OPTIMIZATION MODELS FOR ROUTING AND FREQUENCY ASSIGNMENT IN WIRELESS MESH NETWORKS

by
GÜLTEN BÜŞRA KARKILI

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Approved by:

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ABSTRACT

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GÜLTEN BÜŞRA KARKILI

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Thesis Supervisor: Prof. Tonguç ÜNLÜYURT

Keywords: Frequency assignment problem, Wireless mesh networks, Multi-path routing, Interference, Optimization

With the first mobile networks developed, frequency channel assignment has become a significant problem due to the limited number of licensed frequencies and cost-related concerns. The minimization of the number of frequencies assigned has become the main objective of the frequency channel assignment problems, and today this problem is applicable and relevant for wireless networks as well. In this study, we focused on routing and frequency assignment models for wireless mesh networks and proposed an integrated approach that combines these two aspects of frequency assignment problems. We modified our approach with respect to different interference models such as protocol-based or SIR-based interference. The integrated model was run for different sizes of randomly generated networks, and the results were compared with the sequential approach proposed in the literature. The impact of the size of the network and the interference model on the number of frequencies assigned were studied. It was observed that the integrated approach results in smaller numbers of frequencies assigned, yet the computation time also increases considerably. The performances of sequential and integrated models were also compared with the heuristic in the literature. Finally, the effect of the distribution of the wireless devices in the network was studied for the sequential approach.

ÖZET

KABLOSUZ ÖRGÜ AĞLARDA ROTALAMA VE FREKANS ATAMA OPTIMIZASYONU

GÜLTEN BÜŞRA KARKILI

ENDÜSTRİ MÜHENDİSLİĞİ YÜKSEK LİSANS TEZİ, ARALIK 2020

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Anahtar Kelimeler: Frekans atama problemi, Rotalama, Kablosuz örgü ağlar, Eniyileme

Mobil ağların ilk geliştirilmesinden bu yana frekans atama problemleri lisanslı frekansların sayısının sınırlı olmasından dolayı önemini korumuş, atanan frekans sayısının enküçüklenmesi frekans atama problemlerinin başlıca amacı haline gelmiştir. Günümüzde frekans atama problemleri kablosuz ağlar için de önemli bir araştırma konusu olarak çalışılmaya devam etmektedir. Bu tezde, kablosuz örgü ağlarda rotalama ve frekans atama modelleri çalışılmış, bu iki tip yaklaşım entegre tek bir modelde ifade edilmiştir. Bu entegre model, protokol bazlı ve sinyal-girişim oranı (SIR) bazlı girişim modellerine uyumlu olacak şekilde dizayn edilmiş, farklı büyüklükteki kablosuz örgü ağlar için çalıştırılmıştır. Entegre modelin sonuçları literatürde daha önce sunulmuş sıralı modellerin sonuçları ile karşılaştırılmış, ağ büyüklüğü ve girişim modeli tipinin sonuçlara etkisi incelenmiştir. Bu incelemeler sonucunda, geliştirilen entegre modelin atanan frekans sayısı bakımından daha iyi sonuçlar verdiği, ancak modelin çözülme süresinin fazlasıyla yüksek olduğu gözlenmiştir. Sıralı ve entegre modellerin performansları literatürdeki sezgisel model ile de karşılaştırılmış, son olarak kablosuz ağdaki dağılımın sıralı modelin performansı üzerindeki etkisi araştırılmıştır.

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I would like to thank my friend and future colleague Rabia for always encouraging me to work harder throughout my graduate school journey. I would also like to thank my family and Hilal for all the emotional support and always being there for me.

To Hilal

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1. INTRODUCTION

The digital age that we live in calls for constant improvement in the ways of communication: faster transmission of data, no interruption during the transferring process, higher capacity and so on. With the mobile and wireless network being regular aspects of our lives, researchers have been focusing on the ways that could enhance this experience over the last a few decades. Frequency assignment problems in wireless networks have been a major issue that could be addressed in this manner. According to the comprehensive survey by Aardal, Van Hoesel, Koster, Mannino & Sassano (2007), with the first mobile networks developed, the frequency assignment problem has become an important issue during 1970s since the licensed frequencies were in limited numbers and the operators during that time were asked to pay for the number of frequencies used. Hence, the minimization of the number of frequencies has become the main objective of frequency assignment problems which started as a cost-oriented problem and then led to other concerns such as minimizing/eliminating interference between communication links or allowing high capacity for the data transfer. The web bibliography maintained by Eisenblätter & Koster (2000) is a rich source that the reader can reach most of the studies conducted since the 1970s in this frequency assignment research field and have a fundamental idea about the constraints and the various objectives of the problem. There are many different scenarios to work on under the title of frequency assignment problems, such as single-hop versus multi-hop networks or static vs. dynamic demands for frequencies. With Hale (1980)'s approach to the problem as a graph coloring algorithm, different graph theory algorithms have also joined the branches of the possible approaches to the main issue.

Because of the NP-hardness of minimum coloring (Karp, 1972), various heuristics have been developed that could run faster and still come up with a solution close to the optimal. Chaudhry, Chinneck & Hafez (2016) came up with fast heuristics that could be used for frequency assignment problems in multi-hop wireless networks which focus on avoiding interference between the links. In multi-hop wireless networks, only some of the access points (APs) are connected to the wired network, which are called the gateways

(GW). (Chaudhry, 2015) They also assumed the wireless devices to be half-duplex, which essentially means they can either receive or transmit the signal at a time. (Chaudhry, 2015).

According to their study, there are four main steps to work on in order to reach a solution for a wireless network frequency assignment problem. The first step is to build a connectivity graph for the network, in which the vertices represent the wireless devices in a multi-hop arrangement. For this initial step, they used a topology control algorithm (TCA) for the nodes with given locations which are uniformly distributed (Chaudhry, Ahmad & Hafez, 2012). In this TCA, for each node, the closest neighbors are selected as possible devices that there can be a direct communication link in between, and the number of these "candidate" neighbors is the main parameter of the algorithm. This logic suits to the relation between the distance, transmit power and interference, since shorter distance between the wireless devices results with lower transmit power which causes less interference between two communication links. After this initial step which results in a final topology, it should be decided which of these candidate neighboring devices will actually share a direct communication link, which was approached as a multi-path routing problem (Chaudhry, Chinneck & Hafez, 2013). In this routing model, a lower bound for all the sources generating a signal was maximized so that equity was ensured throughout the network. The third step is to create the conflict matrix so that the interference information could be stored to be used as a parameter for the following frequency assignment problem (Chaudhry, Hafez & Chinneck, 2014). This conflict matrix is constructed according to various interference models such as protocol-based or SIR (Signal-to-Interference-Ratio)-based interference. The fourth and final step is to solve the frequency assignment problem on this final topology (part 1), for the wireless links decided by the routing model (part 2), while avoiding interference between the links (part 3). Because this problem remains NP-hard and also the way it was formulated for SIR-based interference was non-linear, Chaudhry et al. (2016) introduce fast heuristics that could come up with near-optimal solutions in much shorter computational time.

In the light of this comprehensive study, we were motivated to see how these separate parts of the solution method could be combined, and how the routing model would work if the lower bound that was being maximized was relaxed. Another goal was to see if the optimal frequency assignment could be achieved by re-arranging the final model by linearization. Since run time was an issue when the network consisted of 35 wireless devices and a single gateway in the study proposed by Chaudhry et al., we were curious if we could reach optimal solutions for smaller wireless networks. Finally we aimed to adopt a version of these introduced fast heuristics to be able to compare them with the optimal solutions in terms of the number of frequencies assigned and the execution time.

With these motivations, our contributions to the research in the field of frequency assignment problems can be listed as follows:

- We propose an integrated approach that combines the routing and frequency assignment models with the aim of providing better results in terms of the number of frequencies assigned. We compare the original sequential approach with our integrated model for small and large networks while using different kinds of interference models.
- We rewrite the routing model so that the focus of the model shifts from ensuring the equity throughout the network to minimizing the number of links used, which indirectly leads to the minimization of the frequencies assigned. This new routing model is again compared with the original sequential approach.
- We implement the heuristic described by Chaudhry et al. (2016) and compare it with the results of the sequential approach and our integrated model.
- We observe the performance of the sequential models for clustered networks instead of uniformly distributed devices.

Here is how the rest of this thesis is organized: In the following chapter, the previous works in the literature are introduced and their relation to our study is discussed. In Chapter 3, the problem definition is given and the main network establishment is introduced, namely the topology algorithm used and the interference models are presented. In Chapter 4, the sequential approach of Chaudhry et al. has been described and our integrated model is introduced. In Chapter 5, the results of this comparison between two approaches, the re-arranged routing model, the heuristic comparison and network re-arrangement are presented. The thesis concludes with a discussion and further research recommendations.

2. LITERATURE REVIEW

The frequency assignment problem can be handled by adopting many perspectives with different objectives. Dai, Chen & Huang (2011) work on frequency assignment problem in GSM systems and instead of a single-objective approach which focuses on, for instance, minimizing the total interference, they use a bi-objective model instead. The first part is mainly about the summation of all the interference while representing it as a cost function to be minimized, and the second part focuses on minimizing the total number of co-channel and adjacent-channel violations, while using a compatibility matrix to denote these violations. This study shows that various goals of the frequency assignment problem can be combined, which is a parallel mindset with our integrated model approach. However in our study, co-channel and adjacent-channel violations have not been considered and there is an emphasis on the network throughput alongside the interference.

Although edge coloring is a frequently used technique to solve frequency assignment problems, it can still be reinterpreted to accommodate the conditions the main problem calls for. For example in Hsu, Liu, Wang & Wu (2006)'s study, the edge coloring algorithm is re-designed as generalized edge coloring (g.e.c) so that it satisfies the constraint which ensures that two devices that communicate will share at least one frequency channel together. The new approach allows a vertex to be a neighbor to a certain number of vertices to make sure this constraint is satisfied. This number also denotes the number of devices that the vertex can communicate with, and the possibility of finding an optimal solution heavily depends on this value. In our study, this requirement of having common frequency channels for communication is not considered. The routing model ensures that the communication is continuous and possible for all the devices, and the coloring algorithm is used merely to make sure that the links interfering with each other are not assigned with the same frequency.

The interference models utilized in this study are mainly based on Chaudhry et al. (2012)'s introduction to the subject. However, there are various ways to approach interference modelling. For instance, in their study Cheng, Huang, Huang & Wu (2005) consider two types of interference as well, but those are classified as the interference due to simultaneous transmission and interference due to devices transmitting to the same sink device. The proposed channel assignment algorithm focuses on eliminating both. Another point that stands out in this study is that disk graphs are used to represent the network, and a new class of disk graph is introduced that could correctly display the significance of overlapping hosts. With this type of disk graph representation, the channel assignment problem remains NP-hard and an approximation for the number of channels needed is introduced. With the help of this study, it can be said that the network representation and interference modelling can vary, yet it is still necessary to tackle the frequency assignment problem since its NP-completeness remains.

The main proposal of our study has been the integration of routing problem with frequency assignment. There has been other integrated approaches to overcome different issues in this field as well. For example, Eisenblatter, Geerdes & Siomina (2007) introduce a sequential approach that first chooses locations for access points by a capacitated facility location model. Then, in the second part a frequency assignment model is introduced that minimizes the total interference. The authors then have come up with a joint approach that seeks for the optimal solution while combining both models. Similar to our study, they also observed in their experiments that the integrated model could take hours to be solved optimally, which clarifies the run time issues for integrated models in this field. In our study, device placement is not considered as a problem as it was in this paper, and the locations are accepted as given parameters for the model. The authors also presented numeric results by solving their models for a realistic office environment, which can be an interesting further step for our integrated approach as well.

When graph coloring algorithm is applied for the frequency assignment problems, the frequencies assigned are non-overlapping. However, in a realistic setting, the number of available non-overlapping channels is bounded and this becomes an issue for the non-overlapping channel assumption of graph coloring algorithm. In their study, Mishra, Banerjee & Arbaugh (2005) handle this issue by proposing the assignment of partially overlapping frequency channels to the devices. A weighted graph coloring algorithm is introduced by the authors that would minimize the impact of these partially overlapping frequencies, and their proposed algorithms result in a reduction in interference as these partially overlapping channels are benefited from. In our study, the frequencies used remain to be completely independent (non-overlapping) and the standard graph coloring algorithm is preferred.

In our study, the assignment of the frequencies is fixed and does not change with respect to the alterations in the network. However, in certain real life scenarios it is possible for these locations to be moving, which calls for a dynamic frequency assignment approach. In their study, Hassan, Funabiki & Nakanishi (2008) formulate this dynamic assignment problem for wireless internet-accessing mesh networks, with a decision function designed to reduce the number of these reassignments to avoid frequent disruptions in the system. This type of reassignment is not allowed in our study, and the locations of the devices and the final frequency assignments are assumed to be permanent.

It is very common in frequency assignment problems to focus on the minimization of the interference while keeping the frequencies fixed instead of minimizing the number of frequencies assigned, namely fixed spectrum frequency assignment problem. Zhang, Zhao & Xiong (2009) study this particular type of channel assignment and improve the widely used tabu search algorithm to come up with a higher running speed. A greedy algorithm combined with a penalty threshold is introduced to improve the running speed, and experimental results show that the solution stays very close to optimal while the execution time gets lower compared to genetic and tabu search algorithms. This study is an example of the significance of run time in frequency assignment problems, as the heuristics adopted to fix this issue can be improved even further. Run time is a critical component of our study as well, since our integrated approach takes a remarkably long time to get solved.

Alongside the tabu search algorithm, many heuristic algorithms, as well as their combinations, have been utilized to come up with near-optimal solutions for the frequency assignment problem. For instance, in their study Vieira, Gondim, Rodrigues & Bordim (2008) come up with a hybrid algorithm that combines GRASP and a sequential allocation heuristic for channel allocation problem in wireless networks. Column generation algorithms are also shown to be useful for graph coloring problem in the study by Yüceoğlu, Şahin & van Hoesel (2017), which makes these algorithms suitable methods for the frequency assignment problem as well. In fact, a column generation algorithm was used by Koster & Tieves (2012) to handle the co-channel interference minimization in GSM networks. Utilization of these heuristics and algorithms in the literature shows that a possible further step for our integrated approach could be adopting these methods so that the run time issue would be resolved.

3. PROBLEM DEFINITION AND THE NETWORK MODEL

3.1 Problem Definition

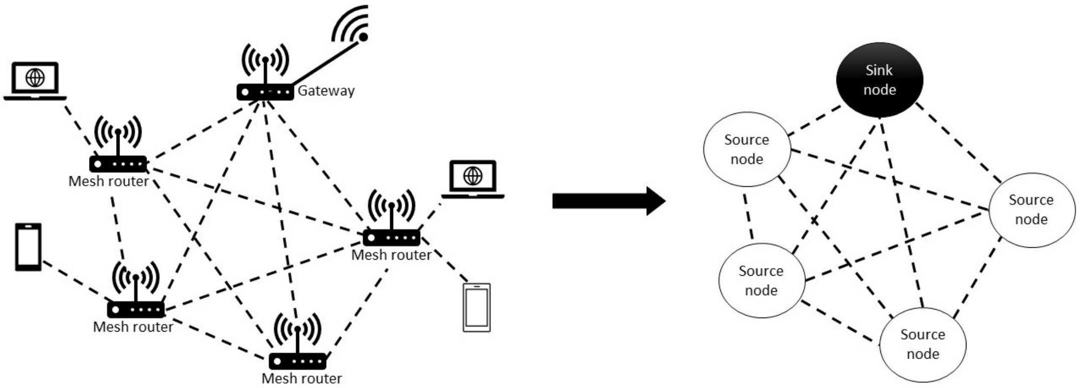


Figure 3.1 : A wireless mesh network and corresponding graph representation

In our analysis, we consider a wireless mesh network with a single gateway (GW). The GW is the only access point in the network connected to the wired network, meaning all the signals generated by other access points (wireless devices) in the network should be arriving at the GW. Since each access point generates its own signal and it is transmitted to the GW, this problem becomes a multi-path multi-commodity problem, where each signal is treated as an identical commodity and the GW acts like a sink node while all the other devices are the source nodes (Figure 3.1). This transmission is carried through the wireless links between the devices and as long as it does not cause interference due to transmission power and/or distance between the links, the links can be assigned with the same frequency (Chaudhry, 2015). The frequency here represents the scarce source that comes in a limited number, that is why the main objective of these models is to minimize the number of frequencies with respect to the interference between the wireless links. It is also essential that the link capacity is fully utilized, which makes the total network throughput a crucial part of these models. In Chaudhry et al. (2016)'s work this was ensured by maximizing a lower bound for each flow generated by each source node, which provides a fairness throughout the network since each signal transmitted to the GW is equally important. Hence, in their routing model this lower bound is maximized, and in the frequency assignment model the number of frequencies is minimized. In our case, we wanted to combine these two models in one joint approach so that the model could yield better results in terms of the number of frequencies assigned. This analysis constitutes the main proposal of this thesis. Alongside this study, a further analysis on the lower bound mentioned earlier was conducted. Instead of maximizing this lower bound, which leads to the maximization of minimum network throughput, a different approach that minimizes the number of links used was adopted, so that the number of frequencies assigned could be decreased further at the expense of losing fairness. Moreover, we also compared the results of these sequential and integrated models with the results of the heuristic proposed by Chaudhry et al. (2016), and finally we investigated the impact of the network distribution on the number of frequencies assigned. These additional examinations were presented alongside the comparison of the results obtained by using sequential and integrated models.

For the rest of this section, the interference models and the topology control algorithms used in the upcoming optimization models will be introduced. The topology control algorithm decides on the candidate neighbors for each wireless device in the network and draws temporary links between them, creating a final topology. The routing problem is then solved based on these temporary links, and this is how permanent and direct communication links are determined. The interference models are used so that before running the routing and frequency assignment problems, the interference for each link combination could be determined. This interference information and the final topology are then used in the optimization models as parameters.

3.2 Interference Models

Two interference models in wireless networks will be introduced, namely protocol and Signal-to-Interference-Ratio (SIR) models. The protocol model is a straightforward model that decides on the interference merely based on the distance between the devices while the SIR model is seen as a more accurate and realistic representation.(Chaudhry et al., 2014)

3.2.1 Protocol-based Interference Model

This interference model essentially checks the distance between all four of the wireless devices forming two separate wireless links, and compares each distance with the interference range of the corresponding node. (Chaudhry, 2015) If a neighbor device is within the interference range of another, there is said to be an interference between these two links. This interference is used in the optimization models as a parameter that takes the value of 1 when there is an interference.

In other words, for wireless devices/nodes x,y,z,t where there are direct links between the nodes $x-y$ and also $z-t$; if *any* of the conditions below is satisfied, then there is interference between those two wireless links.

d_{ab} : The Euclidean distance between the nodes a and b.
R_a : The interference range of the node a.

$$d_{xz} \leq R_x \tag{3.1}$$

$$d_{xt} \leq R_x \tag{3.2}$$

$$d_{yz} \leq R_y \tag{3.3}$$

$$d_{yt} \leq R_y \tag{3.4}$$

$$d_{zx} \leq R_z \tag{3.5}$$

$$d_{zy} \leq R_z \tag{3.6}$$

$$d_{tx} \leq R_t \tag{3.7}$$

$$d_{ty} \leq R_t \tag{3.8}$$

For the interference range of each wireless device, a random parameter proportional to the whole area covered by the devices was used in the optimization models.

3.2.2 SIR-based Interference Model

Signal to Interference Ratio (SIR) of two wireless links, between the nodes $x-y$ and $z-t$ is defined as the ratio of the power threshold needed to process the received signal at link $x-y$ ($RxThresh$) to the maximum power received at link $x-y$ from the link $z-t$. (Chaudhry et al., 2016) Comparing this ratio to the SIR threshold will provide the interference information between these two links. If this ratio is lower than the SIR threshold value, then it is decided that there is interference between two links. In a network where the interference of a link with all the other links sharing the same frequency must be considered, the denominator of the SIR ratio becomes the summation of the maximum power received from all the other links sharing the same frequency. In this section and throughout this study, the notation introduced by Chaudhry (2015) was used for the parameters so that further research on this topic could be more convenient for the reader.

$RxThresh$: Power threshold needed to process the received signal
$SIRThresh$: SIR threshold
$P_{r,xt}$: Power received at node x from node t
P_{ij} : Maximum power received at link i (between $x-y$) from link j (between $z-t$)
K_i : The set of all the links using the same frequency channel as link i

$$P_{ij} = \max\{P_{r,xz}, P_{r,xt}, P_{r,yz}, P_{r,yt}\} \tag{3.9}$$

$$SIR_{xy-zt} = \frac{RxThresh}{\sum_{k\in K_i} P_{ik}} \tag{3.10}$$

$SIR_{xy-zt} < SIRThresh \Rightarrow$ The links between the nodes $x-y$ and $z-t$ are interfering with each other.

Calculating the maximum power received is a two-step process: First the transmission power is calculated, and then this value is plugged in the formula used for calculating the received power. The decision about which formulas to use for these calculations depends on the relation between the distance between the nodes and the cross over distance.

d : The Euclidean distance between each pair of the nodes (in connectivity graph).
h_t : Antenna height of the transmitter
h_r : Antenna height of the receiver

λ : Signal wavelength
G_t : Transmitter antenna gain
G_r : Receiver antenna gain

$$\text{Cross\_over\_distance} = \frac{4\pi h_t h_r}{\lambda} \tag{3.11}$$

$d \leq Cross\_over\_distance \Rightarrow$ The following formulas are used:

$$P_t \text{ (Transmission power)} = \frac{RxThresh(4\pi d)^2}{G_t G_r \lambda^2} \tag{3.12}$$

$$P_r \text{ (Received power)} = \frac{P_t G_t G_r \lambda^2}{(4\pi d)^2} \tag{3.13}$$

$d > Cross\_over\_distance \Rightarrow$ The following formulas are used:

$$P_t \text{ (Transmission power)} = \frac{RxThresh(d)^4}{G_t G_r h_t^2 h_r^2} \tag{3.14}$$

$$P_r \text{ (Received power)} = \frac{P_t G_t G_r h_t^2 h_r^2}{(d)^4} \tag{3.15}$$

When the links between the nodes $x - y$ and $z - t$ are considered, to calculate the power received at node x from node z, $P_{r,xz}$, first the transmission power required when z is transmitting to t ($P_{t,zt}$) is calculated. This process is repeated until all the values for $P_{r,xz}, P_{r,yz}, P_{r,xt}, P_{r,yt}$ are obtained. Then, the maximum of these could be used for the comparison to the SIR threshold.

Similar to the previous protocol model, this interference model is utilized in the optimization models by introducing a parameter that takes the value of 1 if there is an interference. Unlike the previous model, if there is no interference, instead of taking the value of 0 the parameter takes the value of maximum power received to allow the cumulative maximum power received to be calculated for each set of links using the same frequency.

3.3 Topology Control Algorithm

Before proceeding to the optimization stage, the final network topology must be determined in order to have a structure that the routing model can be based upon. Various topology control algorithms (TCAs) have been studied in the literature which maintain the connectivity in the network and reduce the number of available links (Li & Hou, 2004; Shen, Cai & Xu, 2007). One of the proposed topology control algorithms, *Select x for less than x*, adopts the logic of establishing a link between a node and its at least x nearest neighbors (Chaudhry et al., 2012). The authors have suggested that this distance-based approach helps minimizing the interference between the wireless links, since the transmission power and the interference are directly related to the distance between the devices.

Motivated by this approach, in our topology, we selected exactly t nearest neighbors for each of the nodes instead of using this number as a lower bound.(Algorithm 1) For each network, if the (routing) model could not come up with an optimal routing structure before proceeding to the frequency assignment stage, this t value was changed accordingly so that the connectivity could be maintained in the network. The results were presented with respect to the changing t values so that the impact on the final results could be observed. From an optimization-based point of view, this stage also helps the routing model to be based on a smaller and limited number of links which results in computational convenience.

Algorithm 1: Topology Control Algorithm

```
Result: $E$: List of edges
Input: $N$: Number of vertices, $d$: N×N matrix of distances, $t$: Selection
  parameter;
Begin:
Sort the matrix $d$ and return the indices that would sort row $i$ of $d$.
Matrix $S \leftarrow$ Sorted indices for each row $i$, excluding $i$.
Initialize an empty list, $E$.
for $i = 1,2, \ldots, N$ do
    for $j = 1,2, \ldots, t$ do
        Add $(i, S[i,j])$ to the list $E$ as an edge;
        Add $(S[i,j]), i)$ to the list $E$ as an edge;
    end
end
Clean the repetitive edges in the list $E$.
End of algorithm
```

4. OPTIMIZATION MODELS

4.1 Sequential Approach: Routing and Frequency Assignment Models

In this section, the sequential models that were used by Chaudhry (2015) will be introduced as the baseline for the subsequent analysis conducted. Once the candidate arcs are selected by the topology control algorithm, they are used in the first model to obtain the optimal routing. Next, the second model takes these arcs as input and assigns frequencies by using a graph coloring model.

4.1.1 Routing Model

The graph $G = (V, E)$ is built with V as the set of vertices (wireless devices) and E as the set of edges (links between devices) that are determined by the topology control algorithm. The set of sink nodes (GW) that is selected randomly denotes the gateway nodes that all the signals will be transmitted to. The set of commodities (P) are generated by the set of their source nodes (S_p), and they correspond to individual signals generated by each source. In this setting, all nodes other than the ones in the set GW are *source nodes*, trying to send data to the gateway. Moreover, since all the source nodes are equally significant and also they are distinct, the problem becomes a multi-commodity problem with the number of commodities being equal to the number of source nodes. The output of the model is the list of arcs in the set E that will be used for the optimal routing.

Parameters

capacity : Link capacity.
n : Node degree constraint
c : Small cost added to the objective function to prevent reoccurring flow-loops (in practice it was taken as 0.0001).

Variables

f_{ij}^{p} : The amount of flow of commodity p on link $(i,j) \in E$
f_{p}^{d} : The total flow of commodity p arriving at gateway $d \in GW$
f_{p}^{s} : The total flow of commodity p leaving its source $s \in S_p$
z_{ij} : Binary variable taking value of 1 if link (i,j) is used, and 0 otherwise.
y_{max} : Lower bound for the network throughput for each commodity/source.

$$\max\ y_{max} - c \sum_{(i,j)\in E} z_{ij} \tag{4.1}$$

$$\text{s.t.} \sum_{i\in V,(i,d)\in E} f_{id}^{p} - \sum_{i\in V,(d,i)\in E} f_{di}^{p} = f_{d}^{p} \qquad ;\forall p \in P, \forall d \in GW, \tag{4.2}$$

$$\sum_{j\in V,(s,j)\in E} f_{sj}^{p} - \sum_{j\in V,(j,s)\in E} f_{js}^{p} = f_{s}^{p} \qquad ;\forall p \in P, \forall s \in S_p \tag{4.3}$$

$$\sum_{j\in V,(i,j)\in E} f_{ij}^{p} - \sum_{j\in V,(j,i)\in E} f_{ji}^{p} = 0 \qquad ;\forall p \in P, \forall i \in V/(S_p \cup GW) \tag{4.4}$$

$$\sum_{j\in V,(j,i)\in E} z_{ji} - \sum_{j\in V,(i,j)\in E} z_{ij} \leq n \qquad ;\forall i \in V \tag{4.5}$$

$$f_{s}^{p} \geq y_{max} \qquad ;\forall p \in P, \forall s \in S_p \tag{4.6}$$

$$z_{ij} + z_{ji} \leq 1 \qquad ;(i,j),(j,i) \in E \tag{4.7}$$

$$\sum_{p\in P} f_{ij}^{p} \leq capacity \times z_{ij} \qquad ;(i,j) \in E \tag{4.8}$$

$$f_{s}^{p}, f_{d}^{p}, f_{ij}^{p}, y_{max} \geq 0 \qquad ;\forall p \in P, \forall d \in GW, \forall s \in S_p, \forall (i,j) \in E \tag{4.9}$$

$$z_{ij} \in \{0,1\} \qquad ;\forall (i,j) \in E \tag{4.10}$$

In the objective function (4.1), the lower bound for the network throughput (y_{max}) is maximized, which ensures that fairness is ensured throughout the network while the arcs that will be used for optimal routing are being determined.

With constraints (4.2), (4.3) and (4.4), it is ensured that the flow is balanced for the set of sink nodes, the set of source nodes, and the intermediate nodes, respectively. Constraints (4.5) are added so that for each node in the network, the summation of incoming and outgoing arcs will not exceed the node degree parameter. Constraints (4.6) are added since the flow on each arc leaving a source node for the commodity p should be greater than or equal to the y_{max} bound. Constraints (4.7) ensure that there can be either transmission or receipt happening at a time on a wireless link, since all wireless devices are assumed to be half-duplex. Constraints (4.8) are added because if the arc is used for the routing, the total flow on that arc cannot exceed the flow capacity. The final two constraints include the non-negativity and binary variable definitions.

Once the arcs that will be utilized during transmission between the wireless devices are permanently determined with the routing model, they are used as input for the second model which solves the frequency assignment problem.

4.1.2 Frequency Assignment Model

The second step of the sequential approach includes a graph coloring model that assigns frequencies to the arcs that were decided by the previous model, defined as the set U. The set F denotes the set of available frequencies. If it is not possible to decrease the number of frequencies, the model is expected to assign each used link a different frequency. Hence, in practice, the set F was generated the same size as the set of used links, U.

Parameters

inf_{ijpq} : The interference value that takes the value of 1 if there is an interference between arcs (i, j) and (p, q), and 0 otherwise, based on the protocol model for interference.

Variables

v_{ij}^k : Binary variable taking value of 1 if the frequency $k \in F$ is assigned to the link $(i, j) \in U$, and 0 otherwise.
col_k : Binary variable taking value of 1 if the frequency $k \in F$ is used, and 0 otherwise.

$$
\begin{aligned}
\min \ & \sum_{k \in F} col_k && && (4.11)\\
\text{s.t.} \ & \sum_{k \in F} v_{ij}^k = 1 && ;\forall (i,j) \in U && (4.12)\\
& v_{ij}^k \leq col_k && ;\forall k \in F, (i,j) \in U && (4.13)\\
& v_{ij}^k + v_{pq}^k - 1 \leq 1 - inf_{ijpq} && ;\forall k \in F, (i,j),(p,q) \in U && (4.14)\\
& col_k, v_{ij}^k \in \{0,1\} && ;\forall k \in F, (i,j) \in U && (4.15)
\end{aligned}
$$

The objective function (4.11) minimizes the number of colors assigned. Constraints (4.12) ensure that each used arc is assigned with one frequency. Constraints (4.13) are added since a frequency cannot be assigned to any arc if the frequency is not used in the first place. Constraints (4.14) are written so that if there is an interference between (i,j) and (p,q), they will not be assigned with the same frequency. Finally, binary variables are defined in the last part.

If SIR model is used for interference instead of the protocol model, the only constraints changing in the frequency assignment model will be (4.14), and the following constraint will be replacing it:

$$
\sum_{q \in V} \sum_{p \in V} P_{ijpq} v_{ij}^k v_{pq}^k \leq \frac{RxThresh}{SIRThresh} \qquad ;\forall (i,j) \in U, \forall k \in F \tag{4.16}
$$

Here the parameters $RxThresh$ and $SIRThresh$ correspond to the receiver and SIR thresholds, and P_{ijpq} is the parameter that denotes the maximum power received, as they were introduced in the previous section.

Since this new version (4.16) includes the product of two binary variables, the SIR-based frequency assignment model becomes quadratic. Therefore the following constraints were used for linearization, with the additional binary variable n_{ijpq}^k introduced to the model:

$$\sum_{(p,q)\in U,(i,j)\neq(p,q)} P_{ijpq} n^k_{ijpq} \leq \frac{RxThresh\_mwatts}{SIRThresh} \qquad ;\forall k \in F, \forall (i,j) \in U \tag{4.17}$$

$$n^k_{ijpq} \leq v^k_{ij} \qquad ;\forall k \in F, \forall (i,j),(p,q) \in U, (i,j) \neq (p,q) \tag{4.18}$$

$$n^k_{ijpq} \leq v^k_{pq} \qquad ;\forall k \in F, \forall (i,j),(p,q) \in U, (i,j) \neq (p,q) \tag{4.19}$$

$$n^k_{ijpq} \geq v^k_{ij} + v^k_{pq} - 1 \qquad ;\forall k \in F, \forall (i,j),(p,q) \in U, (i,j) \neq (p,q) \tag{4.20}$$

4.2 Integrated Approach

Solving the frequency assignment problem with a sequential approach is beneficial in terms of the computation time. However, it is reasonable to seek for an approach that combines these two models so that a global optimal solution can be achieved. Hence, an integrated model is introduced.

The logic behind using y_{max} as a lower bound is kept in this approach as well. Before solving the integrated model, the routing model of the sequential approach is solved with the same setting and for the same network so that the y_{max} value can be obtained. Then, this value is used as an input that works as a lower bound parameter in the integrated model.

Since in this model the routing and frequency assignment are conducted simultaneously, the set of used arcs U is not an input anymore. Apart from y_{max}, all the other variables in the previous two models, f^p_{ij},f^d_p,f^s_p,z_{ij},v^k_{ij} and col_k, are also used in this approach. Parameters inf_{ijpq} for the protocol-based interference and $P_{ijpq}, RxThresh, SIRThresh$ for SIR-based interference are introduced to the model.

$$\min \sum_{k \in F} col_u \tag{4.21}$$

$$\text{s.t.} \sum_i f_{id}^p - \sum_i f_{di}^p = f_d^p \qquad ;\forall p \in P, \forall d \in GW, (i,d), (d,i) \in E \tag{4.22}$$

$$\sum_j f_{sj}^p - \sum_j f_{js}^p = f_s^p \qquad ;\forall p \in P, \forall s \in S_p, (s,j), (j,s) \in E \tag{4.23}$$

$$\sum_j f_{ij}^p - \sum_j f_{ji}^p = 0 \qquad ;\forall p \in P, \forall i \in V/(S_p \cup GW), (i,j), (j,i) \in E \tag{4.24}$$

$$\sum_j z_{ji} - \sum_j z_{ij} \leq n \qquad ;\forall i \in V, (i,j), (j,i) \in E \tag{4.25}$$

$$f_s^p \geq y_{max} \qquad ;\forall p \in P, \forall s \in S_p \tag{4.26}$$

$$z_{ij} + z_{ji} \leq 1 \qquad ;(i,j), (j,i) \in E \tag{4.27}$$

$$\sum_{p \in P} f_{ij}^p \leq capacity \times z_{ij} \qquad ;(i,j) \in E \tag{4.28}$$

$$v_{ij}^k \leq col_k \qquad ;\forall k \in F, (i,j) \in E \tag{4.29}$$

$$v_{ij}^k \leq z_{ij} \qquad ;(i,j) \in E, \forall k \in F \tag{4.30}$$

$$z_{ij} - \sum_{k \in F} v_{ij}^k = 0 \qquad ;(i,j) \in E \tag{4.31}$$

$$v_{ij}^k + v_{pq}^k - 1 \leq 1 - inf_{ijpq} \qquad ;\forall k \in F, (i,j), (p,q) \in E \tag{4.32}$$

$$f_s^p, f_d^p, f_{ij}^p \geq 0 \qquad ;\forall p \in P, \forall d \in GW, \forall s \in S_p, \forall (i,j) \in E \tag{4.33}$$

$$z_{ij}, col_k, v_{ij}^k \in \{0,1\} \qquad ;\forall (i,j) \in E, \forall k \in F \tag{4.34}$$

The objective function (4.21) minimizes the number of colors assigned. With constraints (4.22), (4.23) and (4.24), it is ensured that the flow is balanced for the set of sink nodes, the set of source nodes, and the intermediate nodes, respectively. Constraints (4.25) are added so that for each node in the network, the summation of incoming and outgoing arcs will not exceed the node degree parameter. Constraints (4.26) are added since the flow on each arc leaving a source node for the commodity p should be greater than or equal to the y_{max} bound obtained first with the routing model. Constraints (4.27) ensure that there can be either transmission or receipt happening at a time on a wireless link, since all wireless devices are assumed to be half-duplex. Constraints (4.28) are added because if the arc is used for the routing, the total flow on that arc cannot exceed the flow capacity. Constraints (4.29) are added since a frequency cannot be assigned to any arc if the frequency is not used in the first place. Constraints (4.30) are written so that the frequency will be assigned if the link is being used. Constraints (4.31) ensure that each used arc is assigned with one frequency. Constraints (4.32) are written so that if there is an interference between (i,j) and (p,q), they will not be assigned with the same frequency. Finally, binary and non-negative variables are defined in the last part.

When the interference is chosen as SIR-based, the same linearization process that was used for the sequential approach will be utilized for the integrated model as well. Hence, instead of the constraints in (4.32), the following constraints will be added to the model while switching from protocol-based interference model to SIR model.

$$\sum_{(p,q)\in E,(i,j)\neq(p,q)} P_{ijpq} n_{ijpq}^{k} \leq \frac{RxThresh}{SIRThresh} \qquad ;\forall k \in F, \forall (i,j) \in E \tag{4.35}$$

$$n_{ijpq}^{k} \leq v_{ij}^{k} \qquad ;\forall k \in F, \forall (i,j),(p,q) \in E, (i,j) \neq (p,q) \tag{4.36}$$

$$n_{ijpq}^{k} \leq v_{pq}^{k} \qquad ;\forall k \in F, \forall (i,j),(p,q) \in E, (i,j) \neq (p,q) \tag{4.37}$$

$$n_{ijpq}^{k} \geq v_{ij}^{k} + v_{pq}^{k} - 1 \qquad ;\forall k \in F, \forall (i,j),(p,q) \in E, (i,j) \neq (p,q) \tag{4.38}$$

Unlike the linearization constraints introduced in the previous section (4.17 - 4.20), the arcs are now defined in set E since the set of used arcs U does not exist in the integrated model.

5. RESULTS

In this chapter, the results of the models introduced are presented. The models were written in Python, solved by Gurobi solver and executed in Spyder environment. In Section 5.1, the performances of the sequential models and the integrated approach are compared for different sizes of networks in terms of the number of frequencies assigned and the execution time. These studies are also classified according to the interference model chosen. In Section 5.2, the effect of the lower bound y_{max} on the number of frequencies assigned is observed. In Section 5.3, the performance of the heuristic proposed by Chaudhry et al. (2016) is compared with the sequential and integrated approaches in terms of the number of frequencies assigned and the computation time. Finally in Section 5.4, the distribution of the wireless devices in the network is altered (uniform to clustered) in order to observe its impact on the performance of the models.

5.1 The Comparison of Sequential and Integrated Models

In this section, the number of frequencies assigned to the wireless links by the sequential and integrated models is presented for comparison. These results are classified based on the interference model used, and different sizes of networks are studied. For each node degree and TCA parameter combination, 25 different networks were generated and the average number of frequencies was calculated. The wireless link capacity was chosen as 24 Mbps.

5.1.1 Protocol Interference

As the first step of the analysis, a small network containing a single gateway (GW) and 5 wireless devices was generated. In Chaudhry et al. (2016) 's study, the wireless networks were consisting of 36 wireless devices in total, uniformly distributed in 500 x 500 units of area. To be able to keep the density similar to this setting, smaller networks were generated proportionally. Hence, the networks made up of 5 nodes and one GW were uniformly distributed in 200 x 200 units of area. This data generation was conducted by creating uniformly distributed random values between 0 and the width of the area that the wireless devices were assumed to be placed in. This process was repeated for both x and y coordinates of each node. Then, these coordinates were utilized for the interference models as the Euclidean distance between each pair of nodes was being calculated. The interference range parameter used in protocol-based interference model was also a uniformly distributed random variable between 0 and 50.

The results are presented in Table 5.1. The node degree and TCA parameters were changed so that different scenarios of frequency assignment could be observed. The third column shows the average number of frequencies assigned by the sequential models, and on the fourth one the number of frequencies assigned by the integrated model was listed. The fifth column shows the improvement ratio between these two average values, and the last column represents the percentage of the number of networks in which the integrated model came up with a strictly smaller number of frequencies assigned (out of 25 networks in total). On average, the integrated model performed better than the sequential models in terms of the number of frequencies assigned, and since the networks were relatively small, the difference between the total run times for each was negligibly small.

Table 5.1 : Results for the network with 5 devices and a single GW, with protocol-based interference

Node degree (n)	TCA parameter (t)	Avr. # of freq (sep)	Avr. # of freq (int)	Improvement ratio (%)	% of inst. improved
2	2	3.44	3.32	3.49	8
2	3	3.68	3.52	4.35	12
2	4	3.64	3.44	5.49	16
3	2	4.44	4.24	4.50	16
3	3	4.52	4.04	10.62	44
3	4	4.56	3.8	16.67	56
4	2	5	4.48	10.40	40
4	3	4.96	4.36	12.10	48
4	4	4.84	4.36	9.92	36
5	2	5.2	4.68	10	44
5	3	4.8	4.36	9.17	40
5	4	4.88	4.76	2.46	8

The same analysis was conducted for the networks containing 10 wireless devices and a single gateway as well. This time the wireless devices were distributed randomly in 276 x 276 units of area and the interference range parameter was distributed uniformly between 0 and 75. The results are presented in Table 5.2. Same as the previous analysis, this time again for all instances the integrated approach performed better in terms of the number of frequencies assigned. Higher levels of improvement were observed compared to the networks with 5 devices, for instance for $n = 4$ and $t = 3$, wireless links in 22 out of 25 networks were assigned with fewer frequencies when the integrated model was preferred.

Table 5.2 : Results for the network with 10 devices and a single GW, with protocol-based interference

Node degree (n)	TCA parameter (t)	Avr. # of freq (sep)	Avr. # of freq (int)	Improvement ratio (%)	% of inst. improved
2	2	5.35	5	6.54	20
2	3	5.32	4.44	16.54	68
3	2	6.09	5.27	13.38	48
3	3	6.4	4.92	24.07	76
4	2	6.26	5.32	15.06	56
4	3	6.76	5.2	23.08	88
5	2	6.3	5.32	15.64	56
5	3	6.92	5.48	20.81	68

When the size of the network was increased to 35 devices and one GW, the execution time has become an issue for the integrated approach, unlike the previous scenarios. In order to be able to obtain a solution by using the integrated model, both the node degree and the TCA parameters were kept as 3 instead of getting changed repeatedly. First, 2 hours were given for each network, summing up to 50 hours in total. The integrated model was able to assign frequencies in only one network, while the sequential models were able to solve 21 out of 25 instances in less than 6 minutes. Then, the time limit was set as 4 hours for each, summing up to 100 hours in total. This time the integrated model was able to solve 4 instances compared to the 21 solved by the sequential models. The results of this trial can be seen in Table 5.3. The average number of frequencies assigned by the sequential models was 21.33, whereas this average was 16.5 for the integrated model. The wireless links in all 4 of these instances were assigned with a strictly smaller number of frequencies when the integrated model was preferred, and on average the improvement ratio was 22.66%. However, it is worth noting that the run time was more than 10 times higher compared to the sequential models and the number of results obtained decreased by 80.95%. When the network size gets larger, it is clear that there is a noticeable trade-off between the execution time and the performance of the model preferred.

Table 5.3 : Results for the network with 35 devices and a single GW, with protocol-based interference

Node degree (n)	TCA parameter (t)	Avr. # of freq (sep)	Avr. # of freq (int)	Improvement ratio (%)
3	3	21.33	16.5	22.66

5.1.2 SIR Interference

In this section, performances of the sequential and integrated models are compared when the interference model is chosen to be SIR-based. G_t, G_r, h_t, h_r values were chosen as 1 and the frequency was assumed as 5.805 GHz, similar to Chaudhry, Hafez & Chinneck (2015)'s work. Additional to the networks of sizes 5, 10 and 35 (+GW) generated in the previous section, networks of 15 and 20 were also created. One of the nodes was randomly chosen as the gateway. The network consisting of 15 wireless devices and a single GW was uniformly distributed in 333 x 333 units of area and the network with 20 wireless devices and a single GW was generated in 382 x 382 units of area. This data generation was conducted by creating uniformly distributed random values between 0 and the width of the space that the devices were assumed to be placed in. This process was repeated for both x and y coordinates of each node. All 25 networks were generated accordingly.

As it can be observed in Tables 5.4 and 5.5, the improvement in the number of frequencies assigned becomes even clearer as the number of devices increases from 5 to 10. Moreover, run time is not a problem for those network sizes as both the sequential and integrated models solve the problem for 25 instances under 6 minutes. However, as the number of devices increases to 15, computation time becomes much more longer. This is why for the network sizes 15, 20 and 35, the analysis based on different t and n parameters was not preferred, and the parameters were fixed as $t = 4$ and $n = 3$ for each network instead.

Table 5.4 : Results for the network with 5 devices and a single GW, with SIR-based interference

Node degree (n)	TCA parameter (t)	Avr. # of freq (sep)	Avr. # of freq (int)	% of inst. improved
2	2	5.2	5.24	0
2	3	6	6	0
2	4	6	6	0
3	2	6.36	6.36	0
3	3	7.12	7.12	0
3	4	7.08	7	8
4	2	6.52	6.48	4
4	3	7.48	7.48	0
4	4	8	8	0
5	2	6.64	6.6	4
5	3	7.04	6.96	8
5	4	6.2	6.2	0

Table 5.5 : Results for the network with 10 devices and a single GW, with SIR-based interference

Node degree (n)	TCA parameter (t)	Avr. # of freq (sep)	Avr. # of freq (int)	Improvement ratio (%)	% of inst. improved
2	2	9.90	9.85	0.51	5
2	3	9.88	9.8	0.81	8
3	2	10.65	10.43	2.04	21.74
3	3	12.16	11.8	2.96	36
4	2	10.83	10.48	3.21	30.43
4	3	12.44	11.88	4.5	44
5	2	10.74	10.48	2.43	26.09
5	3	12.32	12.08	1.95	24

In Table 5.6, it can be seen that there is a considerable amount of improvement in the number of frequencies assigned when the integrated approach is preferred. However, it is also a fact that the run time also became much longer when this model was chosen. For example, for 20 nodes and a single GW, it takes 140.6 seconds for all 25 instances to be solved by the sequential models, whereas it takes around 6000 seconds for the integrated model to solve the same instances. But at the same time, the wireless links in 22 out of these 25 networks were assigned with a smaller number of frequencies, and the average number of frequencies dropped by almost 8 percent. When the size of the network was chosen as 35, the sequential models managed to solve 23 out of 25 instances in 1033.6 seconds, whereas the integrated model was able to solve only 7 out of 23 instances with 1 hour limitation for each instance. In other words, the execution time increased to 25 hours, the instances with an output dropped by 70 percent and yet, 6 out of 7 of those instances were resulted in smaller numbers of frequencies assigned, with an average improvement of 17.93 percent.

Table 5.6 : Results for the networks in various sizes, with SIR-based interference

# of nodes	Avr. # of freq (sep)	Avr. # of freq (int)	Improvement ratio (%)	% of inst. improved	Increase in run time (%)
15	15.84	14.8	6.57	68	2008.81
20	21.56	19.84	7.98	88	4167.42
35	32.7	26.83	17.93	85.72	8611.98(for 30.44%)

With these results, it is possible to see that better results were obtained in terms of the number of frequencies assigned, even when the interference model was chosen to be more complicated, which caused the optimization model to get more complex as well. However, it is also evident that the run time issue is still relevant when the interference model is SIR-based and the network size is larger.

5.2 A Different Objective for Routing

While the routing part of the sequential models was being introduced in the previous chapter, it was emphasized that the lower bound for the network throughput (y_{max}) was maximized in the objective function so that fairness would be ensured. The arcs are chosen with respect to this fairness concern and this decision eventually effects the frequency assignment in the second part of the sequential models. This situation brings up the question of whether the performance of the frequency assignment model changes if this fairness element of the routing model is relaxed. In order to answer this, the routing model (the first part of the sequential approach) was rewritten so that the lower bound y_{max} would not be maximized anymore while equity is being ensured, but instead it would be an input parameter that is being multiplied by a number between 0 and 1. To ensure this, first the routing model was solved in its original form, then the obtained y_{max} values were plugged in the new version of the routing model as a parameter, and the output of this second step was chosen as the actual routing.

The objective function of the routing model was as follows:

$$\max y_{max} - c \sum_{(i,j) \in E} z_{ij} \tag{5.1}$$

Since now y_{max} is a parameter, the objective function focuses on minimizing the number of arcs used instead:

$$\min \sum_{(i,j) \in E} z_{ij} \tag{5.2}$$

And the lower bound constraint was written as:

$$f_s^p \geq y_{max} \qquad ; \forall p \in P, \forall s \in S_p \tag{5.3}$$

Now to ensure the lower bound was being relaxed, it is updated as:

$$f_s^p \geq y_{max} A \qquad ; \forall p \in P, \forall s \in S_p \tag{5.4}$$

where $A \in [0,1]$. The rest of the routing model stays exactly the same. The performance of this approach was compared to the original version of the routing model, while the frequency assignment part of the sequential approach remained the same for both routing versions.

It was observed in Table 5.6 that the older version of the sequential approach results in 15.32 frequencies assigned on average when the network size is 15. In Table 5.7 it can be seen that as A gets larger, the number of frequencies assigned gets even smaller, which makes the trade-off between the network throughput fairness and the number of frequencies assigned more visible to the reader. Similarly, in Table 5.6 it was seen that the average number of frequencies assigned was 21.08. In Table 5.8 it is possible to see that as A gets larger, the average number of frequencies assigned again gets smaller.

Table 5.7 : Results for the network with 15 nodes, sequential models, and SIR-based interference

A	Avr. # of freq	Improvement ratio (%)	% of inst. improved
0.95	15.32	0	4
0.9	15.16	1.04	16
0.85	15.2	0.78	12
0.8	14.96	2.35	28

Table 5.8 : Results for the network with 20 nodes, sequential models, and SIR-based interference

A	Avr. # of freq	Improvement ratio (%)	% of inst. improved
0.95	21.2	-0.57	12
0.9	21.08	0	20
0.85	20.92	0.76	28
0.8	20.68	1.90	36
0.75	20.64	2.09	32

To conclude this analysis, it can be said that it is important to decide which element of this frequency assignment will be prioritized. The original modelling approach introduced by Chaudhry et al. (2016) ensures that the fairness is considered by maximizing a lower bound for every source generating a signal, but it also prevents the frequency assignment problem from leading results with smaller number of frequencies.

5.3 Heuristic

The heuristic approach introduced by Chaudhry et al. (2016) is implemented alongside the routing model of the sequential approach so that its performance could be compared with the integrated model. The heuristic substitutes for the frequency assignment part of the sequential approach. The comparison was conducted with SIR-based interference since it was much more complex and the run time was significantly higher compared to protocol-based interference as it was presented earlier. The heuristic was again implemented in Python language and the routing part was solved by Gurobi solver. The steps are as follows:

- The routing model is solved in the same way it was handled in sequential approach. The links used for routing are extracted to be input for the heuristic.
- The SIR-based interference is also calculated the same way and the matrix of maximum received power for each pair of nodes is generated as an input for the heuristic.
- Each link is assigned with a key number, since the routing model gives the output as pairs of nodes that the links are placed in between. The total number of links used in routing is m.
- The maximum received power for each link used in routing is extracted from the interference matrix and stored in mxm matrix MP. If two links are interfering with each other, the value of 1 in interference matrix is also carried to the matrix MP.
- The number of links each link interferes with is determined according to the number of "1"s occurring in the corresponding link's row in matrix MP. This information is stored in an array of m, $InfCount$.
- The indices that would sort the array $InfCount$ from the link that interferes with the most to the least are extracted in an array of m, $InfSort$.
- $InfSort$ is updated so that the links that interfere with *all* the other links are excluded and stored in a separate array, $InfAll$.
- Starting from the first link in the updated version of $InfSort$, each link is grouped with all the other links they are not interfering with and classified into lists, forming a list of lists $InfList$.
- Each group (list) in $InfList$ is handled one by one. For each group, starting with the first two links in that group, it is checked whether the cumulative SIR exceeds the SIR Threshold or not. If the next link in the group makes the cumulative SIR exceed the threshold, it is removed from the group. If it does not, it is kept in the group and removed from all the other groups. This way the groups in $InfList$ are

updated in a way that the cumulative SIR for each group does not exceed the SIR threshold.

- Each final group in $InfList$ is assigned with a separate frequency.
- Each link that was stored in $InfAll$, that were interfering with all the other links, are assigned with a separate frequency.
- The heuristic is complete when all the links are assigned with a frequency and the total number of frequencies assigned is equal to the number of final groups in $InfList$ plus the number of links in $InfAll$.

The heuristic was used for the network consisting of 35 devices and a single gateway and the results were compared with the previous assignments conducted by sequential and integrated models. In Figure 5.1 the comparison of the average number of frequencies can be observed. It is clear that the integrated approach still performs the best in terms of the number of frequencies assigned, and the heuristic model assigns more frequencies than the sequential and integrated models, but the run time issue for each approach must again be addressed.

It takes 1033.6 seconds for the sequential models to solve 23 out of 25 instances, whereas the integrated model was able to solve only 7 out of 23 instances with 1 hour limitation for each instance, 25 hours execution time in total. 6 out of 7 of those solved instances were resulted in smaller numbers of frequencies assigned, with a decrease of 17.93 percent in the average number of frequency channels. The heuristic, on the other hand, solves all of the instances that were solved by the sequential models in 951.17 seconds. Although the number of frequencies assigned has increased by almost 6.65 percent, the run time gets shorter by almost 8 percent as well. In terms of execution time, the heuristic method outperforms other models, yet in terms of the average number of frequencies assigned, the integrated approach remains preferable.

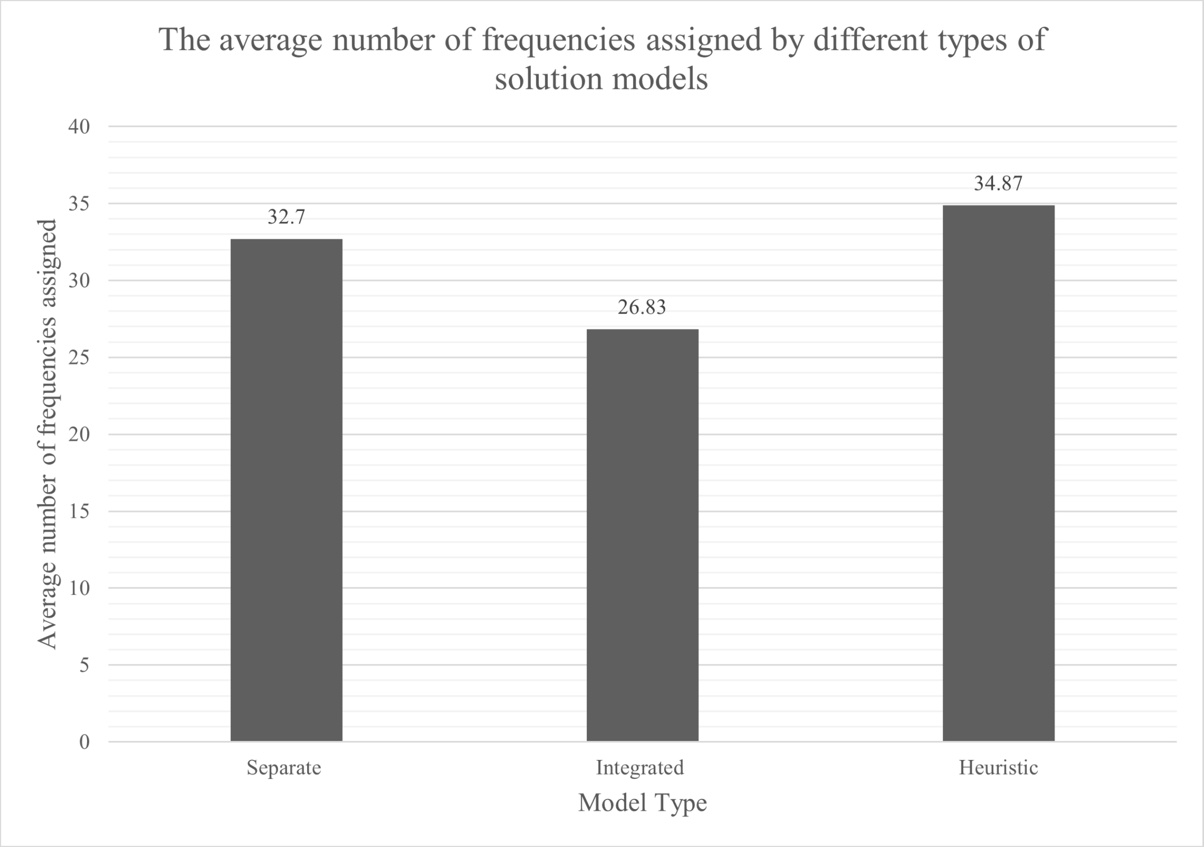


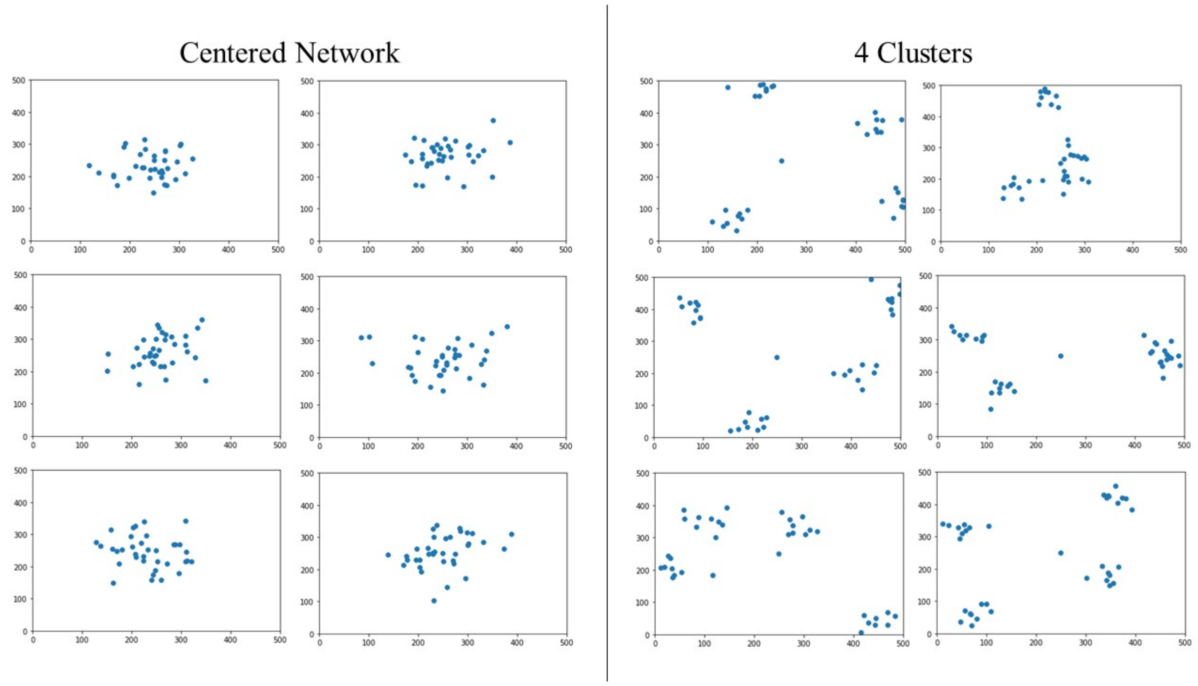
Figure 5.1 : The comparison of the average number of frequencies assigned, SIR interference, 35 nodes

5.4 Clustered Network Analysis

Until now, the networks generated have been consisting of uniformly distributed devices. In this section, we wanted to observe how the model reacts to clustered networks, and generated two types of networks in 500 x 500 units of area.

The first type with a single center is made up of a gateway (GW) that is placed in the center ($x = 250, y = 250$) and all 35 devices are clustered around the GW. Hence their x and y coordinates are normally distributed with mean 250 and standard deviation 50.

The second type of the network consists of a GW in the center again, yet this time 35 wireless devices are clustered around four different centers around the GW. The position of the GW ($x = 250, y = 250$) is assumed to be the origin, and each of these four (pseudo) centers are generated randomly so that each quadrant has a center in it. The rest 31 of the wireless devices are normally distributed around these centers, with the mean of their x and y coordinates being equal to the center's coordinates and the standard deviation equal to 25. 25 networks were generated in both types and 6 examples for each are presented in Figure 5.2.


Centered Network

4 Clusters

Figure 5.2 : Alternative networks generated, 35 devices + GW

The main goal was to compare the sequential and integrated models for these networks. But since it was not possible for the integrated model to come up with an optimal solution without the run time or memory issues, only the results of the sequential models were studied. The sequential models were run for both types of networks, for the parameters $n = 10, t = 10$ and $n = 10, t = 12$. In Figure 5.3 the results are displayed. On average, it took 2059 seconds for the sequential models to come up with optimal solutions for 23.5 out of 25 single-centered networks (on average), and it took 1549 seconds on average to come up with optimal solutions for all 25 of the networks with four clusters. Although the run time is lower and the number of instances with an optimal solution is higher for the network with 4 clusters, when the results are analyzed it can be seen that in most of the cases the sequential models assigned a different frequency for each of the links, since the average number of links is very close to the average number of frequencies. This shows that when the network is clustered, the need for a higher number of frequencies arises, which could be an issue when the number of frequencies to be assigned is limited. It can be seen that for the networks with a single center this issue was not observed, and the model was able to assign the same frequencies for some of the links, since the difference between the number of frequencies assigned and the number of links used is greater.

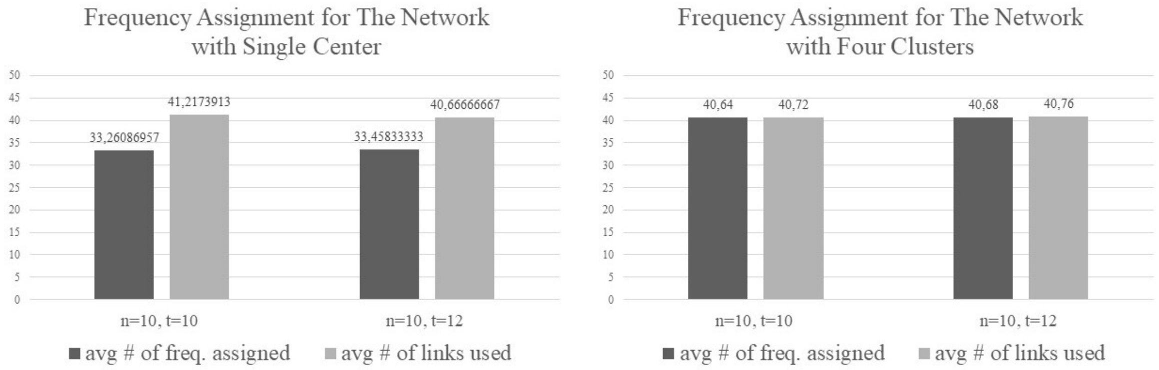


Figure 5.3 : Networks with single center vs. four clusters, frequency assignment comparison, 35 nodes

We believe that this analysis could be improved further so that the performance of the models could be more observable for real-life scenarios where the wireless devices are clustered, such as an office building or a research center. Moreover, if the integrated approach could be implemented for this scenario, better results in terms of the number of frequencies assigned could be expected.

6. DISCUSSION

In this thesis, the frequency assignment problem for wireless mesh networks was considered. In the light of Chaudhry et al. (2016)'s study, the multi-path multi-commodity routing model that controls the network throughput and the frequency assignment model were integrated into a joint model so that better results in terms of the number of frequencies assigned could get achieved. The interference models used in the optimization and also the topology control algorithm that generates the final topology for the network were also introduced. For each wireless device in randomly generated networks, the candidate neighbors were decided by the topology algorithm. The interference for each wireless link combination was calculated based on two different interference models in the literature. The sequential approach that consists of the routing and the frequency assignment model (Chaudhry, 2015) was reported before introducing the integrated version. The integrated model that we propose in this study combines these two models and decides on the direct communication links between wireless devices while simultaneously determining the frequency that they will be assigned with. In experimental results, it was observed that the integrated model performs better in terms of the number of frequencies assigned, for both protocol-based and SIR-based interference models, and for different sizes of networks ranging from 5 to 35 (+GW). However, it was also observed that the computation time is a significant issue for the integrated approach, and it becomes even more evident when the interference model is SIR-based and when the network size gets larger.

The impact of the lower bound for the network throughput which was maximized in the routing model was studied further by relaxing it instead, and it was observed that it was possible to achieve smaller numbers of frequencies assigned when this bound was chosen to be lower. It was also shown that both the sequential and integrated models perform better than the heuristic in terms of the frequency assignment, yet the heuristic remains the fastest solution approach. Finally, it was observed that when the wireless devices in the network are clustered rather than uniformly distributed, it was much harder for both types of models to come up with an optimal solution, and the results provided by the sequential approach showed that the more the network was clustered, it would get more difficult for the model to minimize the number of frequencies assigned.

Since the run time was the main issue of the frequency assignment part of our integrated model, a further research could be conducted so that this joint model could be rearranged in a way that would decrease the computation time. Instead of linearizing the previously quadratic SIR-based frequency assignment model, new variables could be chosen in a way that would eliminate this quadratic approach. Various heuristic algorithms that were mentioned in the literature review could also be integrated in this approach to obtain better results in terms of frequency assignment. New and exact methods that can solve this integrated model could be developed. With these improvements, the study on clustered networks could be repeated and a real-life application could be possible.

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