LIMITED CONSIDERATION AND ASPIRATION BASED DECISION-MAKING IN SIMPLE ALLOCATION PROBLEMS

by

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Abstract

In this thesis, we carry out a revealed preference analysis on a class of solutions to simple allocation problems. We adapt two different models of decision making into the simple allocation problems literature. We introduce and characterize the Limited Consideration Choice Model. In this model, a constraint function generated from the characteristic vector narrows down the feasible alternatives by forming the consideration set. The intuition behind this model is that the decision maker is not aware of all feasible alternatives and a reference point can change the choice of the decision maker. We next define some consistency and fairness properties over the constraint function and analyze their implications over the Limited Consideration Choice Models. We next introduce and characterize an Aspiration Based Choice Model. In this model, a distance minimization procedure between the aspiration point that is endogenously determined within the model and the feasible set of alternatives is used. The intuition behind this model is capturing the notion of resemblance in the mind of the decision maker using a subjective distance function. We prove that using the P-norm metric as in the Yu family used in the bargaining literature with $p > 1$ within this model gives the Constrained Equal Losses Rule.

BASİT DAĞITIM PROBLEMLERİNDE KISITLI DEĞERLENDİRME VE ESİNLENME TEMELLİ KARAR VERME

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Özet

Bu tezde, basit dağıtım problemlerinin bir çözüm kuralları sınıfı üzerinde açıklanan tercih analizi uyguladık. İki farklı karar verme modelini basit dağıtım problemleri literatürüne adapte ettik. Brinci olarak, Kısıtlı Değerlendirme Seçim Modelini sunduk ve karakterize ettik. Bu modelde, karakteristik vektörün bir fonksiyonu olan kısıt fonksiyonu, değerlendirme kümesini oluşturarak, ulaşılabilir alternatifler kümesini küçültür. Bu modelin temelini oluşturan düşünce sistemi, karar vericinin tüm ulaşılabilir alternatiflerinin farkında olmadığı ve bir referans noktasının onun seçimlerini değiştirebileceği varsayımlarına dayalıdır. Daha sonra Kısıtlı Değerlendirme Modeli içinde tutarlılık ve adillik analizleri yapabilmek için kısıt fonsiyonları üzerinde birtakım özellikler tanımladık. İkinci olarak Esinlenme Temelli Seçim Modelini sunduk ve karakterize ettik. Bu modelde, endojen biçimde modelin içinde belirlenen esinlenme noktası ile ulaşılabilir alternatifler arasındaki mesafeyi minimize etme prosedürü kullanılır. Bu modelin temelini oluşturan düşünce sistemi, subjektif bir mesafe fonksiyonu kullanarak, karar vericinin zihnindeki benzerlik nosyonunu yakalamaktır. Bu modelde $p > 1$ sağlanırken P-norm metriklerinin kullanımının Kısıtlandırılmış Eşit Kayıplar Kuralını vereceğini ve bu modelin pazarlık problemleri literatüründeki $p > 1$ parametresine göre Yu çözüm ailesine karşılık geleceğini kanıtladık.

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1 Introduction

Revealed preference theory is based on the idea that the preferences of consumers can be revealed by analysis of observable data on choice behavior. The first description of the concept was by Samuelson (1938). It was followed by an enormous amount of studies departing from the extensions and the refinements of the original idea. The main concern of the theory was to describe the economical environment where the observable data on the choice are consistent with the utility maximization behavior, and to understand the conditions and situations under which one can forecast or reveal additional information on the demand. The concept became one of the most influential idea of the economics.

Revealing information about preferences departing from the choice is applicable to a wide range of choice situations. On the application of the theory to bargaining games, Nash (1950) worked on the bargaining rules that can be said rational in the sense that they are maximizers of an underlying preferences of an impartial arbitrator. Revealed preference literature also focuses on group preferences and group choices motivated by social choice considerations or game-theoretic considerations.

We follow a revealed preference approach on a class of solutions to simple allocation problems. We are interested in the following simple allocation problem. A social endowment of E is to be allocated among N agents. Each agent i has his characteristic value c_i. A social endowment E can be interpreted as a divisible commodity to be divided between the members of the society. Each agent is characterized by c_i amount of that commodity. Simple allocation problems applies to many economic models. They can be interpreted differently according to the related field. Under **permit allocation** literature, we consider the situation where The Environment Protection Agency is to allocate E amount of pollution permits to the N many firms. Each firm i has an emission constraint c_i depending on its location as directed by local authorities. In the study of the **single peaked or saturated preferences**, there is a social planner in charge of allocating E units of a perfectly divisible commodity among members of N. In this case, each agent i have preferences with saturation point c_i. The rest of the preference information is disregarded. In **taxation**, a simple allocation problem of the public authority that collects E amount of tax from the society of N agents is considered. In this framework, c_i indicates the income level of agent i. This problem is very well-known and basic in public finance. A simple allocation problem can be adopted to the **demand rationing** problem where a supplier is to allocate its total amount of production E among the N demanders. Each demander i demands c_i units of the commodity. This problem is analyzed in detail in the context of supply-chain management literature. **Bargaining with quasi linear preferences and claims** can be interpreted as an arbitrator in charge of allocating E units of a numeraire good among the agents that have quasi linear preferences. Each agent i has a claim c_i that he wants to receive. **Surplus sharing** problems can be analyzed as a simple allocation problem. A social planner is to allocate the return of a project between the N many investors. The size of the return is E. Each of the investors is defined by their invested amount s_i. The total return of the company equals or is bigger than the total invested amount by investor. The project is profitable. In this case $c_i = E - \sum_{N \backslash \{i\}} s_j$ is defined as the maximal share of the agent i. Note that $\sum_N c_i \geqq E$. A **consumer choice under fixed prices and rationing** is another possible application field. A consumer has to allocate his income E among a set of N many commodities. In this case the prices are fixed. There are also rationing constraints on how much the consumer can consume of each commodity as in the fixed-price literature. The agent faces a consumption constraint c_i on commodity i.

We emphasize the application to **bankruptcy** problems where a bankruptcy judge decides how to allocate the remaining liquidation value of the bankrupt firm among its N many creditors. Each creditor claims his credited value. We interpret an allocation rule as a positive construct representing the choices of a decision maker or a bankruptcy judge. We take Kıbrıs (2012, 2013) as a framework on the analysis of the conditions under which the bankruptcy judge's decisions are rational. The rationality concept is stated as the existence of a binary relation $B \subseteq \mathbb{R}_+^N \times \mathbb{R}_+^N$ such that maximizers of the binary relation are the same as the allocation rule's choices for each bankruptcy problem. An allocation rule F is rational if and only if it satisfies a standard property called WARP (The Weak Axiom of Revealed Preferences) or equivalently contraction independence. Furthermore, on the paper it was proved that almost every member of the best-known allocation rules family such as the Proportional rule, the Equal Losses rule and the Talmud rule violates contraction independence hence they are not rational in the framework of the standard theory with the exception of the Equal Gains rule. In the light of the facts mentioned above, the rules that are commonly used in practice or had an important place in the literature as well-known examples can not be rationalized by standard revealed preference theory.

To be able to deal with this problem, we extend the basic setup of the standard revealed preference analysis on solutions to simple allocation problems. We propose to adapt two different models of decision making process into the existing literature.

Our first model is an adaptation of the Limited Attention and Status Quo Bias

Choice Theory (Dean et al (2014)) from behavioral choice literature. The central concept is that a decision maker can not evaluate all feasible alternatives and only considers a limited subset of the alternatives. Additionally, a status quo can change the decision between non status quo alternatives.

Our first model incorporates this idea by constructing a mapping showing the allocations that are considered by the decision maker. In contrast to other decision models in hand, our consideration set is a function of the characteristic vector. Then the decision maker selects the allocation that maximizes a strictly monotone preference relation among the feasible alternatives that are in the consideration set.

Theorem 3 states that any allocation rule F can be rationalized with Limited Consideration Model. Proposition 5 states that if an allocation rule F is consistent with Limited Consideration Model and its constraint function satisfies a property that we defined as consistency, then it is rational. We observe that the constraint function that is consistent with Equal Losses, Equal Gains, Serial Dictatorship and Proportional Rule satisfies the property of Equal Treatment of Equals. We define a fairness criteria to order the Limited Consideration Choice Models with the constraint functions that are consistent to some well-known allocation rules in that sense. Proposition 9 states that the constraint function that is consistent to Proportional rule constitutes the most fair limited consideration model according to our scale and it is followed by the model consisting of the constraint function that is consistent with Equal Gains, Equal Losses and Serial Dictatorship allocation rules given respectively to the order.

Our second model is an adaptation of Aspiration-based choice theory (Güney et al (2011)) from the choice theory literature and the concept of Social compromises (Yu (1973), Conley et al (2000), Pfinsgsten et al (2003)) from the bargaining solutions literature. The main idea behind the Aspiration-based choice theory is that decision makers are in an environment such that a possibly unattainable aspiration alternative influences their decisions. A choice problem in this setting is a pair (S, Y) where $S \subseteq Y \subseteq X$ with S being the set of actual alternatives that are available to choose, Y being the potential set of alternatives that can include unavailable alternatives. The grand set of alternatives is X. The decision maker observes the potential set of alternatives but he needs to choose an alternative from the actual set of alternatives. There is a possibly unavailable aspiration point chosen from the set Y. In the decision making process, the decision maker maximizes his preference relation over the potential set of alternatives to form the aspiration alternative. Then the decision maker selects the closest alternative in the actual set of alternatives to the his aspiration point. The closeness is determined by a subjective and endogenous distance function.

Similarly, there is an analogous decision process behind the concept of Social compromises in the bargaining solutions. In bargaining problems, agents can obtain any of the alternatives in the feasible set. They have different preferences over the feasible alternatives. If they come to an agreement on a particular alternative, that is what they get. If there is no agreement, they end up at the disagreement point that is a prespecified feasible alternative. In this context, there is an utopic ideal point that bargaining parties ideally wants to achieve. The ideal point acts like a reference point. The main idea is having a decision rule that selects the closest point to the unachievable reference point among all feasible alternatives. In that sense, bargaining parties minimize their compromises by decreasing the difference between what they actually want and what they get.

In our case, we try to adapt social compromises or aspiration based choice models into the simple allocation problems literature. We model an environment in which the characteristic vector acts like an aspiration or a reference point that influences the decision process. The selection of the claim vector as a reference point is based on the idea of a decision maker maximizing a strictly monotone preference relation over grand set of alternatives. The grand set of alternatives consists of the allocations such that every agent receives less than or equal to their characteristic value. The decision maker selects the closest allocation to the characteristic vector among the feasible alternatives according to a subjective distance function which captures the notion of resemblance in his mind.

Theorem 11 states that the Equal Losses rule is consistent with Aspiration Based Choice model and Yu solutions from bargaining literature with P-norm metric such that $p > 1$ gives Equal Losses allocation rule in our domain.

The rest of the paper is organized as follows. In section 2, we provide a review of the literature. In section 3, we provide the general framework of revealed preference analysis on simple allocation problems. We refer to Kıbrıs (2012) for reminding the properties of rational rules and restate the necessary and sufficient conditions for rationality. We provide the general forms of the well-known allocation rules from the literature. In section 4, we introduce our Limited Consideration Choice Model. We present our main characterization result. We impose some properties over constraint function and analyze the constraint function that are consistent with some well known allocation rules that behaves accordingly to our properties. In section 5, we introduce our Aspiration Based Choice Model. We provide the general framework. Finally, we conclude in Section 6.

2 Literature Review

The revealed preference notion has been introduced by Samuelson (1938). The assumption of the gathering information about preferences based on the choices has been applied in a wide range of the choice situations. Early works on this field aim to characterize the demand and forecast the chosen consumption bundles in any possible budget set using revealed preference analysis on theoretical framework (Samuelson (1953), Yokoyama (1953), Newman (1960)). In the empirical field Koo (1963) used household data to find what can be called rational behavior, Miller (2002) used public good experiments under the same objective. The purpose was characterizing data sets that fits the idea of decision makers maximizing preferences.

Following the main idea behind the revealed preferences theory, Houthakker (1950) tried to find the necessary and sufficient conditions for the existence of a utility function that represents well-behaved preferences. The utility function should also be compatible with the choice behavior of the decision maker. The aim here was to generalize the law of demand. Strong axiom of revealed preference stated by Houthakker provides the possibility of testing the data set to see if it is consistent with the theory of the consumer. Chernoff (1954) and Sen (1969) characterized the axiom of independence of irrelevant alternatives on choice rules in the context of individual choice. A choice rule satisfies this axiom if an option chosen from a set for a decision problem is also chosen from decision problems with the subsets of that original problem. Richter (1966) focused on the conditions that are necessary for the rationality of a consumer. These axioms defined the conditions under which standard economic theory can identify preferences in the mind of the decision maker by looking at the choice behavior of that agent. Varian (2005, 2006) provided a detailed survey on revealed preference theory and analyzed the development of the literature. Chambers et al (2010) generalized the classical revealed preference theory and obtained applications to the theory of group preferences and Nash equilibrium. This concept applies to a vast range of choice situations.

Simple allocation problems literature is devoted to finding a fair way to divide a certain amount of value or asset among its claimants. Each claimant has a nonnegative claim over remaining asset. The remaining value is insufficient to satisfy all the claims. Bankruptcy problems are one of the application field of this sort of simple allocation problems. This literature was originated in a fundamental paper by O'Neill (1982). The objective of O'Neill was finding applicable and well-behaved rules that provide some consistency properties. The purpose was narrowing down the number of the acceptable division rules departing from the Talmud. Aumann and Maschler (1985) provided a game theoretical analysis of bankruptcy problems. They generated a new allocation rule. Dagan (1996) contributed to the literature by making axiomatic characterization of the two Talmudic bankruptcy rules. In this characterization, the property of independence of irrelevant claims is defined. This axiom requires that an allocation rule can only be a function of the set of allocations as a characterization of Equal Gains Rule. Kıbrıs (2012) followed a revealed preference analysis on simple allocation problems. The purpose was finding the necessary conditions to have decision makers with rational and representable choice rules. Rationality of an allocation rule was defined with the existence of a binary relation which has a unique maximizer over its sets of feasible allocations that coincides with the chosen option by the allocation rule. To reach this goal, the axiom of independence of irrelevant alternatives was applied on allocation rules in the domain of simple allocation problems.

In this thesis, to widen the scope of the analysis of revealed preference theory on simple allocation problems and surpass the problem of rationality of allocation rules, we incorporate two different models of decision making from choice theory and bargaining problems literature into the our framework.

Our first decision making model is based on the idea that the decision maker can suffer from limited consideration and a reference point among the alternatives can affect and perturb his choice between other alternatives. Masatlıoğlu and Ok (2005) analyzed the presence of the status quo bias. In their model a status quo option could change the choices among non status quo options. Masatlıoğlu and Uler (2013) studied the environments that creates the reference effect and its influences over the choice of decision maker when they are not chosen. Rubinstein and Salant (2006) set up a choice model in which an ordering over feasible alternatives determines a reference point within the model. The reference point influence the choice by perturbing the preferences. Choice overload and decision avoidance concepts are studied by Tversky and Shafir (1992), Dean (2009) and Buturak and Evren (2014). Masatlıoğlu et al (2010) provided a model with consumer unawareness that overlooks some options and has a limited consideration set. Masatlıoğlu and Nakajima (2013) utilized the concept of consideration sets and analyzed the behavioral search. Dean et al (2014) combined the idea of the limited attention and status quo bias in a model with an attention set and a psychological constraint function. The attention set restricts the options that are considered by decision maker. The psychological constraint function is formed by the status quo option that can affect the choice among feasible alternatives.

In contrast of these models, our model considers a reference dependent constraint function. That is, reference point can change the choice between other alternatives and at the same time limits the attention of the decision maker by narrowing down the considered alternatives.

Our second decision making model proposes a process formed by a distance minimization procedure, in contrast to standard theory. The concerned distance is between an endogenously determined reference or aspiration point and the feasible alternatives.

In social choice theory, the notion of social compromises was introduced into several different areas. This concept constitutes the decision rules selecting closest feasible alternatives to an unattainable ideal point among all alternatives according to a suitable metric or quasi-metric. A social rule satisfy metric rationalizability when the options selected by social choice rule coincide with the options selected with social compromise model. Stehling(1978), Farkas and Nitzan (1979) and Baigent (1987) followed this approach on Arrovian social welfare functions. Nitzan (1981) analyzed metric rationalization of social choice correspondences.

In choice theory literature, Rubinstein and Zhou (1999) proposed a model where the decision rule selects the alternative which minimizes the Euclidean distance between the feasible alternatives and an exogenously given reference point. Güney et al (2011) constructed a model with an environment in which a minimization procedure is used. The process used to find the closest feasible alternative to a possibly unfeasible aspiration point. In this model, the aspiration point was determined by the maximization of a single preference relation over grand set of alternatives. The closeness was determined according to an endogenous and subjective distance function. The idea behind the use of the subjective distance function was capturing the notion of resemblance that is in the mind of decision maker.

Bargaining problems literature investigates the possible and fair ways of sharing the gains from the cooperation among an N person society. Every member of society tries to maximize their own utility within a set of possible payoffs. There exists an exogenously given disagreement point. The first formal treatment of bargaining problems field was made by Nash (1950). Yu (1973) proposed an alternative approach to bargaining problems. In this study, P-norm distance functions were used to find the selected feasible point that minimize the distance between the ideal point and the achievable points that are available to choose by decision process. Conley et al (2000) made characterization of the symmetric and weighted versions of Euclidean compromise solutions for multiobjective optimization problems. Pfingsten and Waganer (2004) constructed the general framework of the social compromises in the bargaining solution literature. They explored and specified some distance functions corresponding to the well-known bargaining solutions. The analysis of the necessary conditions that provide the metric rationalizability of bargaining solutions was also made.

3 Simple Allocation Problems

Let $N = \{1, ..., n\}$ be the set of agents. For $i \in N$, let e_i be the i^{th} unit vector in $\mathbb{R}^N_+$. We use the vector inequalities $\leqq$, $\leq$, $<$. For each $E \in \mathbb{R}_+$, let $\triangle(E) = \{x \in \mathbb{R}^N_+ \mid \sum_N x_i = E\}$.

A simple allocation problem for a society of N is a pair $(c, E) \in \mathbb{R}^N_+ \times \mathbb{R}_+$ such that $\sum_N c_i \geqq E$ where E represents the endowment level and c represents characteristic vector. Let $\mathcal{C}$ be the set of all simple allocation problems for N. Given a simple allocation problem $(c, E) \in \mathcal{C}$, let $X(c, E) = \{x \in \mathbb{R}^N_+ \mid x \leqq c \text{ and } \sum_N x_i \leqq E\}$ be the choice set of (c, E).

An allocation rule $F : \mathcal{C} \to \mathbb{R}^N_+$ assigns each simple allocation problem (c, E) to an allocation $F(c, E) \in X(c, E)$ such that $\sum_N F_i(c, E) = E$. For every allocation rule F, $F(c, E) \leqq c$ has to hold by construction of the choice set. Additionally, $\sum_N F_i(c, E) = E$ can be interpreted as an efficiency property.

The **Proportional rule** allocates the endowment proportional to the characteristic values: for each $i \in N$,

$$PRO_i(c, E) = \frac{c_i}{\sum_N c_j} E.$$

The **Equal Gains rule** (Constrained equal awards rule) allocates the endowment equally, subject to no agent receiving more than its characteristic value: for each $i \in N$,

$$EG_i(c, E) = \min\{c_i, \lambda\} where \lambda \in \mathbb{R}_+ satisfies \sum_N \min\{c_i, \lambda\} = E.$$

The **Equal Losses rule** (Constrained equal losses) equalizes the losses agents incur, subject to no agent receiving a negative share; for each $i \in N$,

$$EL_i(c, E) = \max\{0, c_i - \lambda\} where \lambda \in \mathbb{R}_+ satisfies \sum_N \max\{0, c_i - \lambda\} = E.$$

The **Serial Dictatorship rule** allocates the endowment to the agent with highest priority until he receives his characteristic value and than repeat the same process for each agent following the priority order over agents.

For every rule F, we construct an induced revealed preference relation, $R^F \subseteq \mathbb{R}^N_+ \times \mathbb{R}^N_+$, as follows: for each $x, y \in \mathbb{R}^N_+$, $xR^F y$ if and only if there is $(c, E) \in \mathcal{C}$ such that $x = F(c, E)$ and $y \in X(c, E)$. The strict revealed preference relation induced by F,

P^F, is defined as xP^Fy if and only if xR^Fy and $x \neq y$.

A rule F satisfies WARP (the weak axiom of revealed preferences) if and only if it is rational. WARP can be stated as follows: for each pair $(c, E), (c', E) \in \mathcal{C}$, $F(c, E) \in X(c', E)$ and $F(c, E) \neq F(c', E)$ implies $F(c', E) \notin X(c, E)$. So that WARP is a necessary and sufficient condition for rationality on our domain as in the standard revealed preferences theory. Another well-known property from revealed preference theory is contraction independence. This property is also referred to as independence of irrelevant alternatives (Nash (1950)) or Sen's property α (Sen (1971)). Contraction independence can be stated as follows: for each pair $(c, E), (c', E) \in \mathcal{C}$, $F(c, E) \in X(c', E) \subseteq X(c, E)$ implies $F(c', E) = F(c, E)$. A rule F satisfies contraction independence if and only if it satisfies WARP. This equivalence provides a simple way of checking whether an allocation rule satisfies WARP.

4 Limited Consideration

4.1 Model

In what follows, we keep using the preliminaries and notations coming from base model constructed by Kıbrıs(2012).

The limited consideration model consists of two elements - a preference relation and a constraint function.

The constraint function is used to find out the options that decision maker is prepared to take into consideration among all possible alternatives. It is a function of the characteristic vector. The characteristic vector represents the reference point in our model. It shows its impact on choice through the channel of attention. In every choice problem $(c, E) \in \mathcal{C}$, the decision maker is aware of the characteristic vector. The constraint function captures the fact that a reference point could influence the decision maker's choice by eliminating some options from consideration. It is based on the idea that characteristic vector as a reference point do matter for the decision maker and it can rule out some options out of consideration. The constraint function assigns to every possible characteristic vector $c \in \mathbb{R}_+^N$ to a consideration set. The consideration set is a subset of possible allocations such that every agent receives less than or equal to his characteristic value. The consideration set represents the set of alternatives that the decision maker is aware of under reference point c.

Definition 1 *A constraint function is a mapping $Q : \mathbb{R}_+^N \to \mathbb{R}_+^N$ such that $\forall c \in \mathbb{R}_+^N$ and $\forall x \in Q(c)$ we have $x \leqq c$.*

The preference relation is used to represent the tastes of the decision maker over the feasible alternatives. We denote the preference relation by R.

An allocation x is said to be the maximizer of R in $X(c, E)$ if xRy holds for all $y \in X(c, E)$.

We interpret the simple allocation problem $(c, E) \in \mathcal{C}$ in the following manner: A decision maker observes the characteristic vector of the given simple allocation problem. He then forms his consideration set. The consideration set is formed by his constraint function that is a function of the reference point. The intersection of the consideration set and the choice set of (c, E), that is $X(c, E)$, constitutes the feasible allocations that can be chosen by decision maker under the simple allocation problem (c, E). The options which are ruled out by constraint function are considered as if they were not feasible since the decision maker is no longer aware of those options. The decision maker selects the maximizer of R in $Q(c) \cap X(c, E)$ as the solution of the simple allocation problem (c, E).

Definition 2 *An allocation rule F is consistent with Limited Consideration Model if there exists a preference relation R and a constraint function Q such that $\forall (c, E) \in \mathcal{C}$, we have*

$$F(c, E) = \arg\max_{R} (Q(c) \cap X(c, E))$$

4.2 Results

In this part, we provide the main characterization of the Limited Consideration Choice Model. In contrast to standard revealed preference theory, limited consideration choice model is consistent with any allocation rule F.

Theorem 3 *Any allocation rule F can be rationalized with a (Q, R) pair.*

Proof. Take any allocation rule F and characteristic vector c.

Take any simple allocation problem $(c, E) \in \mathcal{C}$.

Construct Q such that $Q(c) = \{x \in \mathbb{R}^N_+ \mid x = F(c, E) \text{ for all } E \text{ such that } 0 \leq E \leq \sum_N c_i\}$.

Take R to be represented by the utility function $U(x) = \sum_N x_i$.

We need to show that $F(c, E) = \arg\max_R (Q(c) \cap X(c, E))$ for all $(c, E) \in \mathcal{C}$.

Suppose $x = \arg\max_R (Q(c) \cap X(c, E))$.

We know that $(Q(c) \cap X(c, E))$ is not empty since $Q(c)$ is not empty. For all $x \in Q(c)$, there exists $E = \sum_N x_i$ such that $x = F(c, E) \in X(c, E)$.

Under this construction $x \in (Q(c) \cap X(c, E))$ implies that $x = F(c, E')$ for some level of endowment E' such that $0 \leq E' \leq E$. The maximization process on this set according to R selects the allocation x such that $\sum_N x_i = E$ for all $(c, E) \in \mathcal{C}$ since our preference relation R is strictly monotone.

The chosen allocation corresponds $x = F(c, E)$ for all $(c, E) \in \mathcal{C}$. Hence we have $x = \arg\max_R (Q(c) \cap X(c, E)) = F(c, E)$ for all $(c, E) \in \mathcal{C}$. ■

This theorem shows that Limited Attention Model in simple allocation problems is not falsifiable and any bankruptcy rule can be rationalized by a (Q, R) pair constructed in suitable manner.

We next define some properties over the constraint function.

Consistency imposes a restriction over the constraint function in the same way that the contraction independence restricts the choice behavior.

Definition 4 ***(Consistency)*** *A constraint function Q is said to satisfy consistency, if for each pair $c, c' \in \mathbb{R}_+^N$ such that $c \leqq c'$, $x \in Q(c')$ and $x \leqq c$ then $x \in Q(c)$.*

Consistency requires that an option taken into consideration by the decision maker in a big set of alternatives must be considered in its subsets if it is feasible. The basic idea is that if an option attracts attention in choice set $X(c', E)$, then it also attract attention in subsets of $X(c', E)$ if it remains attainable under the new simple allocation problem.

Proposition 5 *If an allocation rule F is consistent with the limited consideration choice model and it has a consistent Q, then it is rational.*

Proof. We use the axiom of contraction independence as a sufficient and necessary condition on rationality for this proof.

Suppose we have $F(c', E) \in X(c, E) \subseteq X(c', E)$. We need to show that $F(c', E) = F(c, E)$. We have $c \leqq c'$ since $X(c, E) \subseteq X(c', E)$ holds.

Under our construction $x = F(c', E)$ implies that $x \in Q(c')$. If $x^* = F(c', E) \in X(c, E)$ is true, then $x^* \leqq c$ holds.

By consistency, $\forall c, c' \in \mathbb{R}_+^N$ such that $c \leqq c'$, if $x \in Q(c')$ and $x \leqq c$ holds, then $x \in Q(c)$ thus $x^* \in Q(c)$. By definition of $Q(c)$ this implies $x^* = F(c, E)$ for $E = \sum_N x_i^*$. Hence $x^* = F(c, E) = F(c', E)$. ■

Proposition 5 shows that consistency of a constraint function on the limited consideration model is equivalent to rationality of an allocation rule on standard model on our domain.

In what follows, we construct and define some specific constraint functions that are consistent with the well-known allocation rules.

We explicitly define consideration sets of the commonly used allocation rules such that;

$$Q^{PRO}(c) = \{x \in \mathbb{R}_+^N \mid x = PRO(c, E) forallEsuchthat 0 \leq E \leq \sum_N c_i\}$$

for the Proportional Rule,

$$Q^{SD}(c) = \{x \in \mathbb{R}_+^N \mid x = SD(c, E) forallEsuchthat 0 \leq E \leq \sum_N c_i\}$$

for the Serial Dictatorship Rule,

$$Q^{EL}(c) = \{x \in \mathbb{R}_+^N \mid x = EL(c, E) forallEsuchthat 0 \leq E \leq \sum_N c_i\}$$

for the Equal Losses Rule, and

$$Q^{EG}(c) = \{x \in \mathbb{R}_+^N \mid x = EG(c, E) forallEsuchthat 0 \leq E \leq \sum_N c_i\}$$

for the Equal Gains Rule.

We next define another desired property on the constraint function.

Equal treatment of equals is a condition on the constraint function to check if it is well behaved in a normative sense. It is a quite standard notion of fairness in the literature.

Definition 6 ***(Equal Treatment of Equals)*** *A constraint function Q is said to satisfy equal treatment of equals, if for every $c \in \mathbb{R}_+^N$ such that $c_i = c_j$ for some $i, j \in N$, $x_i = x_j$ holds for all $x \in Q(c)$.*

Consider the consideration sets Q^{PRO}, Q^{EL}, Q^{EA} constructed as described above, they satisfy Equal Treatment of Equals.

Consider the consideration set Q^{PRO}. For all $i \in N$, $PRO_i(c, E) = \frac{c_i}{\sum_N c_j} E$ and $PRO_i(c, E) = PRO_j(c, E)$ since $\frac{c_i}{\sum_N c_j} E = \frac{cj}{\sum_N c_j} E$ when $c_i = c_j$.

Consider the consideration set Q^{EG}. For all $i \in N$, the constrained equal awards rule assigns $EG_i(c, E) = \min(c_i, \lambda)$ where $\lambda \in \mathbb{R}_+$ satisfies $\sum_N \min(c_i, \lambda) = E$ and $c_i = c_j$ implies $\min(c_i, \lambda) = \min(c_j, \lambda)$ thus $EG_i(c, E) = EG_j(c, E)$.

Take Q^{EL}. For all $i \in N$, the constrained equal losses rule assigns $EL_i(c,E) = \max\{0, c_i - \lambda\}$ where $\lambda \in \mathbb{R}_+$ satisfies $\sum_N \max\{0, c_i - \lambda\} = E$ and $c_i = c_j$ implies $\max\{0, c_i - \lambda\} = \max\{0, c_j - \lambda\}$ thus $EL_i(c,E) = EL_j(c,E)$.

Consider the consideration set Q^{SD}. It does not satisfy Equal Treatment of Equals.

Example 7 *Take $N = 2$ and $SD(c,E) = \{(E,0)$ if $E \leq c_1$ and $(c_1, E - c_1)$ if $c_1 < E \leq \sum_N c_i\}$.*

Suppose $c_1 = c_2$. Under this construction $SD_1(c,E) = SD_2(c,E)$ holds only if $E = c_1 + c_2$.

Equal treatment of equals imposes restrictions on constraint function only if there are claimants with the same characteristic values.

We construct a new fairness criteria over the limited consideration models to define a more general and testable notion of fairness that looks at every allocation that is in the consideration set. For each agent i, we look at the ratio of $\frac{x_i}{\sum_N x_j}$ to $\frac{c_i}{\sum_N c_j}$ for every endowment level $E > 0$ for a fixed characteristic vector. We find the minimum value among all agents in the society and all endowment levels under this characteristic vector. We consider all possible characteristic vector and related allocations within each Limited Consideration Choice Model. The fairness criteria evaluates each model according to the possible minimum values under different characteristic vector combinations. The basic idea behind this criteria is that the share of an agent from the remaining value should be as close as to the his share of characteristic vector from the total value of characteristic value as an indicator of fairness.

Definition 8 *Given two limited consideration models (Q^1, R^1) and (Q^2, R^2), we say that the first model is more fair than the second model, denoted by $Q^1 > Q^2$, if $R^1 = R^2$ and for all $c \in \mathbb{R}^N_+$, we have*

$$\min_{\substack{x \in Q^1(c) \\ x \geqslant 0 \\ i \in N}} \frac{\frac{x_i}{\sum_N x_j}}{\frac{c_i}{\sum_N c_j}} \geq \min_{\substack{y \in Q^2(c) \\ y \geqslant 0 \\ i \in N}} \frac{\frac{y_i}{\sum_N y_j}}{\frac{c_i}{\sum_N c_j}}$$

and there exists an $c \in \mathbb{R}^N_+$ such that

$$\min_{\substack{x \in Q^1(c) \\ x \geqslant 0 \\ i \in N}} \frac{\frac{x_i}{\sum_N x_j}}{\frac{c_i}{\sum_N c_j}} > \min_{\substack{y \in Q^2(c) \\ y \geqslant 0 \\ i \in N}} \frac{\frac{y_i}{\sum_N y_j}}{\frac{c_i}{\sum_N c_j}}.$$

We next give an order for some commonly used allocation rules according to or fairness criteria.

Proposition 9 *According to our fairness criteria, $Q^{PRO} > Q^{EG} > Q^{EL} > Q^{SD}$.*

Proof. Take Q^{PRO}. For each $i \in N$, $PRO_i(c, E) = \frac{c_i}{\sum_N c_j} E$. For all $i \in N$ and for all $c \in \mathbb{R}_+^N$, we have

$$\frac{\frac{x_i}{\sum_N x_j}}{\frac{c_i}{\sum_N c_j}} = \frac{\frac{\frac{c_i}{\sum_N c_j} E}{\sum_N x_j}}{\frac{c_i}{\sum_N c_j}} = \frac{\frac{\frac{c_i}{\sum_N c_j} E}{E}}{\frac{c_i}{\sum_N c_j}} = \frac{\frac{c_i}{\sum_N c_j}}{\frac{c_i}{\sum_N c_j}} = 1 \tag{1}$$

Thus

$$\min_{\substack{x \in Q^{PRO}(c) \\ x \geqslant 0 \\ i \in N}} \frac{\frac{x_i}{\sum_N x_j}}{\frac{c_i}{\sum_N c_j}} = 1. \tag{2}$$

Take Q^{SD}.Suppose i^{th} person in the society has the highest priority. Take $j \in N$ such that $j \neq i$. For all E such that $E \leq c_i$ we have $SD_j(c, E) = 0$.

Thus

$$\min_{\substack{x \in Q^{SD}(c) \\ x \geqslant 0 \\ i \in N}} \frac{\frac{x_i}{\sum_N x_j}}{\frac{c_i}{\sum_N c_j}} = \frac{\frac{0}{E}}{\frac{c_i}{\sum_N c_j}} = 0. \tag{3}$$

Take Q^{EL}. For each $i \in N$, $EL_i(c, E) = \max\{0, c_i - \lambda\}$ where $\lambda \in \mathbb{R}_+$ satisfies $\sum_N \max\{0, c_i - \lambda\} = E$. Take $c \in \mathbb{R}_+^N$ such that $c_i = c$ for all $i \in N$. In this case $\forall i \in N$, $EL_i(c, E) = c_i - \lambda = \frac{E}{N}$ since $N(c_i - \lambda) = E$. In this case for each $i \in N$, we have

$$\frac{\frac{x_i}{\sum_N x_j}}{\frac{c_i}{\sum_N c_j}} = \frac{\frac{\frac{E}{N}}{E}}{\frac{c_i}{\sum_N c_j}} = \frac{\frac{1}{N}}{\frac{c_i}{Nc_i}} = \frac{\frac{1}{N}}{\frac{1}{N}} = 1. \tag{4}$$

Take $c \in \mathbb{R}_+^N$ such that $c_i = c$ for all $i \in N$ does not hold. Take the person with the lowest characteristic value. Say it is the i^{th} person in the society. For Equal Losses rule, $EL_i(c, E) = 0$ when $\lambda \geq c_i$ where $\sum_N \max\{0, c_i - \lambda\} = E$. Under this allocation rule, λ is determined with $\sum_N c_j - N\lambda = E$ so $\lambda = \frac{\sum_N c_j - E}{N}$. This implies that if $\frac{\sum_N c_j - E}{N} \geq c_i$ then $EL_i(c, E) = 0$. For all E such that $E \leq \sum_N c_j - Nc_i$ we have $x_i = EL_i(c, E) = 0$.

Thus for all $c \in \mathbb{R}_+^N$ such that $c_i = c$ for all $i \in N$ we have

$$\min_{\substack{x \in Q^{EL}(c) \\ x \geqslant 0 \\ i \in N}} \frac{\frac{x_i}{\sum_N x_j}}{\frac{c_i}{\sum_N c_j}} = 1 \tag{5}$$

and otherwise we have

$$\min_{\substack{x \in Q^{EL}(c) \\ x \geqslant 0 \\ i \in N}} \frac{\frac{x_i}{\sum_N x_j}}{\frac{c_i}{\sum_N c_j}} = \frac{\frac{0}{E}}{\frac{c_i}{\sum_N c_j}} = 0 \tag{6}$$

.

Take Q^{EG}. For each $i \in N$, $EG_i(c, E) = \min\{c_i, \lambda\}$ where $\lambda \in \mathbb{R}_+$ satisfies $\sum\limits_N \min\{c_i, \lambda\} = E$. Take $c \in \mathbb{R}_+^N$ such that $c_i = c$ for all $i \in N$. In this case $\forall i \in N$, $EG_i(c, E) = \frac{E}{N}$ since $N\lambda = E$. In this case for each $i \in N$, we have

$$\frac{\frac{x_i}{\sum_N x_j}}{\frac{c_i}{\sum_N c_j}} = \frac{\frac{\frac{E}{N}}{E}}{\frac{c_i}{\sum_N c_j}} = \frac{\frac{1}{N}}{\frac{c_i}{Nc_i}} = \frac{\frac{1}{N}}{\frac{1}{N}} = 1. \tag{7}$$

Take $c \in \mathbb{R}_+^N$ such that $c_i = c$ for all $i \in N$ does not hold. Take the person with the lowest characteristic value. Say it is the i^{th} person in the society. For all E such that $\frac{E}{N} < c_i$ we have $EG_i(c, E) = \frac{E}{N}$ for each $i \in N$. Thus

$$\frac{\frac{x_i}{\sum_N x_j}}{\frac{c_i}{\sum_N c_j}} = \frac{\frac{\frac{E}{N}}{E}}{\frac{c_i}{\sum_N c_j}} = \frac{1}{N}.\frac{\sum_N c_j}{c_j}. \tag{8}$$

The ratio gets smaller when the share of c_i in $\sum_N c_j$ gets bigger and the number of agents in the society increases. But it is always bigger than 0 with finite number of agents.

Thus for all $c \in \mathbb{R}_+^N$ such that $c_i = c$ for all $i \in N$ we have

$$\min_{\substack{x \in Q^{EG}(c) \\ x \geqslant 0 \\ i \in N}} \frac{\frac{x_i}{\sum_N x_j}}{\frac{c_i}{\sum_N c_j}} = 1 \tag{9}$$

and otherwise we have

$$\min_{\substack{x \in Q^{EG}(c) \\ x \geqslant 0 \\ i \in N}} \frac{\frac{x_i}{\sum_N x_j}}{\frac{c_i}{\sum_N c_j}} > 0 \tag{10}$$

Hence we have $Q^{PRO} > Q^{EG} > Q^{EL} > Q^{SD}$ as a scale of fairness. ■

5 Aspiration Based Choice Model

5.1 Model

In what follows, we define a potential set of alternatives, denoted by $Y(c)$ given $c \in \mathbb{R}_+^N$, such that $Y(c) = \{x \in \mathbb{R}_+^N \mid x \leqq c \text{ and } \sum_N x_i \leq \sum_N c_i\}$. The potential set contains all alternatives that are potentially available to choose. For a given simple allocation problem $(c, E) \in \mathcal{C}$, $Y(c) \setminus X(c, E)$ is defined as the set of phantom alternatives in the model. For a given simple allocation problem $(c, E) \in \mathcal{C}$, there is no phantom alternatives at endowment level $E = \sum_N c_i$.

The aspiration based choice model consists of two elements - a continuous metric d and a linear order R.

The linear order R represents the decision maker's tastes over potential set of alternatives. It is used to form the ideal point or aspiration point for a given simple allocation problem $(c, E) \in \mathcal{C}$. Given a simple allocation problem, the maximizer of R in potential set of alternatives is called the aspiration point. The main idea behind the concept of aspiration point is that an option that is potentially unavailable to choose can influence the choice among feasible alternatives even if it is not attainable. Possibly unavailable aspiration points can influence and change the decisions.

The continuous metric $d : X^2 \to \mathbb{R}_+$ is used to capture the notion of resemblance in the mind of the decision maker via subjective distance functions. It is endogenously defined. One can say that this metric acts like a psychological distance definer between different alternatives.

We interpret the simple allocation problem $(c, E) \in \mathcal{C}$ in the following manner: When confronted with a simple allocation problem $(c, E) \in \mathcal{C}$, a decision maker first forms his ideal aspiration point according to linear order R. The maximizer of R in $Y(c)$ is the aspiration point in our model. We represent aspiration point of $Y(c)$ with $a(Y(c))$. On the next step, the decision maker uses his subjective metric d for selecting the closest alternative in $X(c, E)$ to his aspiration point $a(Y(c))$. The aspiration point is the unique element in $Y(c)$ that maximizes the decision maker's utility. Under the construction of our model, we can have two decision makers with the same aspiration point for a given $(c, E) \in \mathcal{C}$ selecting different alternatives from the choice set of $X(c, E)$ since the distance function is determined according to a subjective metric that can change from decision maker to decision maker. The idea behind this construction is capturing the notion of resemblance for each decision maker.

Definition 10 *An allocation rule F is consistent with the Aspiration Based Choice Model if there exists a continuous metric $d : X^2 \to \mathbb{R}_+$ and a continuous linear order R such that*

$$F(c, E) = \arg \min_{x \in X(c,E)} d(a(Y(c)), X(c, E))$$

for all $(c, E) \in \mathcal{C}$ where $a(Y(c))$ is the maximizer of R in $Y(c)$.

We note that this minimization procedure is well defined since the metric d is continuous and the domain is compact.

In our model, we impose to have a strictly monotone preference relation R so that for all $(c, E) \in \mathcal{C}$, $a(Y(c)) = c$ meaning that the characteristic vector is the ideal point for all possible simple allocation problems. This reflects the idea that for any given simple allocation problem, the characteristic vector is the ideal point for all the parties involved. From now on, we take the preference relation R to be represented with the utility function $U(x) = \sum_N x_i$.

Our model is an adaptation of the Yu solutions family from bargaining literature. Yu (1983) stated the bargaining problem as follows: There is a group of n persons, each of them having a utility function defined over a set of alternatives for a decision to be made by the group. The aim was to find a solution option such that the group regret is minimized according to their utility function. P-norm distance function is defined as: $D_p(U(x)) = [\sum_{i=1}^{n} (U_i^* - U_i(x))^p]^{\frac{1}{p}}$. The compromise (Yu) solution with respect to parameter p is stated as the minimizer of $D_p(U(x))$ in set of feasible alternatives such that each agent has a nonnegative utility.

5.2 Results

We provide our main theorem under the Aspiration Based Choice Model. We show that the Aspiration Based Choice Model with P-norm metric such that $p > 1$ corresponds to Yu solution family with respect to parameter $p > 1$ and it gives the Constrained Equal Losses Rule.

Theorem 11 *The constrained equal losses rule is consistent with the aspiration based choice model and corresponds to Yu solutions family with respect to parameter $p > 1$.*

Proof. Let $p > 1$. When $0 \leq p < 1$, we have a quasi-metric since triangular inequality is not satisfied. When $p = 1$ we could have multiple solutions for a minimization process and it is not consistent the unique selection of an allocation rule.

Take R represented by $U(x) = \sum_N x_i$ and form $a(Y(c)) = c$.

Take continuous metric $d = (\sum_N (c_i - x_i)^p)$.

We need to show that

$$EL(c, E) = \arg \min_{x \in X(c,E)} d(c, X(c, E))$$

for all $(c, E) \in \mathcal{C}$.

We have constrained optimization process with inequality constraints at the hand since we search for the closest option to the aspiration point c in $X(c, E)$ so that such an alternative should satisfy $0 \leqq x \leqq c$ and $\sum_N x_i \leq E$. We state the Kuhn-Tucker sufficient conditions for a minimum and we maximize the negative of the distance function to do a minimization since this is a maximum only problem.

Our maximization problem is defined as;

$$\min_{x\in X(c,E)} d\left(c, X\left(c,E\right)\right) = \max_{x\in X(c,E)} -d\left(c, X\left(c,E\right)\right) = \max_{x\in X(c,E)} -\left(\sum_{N}\left(c_i - x_i\right)^p\right)^{\frac{1}{p}} \tag{11}$$

subject to

$$\sum_{N} x_i \leq E, \tag{12}$$

$$x_i \geq 0 \forall i \in N \tag{13}$$

and

$$x_i \leq c_i \forall i \in N. \tag{14}$$

We have the Lagrangian function:

$$\mathcal{L}\left(x_i, \lambda, \mu_i\right) = -\left(\sum_{N}\left(c_i - x_i\right)^p\right)^{\frac{1}{p}} + \lambda\left(E - \sum_{N} x_i\right) + \sum_{N} \mu_i \left(c_i - x_i\right) \tag{15}$$

We state Kuhn-Tucker conditions as:

$$\mathcal{L}_{x_i} = \frac{1}{p}\left(\sum_{N}\left(c_i - x_i\right)^p\right)^{\frac{1-p}{p}} .p.\left(c_i - x_i\right)^{p-1} - \lambda - \mu_i \leq 0 \tag{16}$$

for all $i \in N$

$$\sum_{N} x_i \leq E \tag{17}$$

$$x_i \leq c_i \tag{18}$$

$$x_i \geq 0 \tag{19}$$

$$\lambda \geq 0 \tag{20}$$

$$\mu_i \geq 0 \tag{21}$$

$$x_i[\frac{1}{p}\left(\sum_{N}\left(c_i - x_i\right)^p\right)^{\frac{1-p}{p}} .p.\left(c_i - x_i\right)^{p-1} - \lambda - \mu_i] = 0 \tag{22}$$

$$\lambda\left(E-\sum_N x_i\right)=0 \tag{23}$$

$$\mu_i\left(c_i-x_i\right)=0 \tag{24}$$

for all $i \in N$

We have either $\lambda=0$ or $E=\sum_N x_i$ and either $\mu_i=0$ or $c_i=x_i$ for all $i \in N$.

Suppose we have $c_i=x_i$ and $\mu_i>0$ for some $i \in N$. When $x_i=c_i$ holds, we have $\mu_i=-\lambda$ since $x_i[\frac{1}{p}\left(\sum_N\left(c_i-x_i\right)^p\right)^{\frac{1-p}{p}}.p.\left(c_i-x_i\right)^{p-1}-\lambda-\mu_i]=0$ must holds. This constraint contradicts with the fact that $\lambda \geq 0$ so this is not the case for any $i \in N$. This implies we have $\mu_i=0$ for all $i \in N$ meaning that either $x_i=0$ or $\frac{1}{p}\left(\sum_N\left(c_i-x_i\right)^p\right)^{\frac{1-p}{p}}.p.\left(c_i-x_i\right)^{p-1}=\lambda$ holds for all $i \in N$ and either $\lambda=0$ or $E=\sum_N x_i$ holds .

Suppose we have $\lambda=0$ then $c_i=x_i$ for all $i \in N$ and $E=\sum_N c_i$ since

$$x_i[\frac{1}{p}\left(\sum_N\left(c_i-x_i\right)^p\right)^{\frac{1-p}{p}}.p.\left(c_i-x_i\right)^{p-1}-\lambda-\mu_i]=0 \tag{25}$$

must holds.

Suppose $\lambda>0$, the we have either $\frac{1}{p}\left(\sum_N\left(c_i-x_i\right)^p\right)^{\frac{1-p}{p}}.p.\left(c_i-x_i\right)^{p-1}=\lambda$ for all $i \in N$ implying that $c_i-x_i=\lambda$ for all $i \in N$ and $E=\sum_N x_i$ or $x_i=0$ for some $i \in\{1,..,k\}$ and $\frac{1}{p}\left(\sum_N\left(c_j-x_j\right)^p\right)^{\frac{1-p}{p}}.p.\left(c_j-x_j\right)^{p-1}=\lambda$ for some $j \in\{k+1,...,n\}$ where $N=\{1,..,n\}$ and $E=\sum_{j \in\{k+1,...,n\}} x_j$.

Any solution x^* of the constrained maximization problem gives the constrained equal losses rule solutions.

Any constrained equal losses rule solution selects an option x with following conditions; either $c_i-x_i=\lambda$ for all $i \in N$ and $E=\sum_N x_i$ if such an λ exits or $x_i=0$ for some $i \in\{1,..,k\}$ and $c_j-x_j=\lambda$ for some $j \in\{k+1,...,n\}$ where $N=\{1,..,n\}$ and $E=\sum_{j \in\{k+1,...,n\}} x_j$ if such an λ exists.

The proof shows that

$$EL(c, E)=\arg \min_{x \in X(c, E)} d(c, X(c, E))$$

for all $(c, E) \in \mathcal{C}$ where

$$d=\left(\sum_N\left(c_i-x_i\right)^p\right)^{\frac{1}{p}}$$

.

Hence it corresponds to Yu solutions family with respect to parameter $p > 1$. ■

Under the construction of Aspiration Based Choice Model, we have a decision maker that forms a single subjective distance function that is independent of the choice problem. We do not allow a metric that is a function of c meaning that the notion of resemblance perceived by the decision maker does not change from simple allocation problem to simple allocation problem. This constraint can be interpreted as a consistency requirement in the model.

Remark 12 *Aspiration Based Choice Model with a single continuous metric on simple allocation problems is not consistent with Proportional Rule. Consider the decision maker with subjective distance function* $d = \max(\alpha_i (c_i - x_i))$ *where* α_i *represent the weight of* i^{th} *person in the society such that* $\sum_N \alpha_i = 1$. *For an weight vector* $\alpha \in \mathbb{R}_+^N$ *there exists a suitable characteristic vector* c *such that* $PRO(c, E) = \arg\min_{x \in X(c,E)} (\max(\alpha_i (c_i - x_i)))$ *for all simple allocation problems with* $(c, .) \in \mathcal{C}$ *but for a given* c' *such that* $c' \neq c$ *we have* $\arg\min_{x \in X(c',E)} (\max(\alpha_i (c_i' - x_i))) \neq PRO(c', E)$.

6 Conclusion

This thesis proposes a characterization for limited consideration and aspiration based choice models on simple allocation problems.

Our first model uses a constraint function generated from the characteristic vector to narrow down the allocations which are available to choose instead of rationalizing choice through a utility maximization procedure over choice set of a simple allocation problem. This approach captures the influence of the characteristic vector over the choice of a decision maker via limited consideration. Theorem 3 states that any allocation rule can be rationalized by the Limited Consideration Choice Model. This theorem shows that any choice behavior can be explained by limited consideration concept by identifying the underlying attention set of a decision maker. Proposition 5 shows that the consistency property over the constraint function is equivalent to contraction independence axiom so that any allocation rule that is consistent with our model and has a consistent constraint function is rational. This property characterizes the necessary and sufficient requirement for a well behaved constraint function. Identifying a weaker version a consistency property may be useful for analyzing different choice behaviors departing from the properties of the constraint function. We next show that some well known allocation rules that are consistent with our model have constraint functions that satisfy equal treatment of equals property. Analyzing the constraint functions that are consistent with different allocation rules and that satisfy equal treatment of equals property may be interesting to see the ones that provide this desired property. Proposition 9 provides an order over the limited consideration models with constraint functions that are consistent with some well known allocations rules. Extending this analysis over other limited consideration models remains as a study of this field.

Our second model proposes a distance minimization procedure to capture the notion of resemblance in the mind of the decision maker via subjective distance functions in contrast to standard choice model with a utility maximization procedure. It provides an analysis over the selection of an ideal point and a geometrical interpretation to standard problems. Proposition 11 states that aspiration based choice model with P-norm metrics where $p > 1$ gives the Constrained Equal Losses rule and corresponds to Yu solution family with respect to parameter $p > 1$ from the bargaining literature. Identifying different distance functions that accommodate to other allocation rules may be useful to expand the scope of this study.

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