EXCESS CAPACITY IN A MIXED OLIGOPOLY

by

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ABSTRACT

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Keywords: Mixed Oligopoly; Capacity Choice; Excess Capacity; Price Competition; Parking.

I consider a two-stage mixed duopoly game where a public firm proposes a capacity allocation and a private firm has the option to either accept the allocation and enter the market or reject it and not enter the market in the first stage and the firms engage in a price competition in the second stage. The private firm aims to maximize its profit whereas the public firm aims to maximize social welfare. I show that while the private firm operates at full capacity, the public firm bears excess capacity in the equilibrium even though capacity investment is costly. The theoretical model is highly relevant to the parking literature where the public firm represents on-street parking and the private firm represents a private parking garage. My finding provides alternative rationale for the advocated vacancy rate at on-street parking in the parking literature.

ÖZET

KARMA OLIGOPOLDE ATIL KAPASİTE

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Anahtar Kelimeler: Karma Oligopol; Kapasite Seçimi; Atıl Kapasite; Fiyat Rekabeti; Parklanma.

Bu çalışmada, ilk aşamada kamu firmasının her iki firma için bir kapasite alokasyonu yapıp özel firmanın kendisine teklif edilen kapasiteyi kabul ettiği veya redderek marketten çıktığı, ikinci aşamada ise firmaların fiyat rekabetine girdikleri iki aşamalı karma bir oligopol piyasası modeli ele alınmaktadır. Özel firma karını maksimize etmeyi amaçlarken, kamu firmasının amacı toplumsal refahı maksimize etmektir. Bu çalışmada kapasite yatırımı maliyetli olmasına rağmen, Nash Dengesi'nde özel firma tam kapasitede çalışırken, kamu firmasının atıl kapasiteye sahip olduğu gösterilmektedir. Çalışmada kullanılan kuramsal model, kamu firmasının yolüstü park yerine ve özel firmanın ise özel otoparka karşılık geldiği parklanma yazınıyla yakından alakalıdır. Bu çalışmada elde edilen sonuç, park literatüründe savunulan yolüstü park yerinde boş yer tezine alternatif bir açıklama sunmaktadır.

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1. Introduction

Analyzing capacity choice of firms in oligopolies has been drawing increasing attention throughout the years. Kreps and Scheinkman (1983) show the equivalence of a single-stage Cournot duopoly and a two-stage game in which firms first choose capacities to later engage in a price competition; their finding indicate that none of the firms choose excess capacity in the equilibrium. Yet, excess capacity is commonly observed in the market. In pure oligopoly markets, existence of excess capacity has been justified by modifying the originial model to account for additional factors. Benoit and Krishna (1987) show that firms generally have excess capacity in equilibrium as a punishment device when the price game is repeated infinitely. Deneckere and Kovenock (1996) allow for efficiency differences between firms and reach the conclusion that low cost firm can build excess capacity to place its less efficient rival out of the market. Allen et al. (2000) show that incumbent firms use capacity as a barrier to entry and some of this capacity can be left unused at the equilibrium. The literature on capacity choice of firms in a mixed oligopoloy environment, on the other hand, is not as rich as the pure oligopoly market. The focus of this paper is to analyze the capacity choice in a mixed duopoly under price competition.

There are contradicting results on the capacity choice of public firms in a mixed duopoly under Cournot competition. Lu and Poddar (2005) and Nishimori and Ogawa (2004) assume that firms have a U-shaped cost function that punishes production level deviations from capacity and find that public firm chooses under capacity in the equilibrium. Wen and Sasaki (2001), on the other hand, reach a contradicting conclusion in a mixed oligopoly under repeated Cournot competition in which cost function is not assumed to have a U-shape. Capacity choice in a mixed oligopoly under Bertrand competition has been studied less thoroughly. Barcena-Ruiz and Garzon (2007) show that public firm chooses excess capacity when goods are substitutes in a mixed oligopoly under price competition where firms bear a U-shaped cost function. Merrill and Schneider (1966) show that when one of the many firms that are operating below capacity under price competition is replaced by a social welfare maximizer, private firms change their prices to fully utilize their capacity, while the public firm operates at less than full capacity at equilibrium; this model does not make additional assumptions on the cost function, but it examines the production level choices of firms with exogenous capacities rather than examining the capacity choices of firms.

I aim to analyze the capacity choice of a public firm in a mixed duopoly under price competition using a simple model without complex assumptions and show that the public firm chooses excess capacity in the equilibrium. I consider a two-stage game in which capacity choice is followed by the pricing decision. In the first stage, the public firm chooses its capacity and proposes a capacity for the private firm; the private firm can accept this capacity and bear the associated capacity cost to get the equilibrium profit or reject it to make zero profit by not entering the market. In the second stage, the firms engage in a price competition. Even though the capacity investment is costly, the public firm chooses excess capacity in the equilibrium, which serves as a device to achieve higher social welfare through realization of lower prices in the equilibrium.

The theoretical model used throughout this paper can potentially be applied to the parking literature. According to Shoup (2006), cruising for parking accounts for between 8 to 74 percent of the traffic in downtown areas, which can be eliminated by adjusting the price of parking to ensure the existence of one vacant parking spot per block at on-street parking. Government intervention to increase the price of a public service can be problematic due to political concerns. This model shows that excess capacity can result as a market outcome, rather than a government intervention, when the public firm has the power to determine the capacity levels of itself and the private firm. When planning a new urban area, authorities can allocate on-street and private parking capacities in such a way that Shoup (2006)'s recommendation of always having a vacant parking spot per block can be achieved as a Nash Equilibrium of the price competition between the public and private firm.

2. The Model

I consider a mixed duopoly market consisting of a profit-maximizing private firm and a social welfare-maximizing public firm that are producing homogenous goods. The demand function $D : [0,1] \to [0,1]$ is

$$D(p) = 1 - p. \tag{1}$$

Firms have the same efficiency; for simplicity, I assume that production costs are zero for both firms up to their capacities. Capacity is costly to be installed and firms cannot produce more than their capacities. I assume the capacity cost function $c : [0,\infty) \to [0,\infty)$ to be of the form

$$c(x) = (x^2)/k, \tag{2}$$

where x denotes the capacity, while k is a positive constant and taken as $k = 10$. Capacities of the firms, $x_i \in [0,1]$, are discrete with 0.01 increments, while their prices, $p_i \in [0,1]$, are continuous.

Firms engage in a two-stage competition. In the first stage, the public firm chooses x_1 and x_2, capacities of the private firm and the public firm, respectively. The private firm can accept or reject the capacity proposed by the public firm, x_1. The private firm gets the equilibrium payoff if it accepts the offer and bears the associated capacity cost, $c(x_1)$, whereas it makes zero profit if it rejects the offer and doesn't enter the market. Note that in case the private firm doesn't accept the public firm's capacity allocation in the first stage, its capacity equals zero in the second stage of the game. In the second stage, firms choose their prices, p_1 and p_2, and engage in a Bertrand competition. If firm i charges a price lower than the price of firm j, it faces the whole market demand. Since firms are capacity constrained, low price firm's sales are either equal to $D(p_i)$ or x_i, whichever is smaller in this case. If prices of two firms are equal, then firms share the market demand up to their capacities; any unsatisfied demand due to a firm's insufficient capacity is transferred to the other firm if its capacity permits. Finally, if firm i charges a price higher than the price of firm j, it faces the residual demand in case residual demand is non-negative. Formally, sales of each firm, z_i, is

$$z_i = \begin{cases} \min\{x_i, 1-p_i\} & p_i < p_j \\ \min\{x_i, \max\{\frac{1-p_i}{2}, 1-x_j-p_i\}\} & p_i = p_j \\ \min\{x_i, \max\{0, 1-x_j-p_i\}\} & p_i > p_j \end{cases} . \tag{3}$$

The private firm is trying to maximize its profit, π, which is equal to its revenue minus its capacity cost,

$$\pi = p_1 z_1 - c(x_1), \tag{4}$$

while the public firm is trying to maximize social welfare, SW, which is calculated by substracting total capacity costs incurred from total surplus,

$$SW = \int_0^{z_1+z_2} (1-z)\,\mathrm{d}z - c(x_1) - c(x_2). \tag{5}$$

In the second stage of the game, each firm selects a price to maximize its corresponding objective function given capacities of both firms. Following Cremer et al. (1989), I assume that the public firm has the power to enforce the equilibrium with higher social welfare in case there exists multiple Nash Equilibria in the second stage. In the first stage, the public firm uses backward induction to choose the capacity levels that yield the highest social welfare.

3. Pricing Game

In the second stage of the game, capacities are known by both firms. Given capacities, the private firm chooses the price that maximizes its profit, while the public firm chooses the price that maximizes social welfare. Section 3.1 calculates the possible best responses of the firms under different scenarios and Section 3.2 shows the best responses that coincide.

3.1. Best Response Analysis

The firms' best response functions in general form are

$$BR_1(p_2,x_1,x_2) = \underset{p_1}{\operatorname{argmax}}(\pi | p_2,x_1,x_2) \tag{6}$$

$$BR_2(p_1,x_1,x_2) = \underset{p_2}{\operatorname{argmax}}(SW | p_1,x_1,x_2), \tag{7}$$

where BR_1 (BR_2) is the price that maximizes private firm's profit (social welfare) given capacities, x_1 and x_2, and the public (private) firm's price, p_2 (p_1).

The private firm's best response function can be written as

$$BR_1(p_2,x_1,x_2) = \begin{cases} p^R & p_2 < p^u \\ p_2 & p^u \leq p_2 \leq p^* \\ \varnothing & \max\{p^u, p^*\} < p_2 \leq p^M \\ p^M & p_2 > p^M \end{cases}, \tag{8}$$

where $p^R = \underset{p_1}{\operatorname{argmax}}(\min\{x_1, \max\{0, 1-x_2-p_1\}\} \times p_1)$ is the price that maximizes the profit when the private firm faces residual-demand, p^u is derived from $\min\{x_1, 1-p^u\} \times p^u = \min\{x_1, \max\{0, 1-x_2-p^R\}\} \times p^R$ and is the threshold price such that when $p_2 > p^u$, private firm is better off by undercutting the public firm, p^* is derived from $(1-p^*)/2 = x_1$ and is the threshold price such that when $p_1 \leq p_2 \leq p^*$, the private firm's sales is bounded by its capacity, and $p^M = \underset{p_1}{\operatorname{argmax}}(\min\{x_1, 1-p_1\} \times p_1)$ is the threshold price such that when $p_2 > p^M$, the private firm acts as a monopoly.

The public firm's best response correspondence becomes

$$BR_2(p_1, x_1, x_2) = \begin{cases} [0, 1-x_1-x_2] & p_1 \leq 1-x_1-x_2 \\ [0, p_1] & 1-x_1-x_2 < p_1 < 1-\max\{x_1, x_2\} \\ [0, 1] & 1-x_1 \leq p_1 \leq 1-x_2 \\ [0, 1-x_2] & p_1 > 1-x_2 \end{cases} . \quad (9)$$

Section 3.1.1 examines the best response function of the private firm for different prices charged by the public firm and Section 3.1.2 examines the best response correspondence of the public firm for different prices charged by the private firm.

3.1.1. Analysis of the private firm's best response

When $p_2 \leq 1-x_1-x_2$, public firm's sales is equal to its capacity no matter what p_2 is. Even when the public firm charges a price higher than the private firm's price, the minimum possible demand it can face is $1-x_1-p_2 \geq x_2$. Under this condition, the private firm faces residual demand for any $p_1 \in [0, 1]$ and its best response is to charge p^R by definition. Figure 1 shows the profit of the private firm when $p_2 \leq 1-x_1-x_2$ for different levels of p_1. The private firm's profit, π, is represented in the vertical axis, while its price, p_1, is represented in the horizontal axis. As can be seen in Figure 1, the private firm's profit is maximized when it charges $p_1 = p^R$ in this case.

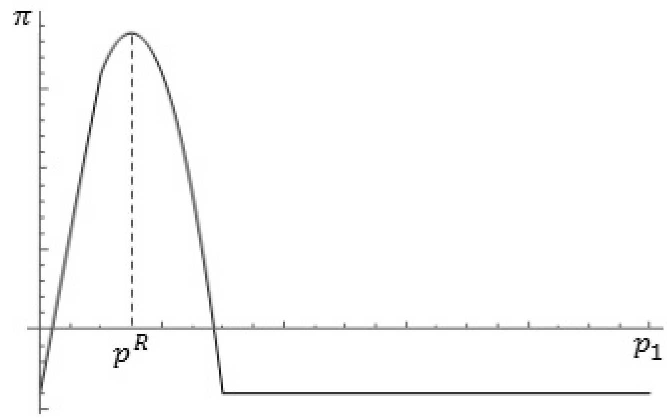

Figure 1: Profit of the private firm when $p_2 \leq 1 - x_1 - x_2$

When $1 - x_1 - x_2 < p_2 < p^u$, price of the public firm is below the p^u threshold and as shown in Figure 2, the private firm is again better off by facing residual demand rather than undercutting the public firm.

Lemma 1. When $p_2 < p^u$, the best response of the private firm is to charge p^R.

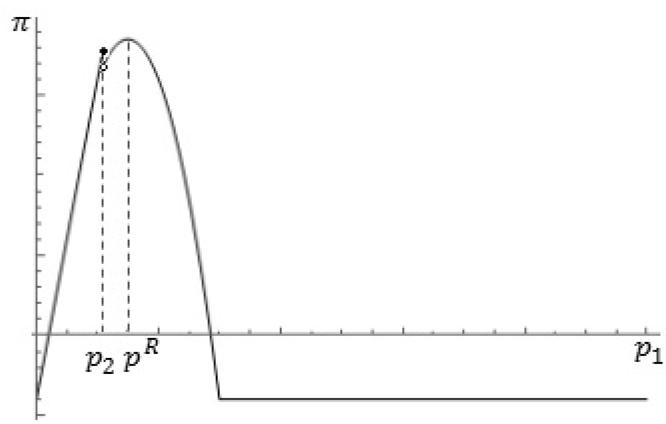

Figure 2: Profit of the private firm when $1 - x_1 - x_2 < p_2 < p^u$

When $p^u \leq p_2 \leq p^*$, the price set by the public firm is above the p^u threshold and as shown in Figure 3, the private firm has a higher profit when it undercuts the public firm's price rather than facing the residual demand. Moreover, since the price of the public firm is below the p^* threshold, the private firm's capacity is binding when $p_1 \leq p_2$. Given that it is optimal for the private firm to charge $p_1 \leq p_2$ in this case and that the sales of the private firm is equal to x_1 for any $p_1 \leq p_2 \leq p^*$, profit is maximized when $p_1 = p_2$.

Lemma 2. When $p^u \leq p_2 \leq p^*$, the best response of the private firm is to charge the public firm's price, p_2.

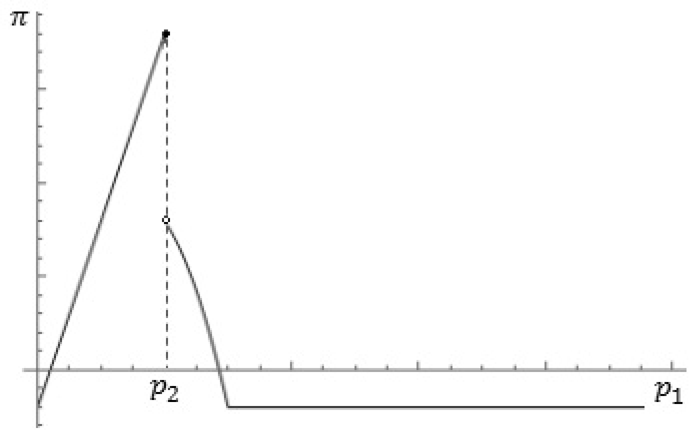

Figure 3: Profit of the private firm when $p^u \leq p_2 \leq p^*$

When $\max\{p^u, p^*\} < p_2 \leq p^M$, the price of the public firm is above the p^u threshold and as shown in Figure 4, the private firm has a higher profit when it undercuts the public firm rather than facing residual demand. Moreover, since the price of the public firm is above the p^* threshold, the private firm's capacity is not binding at $p_1 = p_2$. Given that it is optimal for the private firm to charge $p_1 \leq p_2$ in this case and that it has sufficient capacity to increase its profit by charging a price slightly less than p_2, the private firm's best response is not well-defined.

Lemma 3. When $\max\{p^u, p^*\} < p_2 \leq p^M$, the best response of the private firm is not well-defined.

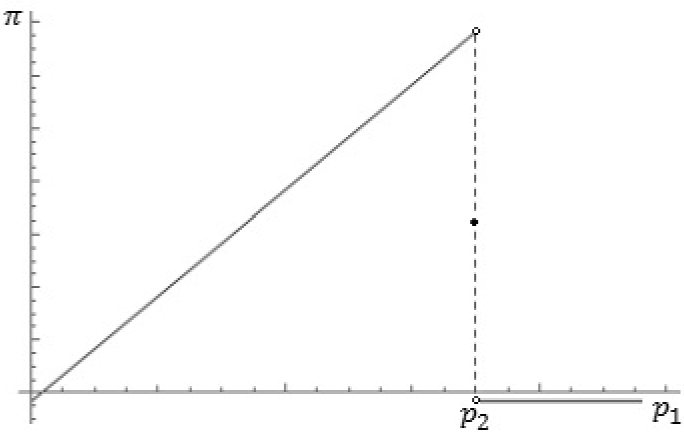

Figure 4: Profit of the private firm when $\max\{p^u, p^*\} < p_2 \leq p^M$

When $p_2 > p^M$, as seen in Figure 5, the private firm's profit is maximized when it charges the monopoly price, p^M.

Lemma 4. When $p_2 > p^M$, the best response of the private firm is to charge p^M.

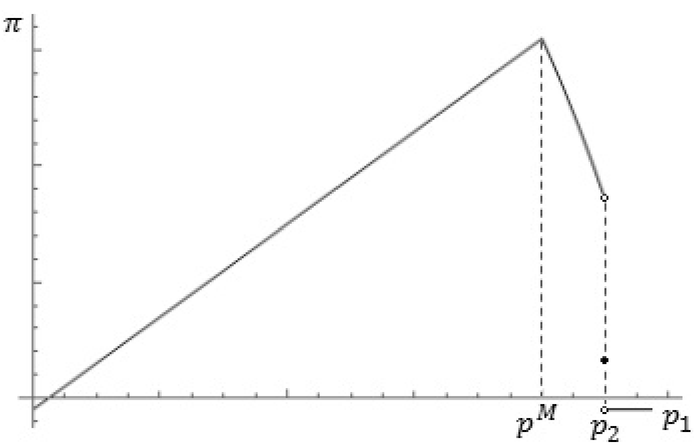

Figure 5: Profit of the private firm when $p_2 > p^M$

3.1.2. Analysis of the public firm's best response

At this stage, since capacity costs are already incurred, social welfare increases as the total output increases. Figure 6 shows social welfare when $p_1 \leq 1 - x_1 - x_2$ for different levels of p_2. Social welfare, SW, is represented in the vertical axis, while the public firm's price,

p_2, is represented in the horizontal axis. When $p_1 \leq 1 - x_1 - x_2$, charging $p_2 \leq 1 - x_1 - x_2$ results in both firms selling at their capacities, which is the maximum total output achievable.

Lemma 5: When $p_1 \leq 1 - x_1 - x_2$, the best response of the public firm is to charge any $p_2 \leq 1 - x_1 - x_2$.

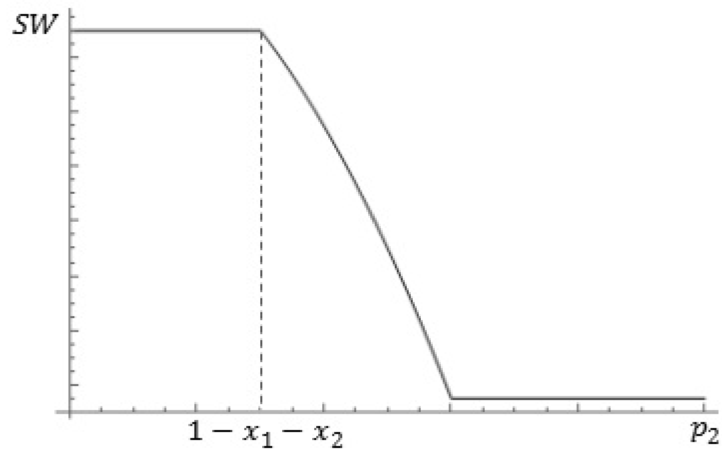

Figure 6: Social welfare when $p_1 \leq 1 - x_1 - x_2$

When $1 - x_1 - x_2 < p_1 < 1 - \max\{x_1, x_2\}$, as shown in Figure 7, maximum total output is reached when the public firm charges $p_2 \leq p_1$. When $p_2 < p_1$, the public firm faces $1 - p_2 > 1 - p_1 > x_2$, meaning that the public firm is capacity constrained for this price interval and $z_2 = x_2$. Then, the private firm faces $1 - x_2 - p_1 < x_1$, meaning that it has sufficient capacity to meet the residual demand. Thus, total output is $1 - p_1$ in this interval when $p_2 < p_1$. When $p_2 = p_1$, total demand is $1 - p_1 < x_1 + x_2$, meaning that there exists sufficient capacity in the market to meet demand; so, total output is the same as the case when $p_2 < p_1$. Total output, hence social welfare, starts to decrease as the price of the public firm exceeds the price of the private firm. When $p_2 > p_1$, the private firm faces $1 - p_1 > x_1$, so $z_1 = x_1$. Then, public firm faces $1 - x_1 - p_2 < x_2 + p_1 - p_2 < x_2$, meaning $z_2 = 1 - x_1 - p_2$. When $p_2 > p_1$, total output is less than the total output reached when $p_2 \leq p_1$ since $1 - p_2 < 1 - p_1$.

Lemma 6: When $1 - x_1 - x_2 < p_1 < 1 - \max\{x_1, x_2\}$, the best response of the public firm is to charge any $p_2 \leq p_1$.

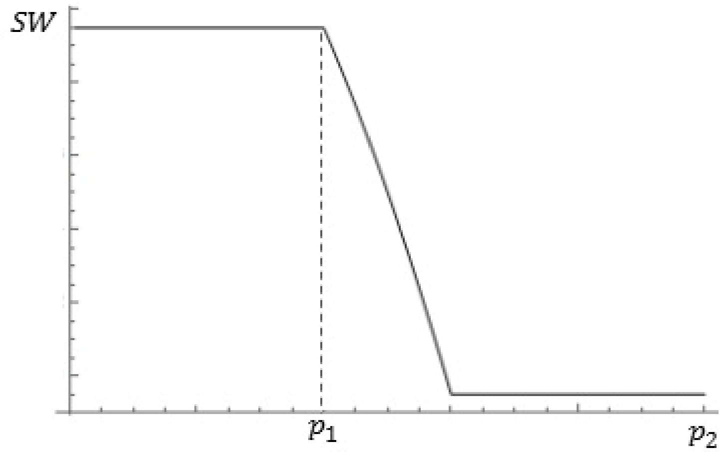

Figure 7: Social welfare when $1 - x_1 - x_2 < p_1 < 1 - \max\{x_1, x_2\}$

When $1 - x_1 < p_1 < 1 - x_2$, the public firm is indifferent between charging any $p_2 \in [0, 1]$, as shown in Figure 8. In this interval, total output is $1 - p_1$ independent from the value of p_2. When $p_1 < p_2$, the private firm faces $1 - p_1 < x_1$ and meets the whole market demand by itself. When $p_1 = p_2$, firms share the market demand; even if there is any unsatisfied demand due the public firm's capacity constraint, the private firm is guaranteed to satisfy the leftover demand since $x_1 > 1 - p_1$. Finally, when $p_1 > p_2$, the public firm faces $1 - p_2 > 1 - p_1 > x_2$ and sells $z_2 = x_2$. Then, the private firm faces $1 - x_2 - p_1 \leq 1 - p_1 < x_1$. Hence, total market output is $1 - p_1$ for all $p_2 \in [0, 1]$.

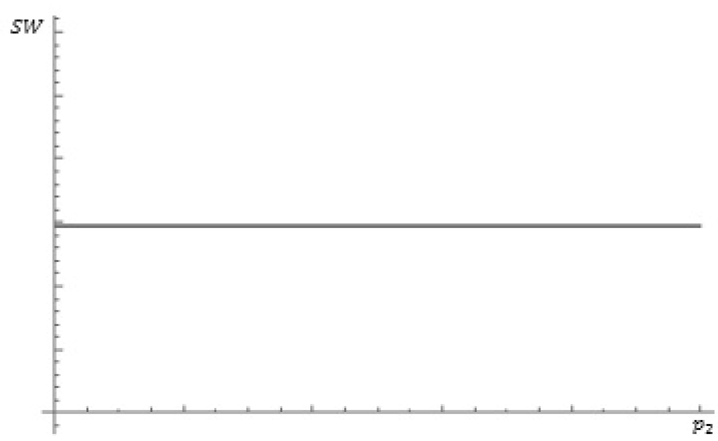

Figure 8: Social welfare when $1 - x_1 < p_1 < 1 - x_2$

Lemma 7: When $1 - x_1 < p_1 < 1 - x_2$, the best response of the public firm is to charge any $p_2 \in [0,1]$.

When $p_1 > 1 - x_2$, social welfare is maximized when $p_2 \leq 1 - x_2$, as shown in Figure 9. In this range, $p_2 \leq 1 - x_2 < p_1$, hence the public firm faces $1 - p_2 \geq x_2$, meaning that $z_2 = x_2$ and the private firm faces the residual demand. Instead, if the public firm charges $p_2 > 1 - x_2$, total market output is $z \leq 1 - \min\{p_1, p_2\} < x_2$, which is less than the sales of the public firm alone when $p_2 \leq 1 - x_2$.

Lemma 8: When $p_1 > 1 - x_2$, the best response of the public firm is to charge any $p_2 \leq 1 - x_2$.

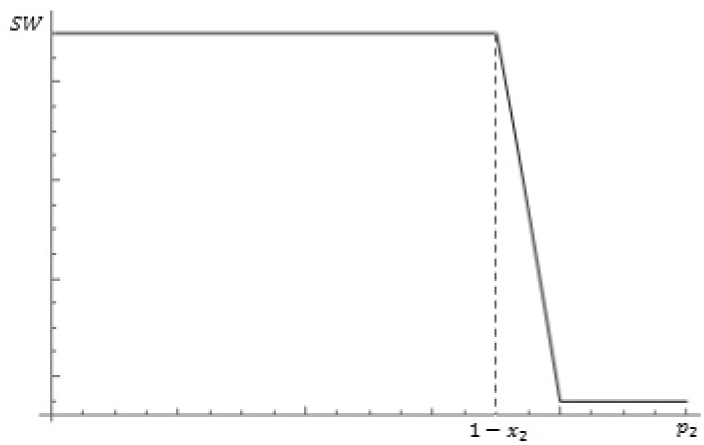

Figure 9: Social welfare when $p_1 > 1 - x_2$

3.2. Types of Nash Equilibria

In the second stage of the game, I show that three types of Nash Equilibria may emerge.

3.2.1. Residual-Maximizing Equilibrium

Residual-Maximizing Equilibrium is the equilibrium in which $p_1 = p^R$ and $p_2 \in [0, p^u]$ are best responses to each other. In the Residual Maximizing Equilibrium, the public firm always sells at capacity, while the private firm may have excess capacity. Since $p_1 = p^R$ and $p_2 \in [0, p^u]$ are always best responses to each other, this type of equilibrium always exists. The best responses of the firms in the Residual-Maximizing Equilibrium is illustrated in Figure 10.

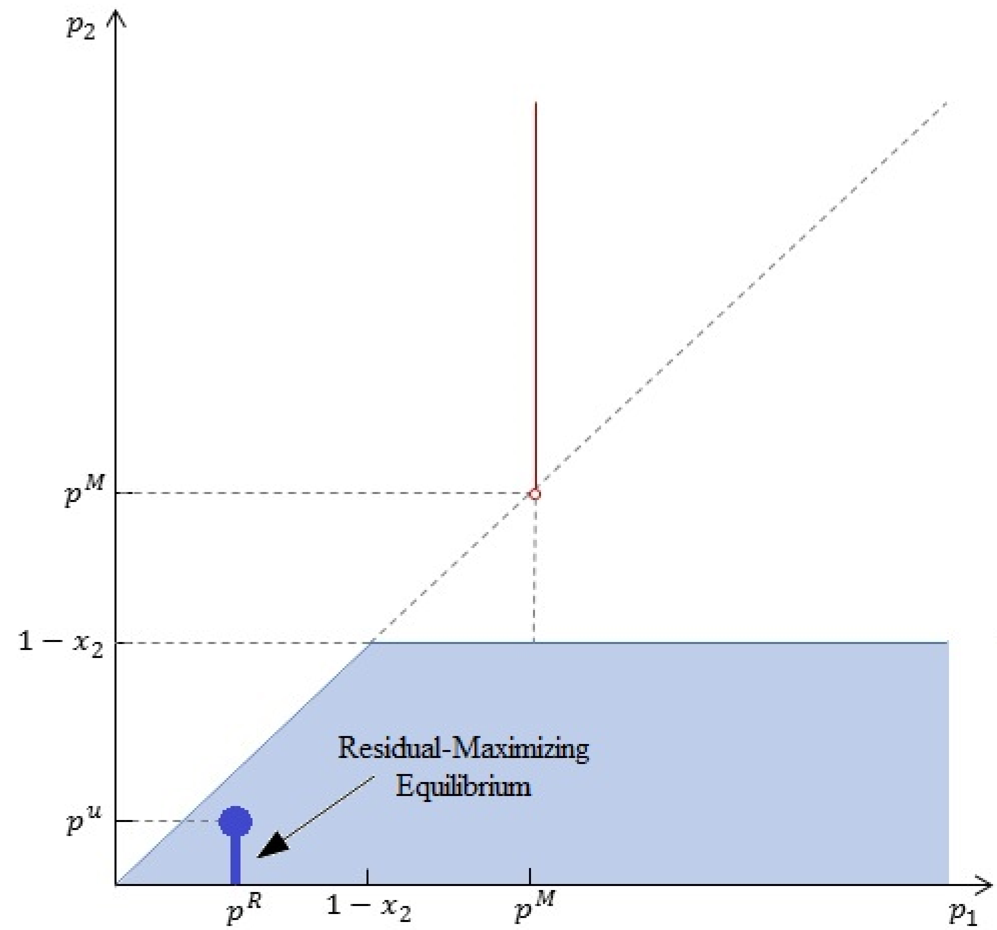

Figure 10: Best responses in the Residual-Maximizing Equilibrium

Showing that $p_1 = p^R$ is the private firm's best response to $p_2 \in [0, p^u]$ is trivial, as the private firm's best response is to charge $p_1 = p^R$ when $p_2 \leq p^u$. To show that $p_2 \in [0, p^u]$ is a best response for the public firm when $p_1 = p^R$, I will revisit Section 3.1.2 for the case $p_1 = p^R$.

If $p^R = p_1 \leq 1 - x_1 - x_2$, the public firm's best response is to charge $p_2 \in [0, 1 - x_1 - x_2]$. Definition of $p^R = \underset{p_1}{\operatorname{argmax}}(\min\{x_1, \max\{0, 1 - x_2 - p_1\}\} \times p_1)$ combined with the fact that

$x_1 > 0$ and $1 - x_2 - p_1 \geq x_1$ results in $p^R = \underset{p_1}{\text{argmax}}(x_1 \times p_1)$ in this region. Maximum possible price to satisfy the given conditions makes $p_1 = p^R = 1 - x_1 - x_2$. Then, public firm's best response is equivalent to charging $p_2 \in [0, p^R]$. By definition, $p^u \leq p^R$; thus, $p_2 \in [0, p^u]$ is a best response to $p_1 = p^R$.

If $1 - x_1 - x_2 < p_1 = p^R < 1 - \max\{x_1, x_2\}$, the best response of the public firm becomes $p_2 \in [0, p_1]$. Since $p^R = p_1 \geq p^u$, $p_2 \in [0, p^u]$ is a best response to $p_1 = p^R$.

If $1 - x_1 \leq p_1 = p^R \leq 1 - x_2$, public firm is indifferent between charging any $p_2 \in [0, 1]$; thus, $p_2 \in [0, p^u]$ is a best response to $p_1 = p^R$.

Finally, $p_1 = p^R > 1 - x_2$ is not possible. Charging $p^R > 1 - x_2$ results in the private firm making zero sales, while charging $p_1 < 1 - x_2$ results in strictly positive revenues; contradicting with the definition of $p^R = \underset{p_1}{\text{argmax}}(\min\{x_1, \max\{0, 1 - x_2 - p_1\}\} \times p_1)$.

$p_1 = p^R$ and $p_2 \in [0, p^u]$ are best responses to each other in every possible case; thus this type of Nash Equilibrium always exists.

Proposition 1. $\{p_1 = p^R,\ p_2 \in [0, p^u]\ \}$ is a Nash Equilibrium.

3.2.2. Follower Equilibrium

Follower Equilibrium is the equilibrium in which $p_1 = p_2$ and $p_2 \in [p^u, \min\{p^*, 1 - x_2\}]$ are best responses to each other. I call this equilibrium as the Follower Equilibrium, since the private firm's profit is maximized when it mimics the public firm's price in this equilibrium. The best responses of the firms in the Follower Equilibrium is shown in Figure 11. In the Follower Equilibrium, the private firm always operates at full capacity, while the public firm always has excess capacity. The excess capacity of the public firm acts as an incentive for the private firm to charge lower prices and operate at full capacity. Had the public firm not have this excess capacity, the private firm would have an incentive to deviate to a higher price; when the public firm is capacity constrained, it no longer has the strategic power to lower the sales of the private firm by charging a lower price. The follower equilibrium can only exist when $p^* \geq p^u > 0$. By the definition of p^*, this indicates that excess capacity can not exist when $x_1 \geq 0.5$.

Showing that $p_1 = p_2$ is the private firm's best response to $p^u \leq p_2 \leq p^*$ is trivial, as the private firm's best response is to charge $p_1 = p_2$ when $p^u \leq p_2 \leq p^*$. Additionally, notice that $p_2 = p_1$ is always in the public firm's best response correspondence given that $p_2 \leq 1 - x_2$, which indicates that $p_2 \in [p^u, \min\{p^*, 1 - x_2\}]$ is in the best response correspondence of the public firm for $p_1 = p_2 \in [p^u, \min\{p^*, 1-x_2\}]$. Hence, $p_1 = p_2$ and $p_2 \in [p^u, \min\{p^*, 1-x_2\}]$ are best responses to each other given that $p^* \geq p^u > 0$.

Proposition 2. $\{p_1 = p_2,\, p_2 \in [p^u, \min\{p^*, 1 - x_2\}]\ \}$ is a Nash Equilibrium.

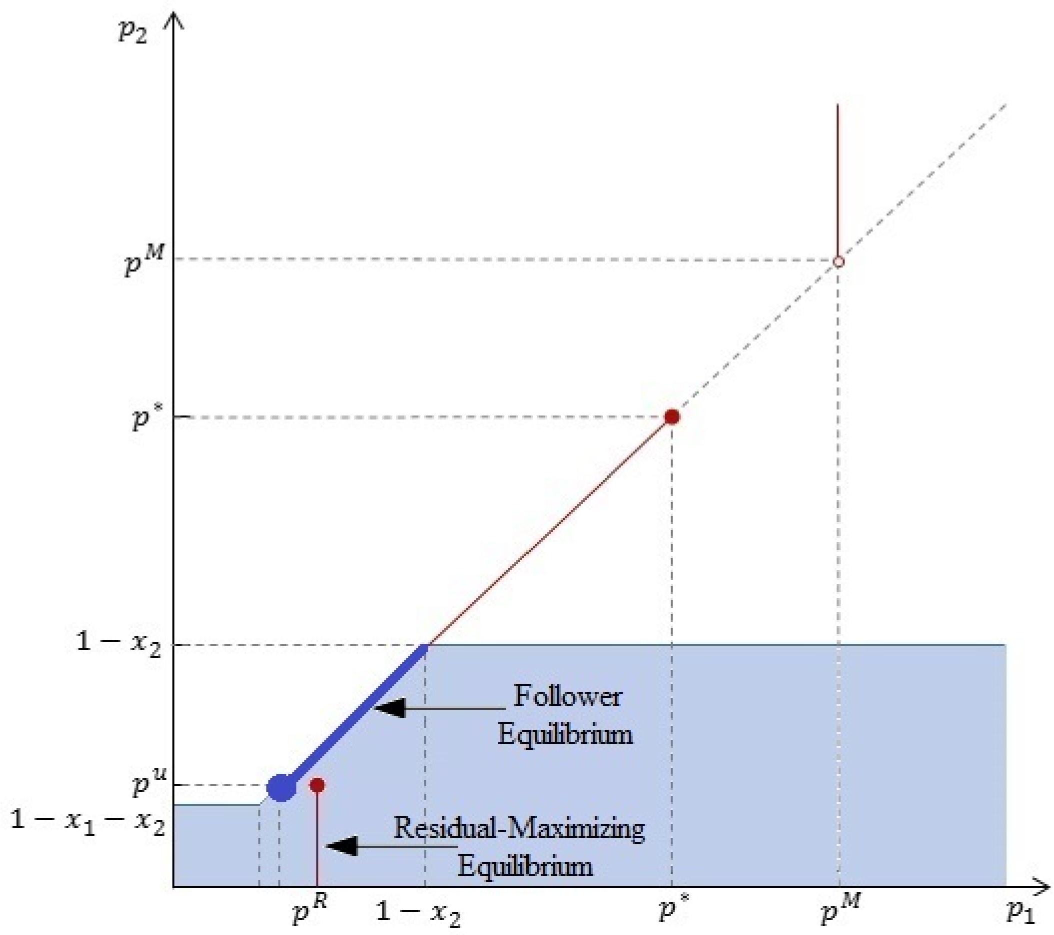

Figure 11: Best responses in the Follower Equilibrium

3.2.3. Monopoly Equilibrium

Monopoly Equilibrium is the equilibrium in which $p_1 = p^M$ and $p_2 \in (p^M, 1]$ are best responses to each other. The best responses of the firms in the Monopoly Equilibrium is illustrated in Figure 12. In the Monopoly Equlibrium, the private firm acts as a monopoly. The private firm may or may not have excess capacity, while the public firm always has excess capacity as its sales always equals to zero. This type of an equilibrium only exists if $1 - x_2 \geq p^M$. By the definition of p^M, this indicates that $x_2 \leq x_1$ must hold for the monopoly equilibrium to exist.

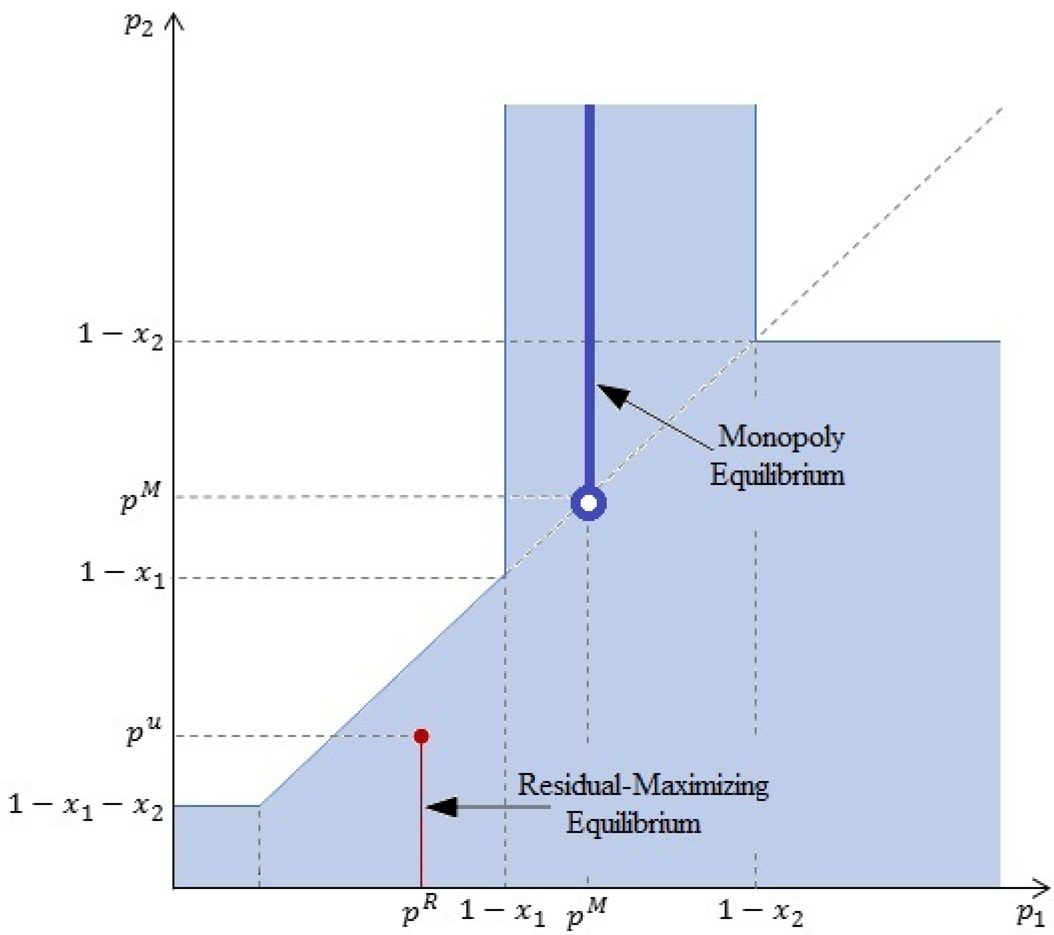

Figure 12: Best responses in the Monopoly Equilibrium

Showing that $p_1 = p^M$ is the private firm's best response to $p_2 > p^M$ is trivial, as the private firm's best response is to charge $p_1 = p^M$ when $p_2 > p^M$. To see that $p_2 \in (p^M, 1]$ is a best response to $p_1 = p^M$, consider the two possible values of p^M separately. If $x_1 > 0.5$, this indicates that $p^M = 0.5$. Additionally, $1 - x_2 \geq p^M$; hence, $1 - x_1 < p^M \leq 1 - x_2$ holds in this case. On the other hand, if $x_1 \leq 0.5$, then $p^M = 1 - x_1$. In this case, $1 - x_1 = p^M \leq 1 - x_2$. In either of the cases, the public firm is indifferent between charging any $p_2 \in [0, 1]$. So, $p_2 \in (p^M, 1]$ and $p_1 = p^M$ are best responses to each other when $1 - x_2 \geq p^M$.

Proposition 3. $\{p_1 = p^M, p_2 \in (p^M, 1]\}$ is a Nash Equilibrium.

4. Capacity Game

In the first stage of the game, the public firm chooses x_1 and x_2 to maximize social welfare. In the game where $x_i \in [0, 1]$ with 0.01 increments, the resulting game table is a 100×100 table, which is left out from the paper for spacial constraints and is available upon request. Table 1 contains the 10×10 compacted version of the original game. The capacities that maximize the social welfare, i.e. the capacities chosen by the public firm, are $x_1 = 0.35$ and $x_2 = 0.53$ when capacities are discrete with 0.01 increments.

In this stage, the private firm can either accept the capacity level allocated by the public firm or it can reject the offer and not enter the market. When the public firm offers to allocate the capacities as $x_1 = 0.35$ and $x_2 = 0.53$, the private firm accepts this capacity allocation, since it makes a positive profit when it decides to enter the market.

Table 1: Profit and social welfare for varying capacity levels

	x_2										
x_1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	(0.000, 0.000)	(0.000, 0.094)	(0.000, 0.176)	(0.000, 0.246)	(0.000, 0.304)	(0.000, 0.350)	(0.000, 0.384)	(0.000, 0.406)	(0.000, 0.416)	(0.000, 0.414)	(0.000, 0.400)
0.1	(0.089, 0.094)	(0.079, 0.178)	(0.069, 0.250)	(0.059, 0.310)	(0.049, 0.358)	(0.039, 0.394)	(0.029, 0.418)	(0.019, 0.430)	(0.009, 0.430)	(0.002, 0.418)	(-0.001, 0.399)
0.2	(0.156, 0.176)	(0.136, 0.250)	(0.116, 0.312)	(0.096, 0.362)	(0.076, 0.400)	(0.056, 0.426)	(0.036, 0.440)	(0.019, 0.441)	(0.006, 0.431)	(-0.002, 0.415)	(-0.004, 0.396)
0.3	(0.201, 0.246)	(0.171, 0.310)	(0.141, 0.362)	(0.111, 0.402)	(0.081, 0.430)	(0.054, 0.444)	(0.031, 0.446)	(0.014, 0.439)	(0.001, 0.426)	(-0.007, 0.401)	(-0.009, 0.391)
0.4	(0.224, 0.304)	(0.184, 0.358)	(0.144, 0.400)	(0.107, 0.414)	(0.074, 0.423)	(0.047, 0.447)	(0.024, 0.443)	(0.007, 0.433)	(-0.006, 0.420)	(-0.014, 0.403)	(-0.016, 0.384)
0.5	(0.225, 0.350)	(0.178, 0.373)	(0.135, 0.391)	(0.098, 0.405)	(0.065, 0.414)	(0.038, 0.419)	(0.015, 0.419)	(-0.003, 0.415)	(-0.015, 0.406)	(-0.023, 0.393)	(-0.025, 0.375)
0.6	(0.214, 0.339)	(0.167, 0.362)	(0.124, 0.380)	(0.087, 0.394)	(0.054, 0.403)	(0.027, 0.408)	(0.004, 0.408)	(-0.014, 0.404)	(-0.026, 0.382)	(-0.034, 0.382)	(-0.036, 0.364)
0.7	(0.201, 0.326)	(0.154, 0.349)	(0.111, 0.367)	(0.074, 0.381)	(0.041, 0.390)	(0.014, 0.395)	(-0.009, 0.395)	(-0.027, 0.391)	(-0.039, 0.382)	(-0.047, 0.369)	(-0.049, 0.351)
0.8	(0.186, 0.311)	(0.139, 0.334)	(0.096, 0.352)	(0.059, 0.366)	(0.026, 0.375)	(-0.002, 0.380)	(-0.024, 0.380)	(-0.042, 0.376)	(-0.054, 0.367)	(-0.062, 0.354)	(-0.064, 0.336)
0.9	(0.169, 0.294)	(0.122, 0.317)	(0.079, 0.335)	(0.042, 0.349)	(0.009, 0.358)	(-0.019, 0.363)	(-0.041, 0.363)	(-0.059, 0.359)	(-0.071, 0.350)	(-0.079, 0.337)	(-0.081, 0.319)
1	(0.150, 0.275)	(0.103, 0.298)	(0.060, 0.316)	(0.023, 0.330)	(-0.010, 0.150)	(-0.038, 0.344)	(-0.060, 0.344)	(-0.078, 0.340)	(-0.090, 0.331)	(-0.098, 0.318)	(-0.100, 0.300)

5. Results

Combining the capacity choice and the best response analysis explained in Sections 3 and 4 with the assumption that the public firm has the power to enforce the equilibrium which has the highest social welfare when multiple equilibria exists, the resulting equilibrium is a Follower Equilibrium in which $x_1 = 0.35$, $x_2 = 0.53$, $p_1 = p_2 = p^u = 0.158$, $z_1 = 0.35$, and $z_2 = 0.492$. In the equilibrium, the private firm operates at full capacity while the public firm has unused capacity. Notice that the public firm chooses to over-invest in capacity, even though capacity investment is costly. The intuition behind this is that the public firm can enforce a lower equilibrium price using this extra capacity as a threat. As the capacity of the public firm increases, profit of the private firm when it faces residual demand decreases; hence, the private firm has a stronger incentive to cooperate with the public firm when the public firm has higher capacity.

One thing to note is that the Nash Equilibrium would be different if the private firm were to choose x_1 in the first stage of the game, rather that the public firm choosing both capacities. In this scenario, there would be two Nash Equilibria; $x_1 = 0.13$, $x_2 = 0.72$, which I call Equilibrium A and $x_1 = 0.12$, $x_2 = 0.73$, which I call Equilibrium B. Both of these equilibria are Residual Maximizing Equilibria and none of the firms have excess capacity in the equilibrium. An important finding is that both of the firms are worse off when firms choose their own capacities rather than the public firm choosing capacities for both firms. In the original setting where the public firm chooses both x_1 and x_2, the equilibrium profit and social welfare are $\pi = 0.043$ and $SW = 0.447$; whereas the equilibrium profit and social welfare are $\pi = 0.018$ ($\pi = 0.017$) and $SW = 0.435$ ($SW = 0.434$) in Equilibrium A (Equilibrium B) when firms choose their own capacities. Hence, not only the public firm, but also the private firm is better off by letting the public firm choose both of the capacities in the first stage.

6. Conclusions

I consider a two-stage mixed oligopoly model in which the public firm chooses capacities on behalf of both firms in the first stage and firms engage in a price competition in the second stage. The private firm has the option to accept the capacity chosen by the public firm on behalf of the private firm and get the equilibrium profit or to reject the allocation and not enter the market. Assuming that the public firm has the power to enforce the equilibrium with the higher social welfare in case there are multiple equilibria in any stage of the game, the private firm accepts the capacity allocation and operates at full capacity, while the public firm has excess capacity in the resulting equilibrium. Moreover, both firms are better off compared to the model in which firms choose their own capacities in the first stage of the game.

The theoretical model used in this paper is suitable to be applied to the parking literature. Parking market can be considered as a mixed oligopoly; usually, on-street parking is operated by a social welfare maximizer public firm, while off-street parking is operated by private garages. It is reasonable to assume that the public firm is the decisive party for the on-street and off-street parking capacities allowed. Furthermore, even when the public firm has the power to manipulate prices, this may be impractical for political reasons. In a setting where the government decides on the on-street and off-street parking capacities in the city-planning phase, there will be unused on-street parking capacity in the equilibrium when the public and the private firm engage in a price competition afterwards.

Shoup (2006) advocates adjusting the parking prices to ensure the existence of one vacant parking spot per block. Arnott (2014) recently shows that the optimal vacancy rate may be larger or smaller depending on the traffic level of the on-street parking location. In these studies, the optimal vacancy rate is calculated with the aim of minimizing the congestion rate caused by cruising for parking. I show that even when congestion and search costs are not accounted for, there will be unused on-street parking capacity in the equilibrium of this setting;

furthermore, the excess capacity is obtained without government intervention on pricing.

Potential extensions to the model to reach more genralizable results are allowing for continuous capacities, relaxing the rule on picking the equilibrium that yields the higher social welfare, and having multiple private firms. These are left for future research.

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