An Acceleration Based Hybrid Learning-Adaptive Controller for Robot Manipulators

Sanem Evren Han[1,2] and Mustafa Unel[*,1,3]

Abstract

The robust periodic trajectory tracking problem is tackled by employing acceleration feedback in a hybrid learning-adaptive controller for n-rigid link robotic manipulators subject to parameter uncertainties and unknown periodic dynamics with a known period. Learning and adaptive feedforward terms are designed to compensate for periodic and aperiodic disturbances. The acceleration feedback is incorporated into both learning and adaptive controllers to provide higher stiffness to the system against unknown periodic disturbances and robustness to parameter uncertainties. A cascaded high gain observer is used to obtain reliable position, velocity and acceleration signals from noisy encoder measurements. A closed-loop stability proof is provided where it is shown that all system signals remain bounded and the proposed hybrid controller achieves global asymptotic position tracking. Results obtained from a high fidelity simulation model demonstrates the validity and effectiveness of the developed hybrid controller.

Keywords

Acceleration Feedback, Hybrid Control, Adaptive Control, Learning Control, High Gain Observer

1. Introduction

Learning based controllers have gained remarkable importance for robotic manipulators that perform the same task repeatedly. This type of controllers improve system performance by utilizing previous error signals into the control input. However, the standard learning control algorithms may not reject aperiodic disturbances. This motivates the design of hybrid controllers such as adaptive/learning control Dixon et al.

*Corresponding Author: munel@sabanciuniv.edu
[1]Facultyof Engineering and Natural Sciences, Sabancı University, Istanbul, Turkey
sanemevren@sabanciuniv.edu
[2]Ford Otosan Product Development, Istanbul, Turkey
[3]Integrated Manufacturing Technologies Research and Application Center, Sabanci University, Istanbul, Turkey

(2002)-Benosman (2014), adaptive iterative learning control using a fuzzy neural network Wang and Chien (2015), adaptive learning PD (AL-PD) control Ouyang and Zhang (2004), hybrid control based on Fourier series expansion Vecchio et al. (2003)-Delibasi et al. (2010), backstepping adaptive iterative learning control Wang et al. (2013).

Dixon et al. (2002)-Dixon et al. (2003) proposed a hybrid adaptive/learning control scheme to achieve global asymptotic link position tracking despite unknown robot dynamics with periodic and aperiodic components. The authors applied the saturation function to the standard learning control law and solved the boundedness problem by showing that the proposed learning feedforward term is bounded for all times. Ngo et al. (2012) also designed an adaptive iterative learning control (AILC) of uncertain robot manipulators in task space for trajectory tracking. Benosman (2014) concentrated on the use of well-known extremum seeking (ES) theory in the learning based adaptivecontrol structure. The local integral input-to-state stability(iISS) feedback controller with a model-free ES algorithmis combined to obtain a learning-based adaptive controller.Wang and Chien (2015) developed an observer-basedadaptive iterative learning control using a filtered fuzzy neural network. A state tracking error observer is introduced to design the iterative learning controller using only the measurement of joint position. An observation error modelis derived based on the state tracking error observer. Then, by introducing some auxiliary signals, the iterative learning controller is proposed based on the use of an averaging filter. Ouyang and Zhang (2004) developed a new control method called adaptive learning PD (AL-PD) control. While PD control acts as a basic feedback control part, learning feedforward control is an iteratively updated term to cope with the unknown robot dynamics. When the number of iterations increase, AL-PD control guarantees the tracking errors converge arbitrarily close to zero.

Vecchio et al. (2003) proposed a hybrid adaptive learning control scheme to solve the periodic tracking problem for single-input, single-output uncertain feedback linearizable systems with maximal relative degree and matching unstructured uncertainties, i.e. no parametrization is available for uncertain nonlinearities. The authors have developed the unknown periodic reference input signal witha known period in Fourier series expansion. The proposed controller learns the reference control signal and identifies the Fourier-coefficients of any truncated approximation. Liuzzo and Tomei (2008) developed the input reference signals as Fourier series expansion and designed AL-PD control that learns the input reference signals by identifying their Fourier coefficients. When the Fourier series expansion of each input reference signal is finite, global asymptotic tracking and local exponential tracking of both the input andthe output reference signals is obtained. Delibasi et al. (2010) proposed a self tuning, desired compensation adaptation law based adaptive controller with disturbance estimation based on Fourier Series Expansion. The proposed hybrid controller guarantees global asymptotic link tracking.

Wang et al. (2013) designed a backstepping adaptive iterative learning control (AILC) where the backstepping like procedure is used to design the main structure of the AILC. The developed controller has two parts; a fuzzy neural network (FNN) is utilized to approximate unknown certainty equivalent controller, and a robust learning term is used to compensate for uncertainty from the network approximation error. Thus, the boundedness of internal signals is guaranteed. Tracking error asymptotically converges to zero.

In this paper, a new hybrid acceleration based learning-adaptive controller is developed to achieve global position tracking for n-rigid link robotic manipulators despite the parameter uncertainties and unknown periodic dynamics. It is known that the use of acceleration feedback is effective for the disturbance rejection in industrial applications such as servo control machines and robot arms which continuously interact with the environment and work under different loads. Therefore, acceleration feedback is incorporated into both learning and adaptive controllers to improve the robustness of the system against periodic and aperiodic disturbances, respectively. Since it is difficult to obtain reliable velocity and acceleration signals from noisy encoder measurements, a cascaded high gain observer (CHGO) is utilized to estimate reliable position, velocity and acceleration feedback signals. The proposed hybrid controller uses these estimated signals as feedback in a high fidelity simulation model to achieve periodic trajectory tracking for a pan-tilt system. The main contributions are as follows:

- A new linear parametrization property is introduced where the unknown parameter vector includes both actuator moment of inertia and friction parameters and the regressor matrix depends not only on link velocities but also accelerations. Thus, acceleration feedback is incorporated into the adaptive controller to improve the robustness of the system against unknown aperiodic disturbances.

- A hybrid learning based adaptive controller has been developed by integrating acceleration based adaptive and learning controllers. The hybrid controller increases the robustness of the system against aperiodic and periodic disturbances.
- Closed-loop stability proof of the proposed hybrid controller is provided to show that all system signals remain bounded and global asymptotic position tracking is ensured.

The remainder of this paper is organized as follows:Section II presents encoder modeling and a cascadedhigh gain observer (CHGO) to estimate reliable position,velocity and acceleration signals. In Section III, a hybridacceleration based learning-adaptive controller is developedand the closed-loop stability proof is obtained. Section IVdemonstrates simulation results where the performance of the proposed hybrid control is shown on a high fidelity pan-tiltmodel. Finally, Section V concludes the paper with someimportant remarks.

2. Link Position, Velocity and acceleration Estimation by A Cascaded High Gain Observer

A cascaded high gain observer is developed to estimate reliable link velocities, $\hat{z}_{o_1}$, and accelerations, $\hat{z}_{o_2}$, inaddition to link positions, $\hat{x}_{o_1}$, by utilizing noisy position measurements from an encoder. This observer consists of two high gain observers in a cascaded structure as depicted in Figure 1.

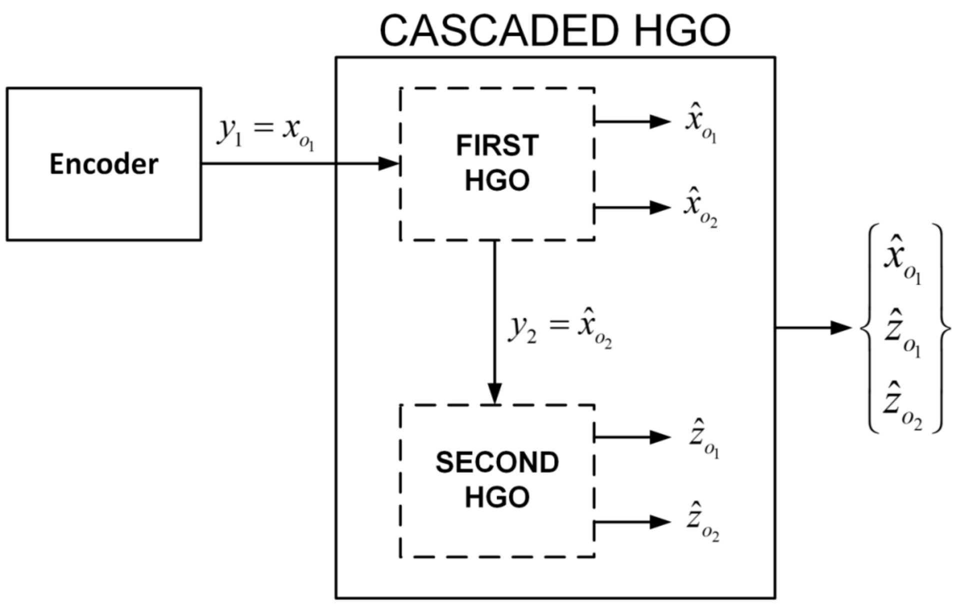

Figure 1. Block diagram of Cascaded HGO Structure

The first HGO uses position measurements from an encoder to estimate position and velocity signals. The second HGO, on the other hand, utilizes estimated velocities by the first HGO to provide estimates of link accelerations. The first HGO is designed as

$$\begin{aligned}\dot{\hat{x}}_{o_1} &= \hat{x}_{o_2} + L_1\left(y_1 - \hat{x}_{o_1}\right)\\ \dot{\hat{x}}_{o_2} &= L_2\left(y_1 - \hat{x}_{o_1}\right)\end{aligned} \tag{1}$$

where $\hat{x}_{o_1} \in \mathbb{R}^n$ and $\hat{x}_{o_2} \in \mathbb{R}^n$ are the estimated link positions and velocities, $\hat{x}_o(t) = [\hat{x}_{o_1} \quad \hat{x}_{o_2}]^T \in \mathbb{R}^{2n}$ denotes the observer state vector, $y_1 = q_m \in \mathbb{R}^n$ is the encoder link position measurement, and the observer gains are defined as

$$L_1 = \frac{\beta_1}{\epsilon_1}, \qquad \text{and} \qquad L_2 = \frac{\beta_2}{\epsilon_1^2} \tag{2}$$

for some positive constants $\beta_1, \beta_2 \in \mathbb{R}$, and $\epsilon_1 \ll 1$. Similarly, the dynamics of the second HGO is given as

$$\begin{aligned}\dot{\hat{z}}_{o_1} &= \hat{z}_{o_2} + L_3\left(y_2 - \hat{z}_{o_1}\right)\\ \dot{\hat{z}}_{o_2} &= L_4\left(y_2 - \hat{z}_{o_1}\right)\end{aligned} \tag{3}$$

where $\hat{z}_{o_1} \in \mathbb{R}^n$ and $\hat{z}_{o_2} \in \mathbb{R}^n$ are the estimated link positions and velocities, $\hat{z}_o(t) = [\hat{z}_{o_1} \quad \hat{z}_{o_2}]^T \in \mathbb{R}^{2n}$ denotes the observer state vector, $y_2 = \hat{x}_{o_2} \in \mathbb{R}^n$ is the estimated velocity by the first HGO, and L_3, L_4 are the observer gains designed as

$$L_3 = \frac{\beta_3}{\epsilon_2}, \qquad \text{and} \qquad L_4 = \frac{\beta_4}{\epsilon_2^2} \tag{4}$$

for some positive constants $\beta_3, \beta_4 \in \mathbb{R}$, and $\epsilon_2 \ll 1$. Those observers are referred as high gain observers because larger observer gains, L_1, L_2, L_3 and L_4, are used in order to achieve zero estimation errors. High gain observers suffer from a peaking phenomenon due to sufficiently small $\epsilon_1$and ϵ_2. This phenomenon is handled by saturating the control input. The readers are referred to Khalil and Praly (2014) for the details.

3. Acceleration Based Hybrid Learning Controller for Robotic Manipulators

This section develops a new hybrid learning based adaptive controller using the acceleration feedback to achieve a global position tracking for a n-rigid link robotic manipulator (e.g.a pan-tilt system where $n = 2$) as in Figure 2 despite the parameter uncertainties and unknown periodic dynamics.The proposed hybrid controller utilizes learning based feedforward terms to compensate for periodic disturbances, and adaptive based feedforward terms to reject aperiodic disturbances.

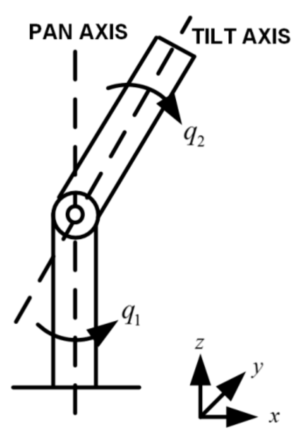

Figure 2. Pan-tilt mechanism

Pan and tilt axes can also be referred as azimuth and elevation axes. The nonlinear model of the pan-tilt system based on the Euler-Lagrange formulation is as follows Tao and Backlash (1999):

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F_v\dot{q} + F_s sgn(\dot{q}) = \tau \tag{5}$$

where

$$q = [q_1 \quad q_2]^T, \quad \tau = [\tau_1 \quad \tau_2]^T,$$

$$M(q) \triangleq D(q) + J = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} + \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix}$$

$$C(q,\dot{q}) = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}, \quad G(q) = [0 \quad 0.5 m_2 g l_2 cos q_2]^T$$

$$F_v(\dot{q}) = [v_1\dot{q}_1 \quad v_2\dot{q}_2]^T$$

$$F_s(\dot{q}) = [k_1 sgn(\dot{q}_1) \quad k_2 sgn(\dot{q}_2)]^T$$

$$D_{11} = \frac{1}{2}m_1 l_1^2 + m_2 l_2^2 + m_2 l_1 l_2 cos q_2 + \frac{1}{3} m_2 l_2^2 cos^2 q_2$$

$$D_{22} = \frac{1}{3} m_2 l_2^2, \;\; D_{12} = D_{21} = 0$$

$$C_{11} = -m_2 l_1 l_2 \dot{q}_2 sin q_2, \qquad C_{12} = -\frac{1}{3} m_2 l_2^2 \dot{q}_1 sin 2q_2$$

$$C_{21} = \dot{q}_1 \left(\frac{1}{2} m_2 l_1 l_2 sin q_2 + \frac{1}{6} m_2 l_2^2 sin 2q_2\right), \quad C_{22} = 0 \tag{6}$$

where $q, \dot{q}, \ddot{q} \in \mathbb{R}^2$ are the joint angles, velocities and accelerations, $M(q) \in \mathbb{R}^{2\times2}$denotes the symmetric and positive-definite inertia matrix, and $D(q) \in \mathbb{R}^{2\times2}$is the robot inertia matrix, $J_1 \in \mathbb{R}$ and $J_2 \in \mathbb{R}$ are motor inertias, $C(q,\dot{q}) \in \mathbb{R}^{2\times2}$is the centripetal-Coriolis matrix, $G(q) \in \mathbb{R}^2$is the gravity vector, $F_v(\dot{q})$ and $F_s(\dot{q}) \in \mathbb{R}^{2\times1}$ are constant, diagonal, positive-definite, viscous and static friction coefficient matrices, $sgn(\dot{q})$ is the signum function applied to the joint velocities, $\tau \in \mathbb{R}^2$is the torque control input vector. $m_1 \in \mathbb{R}$ and $m_2 \in \mathbb{R}$ are the masses of pan and tilt mechanisms, $l_1 \in \mathbb{R}$ is the radius, $l_2 \in \mathbb{R}$ is the length, $v_1 \in \mathbb{R}$ and $v_2 \in \mathbb{R}$ are viscous friction coefficients, and $k_1 \in \mathbb{R}$ and $k_2 \in \mathbb{R}$ are static friction coefficients.

Some dynamical parameters in (5) can change unpredictably due to variations in the environmental conditions.This problem may also occur because the system parameters are slowly time-varying. Unmeasurable changes of the process parameters lead to unsatisfactory control performance. An adaptive controller adjusts itself to tackle unknown parameter uncertainties. Large variations generally occur instatic friction coefficients. However, large variations may also occur in motor inertias. This motivates us to include both motor moment of inertia terms and static friction coefficients in the unknown parameter vector. To this end, a new linear parametrization property is introduced.

For the subsequent control development and stability analysis, the following important properties will be utilized.

Property 1: Symmetric and Positive-Definite InertiaMatrix

The robot inertia matrix, $D(q)$, is symmetric and positive-definite, and satisfies the following inequality:

$$\beta_1 \|\eta\|^2 \leq \eta^T D(q) \eta \leq \beta_2 \|\eta\|^2 \qquad \forall \eta \in \mathbb{R} \tag{7}$$

where $\beta_1, \beta_2 \in \mathbb{R}$ are known positive constants, $\|.\|$ denotes the standard Euclidean norm.

Property 2: Skew-Symmetry

The inertia and centripetal-Coriolis matrices satisfy the following skew-symmetric relationship:

$$\eta^T \left(\frac{1}{2} \dot{D}(q) - C(q, \dot{q}) \right) \eta = 0 \qquad \forall \eta \in \mathbb{R}^n \tag{8}$$

where $\dot{D}$ is the time derivative of the inertia matrix.

Property 3: Bounding Inequalities

The upper bounds for the norms of the centripetal-Coriolis, gravity, and viscous friction terms can be obtained as follows:

$$\|C(q,\dot{q})\|_{i\infty} \leq \sigma_{c1}\|\dot{q}\|\,, \quad \|G(q)\| \leq \sigma_g\;, \; \|F_v\|_{i\infty} \leq \sigma_{f_v} \tag{9}$$

where $\sigma_{c1}, \sigma_g, \sigma_{f_v} \in \mathbb{R}$ represents known positive constants and $\|.\|_{i\infty}$ is the induced infinity norm of a matrix.

Property 4: Linearity in the Motor Moment of Inertia and Static Friction Parameters

The motor moment of inertia terms and static friction coefficients in (5) can be linearly parameterized as

$$J\ddot{q} + F_s sgn(\dot{q}) = W(q,\dot{q})\Phi \tag{10}$$

where unknown parameter vector, $\Phi \in \mathbb{R}^{2n}$, consists of motor moment of inertia terms and static friction coefficients. Regression matrix, $W(q,\dot{q}) \in \mathbb{R}^{n\times 2n}$, includes both known velocities and accelerations.

Using the parametrization property in (10), the robot dynamics given by (5) can be rewritten as

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F_v\dot{q} + W(q,\dot{q})\Phi = \tau \tag{11}$$

Remark 1: Using the assumptions given in (7)-(9), it can be concluded that the torque control input is bounded when all the terms on the left-hand side of (11) are bounded provided that $q(t), \dot{q}(t), \ddot{q}(t) \in \mathcal{L}_\infty$.

3.1. Controller Design

The control objective is to design the torque control input signal, $\tau(t)$, such that the robot link positions will converge to desired trajectories despite the parameter uncertainties in the dynamic model given by (11), i.e. $q(t) \Rightarrow q_d(t)$ as $t \Rightarrow \infty$. To quantify the control objective, the position tracking error, denoted by $e(t) \in \mathbb{R}^n$, is defined as follows:

$$e = q_d - q \tag{12}$$

where $q_d(t) \in \mathbb{R}^n$ is the desired link position. The control objective is based on the assumption that $q(t), \dot{q}(t)$ and $\ddot{q}(t)$ are measurable, and the desired link positions, velocities and accelerations are bounded, periodic functions of time that are defined as follows:

$$q_d(t) = q_d(t-T), \quad \dot{q}_d(t) = \dot{q}_d(t-T), \tag{13}$$

and

$$\ddot{q}_d(t) = \ddot{q}_d(t-T)$$

with a known period of T. To facilitate the subsequent control development and stability analysis, the order of the robot dynamics in (11) is reduced by defining a filtered tracking error variable, $r_h(t) \in \mathbb{R}^n$ as follows:

$$r_h = \dot{e} + \Gamma_1 e + \Gamma_2 \int e\, dt \tag{14}$$

where $\dot{e} \in \mathbb{R}^n$ is the velocity error, i.e. $\dot{e} \triangleq \dot{q}_d - \dot{q}$, and $\Gamma_1, \Gamma_2 \in \mathbb{R}^{n\times n}$ are constant, diagonal and positive-definite controller gain matrices. After taking the time derivative of (14) and multiplying the resulting expression by the inertia matrix, $D(q)$, the open loop error system is obtained as

$$D(q)\dot{r}_h = -C(q,\dot{q})r_h + \vartheta + \xi + W(\dot{q},\ddot{q})\Phi - \tau \tag{15}$$

where the auxiliary expressions $\vartheta, \xi \in \mathbb{R}^n$ are defined as follows:

$$\vartheta = D(q_d)\ddot{q}_d + C(q_d, \dot{q}_d) + G(q_d) + F_v\dot{q}_d \tag{16}$$

and

$$\xi = D(q)(\ddot{q}_d + \Gamma_1\dot{e} + \Gamma_2 e) + G(q) + F_v\dot{q} - \vartheta + C(q,\dot{q})(\ddot{q}_d + \Gamma_1 e + \Gamma_2 \int e\, dt) \tag{17}$$

Since the real system parameters are not exactly known, the auxiliary signal, ϑ, as a function of desired periodic trajectories, is an unknown periodic signal. In light of (7), (9) and (13), it follows that

$$|\vartheta_i| \leq \alpha_i \ \text{ for } \ i = 1,2,\dots,n \tag{18}$$

where $\alpha_i = [\alpha_1 \quad \cdots \quad \alpha_n] \in \mathbb{R}^n$ is a vector of known, positive bounding constants.

By utilizing (7), (9), (12) and (14), and motivated by the result in Dixon et al. (2003), it is obtained that:

$$\|\xi\| \leq \delta(\|Z\|)\|Z\| \tag{19}$$

where the auxiliary signal $Z(t) \in \mathbb{R}^{3n}$ is defined as:

$$Z(t) = [e^T(t) \quad r_h^T(t) \quad \ddot{e}^T(t)]^T \tag{20}$$

and $\delta(.) \in \mathbb{R}$ is a known and positive bounding function. On the basis of the structure of the open-loop error system in (15), the proposed hybrid control law is designed by using an adaptive controller along with a learning based feedforward term as follows:

$$\tau = \Lambda r_h + \kappa \delta^2(\|Z\|) r_h + \hat{\vartheta} + \tau_a \tag{21}$$

where $\hat{\vartheta} \in \mathbb{R}^n$ is an estimate of ϑ in (16) and generated by incorporating acceleration feedback into the standard feedforward term in Dixon et al. (2003):

$$\hat{\vartheta}(t) = sat_\alpha\left(\hat{\vartheta}(t-T)\right) + K_1 r_h + K_2 \ddot{e} \tag{22}$$

and the adaptive controller, τ_a, is designed as follows:

$$\tau_a = W(\dot{q}, \ddot{q})\hat{\Phi} \tag{23}$$

with the update law given by (24):

$$\dot{\hat{\Phi}} = \Upsilon_h W^T(\dot{q}, \ddot{q}) r_h \tag{24}$$

where $\ddot{e} \in \mathbb{R}^n$is the acceleration error, i.e. $\ddot{e} \triangleq \ddot{q}_d - \ddot{q}$, $\Lambda \in \mathbb{R}^{n\times n}$ is a constant, diagonal, positive-definite, controller gain matrix, $\kappa \in \mathbb{R}$ is a constant positive gain, $K_1, K_2 \in \mathbb{R}^{n\times n}$, $\Upsilon_h \in \mathbb{R}^{2n\times 2n}$ represent constant, diagonal, positive-definite, learning control and adaptation gain matrices. Saturation function is denoted by $sat_\alpha(.)$ and defined using the known, positive bounding constants given by (18):

$$sat_{\alpha_i}(\zeta_i) = \begin{cases} \alpha_i\ , \zeta_i \geq \alpha_i \\ \zeta_i\ , \quad -\alpha_i < \zeta_i < \alpha_i \\ -\alpha_i\ , \quad \zeta_i \leq \alpha_i \end{cases} \tag{25}$$

with $\forall \zeta_i \in \mathbb{R}, i = 1,2, \dots, n$. In light of (25), the following inequality will be utilized in the subsequent stability analysis:

$$(\zeta_{1i} - \zeta_{2i})^2 \geq \left(sat_{\alpha_i}(\zeta_{1i}) - sat_{\alpha_i}(\zeta_{2i})\right)^2 \quad (26)$$

where $\forall|\zeta_{1i}| \leq \alpha_i, \zeta_{1i} \in \mathbb{R}, i = 1,2, \ldots, n$.When (21) is substituted into (15), the closed-loop error system for $r_h(t)$ is obtained as:

$$D\dot{r}_h = -Cr_h - \Lambda r_h + W\widetilde{\Phi} + \tilde{\vartheta} + \xi - \kappa\delta^2(\|Z\|)r_h \quad (27)$$

where the parameter estimation error, denoted by $\widetilde{\Phi} \in \mathbb{R}^{2n}$ is defined as:

$$\widetilde{\Phi} = \Phi - \widehat{\Phi} \quad (28)$$

and $\tilde{\vartheta} \in \mathbb{R}^n$ is the learning estimation error:

$$\tilde{\vartheta} = \vartheta - \hat{\vartheta} \quad (29)$$

In light of (13), (16), (18) and (25), the following is derived:

$$\vartheta(t) = sat_\alpha\big(\vartheta(t)\big) = sat_\alpha\big(\vartheta(t-T)\big) \quad (30)$$

$\tilde{\vartheta}$ is obtained by substituting (22) and (30) into (29):

$$\tilde{\vartheta} = sat_\alpha\big(\vartheta(t-T)\big) - sat_\alpha\left(\hat{\vartheta}(t-T)\right) - K_1 r_h - K_2\ddot{e} \quad (31)$$

3.2. Closed-Loop Stability Analysis

Theorem 1: The proposed hybrid controller developed in (21)-(24) can asymptotically drive the position error to zero, i.e.;

$$\lim_{t\to\infty} e(t) = 0 \quad (32)$$

where the controller gains $\Gamma_1, \Gamma_2, \Lambda, K_1$and K_2 givenin (14), (21) and (22) are selected to satisfy the following sufficient condition

$$min\left(\|\Gamma_1\|, \left\|\Lambda + \frac{K_1^T K_1}{2}\right\|, \left\|\frac{K_2^T K_2}{2}\right\|\right) > \frac{1}{4\kappa} \quad (33)$$

where $\|.\|$ is the 2-norm of a matrix, and there exists a first-order differentiable, positive definite function $V_1(e, \dot{e}, \ddot{e}, t) \in \mathbb{R}$ such that

$$\dot{V}_1 \leq -e^T \Gamma_1 e + r_h^T K_1^T K_2 \ddot{e} + \tilde{\vartheta}^T K_2 \ddot{e} + r_h^T (K_1 - I)\tilde{\vartheta} \tag{34}$$

where $I \in \mathbb{R}^{n\times n}$ is the identity matrix.

Proof: To prove the conclusion of Theorem 1, a Lyapunov function candidate, $V(t)$ is defined as

$$V = V_1 + \frac{r_h^T D r_h}{2} + \frac{\tilde{\Phi}^T \Upsilon_h^{-1} \tilde{\Phi}}{2} + \frac{1}{2}\int_{t-T}^{t} \left(sat_\alpha \vartheta(\phi) - sat_\alpha \hat{\vartheta}(\phi)\right)^T \left(sat_\alpha \vartheta(\phi) - sat_\alpha \hat{\vartheta}(\phi)\right) d\phi \tag{35}$$

Taking the time derivative of (35), and using the Leibniz's Rule provided in the appendix and the assumption given in (34) yields

$$\begin{aligned}\dot{V} \leq & -e^T \Gamma_1 e + r_h^T K_1^T K_2 \ddot{e} + \tilde{\vartheta}^T K_2 \ddot{e} + r_h^T (K_1 - I)\tilde{\vartheta} + r_h^T D \dot{r}_h + \frac{r_h^T \dot{D} r_h}{2} - \tilde{\Phi}^T W^T r_h \\ & + \frac{1}{2}\Big[\left(sat_\alpha \vartheta(t) - sat_\alpha \hat{\vartheta}(t)\right)^T \left(sat_\alpha \vartheta(t) - sat_\alpha \hat{\vartheta}(t)\right) \\ & - \left(sat_\alpha \vartheta(t-T) - sat_\alpha \hat{\vartheta}(t-T)\right)^T \left(sat_\alpha \vartheta(t-T) - sat_\alpha \hat{\vartheta}(t-T)\right)\Big]\end{aligned} \tag{36}$$

Using (8) and (27), the following is obtained:

$$\begin{aligned}\dot{V} \leq & -e^T \Gamma_1 e + r_h^T K_1^T K_2 \ddot{e} + \tilde{\vartheta}^T K_2 \ddot{e} + r_h^T K_1 \tilde{\vartheta} - r_h^T \Lambda r_h + r_h^T \xi - r_h^T \kappa \delta^2 r_h \\ & + \frac{1}{2}\left\|\left(sat_\alpha \vartheta(t) - sat_\alpha \hat{\vartheta}(t)\right)\right\|^2 \\ & - \frac{1}{2}\left(sat_\alpha \vartheta(t-T) - sat_\alpha \hat{\vartheta}(t-T)\right)^T \left(sat_\alpha \vartheta(t-T) - sat_\alpha \hat{\vartheta}(t-T)\right)\end{aligned} \tag{37}$$

The expression given in (37) can be rewritten based on (19) and (31) as follows:

$$\dot{V} \leq -e^T \Gamma_1 e + r_h^T K_1^T K_2 \ddot{e} + \tilde{\vartheta}^T K_2 \ddot{e} + r_h^T K_1 \tilde{\vartheta} - r_h^T \Lambda r_h + [\delta \|Z\| \|r_h\| - \kappa \delta^2 \|r_h\|^2]$$
$$+ \frac{1}{2} \left\| \left(sat_\alpha \vartheta(t) - sat_\alpha \hat{\vartheta}(t) \right) \right\|^2$$
$$- \frac{1}{2} \left(\tilde{\vartheta} + K_1 r_h + K_2 \ddot{e} \right)^T \left(\tilde{\vartheta} + K_1 r_h + K_2 \ddot{e} \right) \tag{38}$$

By expanding the last line of (38), and performing cancellations, one obtains

$$\dot{V} \leq -e^T \Gamma_1 e - r_h^T \left(\Lambda + \frac{K_1^T K_1}{2} \right) r_h - \ddot{e}^T \frac{K_2^T K_2}{2} \ddot{e}$$
$$+ \frac{1}{2} \left[\left\| sat_\alpha \vartheta(t) - sat_\alpha \hat{\vartheta}(t) \right\|^2 - \left\| \vartheta(t) - \hat{\vartheta}(t) \right\|^2 \right]$$
$$+ [\delta \|Z\| \|r_h\| - \kappa \delta^2 \|r_h\|^2] \tag{39}$$

By exploiting the property given in (26), completing the square on the bracketed term in the last line of (39), and using (20), (39) can be simplified as:

$$\dot{V} \leq -\left[min \left(\|\Gamma_1\|, \left\| \Lambda + \frac{K_1^T K_1}{2} \right\|, \left\| \frac{K_2^T K_2}{2} \right\| \right) - \frac{1}{4\kappa} \right] \|Z\|^2 \tag{40}$$

where $\|.\|$ is the 2-norm of a matrix.

Signal Chasing: When (33) is satisfied, it follows that $V(t) \in \mathcal{L}_\infty$ based on (35) and (40). Since the signals in $V(t)$ must remain bounded, it can be concluded that $r_h(t), \tilde{\Phi}(t) \in \mathcal{L}_\infty$. If the sufficient condition in (33) is satisfied, then in light of Lemma 1 given in the appendix, it follows that

$$\|Z\| = \sqrt{\int_0^\infty Z^2(t)\, dt} < \infty \tag{41}$$

which in turn implies that $Z(t) \in \mathcal{L}_2$.

Since $\|Z\|_\infty = sup|Z(t)|$, in light of (41) it follows that $\|Z\|_\infty \leq \|Z\| < \infty$, and thus $Z(t) \in \mathcal{L}_\infty$. Therefore, $Z(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty$.

The definition of $Z(t)$ given in (20) implies that $e(t), r_h(t), \ddot{e}(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty$. Since $r_h(t) \in \mathcal{L}_\infty$, it follows from the definition of $r_h(t)$ in (14) that $\dot{e}(t) \in \mathcal{L}_\infty$. Since

$e(t), \dot{e}(t) \in \mathcal{L}_\infty$ and $e(t) \in \mathcal{L}_2$, Barbalat's Lemma in the appendix implies (32) in Theorem 1.

In light of (12) and (13), and using the boundedness of $e(t), \dot{e}(t), \ddot{e}(t)$, it follows that $q(t), \dot{q}(t), \ddot{q}(t) \in \mathcal{L}_\infty$. By exploiting the fact that the learning feedforward term given in (22) is composed of a saturation function, and $r(t), \ddot{e}(t) \in \mathcal{L}_\infty$, it can be concluded that $\hat{\vartheta}(t) \in \mathcal{L}_\infty$. Since Φ represents bounded static friction coefficients and $\tilde{\Phi}(t) \in \mathcal{L}_\infty$, it follows from (28) that $\hat{\Phi}(t) \in \mathcal{L}_\infty$. It is observed that $\tau_a(t) \in \mathcal{L}_\infty$ using $\dot{q}(t), \ddot{q}(t), \hat{\Phi}(t) \in \mathcal{L}_\infty$ in (23). Finally, $\tau_a(t), \hat{\vartheta}(t),\ r_h(t) \in \mathcal{L}_\infty$ implies $\tau(t) \in \mathcal{L}_\infty$ based on (21). Therefore, all system signals remain bounded.

4. Simulation Results

The performance of the developed hybrid learning based adaptive controller given in (21)-(24) is evaluated on the pan-tilt platform and compared with the performance of the hybrid learning based adaptive controller where acceleration feedback signals are not used. The desired trajectories which are presented in Figure 3 are generated based on the following periodic functions:

$$\begin{bmatrix} q_{d_1} \\ q_{d_2} \end{bmatrix} = \begin{bmatrix} (2 + 0.2sin(t))\left(sin(sin(t))\right)\left(1 + e^{-0.6t^3}\right) \\ (1 + 0.2sin(t))\left(sin(sin(t))\right)\left(1 + e^{-0.6t^3}\right) \end{bmatrix} \quad (42)$$

with a period of $T = 6.28\,sec$ and the exponential term isused to provide a "smooth-start" to the system.

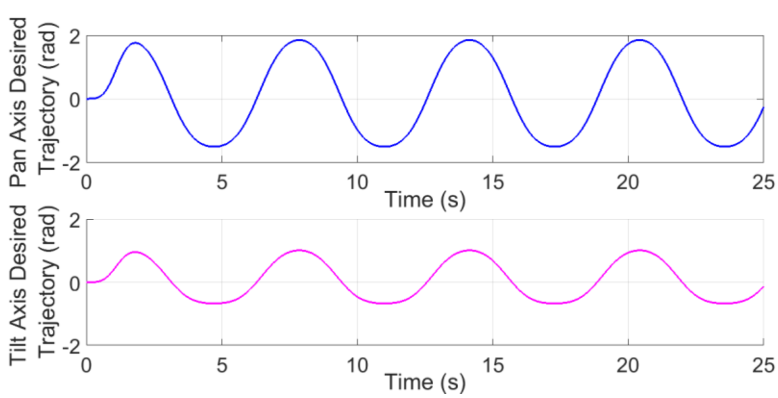

Figure 3. Desired Trajectories

The controller gains are tuned as follows:

$$\Gamma_1 = \begin{bmatrix} 20 & 0 \\ 0 & 14 \end{bmatrix}, \Gamma_2 = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}, \Lambda = \begin{bmatrix} 40 & 0 \\ 0 & 12 \end{bmatrix} \tag{43}$$

$$\mathrm{K}_1 = \begin{bmatrix} 30 & 0 \\ 0 & 10 \end{bmatrix}, \mathrm{K}_2 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, \Upsilon_h = 20 I_{4\times 4} \tag{44}$$

Position and filtered errors reduce after each period of the desired trajectory and globally asymptotically converge to zero as depicted in Figures 4 and 5. Peaks occur in the position errors due to the integration of discontinuities created by signum functions in the static frictions terms of the dynamic model given in (5).

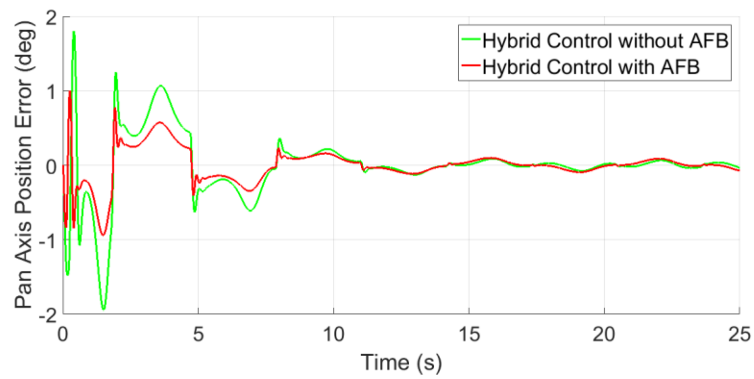

Figure 4. Pan axis position error, $e_1(t)$

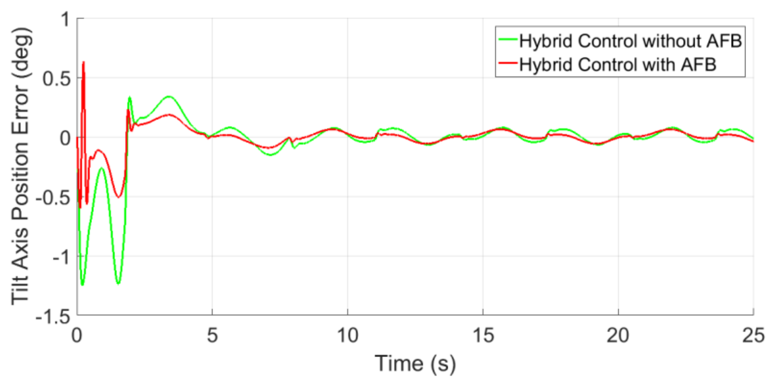

Figure 5. Tilt axis position error, $e_2(t)$

Figures 6-9 depict the torque control inputs and the learning feedforward control inputs. Due to the desired periodic trajectories, control inputs oscillate to reject the unknown periodic disturbances. The proposed controller outperforms the hybrid controller without acceleration feedback as shown in Tables 1 and 2.

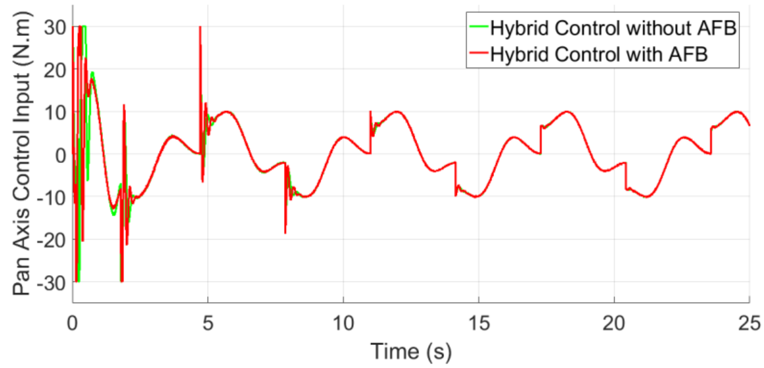

Figure 6. Pan axis control input, $\tau_1(t)$

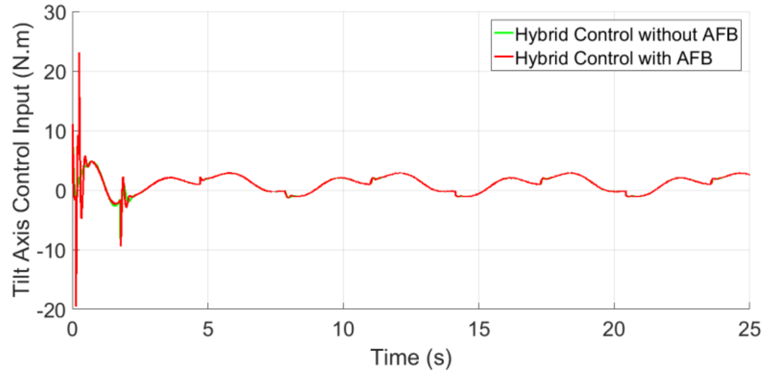

Figure 7. Tilt axis control input, $\tau_2(t)$

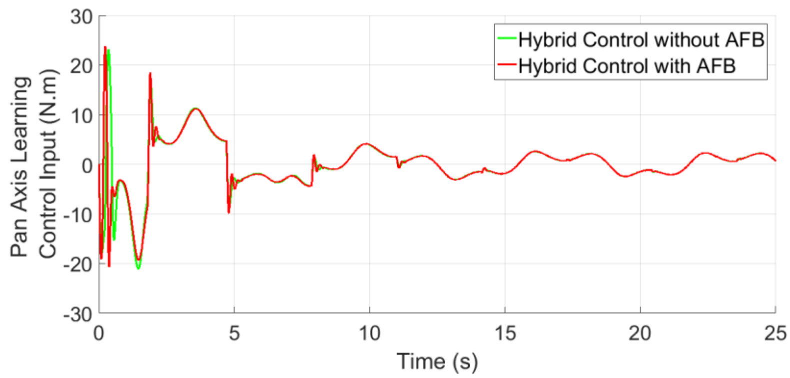

Figure 8. Pan axis learning feedforward control, $\vartheta_1(t)$

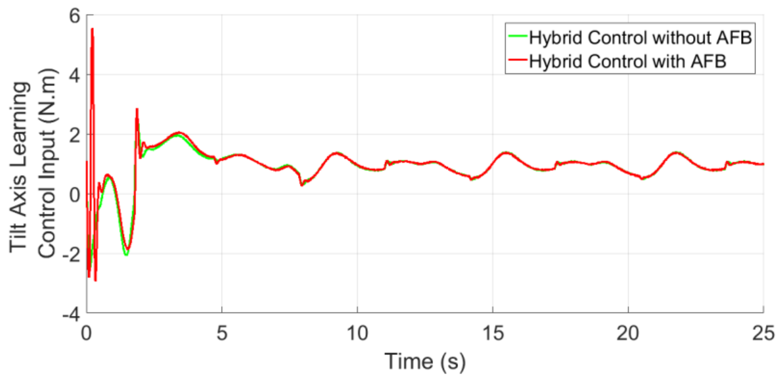

Figure 9. Tilt axis learning feedforward control, $\vartheta_2(t)$

Table 1. Pan axis performance specification

Performance Criteria	Proposed Control	Hybrid Control without AFB
Absolute Worst Case Error (deg)	1.01	1.94
RMS Position Error (deg)	0.22	0.42
RMS Control Input $(N.m)$	7.49	7.47

Table 2. Tilt axis performance specification

Performance Criteria	Proposed Control	Hybrid Control without AFB
Absolute Worst Case Error (deg)	0.63	1.24
RMS Position Error (deg)	0.10	0.23
RMS Control Input $(N.m)$	2.00	1.80

Static friction coefficients are satisfactorily estimated by the adaptive controller. Estimated values of the static friction parameters approximately converge to 3.1 $N.m$ and 0.4 $N.m$as depicted in Figures 10 and 11. Motor moment of inertias are estimated as 2.7 $kg.m^2$ and 1.5 $kg.m^2$ as shown in Figures 12 and 13.

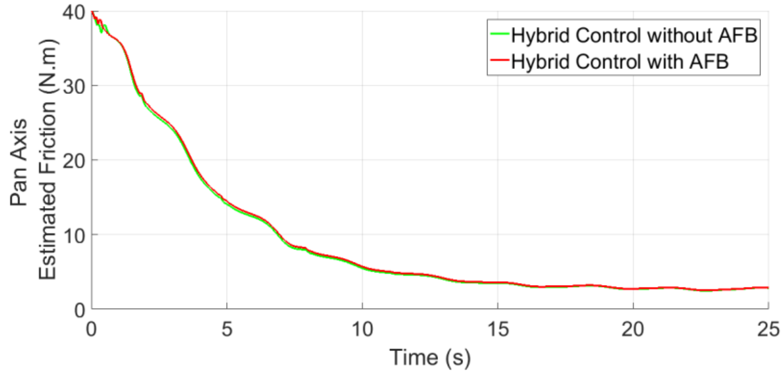

Figure 10. Pan axis estimated friction parameter, $\hat{f}_{s_1}$

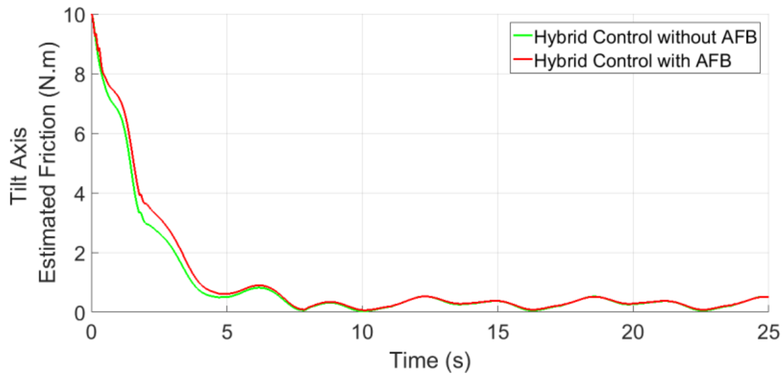

Figure 11. Tilt axis estimated friction parameter, $\hat{f}_{s_2}$

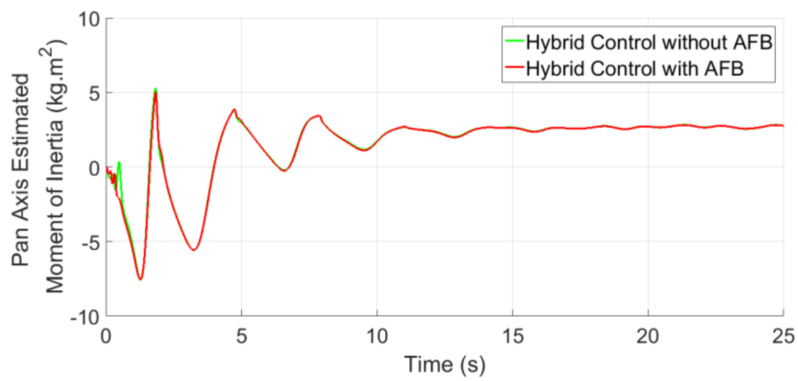

Figure 12. Pan axis estimated friction parameter, $\hat{J}_{m_1}$

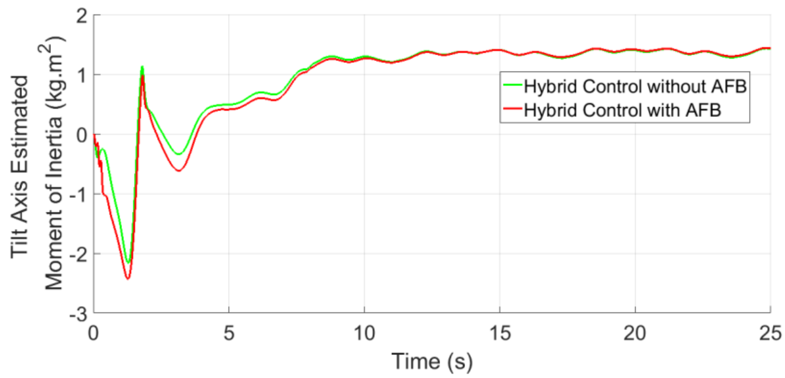

Figure 13. Pan axis estimated friction parameter, $\hat{J}_{m_2}$

5. Conclusion

A new hybrid control method is developed for the trajectory tracking control of robot manipulators where acceleration based learning and adaptive controllers are designed and combined. Utilization of acceleration feedback in the learning control provides more robustness to the system against unknown periodic disturbances with a known period. Adaptive controller, on the other hand, compensates for the uncertainties in the actuator moment of inertias and the static friction parameters. For closed-loop stability analysis, a filtered error is defined where integral of the position error is also included. A cascaded high gain observer is designed to estimate reliable position, velocity and acceleration signals from noisy encoder measurements. Lyapunov based stability analysis show that all system signals remain bounded, and the proposed controller ensures global asymptotic position tracking for a n-rigid link manipulator. The performance of the proposed hybrid controller is tested on a high fidelity simulation model of a pan-tilt platform and it has been found as quite satisfactory.

6. Appendix: Leibniz's Rule and Some Important Lemmas

Leibniz's Rule

Let $f(x,t)$ be a function such that both $f(x,t)$ and its partial derivative $f_x(x,t)$ are continuous in x and t in some region of the (x,t)-plane, including $u(x) \leq t \leq v(x)$ and $x_0 \leq x \leq x_1$. Assuming that the functions $u(x)$ and $v(x)$ are both continuous and have continuous derivatives for $x_0 \leq x \leq x_1$, it follows that

$$\frac{d}{dx}\int_{u(x)}^{v(x)} f(x,t)\,dt = f\big(v(x)\big)\frac{dv}{dx} - f\big(u(x)\big)\frac{du}{dx} + \int_{u(x)}^{v(x)} \frac{\partial}{\partial x} f(x,t)dt \tag{45}$$

Lemma 1

Given a nonnegative function denoted by $V(t) \in \mathbb{R}$ as follows Dixon et al. (2003):

$$V = \frac{1}{2}x^2 \tag{46}$$

with the following time derivative

$$\dot{V} = -k_1 x^2 \tag{47}$$

then $x(t) \in \mathbb{R}$ is square integrable, i.e. $V(t) \in \mathcal{L}_2$.

Proof: If both sides of (47) is integrated, then the following is obtained

$$-\int_0^\infty \dot{V}(t)\,dt = k_1 \int_0^\infty x^2(t)\,dt \tag{48}$$

and

$$k_1 \int_0^\infty x^2(t)\,dt = V(0) - V(\infty) \tag{49}$$

It is known that $V(0) \geq V(\infty) \geq 0$ based on (46) and (47). Therefore, it follows that

$$k_1 \int_0^\infty x^2(t)\,dt = V(0) - V(\infty) \leq V(0) < \infty \tag{50}$$

$$\sqrt{\int_0^\infty x^2(t)\,dt} \leq \sqrt{\frac{V(0)}{k_1}} < \infty \tag{51}$$

Thus, one concludes that $x(t) \in \mathcal{L}_2$.

Barbalat's Lemma

Consider a function $f(t):\ \mathbb{R}_+ \longrightarrow R$. If $f(t), \dot{f}(t) \in \mathcal{L}_\infty$, and $f(t) \in \mathcal{L}_2$, then

$$\lim_{t\to\infty} f(t) = 0 \tag{52}$$

This lemma is often referred to as Barbalat's Lemma Khalil (2002).

7. References

Dixon W. E., Zergeroglu E, Dawson D. M., Costic B. T. (2002) Repetitive learning control: A lyapunov-based approach, IEEE Transactions on Systems, Man, and Cybernetics, Vol. 32, No.4, pp. 538-545, 2002.

Dixon W. E., Behal A, Dawson D. M., Nagarkatti S. P. (2003) Nonlinear Control of Engineering Systems: A Lyapunov-BasedApproach, Birkhauser, 2003.

Ngo T., Wang Y., Mai T. L., Ge J., Nguyen M. H., Wei S.N. (2012) An adaptive iterative learning control for robot manipulator in task space, International Journal of Computers, Communications and Control, Vol. 7, No. 3, pp. 518-529, 2012.

Benosman M. (2014) Learning-based adaptive control for nonlinear systems, European Control Conference, 2014.

Wang Y. C. and Chien C. J. (2015) An observer-based adaptive iterative learning control using filtered-FNN design for robotic systems, Advances in Mechanical Engineering, Vol. 2014, No.471418, 2015.

Ouyang P. R. and Zhang W. J. (2004) Development of an adaptive learning PD control for robotic system applications, IEEE International Conference on Control Applications, 2004.

Del Vecchio D., Marino R., Tomei P. (2003) Adaptive learning control for feedback linearizable systems, European Journal of Control, Vol. 9, No. 5, pp. 483-496, 2003.

Liuzzo S. and Tomei P. (2008) A global adaptive learning controlfor robotic manipulators, Automatica, Vol. 44, No. 5, pp. 1379-1384, 2008.

Delibasi A., Zergeroglu E., Kucukdemiral I. B., Cansever G., Adaptive self tuning control of robot manipulators with periodic disturbance estimation, International Journal of Robotics and Automation, Vol. 25, No. 1, pp. 48-56, 2010.

Wang Y. C., Chien C. J., Chuang C. N. (2013) Backstepping adaptive iterative learning control for robotic systems, Applied Mechanics and Materials, Vol. 284-287, No.3, pp. 1759-1763, 2013.

Khalil H. K., Praly L. (2014) High-gain observers in nonlinear feedback control, International Journal of Robust and Nonlinear Control, Vol. 24, No. 6, pp. 993-1015, 2014.

Tao G and Ma X (1999) Backlash compensation for multivariable nonlinear systems with actuator dynamics, IEEE Conferenceon Decision and Control, pp. 3382-3387.

Khalil H. K. (2002) Nonlinear Systems, Prentice-Hall, New Jersey,2002.