( x \), one can use the previously used fact (9.2) and determine \( \hat{u} \) as;
\[
\hat{u} = \frac{g}{s + g} \cdot u - \alpha \cdot sx + \alpha \cdot \frac{g}{s + g} \cdot sx
\] (17)

where \( \alpha = m \cdot g/T_N \). In real application the velocity is usually calculated using first order filter to obtain \( sx \approx sg_v/(s + g_v) \cdot x \). That approximation leads to the calculation of disturbance as
\[
\hat{u} = \frac{g}{s + g} \cdot u - \alpha \cdot \left( 1 + \frac{g}{s + g} \right) \frac{g_v}{s + g_v} \cdot x
\] (18)
It is now clear that the filter \( g/(s + g) \) could have higher corner frequency that the velocity measurement filter \( sg_v/(s + g_v) \).

IV. CIRCUIT DESIGN

A. Analog Controller

![Fig. 1: Summary of the analog circuit: SMC with DO for position tracking of PEA.](image)

A simplified circuit diagram of the designed circuit for PEA control by analog SMC with DO is shown in Fig. 1. The circuit is based on mathematical operations like summation, differentiation etc. and is realized with op-amp circuits. Therefore, the figure includes the inverting behavior of the op-amps; signal names shown with a minus sign represents inverted-ones. Balloon in the connections, close to the op-amps, represents the weights used in the summations.

The operation of the schematic in Fig. 1 can be described as follows: the position measurement \( x \) is subtracted from the desired position \( x_d \) using op-amp U1 to calculate the tracking error \( e \). U2 has similar operation but it also includes a derivation behavior to calculate \( \dot{e} \). Op-amp U3 is used to sum up those two signals with appropriate weights to calculate the sliding function \( -\sigma \).

In next stage, the signals \( -u \) from the output is fed forward together with \( -\sigma \) to U4 where they are summed and filtered to output the intermediate variable \( \varphi \).

In parallel, the first derivative of the position measurement is calculated using U6 and the summation of this value with \( -u \) is filtered in U7.

As a final stage U5 sums up all calculated values with
appropriate weight to form the control output $-u$ that is fed to the high voltage amplifier (HVA).

B.  High Voltage Amplifier

Piezoelectric actuators require $-60...150V$ for full range operation while the described circuit so far is based on low voltage mathematical operations. For this reason a high voltage amplifier is added to the circuit as a buffer between the calculation part described so far and the actuator. This circuit, bases on Apex Microtechnology’s MP108 power op-amp, is actually a simple inverting amplifier with constant gain as presented in Fig. 2.

![Fig. 2: High voltage amplifier as a final block of the circuit.](image)

V.  EXPERIMENTAL RESULTS

A.  Experimental Setup

For experimental purposes, the setup shown in Fig. 3 has been constructed in the Mechatronics Laboratory at Sabanci University, where the tests were performed. The voltage amplifier is the one described previously, PEA is the piezoelectric actuator with embedded strain gage for position measurement, and the strain gage amplifier is the SCM5B38-03 wide band strain gage amplifier from Dataforth Corporation. Here, SMC is the designed sliding mode control algorithm implemented in DSP (for DSP experiments) or the analog circuit (for analog controller experiments). An actual photo of the setup is presented in Fig. 4 below.

![Fig. 3: Piezoelectric actuator control setup.](image)

Data is captured by Agilent Technologies 54622D digital oscilloscope. Peak to peak values are given at the image captions in metric correspondents: 17.96um (micrometers) corresponds to 1V of the strain gage amplifier reading, or in other words 1um position deflection results 55.68mV.

![Fig. 4: Actual photo of the experimental setup.](image)

B.  Position Tracking Experiments Using DSP

For a comparison of the results, DSP application of the control is realized on dSpace DS1102 platform which possesses TMS320C31 DSP chip running at 40MHz with 50ns cycle time. The platform has two 16-bit ADC (Input) $\pm 10V$ and four 12-bit DAC (Outputs) $\pm 10V$. The algorithm runs at 10kHz, and the discretization is made based on Euler’s method.

The position tracking of 1Hz sinusoidal inputs is studied. First 4.5um peak to peak and then 10.8um peak to peak inputs are tested. The results are shown in Fig. 5 and Fig. 6 respectively. The errors for comparison are 110nm and 200nm, corresponding to 2.5% and 1.9% respectively.

![Fig. 5: DSP tracking experiment for 4.5um pp 1Hz sinusoidal reference.](image)

C.  Analog Circuit, Position Tracking Experiments

Similar experiments are conducted for analog circuit realization of the SMC with a disturbance observer. The tracking of a 35.69um peak to peak 1Hz sinusoidal reference resulted with 50nm peak to peak tracking error corresponding to only 0.14%. Compared to the 2.5% tracking error of the DSP implementation, this result is almost 18 times better.
The tracking of a single sinusoidal pulse of period 1 second and peak to peak amplitude 35.60um is shown in Fig. 8. According to the experiment the peak to peak error value is 54nm corresponding to 0.15% tracking error.

Triangular wave shapes are also studied. As an example tracking of 35.60um pp 1Hz triangular reference is presented in Fig. 9. The resulting peak to peak error is 54nm pp corresponding to 0.15%. Triangular wave shapes are of particular importance since they constitute non-continuous references.

To present the tracking of an abstract but continuous waveform the tracking of a human heart beat signal is presented in Fig. 10. For this waveform with peak to peak amplitude of 21.70um, the tracking resulted with 46nm peak to peak tracking error corresponding to 0.21%.

VI. CONCLUSION

In this work we presented the analog application of a controller, based on SMC with DOB for PEA in the framework of SMC. The designed controller assures a quasi sliding mode motion with exponential convergence to the selected sliding mode manifold. The combination of DOB and the SMC like controller allows very compact analog electronics implementation. Experimental results proved that the analog implementation of the proposed SMC is acceptable and that the controller can track a reference signal with high accuracy. In comparison with DSP realization of the same algorithm the realization in continuous time domain showed at least 2 times performance increase. The addition of the disturbance observer on the other hand, improved this result another 10 times. Resulting controller can track appropriate signals at around 0.05% tracking error corresponding to 50nm for a reference amplitude of 1um.

REFERENCES


