SUCCESS BREEDS SUCCESS LOCALLY: A TALE OF INCUBATOR FIRMS

Eren Inci
Sabanci University

September 2007

Abstract

This paper focuses on the pre-establishment period of start-ups in industrial districts. The industrial architecture is what I call a "rationed agglomeration" in which some entrepreneurs gather around an established firm while other entrepreneurs in the same business stand alone. In a rationed agglomeration, I analyze the effects of relations between established firms, network entrepreneurs, and local financiers on the market prices of loans. I show that such relations improve the match of capital to ideas in the network even though the overall distribution of capital to ideas remains unchanged. This suggests that success breeds success in the networks of established firms. The existence of networks overturns the claim that there are no motives to engage in information gathering in a simple market regime with information asymmetries. In particular, I show that there are market incentives for established firms to decrease the information gap between network entrepreneurs and local financiers.

Keywords: agglomeration; entrepreneur; dispersion; innovation; local financiers; networks; regional economies; project financing; signaling; start-up

JEL Classification: D82; G20; R12; L26

*Tel.: 90-216-483-9340; fax: 90-216-483-9250. Address: Sabanci University - FASS, Orhanli / Tuzla 34956 Istanbul TURKEY. E-mail address: ereninci@sabanciuniv.edu. This paper was initiated while I was visiting the Industrial Economics and International Management Department of the Center for European Economic Research (ZEW) in 2006. I am grateful to their hospitality and financial support. I have greatly benefited from discussions with Richard Arnott, Andy Newman, participants of the Cambridge-MIT Institute’s Workshop on Regional Innovation at the University of Cambridge (2006) and the Annual Congress of the European Economic Association (2007). This research is financially supported by Paula and Daniel Greeley Award and Boston College Dissertation Fellowship Award. All errors are mine.
We [...] show how Cleveland’s initial locational advantages were magnified, perhaps serendipitously, by a small number of successful enterprises that both exemplified the wealth-creation possibilities of these new technologies and served as hubs of overlapping networks of inventors and financiers. Focusing on one of the most important of these hubs—the Brush Electric Company—we show how such enterprises served multiple functions for the inventors who gathered around them. On the one hand, they were places that fostered technological crossfertilization and the exchange of ideas about how to solve particularly difficult problems. On the other hand, they were places where the technological community could pass on—validate—promising ideas and thus perform a useful vetting function for local capitalists.

Lamoreaux, Levenstein, and Sokoloff (2004, pg. 2)

1 Introduction

One important economic phenomenon is that geographic proximity creates positive externalities among firms. Sometimes these are physical spillovers in the form of low transportation costs (Krugman, 1991); sometimes they are intellectual spillovers which are more pronounced among firms that are close to each other (Glaeser et al., 1992). This paper focuses on a different aspect of agglomeration economies. Starting and expanding a business is not easy in a world of intense competition. It is well documented world-wide that many start-ups end up as failures within the first couple of years of starting business (see, for example, Bates (2005), Brandt (2004), Headd (2003), and OECD (2006) for recent evidence). This suggests that the firms in successful industrial locations must have taken alternative avenues that led them to experience less frequent business failures. One such effective avenue is to form informal networks to overcome the stigma of failure. When they are developing ideas and preparing business strategies, potential entrepreneurs not only interact with each other and established firms but also get help from the established firms in obtaining funding. This paper analyzes how these nonmarket institutions (informal networks) form, and then derives the outcome of the market given their presence to see if they lead to better outcomes for the network members and for society in general.

I analyze the effects of ties between start-ups and established firms in close geographic proximity, and their relationships with the local financiers. On one side, for better chances of survival, new firms need enough liquidity, and better technical expertise in production and business plans (such as pricing and marketing). On the other side, the source of successful regional economies can often be tracked down to one or two hub firms which form fertile environments that facilitate the creation of new firms. These hub firms usually breed new firms with lower risk of failure by improving the match of capital to ideas within their networks, and act as seedbeds for new ideas or start-ups by sponsoring the innovative activities of related individuals. They are the places where potential entrepreneurs meet when they are developing their marketable ideas.

Most business failures are the result of lack of access to either sufficient or cheap enough loans. The relationships between established firms and potential entrepreneurs usually end up in financial collaborations. It is well documented by Petersen and Rajan (1994) that ties between firms and their creditors are very important for the availability and cost of funds. The hub

---

1There could be successful and unsuccessful closures. Even with that distinction, there is still a significant number of failures: between 30 to 40 percent of firms experience unsuccessful closures within the first couple of years of business.
firms have good relationships with local financiers which they can use in finding finance for the projects of entrepreneurs in their networks. They may also invest in these start-ups if they see any exploitable profit opportunities. This process results in higher success probabilities in the network.

The general ideas above boil down to a model of the pre-establishment period of start-ups. The stepping stones of the model are based on the work of Lamoreaux, Levenstein, and Sokoloff (2004), who provide a wonderful historical case study of financing innovation in Cleveland, Ohio in the nineteenth century, which was much like the Silicon Valley of its time, and the role of networks in generating and financing innovative ideas. They focus particularly on the Brush Electric Company, the inventors gathered around it, and the firms that were somehow brought to life in that firm’s network. Their paper can also be viewed as the empirical support (or the historical evidence) for the model presented here. I quote passages from this paper wherever necessary to elucidate the assumptions and results.

The pre-establishment period of start-ups is assumed to have two phases. In phase I, potential entrepreneurs collect information about their subject matter. They not only develop innovative ideas but also consider how to market and sell these ideas. They have two options in this phase: they can either stand alone and develop their innovative ideas and business strategies by themselves or they can join the network of a hub firm that provides a collaborative environment with other would-be entrepreneurs. Phase I is similar to the R&D game presented in Inci (2005) with some differences. If an entrepreneur stands alone, his benefit results from his own effort (which has both a deterministic and a random component) and the knowledge that spills over from the nearby entrepreneurs. The benefit that derives from an entrepreneur’s own effort is the same no matter whether he is in or outside the network, and joining the network is costly. However, the degree of knowledge spillover is higher among entrepreneurs who are in the same network.

It turns out that, as a result of network externalities, any stable equilibrium has to be a corner outcome. That is, either all entrepreneurs prefer to join the network or they all prefer to stand alone. The reason for this is that an entrepreneur finds it beneficial to join a network only if sufficient numbers of others are doing so. Nonetheless a hub firm cannot allow just anyone to join its network, which suggests that there will be some sort of rationing process involved in joining the network. This is why we observe firms that are related to each other as well as some others that stand alone in the same industrial district. I call this industrial architecture a "rationed agglomeration." At the end of phase I, the random part of the individual benefit is drawn by the nature, and thus, the types of projects are determined. I assume for simplicity that there can only be two outcomes: good or bad. Hence, there will be entrepreneurs with projects of high and low success probability both in and outside the network. However, I assume that those with a good drawing will be higher in number in the network than outside the network.²

Phase II of the pre-establishment period of start-ups involves seeking funding for the business projects that are already in hand. In this stage, entrepreneurs have already established their networks, come up with their innovative ideas, and prepared the business plans associated with these ideas. Phase II, therefore, focuses on a rationed agglomeration in which there are more high-success probability projects in the network than outside the network. This phase is a variation of the project financing game of Inci (2006). However, the population is now composed of two groups, network entrepreneurs and stand-alone entrepreneurs. It is assumed that all projects are worthy even though some of them have better chances of survival. Efficiency

²This is sometimes called the "network effect" in the literature to refer to the advantages of interacting in a network.
requires that all of these projects should be financed, and thus, credit rationing is not an issue in this model. This helps me highlight the important result that nonmarket institutions may induce inefficient agglomeration in the presence of asymmetric information.

Under normal conditions, entrepreneurs apply for bank loans to finance their projects. There are, however, two important assumptions of the model. First, a hub firm has a belief about the project type of an entrepreneur which may or may not be correct for this particular entrepreneur, but its beliefs on average are informative due to its repeated relationships with entrepreneurs in phase I. It can make useful judgments simply because it has long years of business experience, although there is still some room for incorrect judgments. The network membership of those who are believed to have low-success probability projects expires automatically at this point. Second, a hub firm has close relationships with local financiers to whom it can convey its beliefs about the entrepreneurs in its network. Given the first assumption, this channel can decrease the information gap between the network entrepreneurs and local financiers. Then, if the local financiers trust the information they get from the hub firm, they will provide cheaper loans to those who are labeled as good by the hub firm, and they will decline the applications for privileged loans of those who are labeled as bad.

The declined group, as stand-alone entrepreneurs, may then apply for regular loans that are readily available in the loan market from either the banks or local financiers. Being aware of these relations in the market, lenders (both banks and local financiers) would then change their beliefs about the distribution of types in their loan applicant pool, since a better sample of entrepreneurs is financed with cheaper loans by the local financiers who have an informational advantage on this sample. This means that the price of loans is higher for the stand-alone entrepreneurs and for those who are declined for privileged loans. Therefore, the hub firm’s signaling improves the credit market outcome of those who stay in the network and worsens the outcome of the rest.

It is commonly believed that there are no motives to engage in information extraction in a simple market regime with information asymmetries (see Campbell and Kracaw (1980)). This raises the question of why a hub firm would have any incentive to decrease the information gap between entrepreneurs and local financiers. When there is pooling equilibrium in the credit markets – which is the case in this paper – the market overvalues the start-ups with low success probabilities and undervalues the start-ups with high success probabilities. The existence of networks alters the level of under- and overvaluation. I show that hub-signaling always makes entrepreneurs with high success probability better off by decreasing the level of the market’s undervaluation of their start-ups. However, entrepreneurs with low success probability projects prefer hub-signaling when the network is large enough (or when the signals are not very informative). In such cases, both parties prefer hub-signaling, and side payments promised by them can be sufficient incentives for the hub firm to organize hub-signaling. However, when the network is small (or the signals are sufficiently informative), owners of the low success probability start-ups prefer the status quo while the owners of the high success probability start-ups prefer hub-signaling. Yet, I show that the maximum amount of side payments that the latter group is willing and able to pay to the hub firm is higher than that of the former group. This implies that there are certain incentives for the hub firm to form the hub-signaling mechanism. As I show in the paper, this result can be fairly generalized to a case in which extracting the information about start-ups is costly for the hub firm. Thus, the existence of networks overturns the claim that there will not be costly information extraction. This also suggests that asymmetric information can result in inefficient agglomeration.

I assume throughout the paper that the hub firm conveys its signals honestly to the local financiers. There are a couple of reasons to believe this. First, the hub firm may have some
repeated financial relationship with the local financiers. If so, it may not be in its best interest in the long run to act dishonestly. Moreover, acting dishonestly may jeopardize its credibility in the market as well. Whenever the credibility of signals is a problem, local financiers may finance the start-ups only if the hub firm is also investing in those projects reasoning that if the hub firm believes that its recommended entrepreneurs will have better start-ups on average, it should be more than happy to invest in them. Section 5 formally shows that the signals of the hub firm are credible only if it has sufficiently large stakes in these start-ups. However, this can happen only when the hub firm has sufficient assets. Therefore, the established firms that can credibly organize hub-signaling will be the ones with deep pockets.

The only positive effect of the network in this model comes from phase I, in creating more high success probability projects. In phase II, networks may improve their possible outcomes in the market; however, this may not be useful for society in general since the quality composition of entrepreneurs is still the same. That is, with or without hub-signaling, there will remain the same number of high and low success probability projects in the region, since the lenders will still prefer to finance both types of projects. Arnott and Stiglitz (1991) show that nonmarket institutions may be dysfunctional when they are not informationally advantaged over the market – which is also the case in this paper if the signals of the hub firm are not informative on average. Beyond that, what the model presented here suggests is that even in the case in which the nonmarket institution is informationally advantaged (e.g.; relationships generate useful information), the outcome may not be more useful for society than otherwise. Hub-signaling can create islands of related entrepreneurs that experience less frequent business failures and enjoy cheaper loans even though the overall failure rate in the population remains unchanged. Therefore, an observation of a high number of better types in a network is not sufficient to imply that this network is socially desirable.

This paper is related to the empirical paper by Petersen and Rajan (1994) which finds that banking relationships are valuable, although they conclude that these relationships appear to operate more through quantities than prices. However, they implicitly assume that the decreases in the cost of loanable funds are passed on to the borrowers, which is not necessarily the case in the presence of monopolistic power over information. This is consistent with the results derived here. Suppose there is only one local financier that has access to the signals of the hub firm. Since it has monopoly power on this information it knows that the default rate in its loan applicant pool will now be lower, but it does not need to reflect this change to the borrowers. In that case, it can still finance the same entrepreneurs with the same loan prices available in the market and make positive profits because of its informational advantage.

The paper is organized as follows. Section 2 sketches the pre-establishment period of startups, which has two phases. Phase I – the phase in which entrepreneurs work on innovative ideas and prepare their business plans – is modeled in section 3. Entrepreneurs’ network formation decisions and the resulting industrial architecture in the region are also discussed in this section. Phase II – the phase in which entrepreneurs apply for business loans – is modeled in section 4. In this section, the canonical project-financing equilibrium from banks is derived as a benchmark followed by the analysis of the role of established firms in entrepreneurs’ loan applications to the local financiers. Section 5 discusses the incentive scheme for the hub firm

---

3I focus on relationships between start-ups and an already established firm which has relationships with local financiers. The relationship in Petersen and Rajan (1994) is between lenders and borrowers, not via a hub firm. The hub firm’s ideas about the entrepreneur that it conveys to the lenders should be more credible than an entrepreneur’s signaling of his own type. Moreover, Petersen and Rajan (1994) focus on already established firms whereas I focus on start-up firms for which the data cannot capture the relationship as they define it. They define the relationship as having at least one other financial service from the lender besides borrowing, such as depository services, factoring, or pension fund management.
to organize hub-signaling assumed in the previous sections. Costly signaling and the reliability of signals are also discussed here as extensions of the model. Section 6 concludes. An appendix contains some of the proofs.

2 A model of pre-establishment period of start-ups

I start off with a thumbnail sketch of the model followed by a formal analysis.

2.1 Thumbnail sketch of the model

I consider a simple model of a regional economy to model the pre-establishment period of start-ups. There are two phases in this period. The first phase is the idea-generation stage in which potential entrepreneurs try to develop their business projects. This requires not only engaging in invention projects that might end up with an innovation with some economic value but also developing business plans such as pricing and marketing strategies. In section 3, I model this phase and try to explain why we observe different economic architectures across time and space. The focus of this part of the model is on the aggregate level of a regional economy and it tries to provide some insights into why there are some industrial districts where a number of firms in a network have gathered around established firms while there are other entrepreneurs in the same market who stand alone. Basically, this part analyzes what I call a "rationed agglomeration" (partial agglomeration of start-ups around some established firms).

In the second phase, potential entrepreneurs have already come up with their business ideas which are presumably somewhat risky. They are now in a position to look for financing possibilities for their projects. As I argue in the introduction of the paper, the main obstacle to the formation of new firms is the relentless stigma of failure. Section 4 models this start-up financing game between entrepreneurs and lenders (banks and local financiers) in the presence of a rationed agglomeration around an established firm. I explain the role of this established firm in finding (possibly) cheaper financing for the (potentially) better entrepreneurs in its network by conveying its beliefs about the quality of their projects to the local financiers.

Below I use the words entrepreneur, firm, inventor, individual, and agent interchangeably. Doing so does not make much difference for my purposes since I track the flow of the business project itself rather than its owners at different times in its lifecycle. In a more general framework, inventors come up with an innovative idea; then, an entrepreneur – who may or may not be the same person as the inventor – carries out the project. Where it makes a difference at all, the payoff structure of the model implicitly takes into account the net economics effects of any exchanges of the business project among parties. I also use "entrepreneur" instead of "potential entrepreneur" at some places for brevity. This is harmless since there is no credit rationing in this model, and thus, all potential entrepreneurs will become entrepreneurs no matter what happens.

2.2 Environment and timing

Suppose there is a region at the beginning of the period that can become a thriving industrial district if it experiences a constant formation of successful firms over time. What I have in mind is a would-be agglomeration at the very beginning of its lifecycle that can become a
successful industrial district such as Silicon Valley of the present time or Cleveland, Ohio of the late nineteenth century. As evidenced by Lamoreaux, Levenstein, and Sokoloff (2004), such agglomerations can often be tracked down to one or two firms which act as incubators for new firms with higher success probabilities. For simplicity, I assume that there is one such firm, which I call a hub firm and denote by \( h \). However, the analysis can be generalized to a case in which there are more than one hub firm.\(^4\) If this hub firm is unsuccessful, then this is the end of the story. To model how success breeds success, the rest of the analysis thus focuses on a hub firm which is known to be a successful innovative firm.

There is also a unit mass of entrepreneurs who plan to engage in start-up activities in an innovative sector.\(^5\) This might be the biotechnology or nanotechnology sectors today, and electric light, steel or chemistry sectors at the time of Brush Electric Company. Start-up activities require pre-establishment preparations. These preparations can be anything related to the business project that entrepreneurs plan to employ in the post-establishment period. Innovative idea generation and business plans (such as marketing and pricing strategies) can summarize almost all of these pre-establishment period activities.

All entrepreneurs are assumed to be identical at the beginning. These identical entrepreneurs decide whether to join the network of the hub firm \( h \). Then, they learn their types at the end of phase I. Finally, phase II starts given the distribution of types within and outside the network.

### 3 Phase I: innovative idea creation and business plans

Entrepreneurs are in a position to decide between two options in phase I. They can work on their projects alone in which case they have to come up with their innovative ideas and prepare their business strategies independently (call this a stand-alone entrepreneur). Alternatively, they can approach the hub firm – where there may be other entrepreneurs working on similar projects – and try to make use of the collaborative environment by interacting with the other would-be entrepreneurs in the network (call this a network entrepreneur). Entrepreneur \( i \)'s own effort creates a net benefit with both a deterministic and a random part:

\[
b(e) + \varepsilon_i \quad \forall i \in [0, 1] \quad ,
\]

\(^4\)There is no harm to perceive the hub firm as a representative of all hub firms for my purpose in this paper, just like we do for "representative consumer" in consumption theory.

\(^5\)A continuum of agents is assumed for technical convenience. The analysis can easily be modified to allow for a discrete number of firms in which case, to prevent technical complications, one of the production factors has to be assumed to be infinitely indivisible.
where $e$ is the level of effort and $b(e)$ is the deterministic part of the net benefit from effort with $b'(e) > 0$, and $\varepsilon_i$ is the individual specific random part of the net benefit from effort. This specification allows that some entrepreneurs may end up with better ideas even though all entrepreneurs are \textit{ex ante} identical. A further specification of the random part is given at the end of this section.

In addition to the benefit that comes from effort, entrepreneurs also benefit by observing the other entrepreneurs around them. This is simply the usual story of spillovers, but here spillovers include not only technological knowledge but knowledge about business plans as well. I call them \textit{knowledge spillovers} altogether and denote with $\beta$ the amount of knowledge that flows to an entrepreneur from another entrepreneur. As usual, how much entrepreneurs benefit from knowledge spillovers depends on their ability to value, exploit, and apply the knowledge in their businesses. I denote absorptive capacity of an entrepreneur with $t$. Given a unit mass of entrepreneurs working on business projects in close proximity, the net benefit of an entrepreneur from knowledge spillovers is

$$t\beta \quad \forall \beta \in [0, 1].$$

Making use of (1) and (2), the total net benefit of becoming a stand-alone entrepreneur for entrepreneur $i$ is

$$V^S_i = b(e) + t\beta + \varepsilon_i \quad \forall i \in [0, 1],$$

where superscript $S$ denotes the set of stand-alone entrepreneurs.

The second option for an entrepreneur is to interact with the hub firm. Entrepreneurs who will potentially engage in start-up activities can benefit by interacting with the established firms in the pre-establishment period for various reasons. The first and most important reason is the role played by established firms as the mediums of exchanging ideas. Lamoreaux, Levenstein, and Sokoloff (2004) reported how Brush Electric Company in Cleveland, Ohio fostered exchanging ideas to solve difficult problems and acted as a place for technological cross-fertilization in the era of the Second Industrial Revolution. This exchange is, of course, not and should not be limited to innovative ideas. The firms of today do not have the luxury to learn how to sell innovative ideas by trial and error. To be successful, organizations have to develop marketable ideas that are backed up with strong business plans. For example, stable pricing and marketing strategies play crucial roles in determining the survival chances of start-up firms. Network entrepreneurs can develop those techniques or learn them from other entrepreneurs in the network.

All of these factors create incentives for potential entrepreneurs to gather around established firms to exploit the knowledge existing in them. This may also be beneficial to the established firms since they can increase their stock of knowledge in this process of cross-fertilization of ideas. All in all, we observe that some would-be entrepreneurs are part of a network in which one or two established firms are the crucial nodes. There are, of course, costs and benefits associated with being a member of this network for both parties. To highlight the results without unnecessary complications, this paper models the costs and benefits that accrue to the start-ups but takes the hub firm’s decision as granted. Section 3.2 incorporates the hub firm’s decision and section 5 explores if there are incentives for hub firms to arrange such networks.

There may be rationing in deciding membership to this network. Presumably, established firms would not want to allow any number of entrepreneurs to join their networks. This is not only because they might not have the resources for that, but also because too many entrepreneurs interacting in the network may create congestion in exchanging ideas even when all entrepreneurs are identical (see, for example, Bandiera and Rasul (2006) who show that social effects are positive when there are few agents in the network, and negative when there
are many).

When entrepreneurs join the network, they still need to provide the same effort in their pre-establishment period preparations. However, being a part of the network allows them to benefit more from the knowledge of the other network entrepreneurs. To capture these ideas, I assume that the knowledge spillover between network firms, denoted by $\alpha$, is greater than $\beta$. Nonetheless, being a part of the network is costly. It is much like a club that members have to pay a fee to enter. However, this fee does not have to be pecuniary. In some cases, it may well be the time and congestion costs associated with repeated interactions with other network members.

Let the ratio of entrepreneurs who choose to become a network entrepreneur be $\lambda$. Hence, the total net benefit of becoming a network entrepreneur for entrepreneur $i$ is

\[ V^N_i = b(e) + t[\lambda \alpha + (1 - \lambda)\beta] - c + \varepsilon_i \quad \forall i, \lambda \in [0, 1] \land \alpha > \beta, \tag{4} \]

where superscript $N$ denotes the set of network entrepreneurs and $c$ is the cost of entering the network. This specification implies that knowledge might not spill over to the same extent within and outside the network, which is consistent with Acs et al. (2005) who show that the spillover of knowledge need not occur automatically as has typically been assumed in endogenous growth models.

Some comments on the random part of the benefit from effort are in order. I assume that entrepreneurs – but no one else – learn $\varepsilon_i$ once they finalize their business plans but before they apply for loans. I further assume that $\theta$ of the network entrepreneurs will have a good draw, $\varepsilon_i = \varepsilon_H > 0$, and the rest will have a bad draw, $\varepsilon_i = \varepsilon_L < 0$, such that $\varepsilon^N_i := \mathbb{E}[\varepsilon_i \mid i \in N] = \theta \varepsilon_H + (1 - \theta) \varepsilon_L$, where subscript $H$ is for high type and subscript $L$ is for low type. However, I also assume that $\gamma$ of the stand-alone entrepreneurs will have a good draw and the rest will have a bad draw, such that $\varepsilon^S_i := \mathbb{E}[\varepsilon_i \mid i \in S] = \gamma \varepsilon_H + (1 - \gamma) \varepsilon_L$. Therefore, by the law of large numbers

\[ \Pr(\varepsilon_i = \varepsilon_H) = \begin{cases} \theta & \text{if } i \in N \\ \gamma & \text{if } i \in S \end{cases} \quad \forall i \in [0, 1] \land \gamma < \theta. \tag{5} \]

I assume that $\theta$ and $\gamma$ are common knowledge. $\gamma < \theta$ is assumed to capture what is called a "network effect" in the literature. It is well known that successful networks have disproportionately more high-type individuals even though some stand-alone individuals are able to achieve the same performance by themselves outside the network. Note that $\varepsilon^N_i > \varepsilon^S_i$. For future reference, define the difference between $\varepsilon^N_i$ and $\varepsilon^S_i$ as $\Delta \varepsilon$ for the sake of brevity of the equations. The analysis below focuses on the cases in which $c > \Delta \varepsilon$.

### 3.1 Network formation

I assume that the cost of joining the network is less than its net benefit, which is stated formally in the following assumption.

**Assumption 1** $t(\alpha - \beta) + \Delta \varepsilon > c$.

---

6. Entrepreneurs do not know their type at the point that they are making a decision on whether to join or not to join the network since the types are drawn at the end of phase 1.

7. This assumption is nothing but the conventional assumption in contract theory that the distribution of types is publicly known.

8. The case in which $c < \Delta \varepsilon$ is trivial and the analysis of it is left to the reader.
Given the net benefit scheme specified in the previous section, the network formation equilibrium is the ratio of entrepreneurs in the network, $\lambda^*$, such that none of the entrepreneurs has incentive to change his decision on whether or not to join the network. At this phase, entrepreneurs do not yet have information about the random part of their net benefit from effort. Assuming that they are risk-neutral expected utility maximizers, in any equilibrium of the network formation the following inequality must hold:

$$\max \{EV^S_i(\lambda^*), EV^N_i(\lambda^*)\} \geq \max \{EV^S_i(\lambda), EV^N_i(\lambda)\} \quad \forall i, \lambda \in [0,1].$$

In an interior equilibrium, $EV^S_i(\lambda^*) = EV^N_i(\lambda^*)$ has to be satisfied. This implies that an interior equilibrium is obtained when

$$\lambda^* = \frac{c - \Delta e}{t(\alpha - \beta)}.$$  \hspace{1cm} (7)

However, network externalities prevent this from being a stable equilibrium. Suppose the economy is in the interior equilibrium with $\lambda^*$ entrepreneurs in the network. If an entrepreneur decides to stand alone instead of joining the network, the expected net benefit of each of the other entrepreneurs in the network decreases by $t(\alpha - \beta)$ while the expected net benefit of stand-alone entrepreneurs remains unchanged. If, on the other hand, an entrepreneur does the opposite, the expected net benefit of each of the other entrepreneurs in the network increases by $t(\alpha - \beta)$ while that of stand-alone entrepreneurs remains unchanged. Therefore, the interior solution $\lambda^* = (c - \Delta e)/(t(\alpha - \beta))$ cannot be a stable equilibrium. Then, any stable equilibrium of network formation has to be a corner solution (i.e., either all entrepreneurs prefer to join the network ($\lambda^* = 1$) or none of them prefers to join the network ($\lambda^* = 0$)).

**Proposition 1 (Agglomeration vs. Dispersion)** There are multiple equilibria of network formation and any stable equilibrium has to be a corner solution. Therefore, either all entrepreneurs agglomerate around the hub firm or all of them stand alone.

A graphical characterization of the equilibria is shown in Figure 1. The number of entrepreneurs forming links with the hub firm is given in the $x$-axis and entrepreneurs’ expected payoffs are given on the $y$-axis. $AB$ is the expected payoff of an entrepreneur when he joins the network.
As expected, it increases with the number of network entrepreneurs. On the other hand, the payoff of a stand-alone entrepreneur, which is represented by $CD$ in the figure, is independent of the number of entrepreneurs. Since joining the network is costly, $AB$ starts below $CD$.

Figure 1 also points to the network externality problem. There has to be a sufficiently high number of entrepreneurs in the network (e.g.; at least $(c - \Delta e)/(t(\alpha - \beta))$ entrepreneurs) to make an entrepreneur at least as better off as he can be outside the network. Therefore, there is a coordination problem caused by network externalities: joining the network is individually rational only if enough of the others are doing so. As stated in Proposition 1, there are three different equilibria, one of which is not stable. A stable equilibrium is obtained either when all entrepreneurs stand alone or all join the network, both of which are corner solutions. The former happens at point $C$ and the latter happens at point $D$ in the figure. An interior solution occurs where $AB$ intersects $CD$ at point $E$. However, it is not stable since even a small perturbation or a shock to the system could lead the economy out of this equilibrium because of the snowball effect caused by network externalities.

### 3.2 Network architecture: rationed agglomeration

The previous section makes its analysis under the assumption of an open-club-type of network. Any entrepreneur who is willing to join the network can do so freely. However, this neglects the decision of the hub firm. To incorporate that, assume for the moment that the economic problem of the hub firm prevents it from forming links with all entrepreneurs, and suppose that it is willing to create links only with $\eta$ of them.

If $\eta < (c - \Delta e)/(t(\alpha - \beta))$, then there will be a complete dispersion of entrepreneurs. Such a situation is shown in Figure 2. Suppose the hub firm is willing to form links with $\eta_1$ entrepreneurs. Then, the equilibrium has to happen at point $C$, the unique and stable equilibrium of network formation in this case. Entrepreneurs cannot benefit much from the knowledge base of the network since the number of entrepreneurs allowed in the network is not sufficient for enough knowledge spillovers. Thus, they all prefer to stand-alone and the industry experiences low knowledge spillovers. In that sense, this is a "bad equilibrium" with no collaboration. An industrial district with these characteristics would probably experience lower growth rates.
If \( \eta \geq (c - \Delta \varepsilon)/(t(\alpha - \beta)) \), then \( \eta \) entrepreneurs will be able to form links even though all entrepreneurs would prefer forming links with the hub firm. Figure 2 shows this situation. Suppose the hub firm is willing to form links with \( \eta_2 \) entrepreneurs. Then, a stable equilibrium occurs at point \( D \). This means that \( 1 - \eta_2 \) entrepreneurs are not able to join the network even though they want to do so. This suggests that there has to be some kind of rationing by the hub firm in its selection of entrepreneurs.

**Definition 1 (Rationed Agglomeration)** A rationed agglomeration is an agglomeration of a limited number of entrepreneurs around the hub firm(s) even though all entrepreneurs prefer to do so.

Under our assumption of identical entrepreneurs, the best the hub firm can do is a random rationing like that in the models of credit rationing à la Stiglitz and Weiss (1981). Models of agglomeration usually predict either complete agglomeration or complete dispersion of entrepreneurs whenever there is a corner solution. What I propose here is somewhat different from those predictions and is more consistent with reality. What we observe in reality is there are some entrepreneurs that are linked to each other while others in the same business stand alone. If agglomeration forces make forming links beneficial for an entrepreneur, they must do the same for all the others, as they are assumed to be identical in all respects before nature determines the random part of their net benefit. What is more, even though agglomeration forces lead entrepreneurs to one or the other corner, some sort of rationing mechanism may prevent such outcomes. This makes *ex ante* identical entrepreneurs *ex post* different, and we end up with a corner solution that looks like an interior solution.\(^9\)

Note that whenever \( \eta \geq (c - \Delta \varepsilon)/(t(\alpha - \beta)) \) in addition to the ones I analyze above, point \( E \) would still be an equilibrium, but, as discussed before, it is unstable unless \( \eta = (c - \Delta \varepsilon)/(t(\alpha - \beta)) \) and so is left aside in the analysis below. The results of this section are summarized in the following proposition.

**Proposition 2 (Rationed Agglomeration)** If \( \eta < (c - \Delta \varepsilon)/(t(\alpha - \beta)) \), there is complete dispersion of entrepreneurs. If \( \eta \geq (c - \Delta \varepsilon)/(t(\alpha - \beta)) \), there can be a rationed agglomeration in which only \( \eta \) entrepreneurs can form links with the hub firm even though all of them prefer to do so.

Some comments on the multiplicity of equilibria are in order. The possibility of multiple equilibria can explain why we observe different industrial architectures in different places at different times. Whenever a dispersion equilibrium occurs (point \( C \) in Figure 2), entrepreneurs do not collaborate with each other. In other equilibria (either unstable equilibrium of point \( E \) or stable equilibrium of point \( D \)), there is high collaboration between entrepreneurs. Any unstable equilibrium will sooner or later be broken, which explains why some industrial clusters change their structure in a very short period of time. These equilibria can also be ranked in terms of entrepreneurs’ welfare. Points \( C \) and \( E \) give the same aggregate payoff to the entrepreneurs. However, at point \( D \), network entrepreneurs get higher payoffs and stand-alone entrepreneurs get exactly the same payoffs they would get at points \( D \) and \( E \). Therefore, the aggregate welfare of entrepreneurs is higher at point \( D \) than at points \( C \) and \( E \). In that sense, I conclude

---

\(^9\)This can also be viewed as a gentlemen’s club or an academic alliance. Even though many similar individuals prefer to be a part of them, there will be room only for a limited number of them. Then, the institution applies some rationing rule which may or may not be random.
that points $C$ and $E$ are inefficient equilibria by noting that this analysis neglects the payoff to the hub firm.\footnote{Nonetheless, note that I do not need a hub firm to get this result. Even in the absence of a hub firm, this result says that entrepreneurs might be better off by interacting with each other but they can end up with an inefficient equilibrium of no cooperation due to coordination problems. One might, therefore, be tempted to predict that entrepreneurs can cooperate by solving this coordination problem. Although it seems possible in this simple framework of identical entrepreneurs, such an incentive vanishes in richer environments with heterogeneous agents. The simple reason for this is that coordination might not make all entrepreneurs but only a subgroup of them better off, in which case it is not supported by all (or possibly by the majority).}

The above analysis implicitly assumes a star network structure among the hub firm and entrepreneurs. In a star network, one player (in this case the established firm) is at the center of the network and the others (in this case the entrepreneurs) gather around it. This is not only a plausible network architecture observed in actual industrial clusters but also a theoretically justifiable one. Bala and Goyal (2000) work on the very general payoff structures of noncooperative network formation games. They show that in a model of two-way knowledge spillovers, the strong Nash equilibrium of a network structure tends to be either an empty network in which none of the agents is connected to the others (coinciding with the complete dispersion result of Proposition 2) or a star network (coinciding with the agglomeration result of Proposition 2).

4 Phase II: financing of business projects

Before they would be willing to invest in new technological ventures, wealthy individuals had to be convinced of two things: first and most obviously, that it was indeed possible to earn high rates of return by putting their money in this kind of enterprise; [...] by serving as the hub of overlapping networks of inventors and investors, [hub firms] could both stimulate ongoing inventive activity and provide the expertise needed to assess the economic merits of the resulting discoveries. (Lamoreaux, Levenstein, and Sokoloff, 2004, pg. 14)

In the second phase of the pre-establishment period of start-ups, entrepreneurs seek financing for their risky investment projects. At this stage, they have already developed their business projects and established their network. This means that they now know what kind of a project they have: a promising high success probability project or a not-so-promising low success probability project.

The way I model this is the following. At the end of phase I, entrepreneurs learn the random part of their net benefits from effort\footnote{This is an important assumption that connects phase I to phase II without any problems.}, which is denoted by $\varepsilon_i$ in eq. (1). I assume, without loss of generality, that there is a one-to-one mapping from the net benefits of the entrepreneurs to the success probability of their start-ups. Those who experience a good draw, $\varepsilon_H$, will have a success probability of $p_H$, and those who experience a bad draw, $\varepsilon_L$, will have a success probability of $p_L$, where $p_H > p_L$. Formally,

$$p_i = \begin{cases} 
  p_H & \text{if } \varepsilon_i = \varepsilon_H \\
  p_L & \text{if } \varepsilon_i = \varepsilon_L 
\end{cases} \quad \forall i \in [0, 1]. \quad (8)$$

The base model here thus boils down to a canonical project-financing model with two types: entrepreneurs with high and low success probability projects. However, there is a difference.
The type of a start-up can be affected by its entrepreneur’s interactions in phase I. That is, the distribution of project types differs according to the entrepreneurs’ past decisions on whether to join the network: \( \theta \) of the network entrepreneurs have a high success probability project, whereas only \( \gamma \) of the stand-alone entrepreneurs have a high success probability project.

To focus on the interesting cases, from now on I assume that \( \eta \geq \frac{c}{t(\alpha - \beta)} \) and that the economy is in a rationed agglomeration equilibrium (i.e.; there are \( \eta \) network entrepreneurs and \( 1 - \eta \) stand-alone entrepreneurs). Given this, there are \( \theta \eta \) network entrepreneurs with a high success probability project and \( (1 - \theta) \eta \) network entrepreneurs with a low success probability project. The corresponding numbers for stand-alone entrepreneurs are \( \gamma(1 - \eta) \) and \( (1 - \gamma)(1 - \eta) \), respectively. Therefore, the overall number of entrepreneurs in the whole population with high success probability projects is

\[
\theta \eta + \gamma(1 - \eta),
\]

and the number of those with low success probability projects is

\[
(1 - \theta) \eta + (1 - \gamma)(1 - \eta).
\]

To be able to undertake a project, an entrepreneur has to have \( I \) units of capital. It is assumed for simplicity that entrepreneurs have no wealth. Therefore, they need to borrow \( I \) units of capital from a lender. In the base model, I assume that they finance their projects from one source, which is consistent with the findings of Petersen and Rajan (1994) on small business financing. The cost of loanable funds is equal to the risk-free (gross) interest rate \( R \) in the economy. If an entrepreneur is successful, the project yields \( Y \) units of capital at the end of the period, and if not it yields a smaller return, which is normalized to here zero. I also assume that all projects have a positive net present value, which is stated in the following assumption.

\textbf{Assumption 2 (NPV of Projects)} \( p_H Y > p_L Y > RI \).

Therefore, it is not only the case that all entrepreneurs prefer undertaking their projects had they been able to fully self-finance their projects, but also that lenders prefer financing all projects. Therefore, the focus of this paper is not on the inefficiencies that rise up because of lemons problem in the loan market, but simply the pricing problem of different projects and the resulting incentive scheme that induces certain network structures.\(^{12}\) As is shown later, only pooling contracts can be offered in the loan market. Thus, the start-ups of the entrepreneurs with high success probability projects are undervalued in the market.

\subsection*{4.1 Sequence of events}

The sequence of the events in phase II is as follows. Since every entrepreneur is assumed to have no wealth they all need to borrow to start their firms. At the beginning of phase II, entrepreneurs sign financial contracts with lenders and make their investments. Successful entrepreneurs pay off their loans once their payoffs are realized at the end of the period.

\footnote{\textsuperscript{12}\textsuperscript{Inci (2006) focuses on such inefficiencies by assuming that the low success probability start-ups have negative net present value. Note also that the problem here does not entail any credit rationing.}}
4.2 Lenders (banks and local financiers)

The lenders are risk-neutral lenders in Bertrand competition with each other. They can be either banks or local financiers. They form their beliefs simultaneously and choose the contracts they will offer taking as given the cost of loanable funds, which is equal to the risk-free interest rate of $R$. At this moment, both banks and local financiers are assumed to have the same information set. However, later I incorporate the possibility that local financiers can make use of local information available from the hub firm.

Lenders offer contracts contingent on the announced type and the project outcome (either success or failure) of an entrepreneur. Contracts specify the repayments to the lenders for both outcomes. Let the repayment to the lender be $D^G_i(R)$ in the good outcome and $D^B_i(R)$ in the bad, where $G$ stands for good and $B$ for bad. The general form of the contract offered by lender $l$ is

$$C_l \equiv \begin{bmatrix} C^H_l \\ C^L_l \end{bmatrix} = \begin{bmatrix} D^G_l(R) \\ D^B_l(R) \end{bmatrix}.$$  \hspace{1em} (11)

Here $C^H_l$ is the contract designed for loan applicants with a high success probability project and $C^L_l$ is for applicants with a low success probability project. I assume that there is limited liability, and therefore, contracts cannot leave entrepreneurs with negative end-of-period payoffs:

$$\pi^G_i \geq 0 \quad \text{and} \quad \pi^B_i \geq 0 \quad \forall i = H, L,$$  \hspace{1em} (12)

where $\pi^G_i$ is the realized payoff of an entrepreneur in the good state and $\pi^B_i$ is the realized payoff of an entrepreneur in the bad state.

4.3 Entrepreneurs

The expected payoff of an entrepreneur at the beginning of the period, $\Pi$, is given by

$$\Pi = p_i(Y - D^k_i(R)) - (1 - p_i)D^k_i(R) \geq 0 \quad \forall i = H, L, \forall k = B, G.$$  \hspace{1em} (13)

An entrepreneur is going to be successful with probability $p_i$ in which case he produces $Y$ and gives $D^k_i$ of it to the bank. Thus, the expected net return in the case of a good state is $p_i(Y - D^k_i)$. If he is unsuccessful he produces something less than $Y$ (which is normalized to zero) and gives $D^k_i$ of it to the bank. However, limited liability prevents $D^k_i$ from being higher than what the entrepreneur has. Since the low output is normalized to zero it follows immediately that $D^k_i$ is going to be zero as well, but for the sake of generality of the analysis I keep it.

4.4 Equilibrium definition

I use the standard Bertrand-Nash equilibrium concept. An equilibrium comprises all contracts offered by lenders that are consistent with each other. Each lender offers entrepreneurs a contract that maximizes his profits. Then, among all alternatives, entrepreneurs choose the best contract for them. Formally, an equilibrium in the credit market is defined as follows.

**Definition 2 (Equilibrium Concept)** Assume that lenders are Bertrand-Nash players following pure strategies. Given $R$, a credit market equilibrium is the contract offered by lenders such that all contracts earn nonnegative profits and there are no new contracts that could earn higher profits.
An equilibrium has to be individually rational and incentive compatible for every entrepreneur.
After normalizing the payoff to an entrepreneur in case of inactivity to zero, individual rationality asserts that an entrepreneur $i$ can earn at least as much as he could when he does not participate in the market at all:

$$p_i(Y - D_i^G(R)) - (1 - p_i)D_i^B(R) \geq 0 \quad \forall i = H, L . \quad (14)$$

Incentive compatibility assures that entrepreneur $i$ does not have an incentive to apply for the loan contract aimed at entrepreneurs $j$:

$$p_i(Y - D_i^G(R)) - (1 - p_i)D_i^B(R) \geq p_i(Y - D_j^G(R)) - (1 - p_i)D_j^B(R) \quad \forall i, j = H, L . \quad (15)$$

Under these conditions, it is impossible to design contracts such that entrepreneurs with different projects in terms of success probabilities self-select themselves into different contracts.
In other words, it is impossible to identify which entrepreneurs have high success probability projects since it is always beneficial for an entrepreneur with a low success probability project to misrepresent himself as having a high success probability project. Lemma 1 proves this claim formally.

**Lemma 1** There exists no separating equilibrium in the credit market.

**Proof.** See Appendix A.1. ■

### 4.5 Start-up financing without hub-signaling

As a benchmark begin with the case in which there is a network but the hub firm has no role in start-up financing. In this case, entrepreneurs simply apply for loans by themselves.
From Lemma 1, the only possibility in the loan market is a pooling contract which imposes $D_H^G(R) = D_F^G(R) = D^G$ and $D_H^B(R) = D_L^B(R) = D^B$. Figure 3 derives the pooling equilibrium. $ZP_p$ is the zero-profit condition with both types of projects above which the profit of the lender increases and under which it decreases.

$$\tilde{p}D^G + (1 - \tilde{p})D^B = RI , \quad (16)$$

where $\tilde{p}$ is the average success probability of the projects of the loan applicant pool. The applicant pool is composed of both network and stand-alone entrepreneurs. By making use of (9) and (10), this average success probability can be written as

$$\tilde{p} = [\theta \eta + \gamma(1 - \eta)]p_H + [(1 - \theta)\eta + (1 - \gamma)(1 - \eta)]p_L . \quad (17)$$

To determine the pooling equilibrium, start with an arbitrary contract, say, $C_1$. The iso-profit lines passing through $C_1$ are shown in Figure 3. The steeper one is for a low success probability project and the other is for a high success probability project.
$C_1$ cannot be an equilibrium since there is a deviation contract $C_2$ northwest of it which is attractive to an entrepreneur with a high success probability project but not to an entrepreneur with a low success probability project. There exist such deviation contracts as long as the contract is not on the $y$-axis. However, there is no such deviation contract on the $y$-axis, because contracts have to be in the first quadrant by limited liability. However, any contract on the $y$-axis, such as $C_3$, cannot be an equilibrium if it makes positive profits. Otherwise, it can be undercut by Bertrand
competition. Zero profits are obtained at the intersection of $ZP_p$ and the $y$-axis where the equilibrium pooling contract $C^*$ is obtained. The pair of iso-profit lines passing through $C^*$ are shown by the lines $HH'$ and $LL'$ for a high and a low success probability project, respectively, in the figure.

The equilibrium pooling contract takes the simple debt form with a repayment of $RI/\bar{p}$ in the good state and zero repayment in the bad state:

$$D^G = \frac{RI}{\bar{p}}, \quad D^B = 0.$$  \hspace{1cm} (18a)

The effective interest rate implied by this contract is $R/\bar{p}$.

### 4.6 Local financiers and hub-signaling

The entrepreneurs who organized and promoted [...] new ventures secured investment capital largely by relying on personal connections. [...] [T]hey could be based on the recommendations of men who had established their expertise in the community, as when Brush secured backing for the Linde Air Products Company simply by assuring local businessmen of the merits of the technology. (Lamoreaux, Levenstein, and Sokoloff, 2004, pg. 27)

Throughout phase I, entrepreneurs have close and repeated relationships with the hub firm. The hub firm, thus, has a rough idea of the quality of the projects of these entrepreneurs. For the moment, I assume that this information is costless and comes naturally due to repeated interaction between the parties in phase I. Section 5.1 generalizes the model to a case in which gathering this information is costly. The hub firm has close links with local financiers, too. These links can be the result of ongoing or past financial relationships. It is sometimes the case that these local financiers are organized by the hub firm or by its past employees as reported by Lamoreaux, Levenstein, and Sokoloff (2004).
Potential lenders have to be convinced that the projects they plan to finance are promising. Because the hub firm is known to be a successful firm that has been able to manage very successful business projects in the area local financiers can trust its expertise in evaluating start-ups. Therefore, local financiers can make use of the local information that a bank cannot gather. Assume for the moment that the hub firm communicates its ideas honestly, which allows me to focus on the value of network relationships in isolation. How to guarantee the credibility of the local information is discussed in section 5.2.

Suppose the hub firm sends a signal \( \sigma \) to the local financiers that takes on one of the two values: good or bad. That is, it conveys its beliefs about every entrepreneur in its network by labeling each as good (meaning an entrepreneur with a high success probability project) or bad (meaning an entrepreneur with a low success probability project). It can, of course, make wrong judgments. The probability of a good signal for an entrepreneur with a high success probability project is

\[
\Pr\{\sigma = \text{good} \mid i = H \land i \in N\} = x \quad x \in [0, 1]
\]  

and that for an entrepreneur with a low success probability project is

\[
\Pr\{\sigma = \text{good} \mid i = L \land i \in N\} = y \quad y \in [0, 1]
\]

Then, conditional on a good signal from the hub firm, the Bayesian probability that a loan applicant is an entrepreneur with a high success probability project is

\[
\Pr\{i = H \mid \sigma = \text{good} \land i \in N\} = \frac{\Pr\{i = H \land \sigma = \text{good} \land i \in N\}}{\Pr\{\sigma = \text{good} \land i \in N\}} = \frac{\theta x}{\theta x + (1 - \theta) y}
\]  

and conditional on a bad signal from the hub firm, the Bayesian probability that a loan applicant is an entrepreneur with a high success probability project is

\[
\Pr\{i = H \mid \sigma = \text{bad} \land i \in N\} = \frac{\theta(1 - x)}{\theta(1 - x) + (1 - \theta)(1 - y)}
\]

Similar expressions for the Bayesian probabilities that a loan applicant is an entrepreneur with a low success probability project are given by

\[
\Pr\{i = L \mid \sigma = \text{good} \land i \in N\} = \frac{(1 - \theta)y}{\theta x + (1 - \theta) y}
\]  

\[
\Pr\{i = L \mid \sigma = \text{bad} \land i \in N\} = \frac{(1 - \theta)(1 - y)}{\theta(1 - x) + (1 - \theta)(1 - y)}
\]

Thus, the belief of the hub firm about the projects of the network entrepreneurs can be imperfect. That is, it can label a good project as a bad project with probability \( 1 - x \) and a bad project as a good project with probability \( y \). However, a firm that has engaged in many innovative activities and formulated successful business strategies, such as Brush Electric Company, would on average make valuable judgments about business projects. Given that it has had a continuous relationship with network entrepreneurs in phase I, it is reasonable to assume that the hub firm’s judgments about the network entrepreneurs are useful on average. Technically, this is achieved if the monotone likelihood ratio property (MLRP) holds for the distribution of types. This requires the ratio of the Bayesian probability of a good signal to a bad signal to be increasing with the type of the project. That is, the ratio \( (21)/(22) \) is greater than the ratio \( (23a)/(23b) \) should hold, which boils down to the following assumption.
Assumption 3 (Informativeness of Signals) Signals are informative: \( x > y \).

Suppose that these signals are received by at least two local financiers and that they trust these signals. I assume that signals are private information between the hub firm and the local financiers and cannot be credibly communicated to anyone else. However, local financiers do know that the hub firm has contacts with other local financiers, too. Therefore, the local financiers have access to some local information that the other lenders do not have.

In the case in which there is no hub-signaling, the average success probability of the loan applicant pool is given by \( \bar{p} \) and, as is shown in (18a), the equilibrium lending interest rate is \( R/\bar{p} \) for any loan granted. However, the extra information that the local financiers have give them the ability to price discriminate between network entrepreneurs and stand-alone entrepreneurs. The average success probability of network entrepreneurs with a good signal, \( \hat{p} \), is

\[
\hat{p} = \frac{\theta xp_H + (1 - \theta) yp_L}{\theta x + (1 - \theta) y}.
\]  

(24)

Suppose they grant a loan only if they get a good signal from the hub firm. A similar analysis of section 4.5 with new (Bayesian) incentive constraints and (Bayesian) zero profit conditions shows that local financiers offer a lending interest rate of \( R/\hat{p} \) to any network entrepreneur with a good signal. A simple comparison of (17) and (24) depicts that \( \hat{p} > \bar{p} \) as long as \( x > y \), which holds by Assumption 3. Therefore, the existence of a network allows local financiers to provide cheaper loans to network entrepreneurs with a good signal.

Those labeled with a bad signal are denied the privileged loans provided by the local financiers. From the perspective of the local financiers, the average success probability of the loan applicants that are standing alone, \( \bar{p} \), is given by

\[
\bar{p} = \frac{\gamma p_H + (1 - \gamma)p_L}{\gamma + (1 - \gamma)}.
\]  

(25)

It is easy to show that \( \bar{p} < \hat{p} \). However, note that those who could not get a privileged loan from the local financiers can apply for loans as stand-alone entrepreneurs. This changes the average success probability of the stand-alone loan applicants. Having known this, banks and local financiers would set the price of the loans accordingly. The new average success probability outside the network is now given by

\[
\tilde{p} = \frac{[(1 - x)\theta \eta + \gamma(1 - \eta)]p_H + [(1 - y)(1 - \theta)\eta + (1 - \gamma)(1 - \eta)]p_L}{[(1 - x)\theta \eta + \gamma(1 - \eta)] + [(1 - y)(1 - \theta)\eta + (1 - \gamma)(1 - \eta)]}.
\]  

(26)

It is also easy to show that \( \tilde{p} < \bar{p} \). The reason for this is the following. The average success probability of the whole population is \( \bar{p} \). A sample of this population, which has an average success probability of \( \hat{p} > \bar{p} \), is in the network. Therefore, the average success probability of the remaining population has to be less than \( \tilde{p} \).

Suppose this static game is played at every period. Then, the network of the hub firm – which is known to be a successful firm – incubates start-ups with better chances of survival on average than the rest of the start-ups. This means a better match of capital to ideas in the network. That is, networks of successful established firms give birth to further successful firms. I record this result in the following proposition.

Proposition 3 Success breeds success in the network.
Figure 4: Equilibrium contracts with and without hub-signalling

Figure 4 shows the effect of hub-signaling in the credit market. In the absence of hub-signaling all lenders offer one pooling contract for all borrowers. In this case, the zero profit condition is given by $Zp$ and the equilibrium contract is characterized by $C^*$, which is the same $C^*$ shown in Figure 3. When there is hub-signaling, the local financiers can effectively price discriminate between the two groups of borrowers. The first group is composed of network entrepreneurs that are labeled with a good signal by the hub firm. Given the information structure, local financiers have an informational advantage on the quality of these firms. The zero profit condition is given by $ZPp$ for the loans they provide to the entrepreneurs labeled with good signals. The equilibrium contract for this group is given by $C^*$. This contract gives higher payoffs to the entrepreneurs in case of a good state at the end of the period. The second group is composed of two different kinds of entrepreneurs: stand-alone entrepreneurs and network entrepreneurs who are labeled with a bad signal by the hub firm and thus were denied the privileged loans. The average success probability in this group is $\bar{p}$ and the zero profit condition of the banks is given by $ZP\bar{p}$. The corresponding equilibrium contract is $C^{**}$. This contract provides a smaller payoff to the entrepreneurs in the good state. Table 1 summarizes the lending interest rates offered by lenders.

<table>
<thead>
<tr>
<th></th>
<th>Lending Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>for stand-alone entrepreneurs for network</td>
</tr>
<tr>
<td>without</td>
<td>Banks</td>
</tr>
<tr>
<td>hub-signaling</td>
<td>Local Financiers</td>
</tr>
<tr>
<td>with</td>
<td>Banks</td>
</tr>
<tr>
<td>hub-signaling</td>
<td>Local Financiers</td>
</tr>
<tr>
<td>Note: ( \hat{p} &lt; \bar{p} &lt; \hat{p} ).</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Lending interest rates

The results would still go through even if the entrepreneurs are risk averse. In section 5, I show that the existence of a network makes either both high and low success probability entrepreneurs better off or only the high success probability entrepreneurs better off. Being a
part of the network is preferable by both parties in the former case. In the latter case, low success probability entrepreneurs would not want to be known as network entrepreneurs, but any explicit action they can take (including leaving the network) would perfectly signal their types. Therefore, they prefer staying in the network after learning their types. Nonetheless, they might want to provide side payments to the hub firm to collapse the informative signaling mechanism. As is shown in section 5, even when such side payments are allowed the network still persists. Noting that all entrepreneurs have the freedom to leave the network any time they want, this suggests that the qualitative results are robust even with risk-averse agents.

4.7 Functional and dysfunctional networks

The results imply that the start-ups that are financed by the privileged loans of the local financiers are going to be more successful on average. However, this does not mean that the region benefits from it. As the model shows, entrepreneurs denied the privileged loans can apply for loans as stand-alone entrepreneurs. In the end, although some of the entrepreneurs will be paying lower prices for the loans, all firms will have access to credit, and thus, the distribution of types will be the same as in the case in which there is no hub-signaling. The only benefit of the network to the society is thus the fact that it is an incubator of relatively more high success probability firms (that is $\theta > \gamma$) by serving as a place for social interactions, which happens in phase I. In phase II, the network creates an island of entrepreneurs who have high success probability start-ups on average which allows them to get cheaper loans.

In general the performance of the network is dependent on the informativeness of the signals. Arnott and Stiglitz (1991) show that a nonmarket institution (a network of entrepreneurs in this paper) may be dysfunctional in cases in which it is informationally disadvantaged relative to the market institution. In this paper, I assume that the signals are informative, but, from a social point of view, the outcome of the economy is not any better than the equilibrium without the hub-signaling mechanism. Therefore, nonmarket institutions may not only be dysfunctional when they are informationally disadvantaged as suggested in Arnott and Stiglitz (1991) but may also be useless for the goals of society even when they have superior information about the economy. The benefits of the nonmarket institution accrue only to its privileged members in terms of prices of the loans, but those who are able to enter into entrepreneurship are still the same.

I should underline that the argument I provide here is not trying to show that networks are completely useless. They indeed serve the goals of society in Phase I by creating a disproportionately high number of high success probability projects. However, I do want to attack the partial view that the observation of a group of successful entrepreneurs in a network is sufficient to conclude that this network is socially desirable. As the simple model shows here, it may well be the case that the network has created an island of successful entrepreneurs by selecting them from the population, but it has not increased the number of high success probability projects at all.

4.8 Monopolistic local financier

An important thing to note is that the price of the loan offered to the network entrepreneurs does not necessarily decrease due to hub-signaling. This result is dependent on the structure

\footnote{Remember that entrepreneurs make their decisions on whether to join or not to join the network before they learn their types, which happens in Phase I.}
of the loan market. In the previous sections, I assume that the lenders are in a Bertrand competition. In general, lenders (in particular local financiers) may have monopolistic power that prevents prices from going down. To see that, suppose for the moment an extreme case in which there is only one local financier that has access to the signals of the hub firm. The extra information it has effectively improves its expected non-repayed loans such that if it were to compete with others, its zero-profit condition would be characterized by $Z P \equiv p$ in Figure 4. However, the monopolistic local financier does not need to pass on this cost decrease to the loan applicants. Since other lenders are offering $R \equiv p$ it cannot achieve the complete monopoly profits either. If it asks for higher interest rates than $R \equiv p$, the network entrepreneurs can simply apply for loans as stand-alone firms. Therefore, the monopolistic local financier offers the same loan price that all others are currently offering in the market and enjoys profits of $R(1/p - 1/\hat{p})$ per dollar lent. In general, the outcome depends on how informed the lenders are.

Petersen and Rajan (1994) assume that the decreases in cost of loanable funds are passed on to the borrowers. In the model here, the cost of loanable funds — which is nothing but the risk-free interest rate $R$ — does not change. However, the monopolistic local financier expects to have a lower number of defaults. This effectively decreases its expected losses, but it does not need to reflect this situation to the borrowers because of its monopoly power.

Note that this has nothing to do with the fact that success breeds success in the network. Even though the price of the loans does not go down in the case of a monopolistic local financier, there is still a better match of capital to ideas among those who are financed by the privileged loans of the local financier. That is, the average success probability in the network is higher than that outside the network.¹⁴

## 5 Incentives for hub-signaling

Until now, I have assumed that there are certain incentives for the hub firm to form the signaling mechanism. Here, I analyze the incentives for such an organization. Credit markets undervalue the start-ups of entrepreneurs with high success probability projects while they overvalue the start-ups of entrepreneurs with low success probability projects. From an *ex ante* point of view, in the absence of hub-signaling, the market value of any start-up firm, $\tilde{V}$, is given by

$$\tilde{V} = \hat{p}Y - RI ,$$

(27)

regardless of whether the entrepreneur has a high or a low success probability project. The hub-signaling mechanism changes the levels of under- and overvaluation of start-ups. When there is hub-signaling, the market value of the start-up of a network entrepreneur with a high success probability project, $V_H$, is

$$V_H = [x\hat{p} + (1 - x)\hat{p}]Y - RI ,$$

(28)

and that of a network entrepreneur with a low success probability project, $V_L$, is

$$V_L = [y\hat{p} + (1 - y)\hat{p}]Y - RI .$$

(29)

¹⁴Outside of the network includes stand-alone entrepreneurs and network entrepreneurs who are labeled with a bad signal. The second group is considered to be outside the network since they independently apply for loans.
The following proposition proves that network entrepreneurs with high success probability projects always prefer hub-signaling, but the preference of network entrepreneurs with low success probability projects depends on the network size, the number of high success probability projects in the network, and the informativeness of the signals.

**Proposition 4** A network entrepreneur with a high success probability project always prefers hub-signaling. A network entrepreneur with a low success probability project prefers hub-signaling when the network is large enough (i.e.; given the signal structure \( \eta > \frac{y}{x+(1-\theta)y} \)) or alternatively when signals are not sufficiently informative (i.e.; given the network size \( \frac{x}{y} > \frac{1-(1-\theta)y}{\theta\eta} \)). Otherwise, they prefer the status-quo.

**Proof.** See Appendix A.2. ■

This suggests that, given the size of the network, hub-signaling benefits the network entrepreneurs with low success probability projects only if the signals are not so informative or, given the signal structure, benefits them only if the network is large enough. The reason for this result is the cross-subsidization induced by the contractual structure of the credit market. The start-ups of the network entrepreneurs with low success probability projects are overvalued due to pooling contracts, and this results in a wealth distribution from the owners of undervalued start-ups to the owners of overvalued start-ups. Therefore, the pricing of the projects is not the first-best. Hub-signaling can improve the situation by mitigating the adverse selection problem for some network entrepreneurs.

*Ex ante*, the owners of high success probability projects face the chance of getting a good signal with probability \( x \) while they also face the possibility of a bad signal, in which case their project is going to be even further undervalued. However, it turns out that, as long as signals are informative, the former dominates the latter in expected payoff. The situation is different for the network entrepreneurs with low success probability projects. They always want to misrepresent themselves as having high success probability projects. Given a network of a fixed size, they still have a chance to be labeled with a good signal if the signals are not very informative. This increases the level of overvaluation to even higher levels. However, if the signals are sufficiently informative, they are more likely to be labeled as bad by the hub-signaling mechanism, in which case they have to get loans with an interest rate of \( R/\hat{p} \). From another perspective, given the signal structure of the hub firm, if the network is large enough, they still have a high chance of not being labeled correctly by the signaling mechanism which might make them better off in expected terms. Therefore, the network entrepreneurs with low success probability projects are most likely to be worse off in smaller networks with an effective signaling structure and better off in larger networks with a cumbersome signaling structure.

**Corollary 1** When the network is sufficiently large (e.g.; \( \eta > \frac{y}{x+(1-\theta)y} \)), both high and low success probability project owners prefer hub-signaling.

**Proof.** The result follows directly from Proposition 4. ■

Focus now on the more interesting case in which the network is sufficiently small (e.g.; \( \eta < \frac{y}{x+(1-\theta)y} \)). A network entrepreneur with a high success probability project will be willing to make a side payment to the hub firm to induce it to organize the hub-signaling mechanism. This side payment can at most be \( \mathcal{V}_H - \hat{V} \). On the other hand, a network entrepreneur with a low success probability project will be willing to make a side payment to the hub firm to
prevent it from organizing the hub-signaling mechanism. This side payment can at most be \( V - V_L \).

Whether the hub firm organizes the hub-signaling mechanism depends on the total side payments from both groups of entrepreneurs. Campbell and Kracaw (1980) claim that there would not be incentives to engage in information extraction in a simple market with information asymmetries, which they call the "nonexistence of rational expectations equilibrium." The basic intuition for the nonexistence of rational expectations equilibrium is that, since banks make zero profits, the overvaluation of the firms should exactly match with the undervaluation of firms in equilibrium. However, the existence of networks overturns this claim. In our setting the equalities of overvaluation to undervaluation correspond to the following equalities.

\[
(1 - x)\eta \theta + \gamma (1 - \eta)(p_H - \hat{p}) = [(1 - y)\eta (1 - \theta) + (1 - \gamma)(1 - \eta)](\hat{p} - p_L) \quad (30a)
\]
\[
x \theta (p_H - \hat{p}) = y (1 - \theta)(\hat{p} - p_L) \quad . \quad (30b)
\]

The first one focuses on the cross-subsidization between entrepreneurs with high and low success probability projects that are either stand-alone entrepreneurs or network entrepreneurs who are denied privileged loans. The second focuses on the cross-subsidization among the network entrepreneurs who finance their start-ups with privileged loans. However, these are ex post realizations. From an ex ante point of view, the total amounts of overvaluation and undervaluation among network entrepreneurs are not equal to each other, which suggests that they may be a rational expectations equilibrium. The following proposition formally expresses this claim.

**Proposition 5** When the network is small (e.g.; \( \eta < \frac{y}{x + (1 - \theta)y} \)), the total increase in the market value of the start-ups of network entrepreneurs with high success probability projects because of hub-signaling is higher than the total decrease in the market value of the start-ups of the network entrepreneurs with low success probability projects.

**Proof.** See Appendix A.3.

I am now in a position to state a very important result. In Proposition 4, I showed that when the network is sufficiently small, any network entrepreneur with a high success probability project is willing to offer a side payment to the hub firm to induce it to form the hub-signaling mechanism and any network entrepreneur with a low success probability project is willing to offer a side payment to the hub firm to prevent it from this mechanism. Moreover, in Corollary 1, I show that both parties prefer hub-signaling if the network is sufficiently large. Therefore, there may be incentives to form the hub-signaling mechanism, which is stated in the following corollary.

**Corollary 2** There are market incentives for the hub firm to arrange the hub-signaling mechanism.

**Proof.** The result directly follows from Proposition 4, Corollary 1, and Proposition 5.

### 5.1 Costly signaling and the price of a signal

In the previous sections, I assume that the information required for signals is a natural byproduct of close and repeated interaction between the hub firm and the network entrepreneurs.
This is somewhat realistic; social relationships do occasionally yield useful information. However, in general, extracting this information can be costly to the hub firm. This section, which is based on Campbell and Kracaw (1980), sketches the more general case in which the cost of information extraction is $C$ per network entrepreneur.

Let the side payment that a network entrepreneur with an undervalued project is willing to offer to the hub firm to induce hub-signaling be $S_H$. This side payment can at most be equal to the total amount of market value change to the start-up of the network entrepreneur ($S_H \leq V_H - \bar{V}$). Moreover, it has to cover at least the cost of information extraction ($S_H > C$). Therefore, if all network entrepreneurs with undervalued start-ups in the network offer side payments to the hub firm, $\eta \theta C < \eta \theta S_H < \eta \theta (V_H - \bar{V})$ has to hold, or simply

$$C < S_H < V_H - \bar{V} \quad .$$

In the same way, let the side payment that a network entrepreneur with an overvalued project is willing to pay be $S_L$. This side payment can at most be $\bar{V} - V_L$, and since not producing the signal is costless it should also be greater than zero. Therefore, if all network entrepreneurs with overvalued start-ups in the network offer side payments, $0 < \eta (1 - \theta) S_L < \eta (1 - \theta) (\bar{V} - V_L)$ has to hold, or simply

$$0 < S_L < \bar{V} - V_L \quad .$$

In the previous section, I show that the total amount of side payments by the network entrepreneurs with undervalued projects is greater than that by the network entrepreneurs with overvalued projects. Therefore, if

$$\theta (V_H - \bar{V}) - C > (1 - \theta) (\bar{V} - V_L) \quad ,$$

or simply,

$$C < \theta V_H + (1 - \theta) V_L - \bar{V} \quad ,$$

there can be incentives to form the hub-signaling mechanism. This means that if the cost of acquiring information is small enough, which is most likely to be the case for an experienced hub firm such as Brush Electric Company, there will be hub-signaling.

**Proposition 6** There exists a level of side payments such that the hub firm invests in signal extraction.

The importance of this result is that it shows the possibility that asymmetric information results in inefficient agglomerations. In the previous sections, I show that the match of capital to ideas remains unchanged as a result of a costless hub-signaling mechanism. However, Proposition 6 shows that hub firms may have incentives to engage in information extraction even when it is costly to do so.

### 5.2 Reliability of signals

Some of the men who invested their savings in the new firms were also officers and directors of banks. For example, James J. Tracy, one of the original incorporators of Brush Electric, became vice president of the Society for Savings after a long career in various other Cleveland financial institutions. Similarly, Myron T. Herrick, a member of
the Sperry Syndicate and one of the initial investors in National Carbon, was secretary-treasurer and then president of the Society for Savings, a founder of the Euclid Avenue National Bank, a director of the American Exchange National Bank, and a director of the Garfield Savings Bank. Some of the inventors and other businessmen involved in these startups and spinoffs also helped to organize financial institutions during this period [...]. (Lamoreaux, Levenstein, and Sokoloff, 2004, pg. 29)

The implicit assumption in the above analysis is that the hub firm honestly conveys the information it has to the local financers. In reality, the credibility of the signals is questionable. One way for local financers to overcome this problem is to finance the entrepreneurs in which the hub firm is also a claimant. That is, if the hub firm claims that the entrepreneurs it labels as good have higher average success probabilities, then it should also be willing to invest in them. From the other way around, the signals should be credible if the hub firm holds a sufficient amount of equity in the portfolio of start-ups it labels with good signals.

Suppose the hub firm has \( \eta(\theta x + (1 - \theta) y)W \) units of capital that it can allocate for investment in the start-ups of the network entrepreneurs. I assume that this wealth is observable by local financers and \( W < I \). I also assume that the local financers can verify whether the hub firm in fact invested in the start-ups of the network entrepreneurs. There are \( \eta(\theta x + (1 - \theta) y) \) network entrepreneurs that the hub firm labels with a good signal. An optimal investment strategy for the hub firm is to invest an equal share of its wealth endowment into these start-ups, which means that it invests \( W \) units of capital in each start-up in its portfolio to get \( \alpha < 1 \) share of each of them.\(^{15} \) If it honestly produces the signals, its payoff is

\[
\eta(\theta x + (1 - \theta) y)\left[\alpha \hat{p}(Y - R(I - W)) - RW\right] - \eta C \quad .
\]  

The first term is the net return on investment. It gets \( \alpha \) share of each start-up by paying \( W \) for each. There are \( \eta(\theta x + (1 - \theta) y) \) such entrepreneurs and their average success probability is \( \hat{p} \). Those entrepreneurs need only \( I - W \) units of capital from local financers to start their firms. The second term is the cost to the hub firm of extracting information.

Alternatively, the hub firm can choose \( \eta(\theta x + (1 - \theta) y) \) firms randomly without incurring the cost of extracting information and announce them as the ones with good signals to the local financers. If it does that, the average success probability in this random sample is going to be

\[
\hat{p} = \theta p_H + (1 - \theta)p_L \quad ,
\]  

where \( \hat{p} < \hat{p} \). This time, the payoff to the hub firm is

\[
\eta(\theta x + (1 - \theta) y)\left[\alpha \hat{p}(Y - R(I - W)) - RW\right] \quad .
\]  

A simple comparison of (35) and (37) shows that, given \( W \), honestly reporting signals is preferable by the hub firm if it buys a sufficiently large share of each start-up:

\[
\alpha > \frac{C}{(\theta x + (1 - \theta) y)(Y - R(I - W))(\hat{p} - \hat{p})} \quad .
\]  

This suggests not only that the signals of the hub firm are reliable if it invests in the portfolio of the start-ups for which it sends signals to the local financers but also that a hub firm has

\(^{15}\text{Note that debt is the optimal way of financing in this model. However, here, there is a hub firm which decides first to buy an } \alpha \text{ share of the potential start-up, and then, the owner of this potential start-up looks for financing for the remaining part of the start-up. So, equity financing here can be seen as a device used by the hub firm to credibly send the signals to the local financers.} \)
to have deep pockets to engage in credible hub-signaling. Some of the results of this section are recorded in the following proposition.

**Proposition 7** The signals of the hub firm is reliable if it buys a sufficiently large share of the portfolio of start-ups for which it sends good signals to local financiers.

6 Conclusion

In this paper, I offer some thoughts on how success breeds success locally. I model the pre-establishment period of start-ups. It is shown that a networking structure leads potential entrepreneurs and hub firms to form what I call a rationed agglomeration in which there are both entrepreneurs who interact with each other within the network and other entrepreneurs in the same business outside the network. I also show that established firms can help network entrepreneurs in obtaining financing for their start-ups. Although this help may improve the match of capital to ideas in the network, the overall distribution of capital to ideas remains unchanged. Therefore, the only benefit of networks to the society at large comes in phase I of the pre-establishment period – the period in which entrepreneurs develop innovative ideas and work on business plans. The subsequently formed networks can act as places for cross-fertilization of ideas by generating a disproportionately high number of superior ideas.

It is clear that entrepreneurs have an incentive to join the network of a hub firm, but some of them cannot do so because of the rationing by the hub firm. However, explaining why a hub firm forms such a network is not entirely trivial. In the presence of asymmetric information between the entrepreneurs and financiers, the market overvalues the start-ups with low success probabilities and undervalues the start-ups with high success probabilities. When the hub firm shares its information about the entrepreneurs in its network with the local financiers, the level of under- and overvaluation is altered for the network entrepreneurs. This sometimes makes both parties better off, and sometimes makes only the start-ups with high success probabilities better off. In the former case, when both parties are able to provide side payments to the hub firm, it is obvious that there are incentives for the hub firm to communicate its views to the local financiers.

It turns out that the hub firm also has incentives in the latter case, in which only the start-ups with high success probabilities are benefited so that the interests of the two groups of entrepreneurs conflict. This is simply because the side payments that the start-ups with the high success probabilities are willing and able to offer to the hub firm to induce it to communicate its ideas to the local financiers are greater in amount than the side payments that the start-ups with low success probabilities are willing and able to pay to the hub firm to induce it not to do so. This suggests that there are certain incentives for the hub firm to decrease the information gap between the entrepreneurs and the local financiers. One potential problem in this is that the hub firm might not share its views honestly with the local financiers if collecting this information is costly. Nonetheless, the reliability of the signals is guaranteed whenever the hub firm buys a sufficiently large share of the portfolios of start-ups in its network that it claims are promising, which is consistent with what is observed in the early stages of industrial districts.

I explain how the information generated in the networks of entrepreneurs around established firms can mitigate the adverse selection problems for some privileged members of the network by decreasing the information gap between the network entrepreneurs and local financiers. This is one explanation of successful agglomerations such as those found in Cleveland, Ohio.
in the nineteenth century. It remains unexplained, however, why some agglomerations rise up repeatedly while others rise and then decline.

While writing this paper, I came across the Green Paper: Entrepreneurship in Europe (European Commission, 2003) which stresses that networks between established firms and start-ups have been becoming increasingly important. Having understood the vitality of such networks in 1997 the Federal Ministry of Education and Research in Germany launched a program called EXIST to increase regional cooperation between the economic and scientific actors by forming networks between them. Two of the aims of the program are to increase knowledge spillovers and to foster the pool of potential entrepreneurs by creating networks that would otherwise not be formed. This suggests the feasibility of the idea that the government might improve the structures of the networks. The normative conclusions and public policies are left for future research.

A Appendix

A.1 Proof of Lemma 1

Assume instead that there is a separating equilibrium. The Bertrand competition among the lenders requires that they make zero profits in an equilibrium. Therefore, if there were separating contracts, lenders have to make zero profits from each of these contracts. The zero-profit condition for the contract designed for entrepreneurs with a high success probability projects is given by

$$p_H(Y - \pi_H^G) - (1 - p_H)\pi_H^B = RI$$  \hspace{1cm} (A-1)

and that for an entrepreneur with a low success probability project is given by

$$p_L(Y - \pi_L^G) - (1 - p_L)\pi_L^B = RI$$  \hspace{1cm} (A-2)

The corresponding iso-profit lines are given by

$$p_H\pi_H^G + (1 - p_H)\pi_H^B = \bar{Y}_H$$  \hspace{1cm} (A-3a)

$$p_L\pi_L^G + (1 - p_L)\pi_L^B = \bar{Y}_L$$  \hspace{1cm} (A-3b)

where $\bar{Y}_H$ and $\bar{Y}_L$ are the levels of $Y_H$ and $Y_L$. If there were a separating contract, eqs. (A-1)-(A-3b) would have to be satisfied simultaneously. However, note that (A-1) is parallel to (A-3a) and (A-2) is parallel to (A-3b) in the $\pi^B - \pi^G$ space, and the slope of the former group is smaller than that of the latter group. Given limited liability, these four equations cannot be satisfied at the same time with a separating contract. A contradiction is obtained. Therefore, there are no separating contracts.

A.2 Proof of Proposition 4

If the difference between $\mathcal{V}_H$ and $\bar{V}$ is positive, the market value of the start-up of a network entrepreneur with a high success probability project increases due to hub-signaling. That is, $\mathcal{V}_H - \bar{V} = [x\hat{p} + (1 - x)\bar{p}]Y - \bar{p}Y > 0$. This holds when

$$x\hat{p} + (1 - x)\bar{p} > \bar{p}$$  \hspace{1cm} (A-4)
By substituting for $\tilde{p}$, $\tilde{p}$, and $\tilde{p}$, one can get
\[
\frac{(p_H - p_L)(x - \theta\eta x - (1 - \theta)\eta y)[\theta x - (\theta x + y(1 - \theta))(\eta\theta + \gamma(1 - \eta))]}{[\theta x + (1 - \theta)y][1 - (\theta\eta x + \eta y(1 - \theta))]} > 0 .
\]
It can be easily verified that the terms $(p_H - p_L)$ and $(\theta x + (1 - \theta)y)$ are positive. Then, (A-4) holds when
\[
\frac{(x - \eta[\theta x + (1 - \theta)y)][\theta x - (\theta x + y(1 - \theta))(\eta\theta + \gamma(1 - \eta))]}{1 - \eta(\theta x + y(1 - \theta))} > 0 .
\]
The first term in the numerator is positive because $\theta x + (1 - \theta)y$ is a weighted average of $x$ and $y$, and therefore, it is between $x$ and $y$. Multiplying this with $\eta$, one get a number between $\eta x$ and $\eta y$, which is definitely less than $x$. Moreover, by similar reasoning, the term in the denominator is also positive since $\eta(\theta x + y(1 - \theta))$ is less than one.

Below is the second term in the numerator.
\[
[\theta x - (\theta x + y(1 - \theta))(\eta\theta + \gamma(1 - \eta))] < x
\]
\[
[\theta x + (1 - \theta)y] < \theta
\]
It is obvious that $\eta\theta + \gamma(1 - \eta)$ is between $\theta$ and $\gamma$, and thus, it is less than $\theta$. Above I have already shown that $\theta x + y(1 - \theta) < x$. Then, the second term in (A-5) has to be less than $\theta x$, which implies that the term is positive. Therefore, hub-signaling benefits the network entrepreneurs with a high success probability projects.

Similarly, if the difference between $\bar{V}$ and $\mathcal{V}_{L}$ is positive, the market value of the start-up of a network entrepreneur with a low success probability decreases due to hub-signaling. That is, $\bar{V} - \mathcal{V}_{L} = \bar{p}Y - [\bar{y}\tilde{p} + (1 - y)\tilde{p}]Y > 0$. This holds when
\[
y\tilde{p} + (1 - y)\tilde{p} < \tilde{p} .
\]
By substituting for $\tilde{p}$, $\tilde{p}$, and $\tilde{p}$, one can get
\[
\frac{(p_H - p_L)(y - \theta\eta x - (1 - \theta)\eta y)[\theta x - (\theta x + y(1 - \theta))(\eta\theta + \gamma(1 - \eta))]}{[\theta x + (1 - \theta)y][1 - (\theta\eta x + \eta y(1 - \theta))]} < 0 .
\]
Above I have already shown that $\theta x - (\theta x + y(1 - \theta))(\eta\theta + \gamma(1 - \eta)) > 0$ and $1 - (\theta\eta x + \eta y(1 - \theta)) > 0$. It is left to find out when $y - \eta\theta x - \eta(1 - \theta)y < 0$ holds:
\[
y - \eta\theta x - \eta(1 - \theta)y < 0
\]
\[
x > \frac{1 - (1 - \theta)\eta}{\theta y}
\]
\[
\eta < \frac{y}{\theta x + (1 - \theta)y} .
\]
This means that hub-signaling benefits the network entrepreneurs with a low success probability projects only if $\eta > \frac{y}{\theta x + (1 - \theta)y}$ (given the signal structure) or $x < \frac{1 - (1 - \theta)\eta}{\theta y}$ (given the network size).

### A.3 Proof of Proposition 5

For any start-up of a network entrepreneur with a high success probability project, $\mathcal{V}_{H} - \bar{V} = [x\tilde{p} + (1 - x)\tilde{p}]Y - \tilde{p}Y > 0$, and there are $\theta$ of them. In the same way, for any start-up of a network entrepreneur with a low success probability project, $\bar{V} - \mathcal{V}_{L} = p\tilde{Y} - [y\tilde{p} + (1 - y)\tilde{p}]Y > 0$, and there are $1 - \theta$ of them. Hence, the increase in the market value of the start-ups of entrepreneurs with high success probability projects due to hub-signaling is higher than the total decrease in the market value of the start-ups of entrepreneurs with low success probability projects if $\theta[x\tilde{p} + (1 - x)\tilde{p} - \tilde{p}] > (1 - \theta)[\tilde{p} - y\tilde{p} - (1 - y)\tilde{p}]$
or alternatively if the following inequality holds.

\[ \theta x + (1 - \theta)y \tilde{p} + [\theta(1 - x) + (1 - \theta)(1 - y)] \tilde{p} > \tilde{p} \quad . \tag{A-7} \]

By substituting for \( \tilde{p} \), \( \bar{p} \), and \( p \) into (A-7), one can get

\[
\theta x p_H + (1 - \theta) y p_L + [\theta(1 - x) + (1 - \theta)(1 - y)]
\times \left[ (1 - x)\theta \eta + \gamma(1 - \eta) \right] p_H + \left[ (1 - y)(1 - \eta) \eta + \gamma(1 - \eta) \right] p_L > \bar{p} \quad .
\]

After some manipulation

\[
\theta x p_H + (1 - \theta) y p_L + \frac{[(1 - x)\theta \eta + \gamma(1 - \eta)] p_H + [(1 - y)(1 - \theta) \eta + \gamma(1 - \eta)] p_L}{\eta + \frac{(1 - \eta)}{\theta(1 - x) + (1 - \theta)(1 - y)}} > \bar{p} \]

or equivalently

\[
\frac{(1 - \eta)[\theta x p_H + (1 - \theta) y p_L] + [\theta \eta + \gamma(1 - \eta)] p_H + [(1 - \theta) \eta + \gamma(1 - \eta)] p_L}{\eta + \frac{(1 - \eta)}{\theta(1 - x) + (1 - \theta)(1 - y)}} > \bar{p} \]

However, the second term on the numerator is nothing but \( \bar{p} \). Therefore,

\[
\frac{(1 - \eta)[\theta x p_H + (1 - \theta) y p_L] + \bar{p}}{\eta + \frac{(1 - \eta)}{\theta(1 - x) + (1 - \theta)(1 - y)}} > \bar{p}
\]

or equivalently

\[
(1 - \eta)[\theta x p_H + (1 - \theta) y p_L] + [(1 - x) + (1 - \theta)(1 - y)] \bar{p}
\]

\[ > \bar{p} \left[ \theta(1 - x) + (1 - \theta)(1 - y) \eta + (1 - \eta) \right] \quad . \]

Arranging gives

\[
(1 - \eta)[\theta x p_H + (1 - \theta) y p_L] > \bar{p}[1 - \eta - [\theta(1 - x) + (1 - \theta)(1 - y)](1 - \eta)]
\]

\[
(1 - \eta)[\theta x p_H + (1 - \theta) y p_L] > \bar{p}(1 - \eta)[1 - \theta(1 - x) - (1 - \theta)(1 - y)]
\]

\[
\theta x p_H + (1 - \theta) y p_L > \bar{p} \left[ 1 - \theta \right](1 - x) - (1 - \theta)(1 - y)
\]

\[
\theta x p_H + (1 - \theta) y p_L > \bar{p} \theta x + (1 - \theta) y
\]

\[
\theta x p_H + (1 - \theta) y p_L > \bar{p}
\]

\[ \hat{p} > \bar{p} \quad . \]

Therefore, the right-hand side of (A-7) is always greater than the left-hand side.

References


