A THREE-STAGE SOLUTION APPROACH TO TURKISH DAY-AHEAD ELECTRICITY MARKET

by

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Abstract

Electricity day-ahead market is used for matching the electricity demand and supply of participants according to the market clearing prices. Market clearing prices should be announced fast enough to inform the participants about their bids' acceptance status before the actual transaction. In this thesis, we provide a new method for finding the optimal market clearing prices of the Turkish day-ahead electricity market clearing problem. This method works in three stages: First, an approximation of the main model is solved; secondly, a feasible solution is found close to the solution of the approximation; thirdly, the optimal solution for the problem is searched starting from the solution of the previous step. Each step of the approach and additional enhancements are discussed. Then, we test our methodology with two datasets: one is the generated data which is available to the public, and the other is real data obtained from the Turkish day-ahead electricity market. Our method gives promising results and also its simplicity and adaptability is an advantage for using it with other methods in the literature.

Keywords: Turkish Day-Ahead Electricity Market, Energy Systems, Approximation Methods.

TÜRKİYE GÜN ÖNCESİ ELEKTRİK PİYASASINA ÜÇ AŞAMALI ÇÖZÜM YÖNTEMİ

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Özet

Gün öncesi elektrik piyasası, katılımcıların elektrik arz ve taleplerini piyasa takas fiyatları doğrultusunda eşleştirmekte kullanılır. Piyasa takas fiyatları, katılımcılara, tekliflerinin kabul edilme durumunu teklif gerçekleşmeden öğrenebilmeleri için yeterince hızlı ilan edilmelidir. Bu tez çalışmasında, Türkiye gün öncesi elektrik piyasası eşleme probleminde en uygun piyasa takas fiyatlarını bulmanın daha yeni bir yolunu sunuyoruz. Metot üç aşamada çalışmaktadır: İlk olarak, ana modelin yakınsaması çözülür; daha sonra, yaklaşımdan gelen çözüme yakın bir olurlu çözüm bulunur; ve son olarak, olurlu çözüm ana modele başlangıç çözümü olarak verilerek problemin optimum çözümü aranır. Metodolojimizin her adımı ve ek modifikasyonlar detaylı bir şekilde ele alınmaktadır. Yaklaşımımız iki veri kümesi ile test edilmektedir. Bunlardan ilki üretilmiş veri ve kamuya açıktır, ikincisi ise Türkiye gün öncesi elektrik piyasasından edinilmiş gerçek verilerdir. Önerilen yöntem umut verici sonuçlar vermektedir ve aynı zamanda sadeliği ve uyarlanabilirliği literatürdeki diğer yöntemlerle birlikte kullanılabilme avantajı sağlamaktadır.

Anahtar Kelimeler: Türkiye Gün Öncesi Elektrik Piyasası, Enerji Sistemleri, Yakınsama Yöntemleri.

This work is dedicated to my family.

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1 Introduction

Recently, electricity exchange markets enjoy an increase in audience size worldwide. Furthermore, with the additional customers in the market, satisfying everyone in the market is of utmost importance. An electricity market is the system in which one side is bidding for buying electricity and the other side is bidding for selling their produced electricity. Because the electricity cannot be stored efficiently in today's world, it should be utilized exactly when it is produced. This storage problem increases the need for the day-ahead electricity market (DAEM). In DAEM, prices for every hour are decided and then the market is cleared by matching the demands and supplies one day ahead according to the decided prices. To find this prices to satisfy the bidders in the market, market clearing problem is solved in Turkish DAEM.

In Turkish DAEM, to prevent discrimination between the participants, uniform prices are used for market clearing price (MCP) values. For every 24 hours of the day, participants are informed about the MCP values which are the same for every participant and the resulting acceptance or rejection status of their bids. Turkish DAEM uses uniform pricing while deciding the MCP values. In uniformly priced markets, every participant's bid is accepted or rejected according to the MCP value for that hour, so everyone is paid or pays to the system the MCP value of that hour for each electricity received or supplied. These MCP values are calculated by matching the bids with each other, balancing the demand and supply quantities, and maximizing the daily market surplus of the participants in the process. Of course, each market has its own unique requirements and regulations for deciding which bids to be accepted and which bids to be rejected in that hour. The type of bids that participants can enter may also change in each market.

1.1 Bid Types

In Turkish DAEM, three types of bids can be provided: hourly, block and flexible. Price interval for the bids is 0 to 2,000. The unit of measurement of price values is TL/MWh, and quantities are represented in terms of lots where one lot corresponding 0.1 MWh, in Turkish DAEM.

Hourly bids: Participants give discrete data points consisting of two values, quantity and price for their bids for a single hour. Hourly bids can be in either direction, buy and sell, and basically, the participants declare their preferred electricity quantity and its price value in either direction. Furthermore, we assume that these bids are made by rational decision-makers, for example, supply quantities should either increase or stay the same in case of a price increase and vice versa. Hourly bids are the most common bid type in electricity markets. In Turkish DAEM, linear interpolation is being used for determining the price and quantity matchings that are not at breakpoint levels. The line between two breakpoints is called segment. Table 1.1 illustrates a common single hourly bid. For example, suppose that the MCP value for the corresponding hour is decided as 250 TL. That means the MCP value is between the first data point (0, 2,000) and the second data point (500, 1,600). Then the first segment would be partially covered with quantity of 1,800 because MCP is 250 TL and its in between price values 0 and 500. The interpolation between 2,000 lot and 1,600 lot gives us the quantity value of 1,800 lot.

Table 1.1: Single demand hourly bid

Price(TL/MWh)	0	500	2,000
Quantity(Lot)	2,000	1,600	1,200

Block bids: Participants give discrete data points consisting of quantity and price values similar to hourly bids, in addition to that participants give the interval of consecutive hours of which the block bid spans. Furthermore, block bids are either fully accepted for every hour it spans or they are fully rejected. This kill-or-fill property, not allowing partial acceptance of block bids, makes the mathematical formulation of the market non-convex.

In Turkish DAEM, linked bid type, a special type of block bid is also used. Linked bids have two components, parent and child bids, which are connected to each other. The parent block bid must be accepted in order to accept the child block bid. But the reverse does not necessarily apply, the parent bid can be accepted when the child bid is rejected.

In Table 1.2, Block bid-A covers 5 hours and is linked to the block bid-B which covers 9 hours different than hours covered by block bid A. If A is accepted in this system B should be accepted as well, because they are linked to each other.

Table 1.2: One demand and one supply block bids

Name	Type	Price(TL/MWh)	Quantity(Lot)	Range	Link
Block bid-A	Demand	150	500	1-5	Block bid-B
Block bid-B	Supply	200	200	10-18	-

Flexible bids: Participants give quantity and price information but they do not specify the hour information, so flexible bids can be accepted at any hour of the day. In Turkish DAEM, flexible bids are occasionally observed, but only supply type of flexible bids are supported at the moment.

1.2 Rules of Market Clearing

In Turkish DAEM, aggregation method is used to determine the demand and supply curves of the system for each hour. For every price value of a supply or demand bid, the quantity of the demand and supply bids are summed up at those price points respectively, giving us one demand, one supply curve for each hour. This method gives us one curve rather than multiple curves for each side which is easier to work with, however, because of the aggregation of all the participants' bids, this newly obtained curve has more segments and more variance in slopes between the data points. Varying slopes between the data points may lead to non-convexity of the problem.

Each bid in the system either becomes in the money (ITM) or out of the money (OTM) with respect to the MCP value of the hour. For an hourly bid, it is ITM in that segment if that transaction incurs an income to market. If a demand (supply) side bid price value is greater (less) than to the MCP value of that hour, then that bid is ITM. For example, if a bid consists of quantity and price values of (150, 500) and MCP value is 400, accepting that bid results in surplus to the overall system. If a demand (supply) side bid price value is less (greater) than to the MCP value of that hour, then we call that bid OTM. OTM bids incur loss of welfare to the system if accepted. If the MCP value is between price values of two bid points, it is referred as at the money (ATM) and that hourly bid is partially accepted.

A block bid might be ITM or OTM at different hours it span, therefore we refer to an average income value for them. So, if a demand (supply) block bid's price value is greater (less) than or equal to the average of the MCPs of the hours spanned by the block bid, then that block bid is ITM . It is OTM if the other way around. If a block bid's price value is equal to the average of the MCPs of the hours spanned by the block bid then that block bid is ATM. A flexible bid is ITM if the price of the flexible bid is less than or equal to the maximum MCP of that day. Since flexible bids only on the supply side are considered in Turkish DAEM, we have only one ITM condition. If the maximum MCP value is equal to the flexible bid's price value then that bid is ATM.

According to Turkish DAEM regulations, ITM bids should be accepted and OTM bids may or may not be accepted while ensuring a feasible matching between the participants. Turkish market has paradoxically accepted bids (PABs) similar to the U.S. markets. PABs incur negative income to the system because the bid is accepted although they incur loss to the system. Turkish regulations state that all the money lost in that particular transaction will be paid to that participant, and this payment is called uplift or side payment. Market operator pays those losses to the bidder in case OTM bids are accepted. European DAEM is different than Turkish DAEM, some ITM block bids might be rejected if necessary, resulting in paradoxically rejected bids (PRBs).

Energy Exchange Istanbul (EXIST) is an energy exchange company legally incorporated under the Turkish Electricity Market Law and enforced by the Energy Markets Operation License granted by the Energy Markets Regulator Authority (EMRA) of Turkey. EXIST has developed its own model for solving Turkish DAEM clearing problem. A feasible solution obtained by the help of various heuristics is given to this model as a starting initial solution. The objection function of EXIST's model maximizes the total daily surpluses of demand and supply sides which we will discuss later.

1.3 Contributions

Our contribution to the literature is defining a new way of thinking for Turkish DAEM clearing problem. Our three-stage solution approach speeds up the process quite a lot by approximating the real data and obtaining an initial solution from the approximation. In EXIST's model, large data instances cannot be solved efficiently and within the time limit (10 min.) the model may not obtain even a feasible solution. Our three-stage model works well with those, "hard" instances, in the Turkish DAEM. In our method, we get rid of the heuristics part of the current solution approach of EXIST and design a model which only relies on the optimization tools. Because of the data confidentiality, not many methods are tested for DAEM clearing problems, and we come up with a different idea to solve these problems. Our idea depends on data manipulation and approximation, therefore it can work together with different approaches and ideas, and can work with different and more specialized market rules than those employed by Turkish DAEM.

In Chapter 2, related literature review will be presented. The problem description and formulations will be discussed in Chapter 3. In Chapter 4, we will define our three-stage approach, and in Chapter 5, the results of the different approaches we implemented in each step will be shown. In Chapter 6 we will discuss another formulation for DAEM market clearing problems. Finally, Chapter 7 will outline the conclusion of our studies and the future work that can be established from this work.

2 Literature Review

In the literature, MCP calculation problem in DAEM is not covered thoroughly. One of the reasons for that is the confidentiality of the real data solved in DAEM. In European case [1] and Turkish case [2], there are only a few publications working with real data, and in this thesis, we made use of the ideas presented in Madani and Van Vyve [1] and Derinkuyu [2].

Derinkuyu [2] explains and builds a model to solve Turkish DAEM clearing problem, and develops a heuristic approach. The main structure of our model is very similar to the one that is presented in Derinkuyu [2]. Ideas of giving limits to the MCP values and using the aggregation method for demand and supply bids are also proposed by Derinkuyu [2]. One difference is that, Derinkuyu [2] used the minimization of MCP values in his work, different than welfare maximizing of our work. We worked with EXIST on this thesis, so that generated and real datasets are provided by the company. Our aim is to find a new approach to improve the solution time of their method. One of the contributions of this study is exploiting the linear structure of the hourly bid curves to solve an approximation of the data and coming up with a good feasible solution close to the optimal solution. Our three-stage solution approach is also flexible such that it can be combined with other solution methodologies.

Our second contribution is implementing the ideas and models suggested by Madani and Van Vyve [1] to Turkish DAEM. Madani and Van Vyve [1] develops a new formulation for European DAEM problem without using auxiliary variables to force market equilibrium conditions, using both the primal and dual of the model. Although, their work does not consider the flexible and linked block bids, they came up with a new MIP formulation for the European DAEM clearing problem. We modified their work to Turkish DAEM to obtain a different formulation that can be extended in the future with the addition of flexible and block bids.

Participants of the DAEM agree on the specifications of their countries' market rules. Those rules determine which demand and supply bids are matched with each other and which should be accepted considering the uniform prices of the MCP values for every hour. Non-uniform prices for market clearing problem is discussed in the literature as well [3]. Different countries have different specifications such as accepting ITM bids are forced in Turkish DAEM, rejecting OTM bids are forced in European DAEM, uplifts are paid in some markets by the government, and in some uplifts are nonexistent. Some studies minimize or at least consider the uplift price of the system [4], however in Turkish DAEM, uplift minimization is not considered. Martin et al. [5] focus on the constraints of the U.S. model as well, which has similarities with the Turkish DAEM. Comprehensive guides of electricity market are available in [6] and [7] to check the implementations of different market rules.

In several papers, multiple regions and electricity transferred between them are considered, but in Turkish DAEM and also in our work only single region is being considered at the moment. Usually, electricity trade between the regions have limits due to the infrastructure of the lines carrying the electricity, but in many of the models changing the system from one region to multiple regions can be done without the loss of generality [1, 5]. Even multiple countries can be coupled in the same DAEM clearing problem. For example, in Central West Europe many countries are coupled together and their DAEM clearing problem solved together [8]. With the addition of other European countries in the south and north, the algorithm Euphemia is used to solve their DAEM clearing problem [8]. Turkish DAEM has a single region at the moment, but considerations are made to divide the region into multiple regions in the coming years.

Martin et al. [5] presented a new comprehensive model including flexible bids and used decomposing methods in European DAEM. Two types of cuts are developed in this work: the first one can lead to suboptimal solutions, and the second one ensures finding the optimal solution. The latter cuts are slightly slower than the first cuts. Both of these cuts are very similar to the Bender's [9] type cuts and similar ideas can be added to the Turkish DAEM in the future as well.

In this thesis, we maximize the welfare of the whole system. But in some works, only producer side is considered with incorporating the competition among other producers [10] so that the optimal quantity and price bids are decided on one side of the auction. Additionally, producer side does not have all the necessary information which leads to use of stochastic programming models in the energy sector [11, 12]. Most basic scenario we can observe is, producer side optimizes their bidding considering the MCP values of the previous days. In our work, competition side is not considered. Additionally, we do not use stochastic models because we solve the system knowing all the bids from both sides.

In our work, we solved our problem using the primal model only, but the dual of the problem can be incorporated as well to improve the solution time, especially if strong duality can be proven [13, 14]. Work of Madani and Van Vyve [1] combine the primal and dual constraints to build a new model that can improve the solution time. In the literature, other works combining the primal and dual constraints to build a new model can be found. As an example, the work of Zak et al. [15] solve a non-linear MIP formulation to provide a solution to the market clearing problem. Using the dual problem, the dual variables can be interpreted as the MCP values of the system [16]. Generating cuts and using Lagrangian methods [17]and other state-of-the-art methods are needed to solve the DAEM clearing problem because the integer flexible and block bids make this problem combinatorial. It is known that combinatorial problems cannot be handled by the today's solvers without implementing more complicated methodologies to decrease the solution time of the system [18, 19].

3 Problem Description and Formulation

In this section, the market clearing problem will be discussed further and then the main model will be explained. Decision variables and constraints of the Turkish DAEM model will be explained in detail as well.

3.1 Problem Description

In DAEM, for each location and each hour, an optimization problem is solved to maximize the surplus of the system while finding the MCP values of the system. While maximizing the surplus, certain market regulations should be satisfied. For example, almost in all markets demand and supply sides' quantities should be equal to each other for every hour.

Currently, Turkish DAEM is considered as one location, and all the MCP calculations are uniform across the country. This assumption eases the notation of the model, the location indices in the DAEM models can be ignored. We also do not deal with network constraints in our model some other markets do.

There are two main approaches to deal with the data provided in discrete points. Some markets work with step-wise linear curves for demand and supply sides, and some market rules interpolate the points in between to obtain a piece-wise linear system. In step-wise system, because of the rectangular nature of the system, it is relatively easy to calculate the objective function. On the other hand, piece-wise linear model type results in quadratic objective function terms. Figure 3.1 and Figure 3.2 show a small example for each curve type.

Essentially, we need to find the optimal MCP values, such that the accepted total demand and total supply quantity values should be equal for each hour considered.

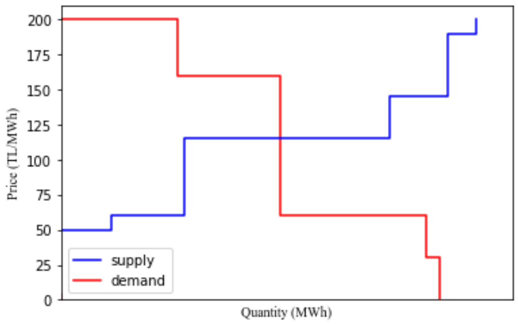

Figure 3.1: Step-wise bid curves

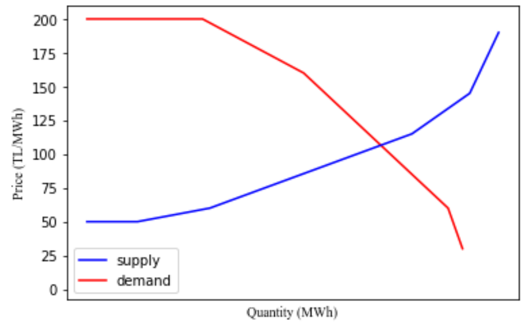

Figure 3.2: Piece-wise bid curves

3.2 Mathematical Formulation

For the aggregation of hourly bids, firstly, a set P is constructed from the different price values for every bid on every segment for each demand and supply side. Secondly, distinct price values of p are selected in set P, and for all selected distinct values of p, the quantity values are summed up. If it does not match, then interpolation is used. At the end for each distinct p value we obtain a q value of aggregated quantity. This way, for every hour, we obtain one demand and one supply curve derived from multiple curves. Each bid curve has at least one segment, each two consecutive data points (Q^l, P^l) and (Q^{l+1}, P^{l+1}) make one segment of our curves, Q and P giving the quantity and price arrays of the curve respectively. Consider the two demand hourly bids for a specific time in Table 3.1. Hourly bid-A has 3 data points and Hourly bid-B has 4. For aggregating them, we need to consider all the distinct price values. Price set P contains the values 0, 500, 1,000 and 2,000. Shared price values of two hourly bids are 0, 500 and 2,000. So, interpolation is needed for the unique price value of 1,000. For hourly bid-A, quantity value corresponding to price value of 1,000 should be calculated.

Table 3.1: Single demand hourly bid

Hourly bid-A	Price(TL/MWh)	0	500	2,000	-
	Quantity(Lot)	2,000	1,600	1,300	-
Hourly bid-B	Price(TL/MWh)	0	500	1,000	2,000
	Quantity(Lot)	2,000	1,200	1,000	500

In Table 3.2, with the summation of quantity values associated with the price values, we obtain our aggregated curve with 4 data points.

Table 3.2: Single demand hourly bid

Aggregated hourly bid	Price(TL/MWh)	0	500	1,000	2,000
	Quantity(Lot)	4,000	2,800	2,400	1,800

Set L_i gives us the segment of the bid i. Furthermore, we define x_{il} as our continuous decision variable showing the acceptance rate of an hourly bid $i \in H$ of segment $l \in L_i$. H is the set of all aggregated continuous hourly bid segments, H^d representing the demand and H^s representing the supply hourly bid segments. T represents the number of hours in the day for which the MCP values will be determined. Usually T is 24 in DAEM, but T may take any positive integer value in general. Value of t_i corresponds to the hour which hourly bid curve $i \in H$ is defined. Because of the aggregation method, every t value in T has one demand and one supply curve only. For example for bid i: t_i = 1 means we select the demand and supply curves of hour 1 and so on. As previously mentioned, at most one segment can be partially accepted at each hour, if there exists a partially accepted bid that means that optimal MCP value is between two real data points.

B is the set of all binary block bids, similarly B^d and B^s represent the set of demand and supply block bids, respectively. Λ^b represents the set of block bids linked to block bid b. Moreover, in Turkish DAEM we only consider supply side of flexible bids, therefore, the set of all flexible bids is F. Finally, MCP_t defines the MCP values of the system for every hour $t \in T$. P_{min} and P_{max} are the minimum and maximum possible values for MCP values. In our model all MCP values are between 0 and 2000. P^t_{min} and P^t_{max} are the lowest and highest possible market clearing prices for each hour t. P^0_{il} and P^1_{il} are the initial and final price values of hourly bid $i \in H$ at segment $l \in L_i$. Similarly, Q^0_{il} and Q^1_{il} are the initial and final quantity values of hourly bid $i \in H$ at segment $l \in L_i$.

N_b shows the total number of time periods where block bid b spans and δ_{bt} is the binary parameter taking the value of 1 if block bid b spans time period t, and 0 otherwise. P_b, Q_b values are the price and quantity values of block bids $b \in B$ and P_f, Q_f values are the price and quantity values of flexible bids. All quantity and price values are being greater than or equal to 0. From segment $l = 0$ to $l = |L_i|$, the quantity of demand and supply bid values are non-decreasing, so $Q_{i(l+1)} \geq Q_{il}$ for $i \in H$ and $l \in \{1, 2, .., |L_i| - 1\}$. From segment $l = 0$ to $l = |L_i|$ supply curves have non-decreasing price values and demand curves have non-increasing price values, which means, $P_{i(l+1)} \geq P_{il}$ for $i \in H^s$ and $l \in \{1, 2, .., |L_i| - 1\}$ and $P_{i(l+1=} \leq P_{il}$ for $i \in H^d$ and $l \in \{1, 2, .., |L_i| - 1\}$. bburay dzelt

In addition to the decision variable x_{il}, we define $y_b \in \{0, 1\}$ which is equal to 1 if block bid $b \in B$ is accepted, and 0 otherwise. $z_f \in \{0, 1\}$ is similarly defined which is equal to 1 if $f \in F$ is accepted, and 0 otherwise. Similarly, decision variable $r_{ft} \in \{0, 1\}$ defines the hour of the day that the flexible bid is accepted. Lastly, the decision variable $w_{il} \in \{0, 1\}$ is defined as 1 if we fully accept a segment of hourly bid $i \in H$ of segment $l \in L_i$, and 0 otherwise. Decision variable w_{il} is used to make sure all ITM hourly bids are accepted and OTM hourly bids are rejected.

Following tables 3.3, 3.4 and 3.5 recapitulates the sets, parameters and the decision variables of the mathematical model of Turkish DAEM clearing problem respectively:

Table 3.3: Sets of the Model

Symbol	Definition
T	Set of time periods
t_i	Time period corresponding to hourly bid $i \in H$
L_i	Set of segments for hourly bid $i \in H$
H^d	Set of demand hourly bids
H^s	Set of supply hourly bids
H	Set of all hourly bids $H^s \cup H^d$
B^d	Set of demand block bids
B^s	Set of supply block bids
B	Set of all block bids $B^s \cup B^d$
Λ^b	Set of block bids to which block b is linked, $b \in B$
F	Set of flexible bids (all supply bids in Turkish DAEM)

Table 3.4: Parameters of the Model

Symbol	Definition
P^t_{min}	The lowest possible market clearing price for period $t \in T$
P^t_{max}	The highest possible market clearing price for period $t \in T$
P_{min}	The lowest valid bid price
P_{max}	The highest valid bid price
P^0_{il}, P^1_{il}	The initial and final price for the hourly bid $i \in H$ of segment $l \in L_i$
Q^0_{il}, Q^1_{il}	The initial and final quantity for hourly bid $i \in H$ of segment $l \in L_i$
P_b, Q_b	The price and quantity of block bid $b \in B$
P_f, Q_f	The price and quantity of flexible bid $f \in F$
N_b	Number of time periods where block bid b spans, $b \in B^s \cup B^d$
δ_{bt}	Binary parameter equal to 1 if block bid $b \in B$ spans period $t \in T$

Table 3.5: Decision Variables of the Model

Symbol	Definition
x_{il}	Accepted fraction of segment l of hourly bid i
w_{il}	1 if segment l of hourly bid i is fully accepted, 0 otherwise
y_b	1 if block bid b is accepted, 0 otherwise
r_{ft}	1 if flexible bid f is accepted in period t , 0 otherwise
z_f	1 if flexible bid f is accepted, 0 otherwise
MCP_t	Market clearing price in period t

Now, we will introduce the objective function and constraints of the market clearing problem in Turkish DAEM. In Turkish DAEM, uplifts are ignored while calculating the MCP values, therefore, only the summation of welfare obtained from hourly, block and flexible bids are considered.

If we consider piece-wise linear curves, welfare summation has additional calculations resulting from interpolation of discrete points. The calculations should include the triangles under the piece-wise linear curves, and because of the partial acceptance of the bids, we need to deal with quadratic terms while calculating the welfare of piece-wise linear curves in DAEM. Total welfare of the system is given by:

$$\sum_{i\in H^d}\sum_{l\in L_i} \Delta Q_{il}P^0_{il}x_{il} + \Delta Q_{il}(\Delta P_{il})\frac{x^2_{il}}{2} - \sum_{i\in H^s}\sum_{l\in L_i} \Delta Q_{il}P^0_{il}x_{il} + \Delta Q_{il}(\Delta P_{il})\frac{x^2_{il}}{2} + \sum_{b\in B^d}\Omega_b y_b - \sum_{b\in B^s}\Omega_b y_b - \sum_{t\in T}\sum_{f\in F}\Omega_f r_{ft},$$

where $\Delta Q_{il} = Q^1_{il} - Q^0_{il}$, $\Delta P_{il} = P^1_{il} - P^0_{il}$, $\Omega_b = Q_b N_b P_b$, and $\Omega_f = Q_f P_f$.
For block bids we accept the bid for every interval it is covering therefore we also multiply the number of hours that the bid is covering.

For every hour, accepted quantity of the demand and supply sides should be equal to each other:

$$\sum_{\substack{i \in H^d: \\ t_i = t}} \sum_{l \in L_i} \Delta Q_{il} x_{il} + \sum_{b \in B^d} y_b Q_b \delta_{bt} = \sum_{\substack{i \in H^s: \\ t_i = t}} \sum_{l \in L_i} \Delta Q_{il} x_{il} + \sum_{b \in B^s} y_b Q_b \delta_{bt} + \sum_{f \in F} Q_f r_{ft} \quad t \in T.$$

The same amount of electricity is used among all the bidders, so the system should accept equal amounts of electricity on both sides to acquire a balance. Whether from hourly, block or flexible bids, the total accepted demand quantity should be equal to the total accepted supply quantity.

For all hourly bids, the system should accept ITM and ATM bids and reject OTM bids. For every hour and each side at most one bid which may be ITM or ATM can be partially accepted. So, if the MCP value of an hour is between two data points, that should be partially accepted with interpolating the acceptance rate according to the MCP value. We introduced constraints called w-constraints for applying these rules to our system:

$$w_{i1} \leq x_{i1} \leq 1 \quad i \in H,$$

$$w_{il} \leq x_{il} \leq w_{i(l-1)} \quad l = 2, ..., |L(t)| - 1, i \in H,$$

$$0 \leq x_{i|L_i|} \leq w_{i(|L_i|-1)} \quad i \in H.$$

For every hour, summation of the demand and supply prices gives us the MCP values:

$$MCP_t = P_{min} + \sum_{\substack{i \in H^s: \\ t_i = t}} \sum_{l \in L_i} \Delta P_{il} x_{il} \quad t \in T,$$

$$MCP_t = P_{max} + \sum_{\substack{i \in H^d: \\ t_i = t}} \sum_{l \in L_i} \Delta P_{il} x_{il} \quad t \in T.$$

Because we need to accept ITM hourly bids, these constraints give us the MCP values of each hour. For supply side, bids with prices that are less than or equal to the MCP values are accepted. For demand side, bids with prices that are more than or equal to MCP values are accepted. These constraints also help in the approximation step that we can obtain meaningful MCP values even if the whole data is not used.

Supply (demand) block bids having bid price less (greater) than or equal to the average MCP of the periods, where the bid is active, must be accepted if they are not child linked bids. Because of that, we write these constraints if that block bid is not also a child bid.

For supply side:

$$\sum_{t\in T}\delta_{bt}MCP_t - N_bP_b \leq (\sum_{t\in T}\delta_{bt}P^t_{max} - N_bP_{min} + \epsilon)y_b - \epsilon \quad b \in B^s\backslash B^{sc}.$$

For demand side:

$$N_bP_b - \sum_{t\in T}\delta_{bt}MCP_t \leq (N_bP_{max} - \sum_{t\in T}\delta_{bt}P^t_{min} + \epsilon)y_b - \epsilon \quad b \in B^d\backslash B^{dc}.$$

While deciding on accepting or rejecting a specific bid we make the decision by calculating average welfares of each bid. Therefore, a block bid is accepted if the average income of that block bid is positive. This means that an accepted block bid does not have to increase welfare for each hour it is covering, but it should increase the overall welfare of the system.

Note that ϵ is needed because we should accept block bids if they are ATM. Therefore, if a price of a block bid is equal to the average MCP values of hours which that block bid spans we should accept that block bid in Turkish DAEM.

If we have linked bids, we need additional constraints for them. A child block bid cannot be accepted unless its parent bid is accepted as well. Linked bid constraints in our model:

$$y_b \leq y_\lambda \quad b \in B, \lambda \in \Lambda^b.$$

Child block bid can be rejected even if it provides positive welfare to the system because its parent might be rejected considering its contribution to the total welfare obtained.

A supply flexible bid must be accepted if the maximum MCP is equal to or greater than the bid price. Meaning, if the bid definitely increases welfare for every hour in that day, we should accept the bid. The model will pick the best and the only hour the bid should be accepted according to the objective function. If the flexible bid does not increase the welfare for every hour, it can be either accepted or rejected.

$$\sum_{t \in T} r_{ft} \leq 1 \quad f \in F,$$

$$\sum_{t \in T} r_{ft} \geq z_f \quad f \in F,$$

$$MCP_t - P_f \leq (P^t_{max} - P_f + \epsilon)(z_f) - \epsilon \quad f \in F, lt \in T.$$

We have two different decision variables here, z_f is for determining whether we should accept the flexible bid or not, and r_{ft} will give the hour that the flexible bid is accepted. If a flexible bid is accepted, it should be assigned to only one hour among the time interval T.

Lastly, our decision variables are binary with the exception of acceptance rate of hourly bids i and the MCP values.

$$x_{il}, MCP_t \geq 0, y_b, r_{ft}, z_f, w_{il} \in \{0, 1\} \quad f \in F, t \in T, i \in H, b \in B, l \in L_i.$$

The full model that we solve for the DAEM clearing problem is as follows:

$$\text{maximize} \quad \sum_{i\in H^d}\sum_{l\in L_i} \Delta Q_{il}P^0_{il}x_{il} + \Delta Q_{il}(\Delta P_{il})\frac{x^2_{il}}{2} - \sum_{i\in H^s}\sum_{l\in L_i} \Delta Q_{il}P^0_{il}x_{il} + \Delta Q_{il}(\Delta P_{il})\frac{x^2_{il}}{2} + \sum_{b\in B^d}\Omega_b y_b - \sum_{b\in B^s}\Omega_b y_b - \sum_{t\in T}\sum_{f\in F}\Omega_f r_{ft} \tag{1}$$

subject to

$$\sum_{\substack{i\in H^d:\\ t_i=t}}\sum_{l\in L_i}\Delta Q_{il}x_{il} + \sum_{b\in B^d} y_b Q_b \delta_{bt} = \sum_{\substack{i\in H^s:\\ t_i=t}}\sum_{l\in L_i}\Delta Q_{il}x_{il} + \sum_{b\in B^s} y_b Q_b \delta_{bt} + \sum_{f\in F} Q_f r_{ft} \quad t\in T \tag{2}$$

$$w_{i1} \le x_{i1} \le 1 \quad i\in H \tag{3a}$$

$$w_{il} \le x_{il} \le w_{i(l-1)} \quad l = 2,...,|L_i|-1, i\in H \tag{3b}$$

$$0 \le x_{i|L_i|} \le w_{i(|L_i|-1)} \quad i\in H \tag{3c}$$

$$MCP_t = P_{min} + \sum_{\substack{i\in H^s:\\ t_i=t}}\sum_{l\in L_i}\Delta P_{il}x_{il} \quad t\in T \tag{4a}$$

$$MCP_t = P_{max} + \sum_{\substack{i\in H^d:\\ t_i=t}}\sum_{l\in L_i}\Delta P_{il}x_{il} \quad t\in T \tag{4b}$$

$$\sum_{t\in T}\delta_{bt}MCP_t - N_bP_b \le (\sum_{t\in T}\delta_{bt}P^t_{max} - N_bP_b + \epsilon)y_b - \epsilon \quad b\in B^s\backslash B^{sc} \tag{5a}$$

$$N_bP_b - \sum_{t\in T}\delta_{bt}MCP_t \le (N_bP_b - \sum_{t\in T}\delta_{bt}P^t_{min} + \epsilon)y_b - \epsilon \quad b\in B^d\backslash B^{dc} \tag{5b}$$

$$y_b \le y_\lambda \quad b\in B, \lambda\in\Lambda^b \tag{6}$$

$$\sum_{t\in T} r_{ft} \le 1 \quad f\in F \tag{7a}$$

$$\sum_{t\in T} r_{ft} \ge z_f \quad f\in F \tag{7b}$$

$$MCP_t - P_f \le (P^t_{max} - P_f + \epsilon)z_f - \epsilon \quad f\in F, t\in T \tag{7c}$$

$$x_{il}, MCP_t \ge 0, y_b, r_{ft}, z_f, w_{il} \in \{0,1\} \quad f\in F, t\in T, i\in H, b\in B, l\in L_i \tag{8}$$

Our problem is maximizing the total welfare surplus (1). At every period, accepted demand and supply quantities must be equal, so constraint (2) gives us our balance equations. Constraint (3) is needed to find the acceptance rates of hourly bids of consumers and suppliers, w-constraints make sure that all ITM hourly bids are fully accepted and all OTM hourly bids are fully rejected. Constraint (4) determines the market clearing prices according to the bid acceptance. Constraint (5) ensures that we accept the block bids which are ITM according to the MCP values. Constraint (6) links the parent and child block bids to each other. Then, constraint (7) are needed to accept a supply flexible bid if it is ITM. The whole model is currently used by EXIST to solve the Turkish DAEM problem.

One slight variation of piece-wise linear curve is step-wise linear curves. If step-wise linear curves are used to calculate the total welfare value, modifying only the objective function is sufficent. As in Figure 3.1, in step-wise curves either the quantity or the price values of the consecutive data points are the same. Therefore, each segment becomes parallel to either x-axis or y-axis, and rectangular shapes are obtained under the step-wise linear curves. Therefore, for calculating the total welfare of the system we need to sum up the rectangles under the bid curves which can be obtained without the quadratic terms in our objective function. If step-wise linear curves are considered our new objective function becomes:

$$\sum_{i \in H^d} \sum_{l \in L_i} \Delta Q_{il} P_{il} x_{il} - \sum_{i \in H^s} \sum_{l \in L_i} \Delta Q_{il} P_{il} x_{il} + \sum_{b \in B^d} \Omega_b y_b - \sum_{b \in B^s} \Omega_b y_b - \sum_{t \in T} \sum_{f \in F} \Omega_f r_{ft}.$$

4 Methodology

We implement the model that the EXIST is using, for solving the data they provided, which we call the generated data. With the main model without changing anything, we solved the 243 intances that we have. The demand and supply curves for each hour are piece-wise linearly represented but not necessarily convex. The non-convexity of the data arises from the fact that, in Turkish DAEM aggregation method is used, and aggregating multiple curves to one curve leads to non-monotonicity. Our 243 intances have 5 features each of which can take 3 different values. These features are:

1) Number of segments for each hourly bid curve which takes the values of 100, 500 and 1,000
2) Number of block bids we have in the system which takes the values of 200, 500 and 1,000
3) Number of flexible bids we have in the system which takes the values of 10, 50 and 100
4) Ratio of the supply block bids to all block bids which takes the values of 0.5, 0.75 and 1
5) The range of the time that block bids cover in the system which takes the values of 1 to 4, 1 to 24 and 16 to 24 in units of hours in a day

Every instance that we consider have three types of bids: hourly, block and flexible. For hourly bids, we have their ID, the segment number, and quantity and price values for each segment number. Block bids do not have any information regarding the segments; however, we have the information of starting and finishing hour of a block bid. Since we can accept flexible bids in any hour in the day, the only information we have on them are the price and quantity values.

In each instance, only one of these features is changed to see the effect of each feature on the efficiency of the solution approach. First we test the model presented in the previous chapter with the generated data obtained from EXIST. All the solution times recorded in this section are performed on 64-bit server with Intel Core i5-5200U processor with a speed of 2.19GHz and 4GB RAM. GUROBI 7.0.1 is utilized for the optimization purposes under the time limit of 10 minutes. We summarize our observations below.

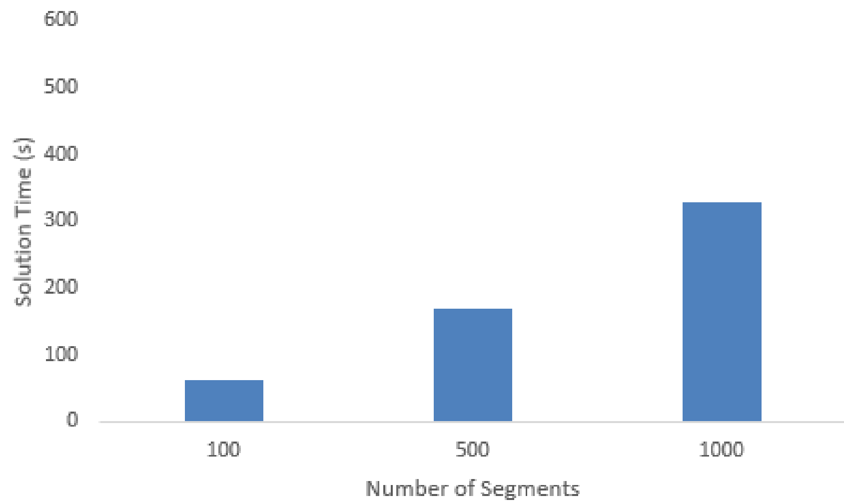

Figure 4.1: The effect of the number of segments on the solution time

As it can be observed from the Figure 4.1, the increase in the number of segments double or triple the average solution time of the problems. Therefore, lowering the number of segments each bid has, leads to much faster solution times for the datasets.

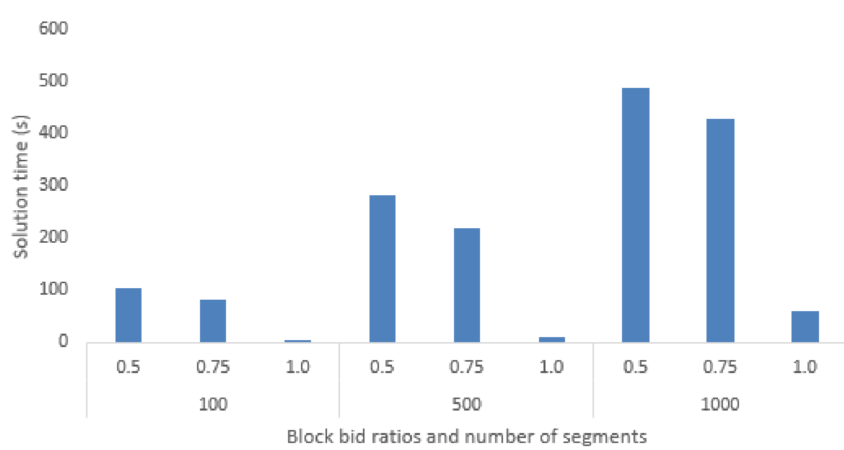

Figure 4.2: The effect of the ratio of block bids on the solution time

As illustrated in Figure 4.2, when the ratio of number of supply block bids to the number of all block bids move from 1 to 0.5, meaning that number of demand and supply block bids are getting closer to each other, instances get harder to solve since the possibility of combining the block bids increase. If the ratio is 1, it means that we have only supply block bids and thus problems become easier to solve.

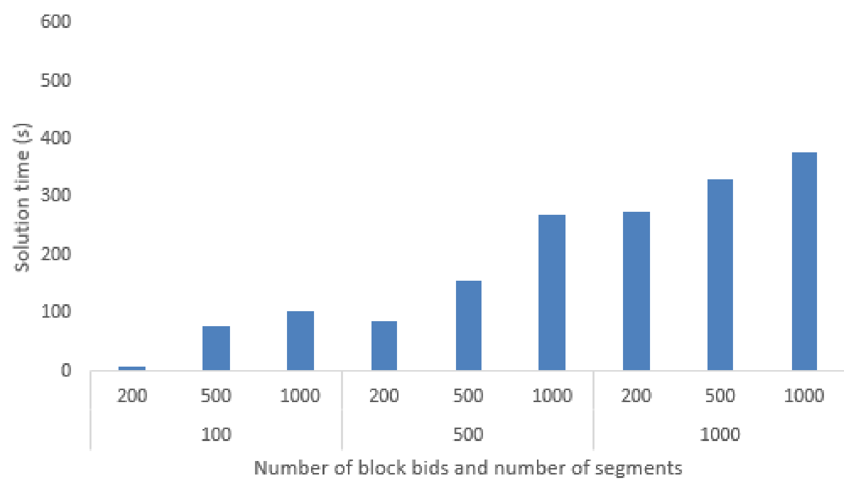

Figure 4.3: The effect of the number of block bids on the solution time

Not surprisingly, other factor that has an effect on run time is the number of block bids in the system. Figure 4.3 reveals that, increasing the number of block bids results in increased solution times, especially if the number of segments of hourly bids is small.

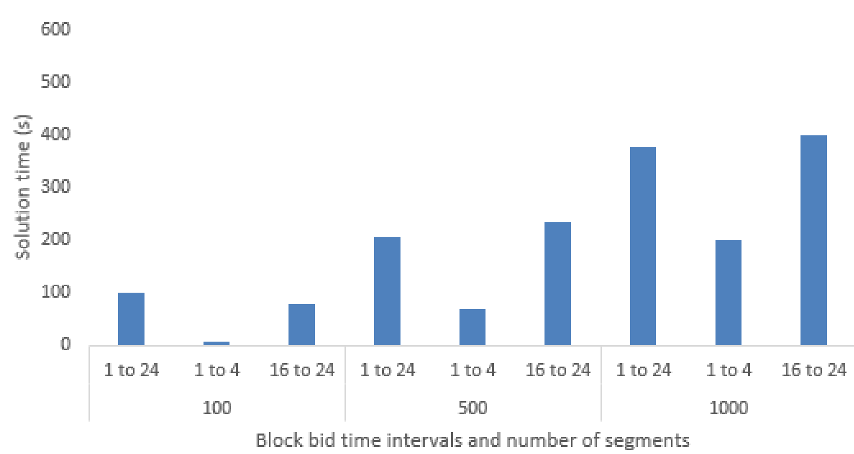

Figure 4.4: The effect of the block bids' spanning interval on the solution time

The time interval covered by the block bids also plays a role at solution times. A block bid affacts the total quantities and consequently the MCP values of the periods it covers. Therefore, we can claim that if we increase the time intervals covered by block bids, problems get much harder to solve. Our claim is supported by 4.4.

In our preliminary computational experiments we observe that even a feasible solution may not be found for some data instances with 1,000 segments at the end of the time limit of ten minutes by the formulation of EXIST. In those instances, solution times are taken as the time limit, ten minutes.

4.1 Three-stage Approach

In the previous section, we discussed the effect of number of segments in the hourly bid curves on the running time of the main model. Additionally, in 1,000-segment cases, we hit the time limit before reaching a feasible solution. This way the idea of figuring out a way to solve the model with fewer segments and still get the same optimal solution as before is obtained. Therefore, our aim becomes finding a good approximation for each demand and supply curves to make a powerful claim about the MCP values of the datasets. We develop a three-stage solution approach which aims to find the optimal MCP values more efficiently.

4.1.1 Approximation Step

In the first step, we solve the approximation of the data. At the end of step 1, we obtain the MCP values from the approximation method and would like to infer the optimal MCP values based on the approximation. In the approximation step, we exploit two features for getting a speed up. First one is, dealing with fewer segment points than before so we can lower the number of decision variables and constraints of our problem by removing some of the data points. And the second one is, we eliminate the non-convexities of the demand and supply curves. Convex curves are obtained since we delete the real data points which make the curves non-convex, and with deleting them we obtain convex curves. When convex curves are obtained, some decision variables that we introduced because of the non-convexity of the original curves can be deleted from the problem. Dealing with convex functions in MIP problems usually, without exception in this case, is much easier and faster than dealing with non-convex functions.

In the generated data, the demand and supply curves have several non-convex components . In our first step, we ignore those non-convexities in the curves, and consider the convex approximations of the supply and demand curves. We consider several approximations as described below.

Using step sizes: One very easy way to diminish the size of the data is to skip them. We introduce a value, step size, to jump from point to point by ignoring the points in between. In the most basic implementation, this is enough for approximating the curves. In the generated data, because of its nice structure, even without doing anything using step sizes of ten works well. We can actually observe that by jumping ten data points at a time, non-convexities of the data are eliminated and consequently, w-binary variables can be excluded from the first step. The number of the step size can play a big role in the solution time of the problem. If the step sizes are too big, we would obtain bad results due to poor approximation of the data. Similarly, if step sizes are too small, the procedure may result a non-convex approximation. We will present the details of these problems in the next sections.

Modified step sizes: One modification can be made for randomly selecting data points. Sometimes, if the variation is high, jumping ten data points ahead can lead to an insufficient approximation of the data. Modified step sizes method overcomes this disadvantage by adding additional data points if they are needed. For example, if we pick our step size as ten, we will consider k data points such as $k \in \{1, 11, 21, ..\}$.Assume from data point 11 to data point 21 there is a huge price difference. Then we can add additional points between 11 and 21 to better approximate the original data. Of course, this depends on the problem instance. For example, in Turkish DAEM's real data, we do not observe a price difference of 50 that easily between two data points. If the user specifies 50 as the threshold number, then data points with price differences more than or equal to 50 are also added to the approximation.

Additional points added each run: Another way to add data points is running the approximation step more than once. We start with several data points and solve the problem, then add new points that are close to the MCP of the previous approximation, and solve the problem again. Because approximation step usually runs very fast, this method might give better results than previous two ideas. After each solution, we will add additional real data points closer to previous MCP optimal values to represent the proximal area of MCP values better. In this way we increase the chance to obtain a feasible solution for the problem. Since the speed of step 2 relies on the quality of the solution of the first step, this may lead to an improvement in the overall solution times.

The disadvantage of this method is, for every 24 hours we need to add additional points each time, so if we have lots of real data points the total solving time of step 1 might increase exponentially. In the worst case scenario, the step 1 would be run until all the data points are added to the system which is a highly time consuming process. Solution time improvement in step 2 may be less than solution time increase in the solution times of step 1.

At the end the first step we obtain a solution to the approximated dataset. The 3-stage method's efficiency is highly correlated with the accuracy of the first step's MCP values.

4.1.2 Feasible Solution Step

In the second step, we want to convert our findings of the first step to an incumbent solution. No matter how good the approximation is, demand and supply accepted quantities may not match with the original dataset. Since we ignore some real data points, the MCP values of first step may be infeasible due to the demand-supply imbalance in the second step. Even if there is feasibility, the first step maximizes the welfare based on the approximated dataset, and thus, the same solution may not be the optimal for the original dataset. In this step, we solve our main problem with all segments and we make use of the solution of the approximation step to obtain a feasible solution for the original problem. So, We consider three methods called as delta, sigma and SC methods:

Delta method: In this method, we introduce a predetermined value delta, and assume that the difference between the MCP values of the second and the first step is not larger than delta. Therefore, we search for a feasible solution in a subset of the original feasible region, which is obtained by introducing additional constraints. For the generated case, even the delta value of 5 works well and finds the optimal solution in 90% of the cases. One drawback of this method is, delta values should be determined carefully. Larger values of delta may lead to excessive computation times, whereas smaller values of delta may result in infeasibility of the second step. One possible solution for this problem is to dynamically update the delta values, but this may also cause additional computational time.

The first step of our problem solves this approximated version and we obtain approximated MCP values, which we define as Approximated Market Clearing Price values $AMCP_t$ at period $t \in T$.

In delta method, only difference we make is that we solve our problem in a subspace of our feasible region. We define this subspace by observing that our first part's AMCP optimal solutions reflect the main problem's optimal MCP values.

Therefore, if we use the delta method, we add the constraints:

$$AMCP_t + delta\_value \geq MCP_t \quad t \in T \quad \text{(9a)}$$
$$AMCP_t - delta\_value \leq MCP_t \quad t \in T \quad \text{(9b)}$$

Because we deal with a subset of the whole feasible region, we can further improve our big-M variables in the corresponding constraints (5a), (5b) and (7c) in our model:

$$\sum_{t \in T} \delta_{bt} MCP_t - N_b P_b \leq (\sum_{t \in T} \delta_{bt}(AMCP_t + delta\_value) - N_b P_b + \epsilon) y_b - \epsilon \quad b \in B^s \backslash B^{sc} \quad \text{(5a\_2)}$$
$$N_b P_b - \sum_{t \in T} \delta_{bt} MCP_t \leq (N_b P_b - \sum_{t \in T} \delta_{bt}(AMCP_t - delta\_value) + \epsilon) y_b - \epsilon \quad b \in B^d \backslash B^{dc} \quad \text{(5b\_2)}$$
$$MCP_t - P_f \leq (AMCP_t + delta\_value - P_f + \epsilon)(z_f) - \epsilon \quad f \in F, t \in T \quad \text{(7c\_2)}$$

Solving the second part, if the subset is not empty for the predetermined delta value, will definitely give us a feasible point for the problem.

Sigma method: Sigma method is the slight variation of delta method which assumes delta as a decision variable rather than a predetermined value. We consider a step-wise objective function to penalize the difference between the MCP values of the second and the first steps. However, our computational experiments show that this method works much slower than the delta method. Additionally, the upper bound of the sigma decision variable highly affects the speed of the second step.

SC method: S-constraint (SC) method is proposed to overcome the feasibility problem of the delta method and solution speed problem of the sigma method. In this method, the solution of the integer decision variables obtained in the approximation step will be given as an initial solution to the problem. Of course, given solution might be infeasible because of the demand and supply imbalance. For that reason, we relax the demand-supply balance constraints for each hour and keep the difference between them by a new decision variable s_t for $t \in T$. In the objective function, we penalize the s_t values with very large weights to make sure they become zero in the optimal solution. Making them zero is a must to obtain a feasible solution without any demand and supply imbalances in the system. Since we try to obtain a feasible solution as close to the optimal solution as possible, welfare of the overall system is maximized while all the s values are forced to become 0 for every hour of the day.

$$\sum_{\substack{i \in H^d: \\ t_i = t}} \sum_{l \in L_i} \Delta Q_{il} x_{il} + \sum_{b \in B^d} y_b Q_b \delta_{bt} - \sum_{\substack{i \in H^s: \\ t_i = t}} \sum_{l \in L_i} \Delta Q_{il} x_{il} - \sum_{b \in B^s} y_b Q_b \delta_{bt} - \sum_{f \in F} Q_f r_{ft} = s_t \quad t \in T \tag{10}$$

If a solution with objective function value 0 is obtained, then we obtain a feasible solution to the original model. On the other hand, the observed feasible solution may result in a very small welfare if we do not incorporate the total welfare to the objective function.

To obtain a high quality feasible solution for the problem we consider the following objective function:

$$\text{maximize} \quad \text{Total Welfare} \quad -M \sum_{t \in T} s_t$$

Big-M values should be sufficiently large to force $s_t = 0$. Additionally, we maximize the total welfare for obtaining solutions closer to the optimal solution at the end of step 2.

4.1.3 Optimal Solution Step

At the end of the second step, we obtain a feasible solution. In the third step, we want to find the optimal solution to the main problem. Therefore, we solve the main problem starting from the second step's feasible solution as the initial solution. If we obtain the optimal solution in the second step, the third step would take less time; and if not, additional computational time would be needed for converging to the optimal solution. We obtain the optimal solution of the main problem by solving the third step.

In the third step we use the model presented in the previous section which EXIST uses currently. Little modifications can be done according to the user preference. For example, changing the focus of the model in GUROBI or CPLEX affects the overall solution time. If the user is confident about the optimality of the initial given solution, the focus can be on the optimality proof. If little improvements are necessary, the focus on improving the initial objective function might be more beneficial.

In the Turkish DAEM, in two minutes, tentative MCP values are shared by the bidders and after additional time the final MCP values are announced. Therefore, the solution of the second step could be the tentative values in this case, and the solution of the third step would be the finalized MCP values.

4.2 Additional Changes

To improve the computation times of our solution method, we make several changes in our formulations.

Bounds on Market Clearing Prices: Note that the bid assignments can be easily decided given the MCP values for each hour. All the hourly bids accepted and rejected can be determined, and the decisions for most of the block and flexible bids may be also determined. Remaining part is accepting or rejecting integer-valued bids while obtaining a balance between the demand and supply sides. Because of this reason, finding a reasonable interval of the MCP values for each hour is important. Additionally, big-M values used in the block and flexible bid constraints depend on the maximum and minimum values of the MCP values, therefore another benefit of obtaining an interval of MCP values is getting tighter big-M values for the problem. To obtain a tighter upper and lower bounds for MCP values we implemented the idea of EXIST [2].

Additional w-constraints: Typically, we have w-constraints for each segment of the demand and supply curves. In the approximation step, we ignore all w-constraints to make the solution time faster, although the solution may not be feasible. However, adding severall w-constraints to the system usually make the overall solution time faster. We can select several segments and include the w-constraints for them. This way, if the data is highly non-convex, we can increase the accuracy of the approximation by including w-constraints to the problem.

We can choose additional $\mathbf{w}$-constraints by observing the slopes of the linear lines between each data points. When the non-convexity is apparent we can select the data points causing the non-convexity and add w decision variables and their respective constraints to those points. This was if that w decision variable becomes 1 all the previous points should be accepted as well and if it is 0 all the points after that should be rejected. Real datasets are usually highly nonconvex at some points, therefore adding this application speeds up the overall solution time. Because of the addition of new constraints and decision variables, approximation step solution time slightly increases, but having a more accurate portrayal of the real data compensates that speed decrease by solving the Feasible Solution Step more efficiently. For generated datasets, additional w-constraints are not needed because approximation of the curves are convex.

Additional $\mathbf{x}$-constraints: Additional x-constraints are added for the same idea with additional w-constraints to the approximation step. In this application, we get rid of all the w-constraints, however, for making a better approximation we add additional constraints to related to x decision variables. Normally, we should accept the bids on one side of the MCP of that hour, and reject the other side. We can have at most one partially accepted bid for each side at each hour. The previous segments of the partially accepted segment should be accepted and the succeeding ones should be rejected. Getting rid of w-constraints overrides this necessity and the solution acceptance rates could become arbitrary. Our new proposed constraints are as follows:

$$x_{i(l+1)} \geq x_{il}, \quad i \in H, l = \{1, ..., |L_i| - 1\}.$$

These types of constraints still do not guarantee feasible solutions for hourly bids, but gives better approximated solutions than not having them. In highly non-convex datasets consecutive bids can become accepted-rejected-accepted, and with additional x-constraints, the new sequence would be accepted-partial-partial.

Fixing block bid values: Because block and flexible bids are substantial reasons for increased solution times, getting them fixed may improve the solution time of the model. We experiment with fixing block bids or giving them as initial solutions, and both methods prove inefficient. Fixing block bids improve the solution times tremendously, however, can lead to infeasibilities or suboptimal solutions frequently. If we directly give block bids' values obtained after approximation step as an initial solution to the system, without using s-constaint method, demand-supply balance constraints may lead to infeasibilities againg and the initial solution may be ignored by the solvers.

Relaxing the quadratic terms: Another improvement that can be made is getting rid of quadratic objective terms. We can have partial acceptance rates for our hourly bids in demand and supply sides. Every hour can have at most two partially accepted segments, one from the demand side and one from the supply side. For example, if we have 500 segments for every hourly bid curves, it makes 24,000 quadratic objective terms while at most 48 of them will get partial values and other 23,952 of them will get binary values. For the today's solvers, dealing with quadratic terms are much slower than dealing with their linear counterparts. To overcome this difficulty, we change the quadratic terms cx^2 with their linear counterpart cx. Our observation is that quite good solution time improvements can be achieved without losing accuracy of the optimal solution. For example, in one dataset, using relaxed linear terms rather than quadratic objective function terms has improved solution time more than 240 seconds.

5 Results

In this section, the results of the proposed method will be presented and discussed.

5.1 Data

In Chapter 4, we discussed the important features of the datasets we have and conclude that the number of segments of the hourly bid curves and the number of block bids are the two main factors that affect the solution times the most.

In this section, we consider the 500-segment datasets because they are the most representative of the real data used in Turkish DAEM. Real dataset demand and supply bid curve segment numbers are between 200 and 400 and because of that we solved the 81 data with 500 segments we have. Furthermore, we disregard the cases with demand-supply ratio equal to 1.0, because their solution times are under 5 seconds most of the time with the model presented in Chapter 3 so that we do not observe any significant time-saving with any of our methods. Disregarding those 27 datasets with ratio of 1.0, we work with the remaining 54 generated datasets with 500 segments and block bid ratio of 0.5 and 0.75.

All the solution times recorded in this section are performed on 64-bit server with Intel Core i5-5200U processor with a speed of 2.19GHz and 4GB RAM. GUROBI 7.0.1 is utilized for the optimization purposes. In this section, step 1, step 2 and step 3, refer to the approximation step, feasible solution step and optimality solution step of the three step approach, respectively.

Our main model is the model used by EXIST at the moment, and when the main model is discussed, it refers to the model without any methodologies used. Figure 5.1 shows the solution times of the data with the main model. Since the time limit is 600 seconds, we cut off the solutions at time 600. However, some of them do not reach to optimal even in 30 minutes.

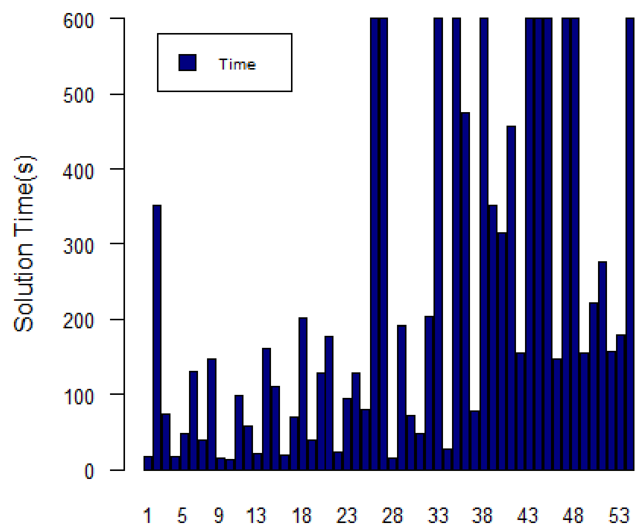

Figure 5.1: Solution times of the main model

5.2 Comparison of Methods

In Figure 5.2, solution times of delta method with delta value 5 is given. In all the cases a solution time improvement is observed. In 3 cases, even with the small value of 5, finding the optimal solution in the subset of feasible region was fairly slow. The main reason for that slow down is that, GUROBI cannot lower the best bound fast enough and reaching the 10^{-6} relative gap was hard because of the tailing effect of MIP.

In Figure 5.3, solution times of delta method with the delta value of 20 are given. As we can observe, some of the solution times of second step increase significantly. One reason for that change is that we increase the feasible region of the second step, so more time is needed to find the optimal solution in that sphere. Increasing the delta value usually makes the step 2 slower but the step 3 faster because we increase the chance of finding the optimal solution at the end of the second step. At the end, if user chooses the delta value of 20, two of the datasets cannot be solved to optimality in 10 minutes.

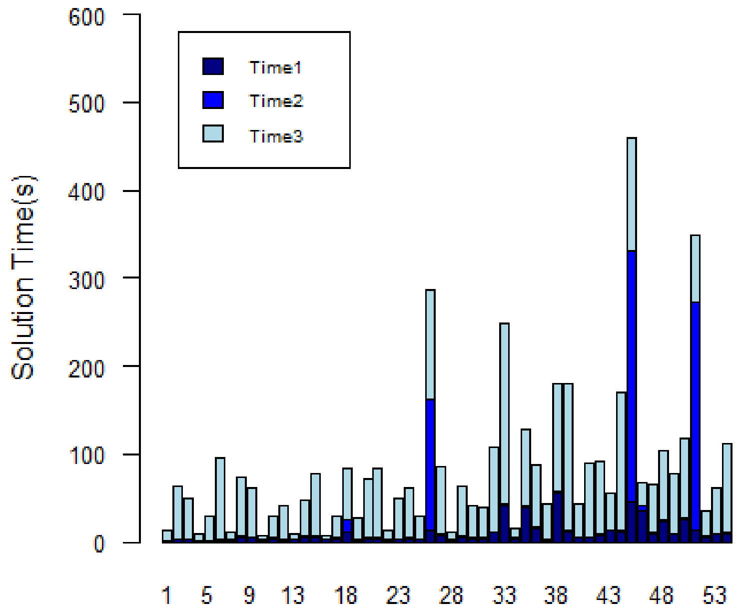

Figure 5.2: Delta method solution times with delta value 5

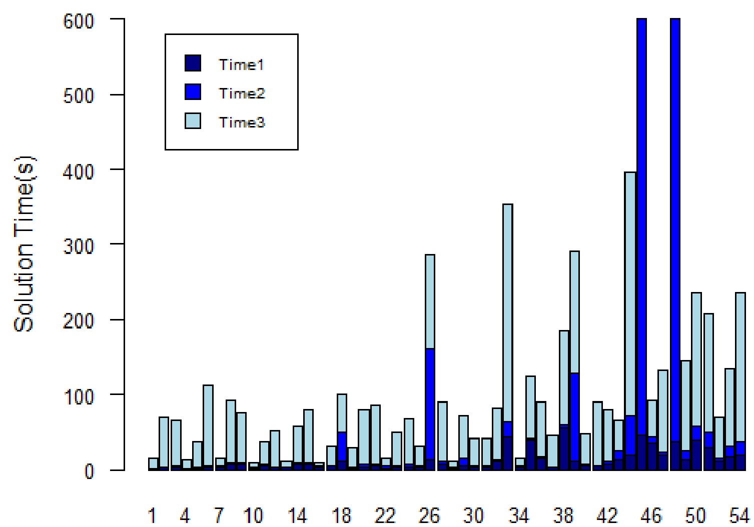

Figure 5.3: Delta method solution times with delta value 20

For the delta method, finding a good delta value is crucial, if too small values are selected then we have infeasibility problem, and if we pick too large values then the solution times increase. For example, choosing 100 for our delta value affects the effeciency of the program significantly.

In Table 5.1 we report average solution times of each step for different values of delta. The final absolute gap of step 2 is the objective function value difference of our solution at the end of step 2 to the optimal value of that data. Solved to optimality at step 2 means that, if the solution of the step 2 is the optimal solution of the system and the step 3 is unnecessary.

We expect to see solution time increase in step 2 and decrease in step 3 while we are increasing our delta value. If we select delta as 5, we can solve all 54 data to optimality at the end, but we find suboptimal solutions at the end of the step 2 and improvements are needed at the step 3. The average of 468 unit distance is observed between the solutions obtained at the end of step 2 and step 3. In contrast to that, if we select 100 as our delta value, we can only solve 88% of all the data, however, at the end of the step 2, if a solution is reached, it is the optimal of the data.

Table 5.1: The results of delta method with different delta values

Delta Values	5	20	100
Solution time of step-1(s)	9.53	11.9	13.2
Solution time of step-2(s)	13.91	31.18	80.97
Solution time of step-3(s)	58.73	55.2	49.62
Solved to optimality at step 3	100%	96%	88%
Solved to optimality at step 2(solved instances)	85%	94%	100%
Final absolute gap of step 2(solved instances)	468	53	0

In Table 5.2, sigma method, SC methods with quadratic terms and relaxed terms are compared with each other. Sigma method gives the best performance in step 2, since it stops when it finds the first feasible solution to the problem. The obtained solution may not be close to the optimal solution. Therefore, step 3 solution times of sigma method are the highest since it needs to reach to the optimal solution from a far away initial solution.

One interesting observation is that changing the quadratic terms to linear terms in the SC method results significant solution time improvement in some of the instances. Because we have quadratic terms in our objective function, our main model is a Mixed Integer Quadratic Program (MIQP). If we remove the quadratic terms from the objective function, GUROBI switches its tactics from solving MIQP to MIP and in some of the instances this switch results decreased solution time. At the end, the average of 8 seconds of improvement can be observed at step 2. Only in 1 of 54 datasets SC-Linear cannot find the optimal solution at the end of the step 2, and that improvement is less than 10^{-5} relative gap from the objective function. With that in mind, if the methodology used is SC, step 3 may become unnecessary for the problem instances for which step 2 ends up with the optimal solution to the original problem.

Table 5.2: The results of sigma, SC-Linear and SC-Quadratic methods

Methods	SC-Quad	SC-Linear	Sigma
Solution time of step-1(s)	8.02	9.01	12.24
Solution time of step-2(s)	16.51	8.52	7.7
Solution time of step-3(s)	58.70	45.7	140.3
Solved to optimality at step 3	100%	100%	96%
Solved to optimality at step 2(solved instances)	100%	98%	0%
Final absolute gap of step 2(solved instances)	0	78	-

In Figure 5.4, we give the average solution times of the SC model with linear terms. Time 1, Time 2, and Time 3 represent the solution times of our three-stage solution approach, approximation step, initial feasible step and optimality check step respectively. This method shows the best performance in terms of the solution time. Although the step 3 solution times are the longest, they are usually for proving the optimality and the improvements observed are insubstantial. Therefore, if a good speedup is needed, the solution found after the step 2 can be used as a good quality solution.

In Figure 5.5, the average solution times of each step under different methods are reported. While the SC method and delta method is competitive, sigma method is not competitive because the initial solution found in the step 2 is not close to an optimal solution, meaning that step 3 has to run for a long time to reach the optimal solution. As we can see, finding a quality initial solution is more important than finding any initial solution.

Table 5.3: The results of SC with linear terms versus main model

Methods	SC-Linear	Main Model
Solution time(s) (solved instances)	17.52	128.05
Solved to optimality	100%	79%

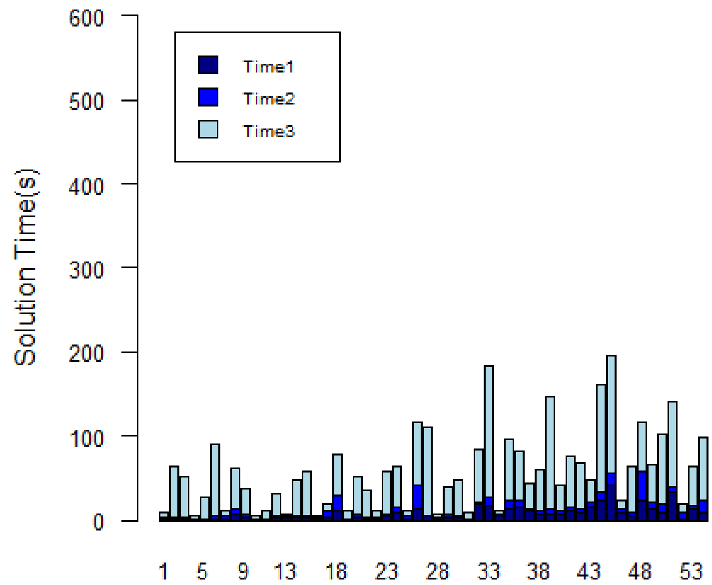

Figure 5.4: Solution times of the SC model

In Table 5.3, we compared the best of our model with these datasets and the main model without any methodologies implemented. Solution time of SC-Linear is the summation of step 1 and step 2 solution times because with the exception of one data point relaxing the quadratic terms did not change the optimal solution. Because of that, we can use the optimal solution of step 2 as the optimal solution of the system. Step 3 solution times can be observed in Figure 5.2. In all of the datasets using SC method with linear terms results in improved solution times and the eleven datasets that cannot be solved to optimality in 10 minutes before, can be solved. Average time improvement is more than 100 seconds for the instances that are solved to optimality, only in one dataset step 3 improved the solution of SC linear terms, and the value of less than 100 can be negligible in that case.

In all the methodologies implemented, we can beat the main model. Delta and sigma methods focus on specific subsets of the feasible region so that big-M values are tightened, and w-constraints are relaxed. In the SC method, we give an initial solution to both step 2 and step 3. In those steps, we ease the hardness of dealing with integer decision variables by fixing them in the initial solution, and this way solving the problem becomes easier. In the end, in our methodologies, the hardness coming from integer decision variables or the non-

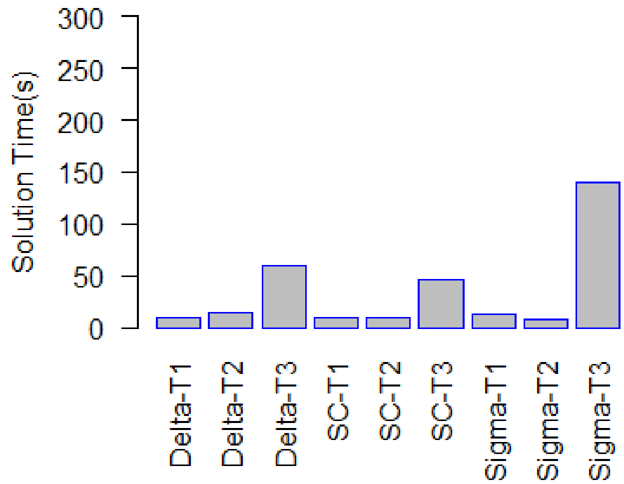

Figure 5.5: Comparing the solution times of three different methods

convexity curves of hourly bids are alleviated and that results in obtaining better solution times for the Turkish DAEM clearing problem.

Another advantage of our methodologies is that we can obtain a good solution at the end of the step 2 with great efficiency. This is particularly important in DAEM because tentative MCP values are shared with the bidders before the finalized MCP values. Because of that, our 3-stage approach may be a good tool to implement and our step 2 solution can be used as the tentative solution of the system and then the finalized solutions can be acquired from the step 3.

6 EMM Method

Mehdi Madani and Van Vyve [1] introduced a cut based approach to solve European DAEM problems. In this thesis, we implemented their approach to provide a time comparison for our three-stage approach method. Additionally, we investigated the feasibility of applying a similar approach to Turkish DAEM. Their method named European Market Model - mixed integer quadratically constrained program (EMM-QUAD-MIQCP) exploits the nature of the dual problem and solves the DAEM problem in European market setting without introducing additional auxiliary variables. Linked and flexible bids are omitted in this approach.

The main model can be written without market specifications, and we will call this model the main primal problem. For the sake of making the notations more concise, quantities of supply bids are counted negatively. This is the main primal model:

$$\text{maximize} \sum_{i \in H} \sum_{l \in L_i} \Delta Q_{il} P^0_{il} x_{il} + \sum_{b \in B} y_b \Omega_b \tag{11}$$

subject to

$$\sum_{\substack{i \in H: \\ t_i = t}} \sum_{l \in L_i} \Delta Q_{il} x_{il} + \sum_{b \in B} y_b Q_b \delta_{bt} = 0 \quad t \in T \quad [MCP_t] \tag{12}$$

$$x_{il} \leq 1, i \in H, l \in L_i \quad [s_{il}] \tag{13}$$

$$y_b \leq 1, b \in B \quad [s_b] \tag{14}$$

$$x_{il} \geq 0, y_b \in Z \quad i \in H, l \in L_i, b \in B \tag{15}$$

When we consider the dual of the main primal problem:

$$\text{minimize} \sum_{i\in H}\sum_{l\in L_i} s_{il} + \sum_{b\in B} s_b \tag{16}$$

subject to

$$s_{il} + \Delta Q_{il} MCP_{t_i} \geq \Delta Q_{il} P^0_{il} \quad i \in H, l \in L_i \tag{17}$$

$$s_b + \sum_{t\in T} MCP_t \delta_{bt} Q_b \geq \Omega_b \quad b \in B \tag{18}$$

$$s_{il}, s_b, MCP_t \geq 0 \quad i \in H, l \in L_i, b \in B, t \in T \tag{19}$$

And the complementarity conditions that follow:

$$s_{il}(1 - x_{il}) = 0 \quad i \in H, l \in L_i \tag{20}$$

$$s_b(1 - y_b) = 0 \quad b \in B \tag{21}$$

$$x_i(s_{il} + \Delta Q_i MCP_{t_i} - \Delta Q_{il} P^0_{il}) = 0 \quad i \in H, l \in L_i \tag{22}$$

$$y_b(s_b + \sum_{t\in T} MCP_t \delta_{bt} Q_b - \Omega_b) = 0 \quad b \in B \tag{23}$$

Madani and Van Vyve's EMM-QUAD-MIQCP formulation aggregates all three forms into one mathematical model, dual constraints of block bids are altered to make it suit to European Electricity Market. In European model, if block bids do not generate welfare they have to be rejected, if they generate welfare then they could be either accepted or rejected. Then the last version of the constraint becomes:

$$s_b + \sum_{t\in T} MCP_t \delta_{bt} Q_b \geq \Omega_b - M_b(1 - y_b) \quad b \in B \tag{24}$$

With replacing the constraint (18) by (24), revised model finds the optimal solution to the European Electricity Market, without flexible and block bids. Constraints (26) and (36) are combining the objective functions of primal and dual models using the strong convexity idea. We have implemented this idea to the Turkish Electricity Market with small changes as it is described in the next subsection.

6.1 EMM-QUAD-MIQCP-TR

$$\text{maximize} \sum_{i \in H} \sum_{l \in L_i} \Delta Q_{il}(P^0_{il} x_{il} + \frac{x^2_{il}}{2} \Delta P_{il}) + \sum_{b \in B} y_b \Omega_b \tag{25}$$

subject to

$$\sum_{i \in H} \sum_{l \in L_i} \Delta Q_{il}(P^0_{il} x_{il} + \frac{x^2_{il}}{2} \Delta P_{il}) + \sum_{b \in B} y_b \Omega_b \geq \sum_{i \in H} \sum_{l \in L_i} s_{il} + \sum_{b \in B} s_b - \sum_{i \in H} \sum_{l \in L_i} \Delta Q_{il} \frac{x^2_{il}}{2} \Delta P_{il} \tag{26}$$

$$\sum_{\substack{i \in H: \\ t_i = t}} \sum_{l \in L_i} \Delta Q_{il} x_{il} + \sum_{b \in B} y_b Q_b \delta_{bt} = 0 \quad t \in T \tag{27}$$

$$x_{il} \leq 1 \quad i \in H, l \in L_i \tag{28}$$

$$y_b \leq 1 \quad b \in B \tag{29}$$

$$s_{il} + \Delta Q_{il} MCP_{t_i} \geq \Delta Q_{il}(P^0_{il} - \Delta P_{il} x_{il}) \quad i \in H, l \in L_i \tag{30}$$

$$\Omega_b - \sum_{t \in T} \delta_{bt} MCP_t Q_b \leq s_b \quad b \in B \tag{31}$$

$$s_b \leq M y_b \quad b \in B \tag{32}$$

$$s_b \geq -M y_b \quad b \in B \tag{33}$$

$$x_{il}, s_{il}, MCP_t \geq 0, y_b \in \{0, 1\}, s_b \quad free, b \in B, i \in H, l \in L_i, t \in T \tag{34}$$

In EMM-QUAD-MIQCP-TR model, we need to ignore the last complementary constraint to force the Turkish DAEM requirements. Dual of quadratic objective function is used to write constraints of the model [20]. Additionally, since the ITM block bids must be accepted in Turkish DAEM, different than European DAEM [1] specifications, modifications are made in block bid constraints. In this new set of constraints we free the decision variable s_b, furthermore, in constraints (32) and (33) we force s_b to 0 if a block bid is rejected. If a block bid is accepted while they are OTM, then s_b takes the value of loss of welfare and compensate the loss in constraint (26), if they are ITM and accepted s_b takes the value of gained welfare and at the end strong duality condition is satisfied.

Similarly we can create the same model with step-wise bid curves. Only changes are due to the change in objective function and from its duality.

6.2 EMM-MILP-TR

$$\text{maximize} \sum_{i \in H} \sum_{l \in L_i} \Delta Q_{il} P_{il}^0 x_{il} + \sum_{b \in B} y_b \Omega_b \tag{35}$$

subject to

$$\sum_{i \in H} \sum_{l \in L_i} \Delta Q_{il} P_{il}^0 x_{il} + \sum_{b \in B} y_b \Omega_b \geq \sum_{i \in H} \sum_{l \in L_i} s_{il} + \sum_{b \in B} s_b \tag{36}$$

$$\sum_{\substack{i \in H: \\ t_i = t}} \sum_{l \in L_i} \Delta Q_{il} x_{il} + \sum_{b \in B} y_b Q_b \delta_{bt} = 0 \quad t \in T, l \in L_i \tag{37}$$

$$x_{il} \leq 1 \quad i \in H, l \in L_i \tag{38}$$

$$y_b \leq 1 \quad b \in B \tag{39}$$

$$s_{il} + \Delta Q_{il} MCP_{t_i} \geq \Delta Q_{il} P_{il}^0 \quad i \in H, l \in L_i \tag{40}$$

$$\Omega_b - \sum_{t \in T} \delta_{bt} MCP_t Q_b \leq s_b \quad b \in B \tag{41}$$

$$s_b \leq M y_b \quad b \in B \tag{42}$$

$$s_b \geq -M y_b \quad b \in B \tag{43}$$

$$x_{il}, s_{il}, MCP_t \geq 0, y_b \in \{0,1\}, s_b \quad free, b \in B, i \in H, l \in L_i, t \in T \tag{44}$$

With current solvers, these problems take a lot of time to find the optimum. Madani and Van Vyve [1] add cuts to the system for acquiring a speed boost. The model finds a feasible solution first, then if it qualifies the restrictions of the European Market we conclude that it is the optimal solution, if it does not qualify, then local cuts are added to the model. Cuts are added such that the optimal solution will be cut from the system. After obtaining the optimal solution, Farkas' lemma is used to check whether the solution satisfies the European market specifications or not. If not, then local cuts added with lazy heuristics callback and until we find an optimal solution satisfying European market specifications we continue adding cuts.

Ultimately, we change their model to the Turkish model and compare the results of both models. EMM-QUAD-MIQCP-TR method cannot be solved without further improvements in today's solvers. We solve the EMM-MILP-TR and EXIST's main model for comparison purposes. Flexible and linked bids are ignored in these results, and step-wise curves are used as the main objective function.

Table 6.1: The results of EMM-MILP-TR and EXIST's main model

Methods	EMM-MILP-TR	Main Model
Solution time(s) (solved instances)	88.2	90.4
Solved to optimality	79%	79%

In Table 6.1, we can observe the solution times of both models are close to each other. We decide to use EXIST's model in our results section because that model has flexible and linked bids. Also, with w-constraints, EXIST's main model can be used to achieve better time saves in the approximation step than EMM-MILP-TR model.

7 Conclusions and Future Research Directions

Finding the MCP values in DAEM more efficiently would lead higher bidder satisfaction and better welfare outcomes. Bidders can participate in more effective decision-making processes if the MCP values are given them earlier. Because the MCP values are calculated every day, coming up with novel ideas to quicken this process is a top priority. We have investigated one idea which can be used to reach better solution times for this purpose.

We examine the approximated data to estimate where the optimal solution might lie in the feasibility region. Although, we do not obtain a feasible solution, we find a place to start for seeking a feasible solution. As long as the approximation is a good estimate of the real data, our solution times are highly competitive. We implement this idea in three steps; approximation step, finding a feasible solution step, and reaching the optimality step. In all datasets, applying this idea yield better solution times. Especially, if the data points are suitable and create "nice" piece-wise linear curves, then our solution finds the optimal solution after the second step and the last step only serves the purpose of proving optimality.

In this thesis, we conclude that SC-linear method works the best comparing the solution time of all the methodologies we implement at step 2. Sigma method gives the quickest feasible solutions, however the quality of the solutions are subpar compared to the other methodologies we implement. Additionally, delta method can be used efficiently if the feasibility problem can be overcome, or in the case that delta values with feasible solutions can be identified.

Apart from the improved solution times, one additional advantage of this method is the flexibility of the idea. The method itself is tremendously adaptable to different or more experimental approaches of the electricity market. Therefore, the addition of new constraints, new cuts, new decision variables and so on would not interfere with the main idea of using approximated data to lead to the optimal solution. Therefore, different models and markets may observe improvement in solution time with implementing the ideas presented in this thesis. This is because our idea changes the way the data is read, but do not change the constraints themselves directly, with the exception of w-constraints.

One future research area could be the addition of cuts to Turkish DAEM. We find the main model as EMM-QUAD-MIQCP-TR, so the addition of flexible and linked bids would be the last step to implement the cuts to the Turkish market. Furthermore, even though we work around the value of delta issue, coming up with tighter values for delta which gives a feasible solution in the respective region would improve the solution time even more. At the moment, we plan to work on these two future research subjects in Turkish DAEM.

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8 Appendix: CPLEX and GUROBI Results

In this section we will give the objective function values and solution times for both GUROBI and CPLEX implementations. Both used the SC method with 10^{-6} optimality gap, and modifications in the methodologies section are implemented. Tables are divided according to their number of block bids, 8.1 has the 18 datasets with 200 block bids, 8.2 has the 18 datasets with 500 block bids and finally 8.3 has the 18 datasets with 1,000 block bids. Both GUROBI and CPLEX reached the optimal value after the second step of our algorithm. Time1 and Time2 values represent the step 1 and step 2 of our solution approach, respectively.

Table 8.1: The results of GUROBI and CPLEX for instances with 500 segments and 200 block bids.

	GUROBI		**CPLEX**	
ID	Time1(s)	Time2(s)	Time1(s)	Time2(s)
1	1	4	2	7
2	4	1	1	6
3	2	2	1	6
4	1	2	1	8
5	1	2	1	7
6	3	4	2	9
7	1	6	1	8
8	7	13	3	8
9	3	5	1	7
10	2	2	1	8
11	1	1	2	7
12	4	2	1	8
13	3	3	3	8
14	5	2	1	8
15	4	2	2	6
16	1	2	1	9
17	4	9	2	8
18	13	19	6	14

Table 8.2: The results of GUROBI and CPLEX for instances with 500 segments and 500 block bids.

	GUROBI		**CPLEX**	
ID	Time1(s)	Time2(s)	Time1(s)	Time2(s)
19	1	1	1	10
20	5	4	3	13
21	2	3	2	11
22	2	3	2	11
23	7	2	1	14
24	12	6	4	13
25	3	5	16	11
26	18	40	6	50
27	6	2	1	7
28	3	2	2	11
29	4	4	2	9
30	4	2	1	8
31	1	1	1	8
32	23	2	1	8
33	22	15	7	11
34	6	3	2	8
35	18	10	4	13
36	22	10	4	17

Table 8.3: The results of GUROBI and CPLEX for instances with 500 segments and 1,000 block bids.

	GUROBI		**CPLEX**	
ID	Time1(s)	Time2(s)	Time1(s)	Time2(s)
37	12	4	1	10
38	10	4	2	11
39	8	9	4	12
40	11	6	2	11
41	14	8	2	10
42	11	7	3	14
43	19	7	4	35
44	29	14	12	75
45	48	15	21	38
46	11	4	7	17
47	7	4	5	12
48	27	49	32	19
49	10	8	3	16
50	7	10	9	14
51	41	7	8	12
52	3	7	27	10
53	16	5	6	12
54	10	18	8	14