

# SAR Imaging of Moving Targets by Subaperture based Low-rank and Sparse Decomposition

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**Abstract**—We propose a subaperture based method for synthetic aperture radar (SAR) imaging of moving targets. It exploits low-rank and sparse decomposition for extraction of moving targets from the complex SAR scene. First SAR raw data are divided into subapertures in the azimuth direction. Subsequently, low-rank and sparse decomposition is applied using the multiple subapertures data to accomplish the separation of moving targets from the stationary SAR background. A full resolution moving target image is reconstructed by combining the spectral information of the sparse subaperture images. Such an image has a high signal to clutter ratio and is well suited for motion estimation and focusing algorithms. This proposed framework extends the applicability of sparsity-driven moving target focusing methods to very low signal to clutter ratio environments. We demonstrate the performance of our approach through experiments with synthetic and real SAR data.

**Index Terms**—SAR imaging, moving targets, low-rank and sparse decomposition, subaperture processing.

## I. INTRODUCTION

Synthetic aperture radar (SAR) imaging of moving targets requires compensation of phase errors caused by these targets to avoid defocusing in the processed image. A major step in this respect is the detection of moving targets by suppression of the static background/clutter. The subsequent steps are motion estimation and focusing of the moving targets.

Conventionally, the single antenna SAR systems employ filtering approaches for clutter suppression [1]. However, considering the typical pulse repetition frequencies of the SAR systems, the performance is degraded by the slow moving targets, higher azimuth compression ratios and the slower SAR platform velocities. More complicated and expensive systems address these limitations by incorporating multiple antennas in order to utilize the phase information. However, we consider single antenna SAR systems in this work.

In comparison to the conventional methods of SAR imaging, sparsity-based methods have achieved improved resolution and reconstruction quality [2], [3]. The authors in [4], [5] proposed a sparsity-based moving target imaging approach, which considers the extra phase terms induced by the moving targets as errors in the observation model of a static scene. Nevertheless, their method requires a relatively high signal

to clutter ratio (SCR) for a good performance. Since most practical SAR systems produce images with a medium to low SCR, static background suppression is an essential step before any focusing could be applied.

Low-rank and sparse decomposition (LRSD) has found extensive applications for background and foreground separation in magnetic resonance imaging (MRI) [6], [7] and computer vision [8]. In order to exploit the LRSD framework for SAR moving target imaging, a data matrix needs to be constructed that contains the temporal SAR images as columns. Such temporal images are readily available in the case of MRI and optical camera videos. However, the temporal images for SAR may not be directly accessible, instead the full resolution SAR image can be split into a number of subaperture images at the cost of a lower azimuth resolution.

Compared to the existing methods, the novel contributions of this paper are: 1) enhancing the applicability of the sparsity-driven moving target imaging/focusing approach [4] to very low SCR scenarios, 2) extension of the LRSD framework proposed in [9] for the moving targets case where background may not be spatially low-rank, and 3) a frequency domain subaperture construction approach which is capable of recovering the original azimuth resolution in the sparse and low-rank images after the LRSD stage.

The final sparse image could be used for motion estimation and focusing of moving targets. The focused moving target image could eventually be overlaid with the corresponding background image to obtain an overall focused and reconstructed SAR image.

## II. PRELIMINARIES

In this section, we provide a brief coverage of the sparsity-driven methods for SAR moving target imaging and the LRSD.

### A. Sparsity-driven Methods for Moving Target Imaging

A SAR signal from a target moving in azimuth direction [1] can be described as:

$$s(\eta) \cong a[(v_p - v_a)\eta] \exp\left(j \frac{2\pi}{\lambda R_0} v_p^2 \eta^2 \left(1 - \frac{v_a}{v_p}\right)^2\right) \quad (1)$$

where  $\eta$  is the slow time,  $v_p$  is the SAR platform velocity,  $R_0$  is the range of closest approach and  $v_a$  represents the azimuth velocity of the target.  $a(\cdot)$  includes the azimuth

weighting, target reflectivity  $f$  and the constant phase terms. The quadratic phase error due to cross-range velocity of the target [10] can be approximated as:

$$\phi(\eta) = \frac{4\pi v_a v_p \eta^2}{\lambda R_0} \quad (2)$$

A sparsity-driven framework was proposed in [4] for spotlight mode SAR moving target imaging. They treat the phase terms induced by the target motion as errors in the imaging model. The SAR observation model considered by this framework is as follows:

$$g = C(\phi)f + n \quad (3)$$

where  $g$  is the noisy observation,  $C$  is the model matrix,  $f$  is the unknown reflectivity of the underlying complex scene, and  $n$  is the observation noise.  $\phi$  represents the phase terms caused by the target motion. Joint estimation of  $f$  and  $\phi$  is sought by solving the following optimization problem:

$$\arg \min_{f, \beta} J(f, \beta) = \arg \min_{f, \beta} \|g - C(\phi)f\|_2^2 + \lambda_1 \|f\|_1 + \lambda_2 \|\beta - \mathbf{1}\|_1 \quad s.t. \quad |\beta(i)| = 1 \quad \forall i \quad (4)$$

where  $\beta$  incorporates the phase errors due to all the points in the scene for every aperture position.  $\lambda_1$ ,  $\lambda_2$  are the regularization parameters and  $\mathbf{1}$  denotes a vector of ones.

### B. Low-rank and Sparse Decomposition

The objective of LRSD is to decompose a matrix  $D$  into its low-rank  $B$  and sparse  $S$  components as follows:

$$\min_{B, S} \text{rank}(B) + \lambda \|S\|_0 \quad s.t. \quad D = B + S \quad (5)$$

where  $\lambda$  is a regularization parameter. However, the rank minimization requires a minimization of non-zero singular values which is an NP-hard problem. It was suggested by [11] and [12] that a convex relaxation could be employed under some conditions by replacing the  $\text{rank}(\cdot)$  and  $l_0$  norm constraints with the *nuclear norm* ( $\|\cdot\|_*$ ) and the  $l_1$  norm, respectively. After considering these convex relaxations, the reformulated problem is given as:

$$\min_{B, S} \|B\|_* + \lambda \|S\|_1 \quad s.t. \quad D = B + S \quad (6)$$

This problem has been solved in a variety of ways by using local optimization algorithms. A bilateral random projections (BRP) scheme was proposed in [13] to significantly accelerate the solution.

## III. PROPOSED METHOD

We propose a subaperture based approach combined with the low-rank and sparse decomposition framework for SAR moving target imaging. Extraction of moving targets from the stationary background is an essential step for their proper imaging. This can be accomplished by LRSD of the complex SAR scene exploiting the subaperture images. A complex SAR image  $I \in \mathbb{C}^{n \times m}$  could be used to construct the subaperture images. Here we consider a band-limited Fourier transform as the forward model to reconstruct the complex SAR image from the raw data  $g$ .

### A. Subapertures and the corresponding data matrix

In order to tailor the LRSD framework for the moving target imaging problem, multiple temporal images of the SAR scene should be stacked to form the matrix  $D \in \mathbb{C}^{p \times q}$  as given below:

$$D = \begin{bmatrix} | & | & \dots & | \\ d_1 & d_2 & \dots & d_q \\ | & | & \dots & | \end{bmatrix} \quad (7)$$

where each column in  $D$  is a vectorized temporal image of the same SAR scene and we have  $p = n \times m$ . This is achieved by constructing  $q$  subaperture SAR images and arranging them as columns of this matrix for LRSD. The first step is to apply the discrete Fourier transform (DFT) in azimuth to the complex SAR image  $I_{(n,m)}$  containing unfocused moving targets. This is followed by a rectangular filtering operation where non-overlapping filtering windows are used to extract the frequency domain subapertures. This frequency domain filtering process can be described as follows:

$$\Omega_{(k,m,i)} = \left( \sum_{n=0}^{N-1} I_{(n,m)} \cdot e^{-j2\pi kn} \right) \times \text{rect} \left( \frac{k - iL}{L} \right) \quad (8)$$

where  $n, k \in \{0, \dots, N-1\}$ ,  $m \in \{0, \dots, M-1\}$

where  $i$  is the subaperture number,  $n$  is the azimuth sample number,  $k$  is the frequency bin number, and  $m$  is the range bin number. Moreover,  $L$  is the length of a subaperture window, i.e., the number of nonzero elements in the rectangular window function. Each subaperture image is then reconstructed by a zero-padded inverse discrete Fourier transform (IDFT) of original image size as given below:

$$d_{(n,m,i)} = \frac{1}{N} \left( \sum_{k=0}^{N-1} \Omega_{(k,m,i)} \cdot e^{j2\pi kn/N} \right) \quad (9)$$

The column vectors for the matrix  $D$  are constructed from the subaperture images as follows:

$$d_i = [d_{(0,0,i)}, d_{(1,0,i)}, \dots, d_{(N-1,M-1,i)}]^T \quad (10)$$

The bandwidth of the azimuth signal is reduced in the process of subapertures formation and consequently the original azimuth resolution is degraded by a factor of  $N/L$  as follows:

$$\rho_s = \rho_o \times \left( \frac{N}{L} \right) \quad (11)$$

where  $\rho_s$  is the subaperture resolution and  $\rho_o$  is the resolution of the original SAR image. However, this loss of resolution is compensated after the LRSD by combining the spectral information of all the subapertures followed by an IDFT. The recovery of original full resolution is a prominent feature of our proposed framework.

### B. LRSD based on subaperture data

Now we turn to the low-rank and sparse decomposition step. A successful LRSD in this scenario is based on the fact that the static parts of the scene do not change position across the subapertures, whereas the moving targets shift their positions in every subaperture. An LRSD framework for SAR

was proposed in [9]. It exploits the spatially low-rank and sparse structure of a stationary scene by making overlapping patches of the SAR image. This LRSD approach produces good results for a SAR scene with a few sparse objects and a slowly varying background. However, it is not well suited for a scene that contains moving targets and where the background is not necessarily spatially low-rank. Here we extend the LRSD framework proposed in [9] for the case of moving targets. Instead of making a patch based data matrix, we form a subaperture-based matrix as described above to exploit the sparse nature of the moving targets. In this case, the background is not strictly required to be spatially low-rank as we enforce low-rank across the subapertures where typically fewer changes are induced by the static components over multiple subaperture data.

The objective of LRSD in this case is to optimize for  $D$ ,  $S$ ,  $B$  and  $\Theta$ . Where  $D$  is the composite part ( $B + S$ ),  $S$  is the sparse component,  $B$  is the low-rank or background component and  $\Theta$  is the random phase of the reflectivity field. The overall optimization problem is formulated as follows:

$$\arg \min_{D, B, S, \Theta} \|g - H\Theta R^*(D)\|_2^2 + \lambda_b \|B\|_* + \lambda_s \|S\|_1 \quad (12)$$

$$s.t. \quad D = B + S, \quad |\Theta_{i,i}| = 1 \quad \forall i$$

where  $g$  is the observed data, and  $H$  is the forward model which takes into account the imaging process. A band-limited Fourier transform was assumed for this purpose.  $R^*$  is the matrix operator which reconstructs a 2D matrix from a column vector. The augmented Lagrangian form of this problem can be solved using alternating direction method of multipliers (ADMM) [9]. The subproblem for the phase matrix  $\Theta$  can be solved by a fixed point algorithm as described in [3].

Once the decomposition is successfully achieved, we use the full aperture data corresponding to the sparse and low-rank components separately to generate full resolution images for both components.

#### IV. EXPERIMENTAL RESULTS

A series of experiments were performed using both synthetic and real SAR data to validate the proposed method. Two subapertures were used in most of our experiments to make the simulations results more convenient for demonstration and limit the number of generated figures. In all of our experiments, we have added synthetic moving targets to the SAR scene with a low SCR. All images are presented on a logarithmic scale for a better dynamic range.

##### A. Synthetic Scene Experiments

First, we performed some experiments on fully synthetic scenes where we simulated the background as well as the moving targets. We constructed a random background and simulated the moving targets by modulating a quadratic phase error in the azimuth direction. Fig. 1 reveals the results of this experiment. Three moving target were placed near the scene center with a very low SCR of -3.2dB as shown in Fig. 1(a). The target motion leads to the spreading of their

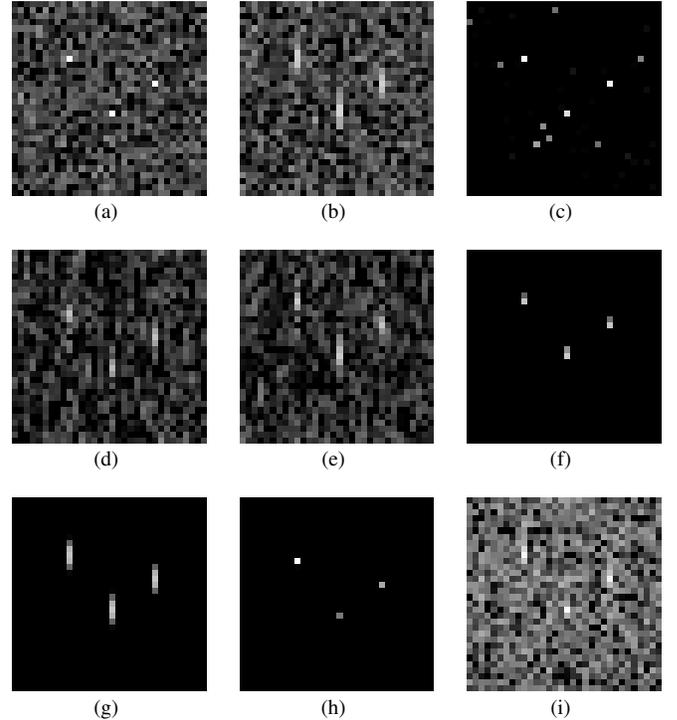


Fig. 1. Results of the synthetic scene experiment. (a) Ground truth scene indicating the locations of three moving targets in a noisy background and low SCR. (b) Conventionally reconstructed scene with quadratic phase errors due to the targets' motions. (c) Focused image produced by the algorithm in [4] without LRSD (d) First subaperture image with a reduced azimuth resolution. (e) Second subaperture image with a reduced azimuth resolution. (f) Sparse image after LRSD, corresponding to the first subaperture. (g) Full resolution conventional image of the sparse part containing the moving targets. (h) Focused moving target image after LRSD and the sparsity-driven focusing. (i) Background image after LRSD and the full resolution conventional reconstruction.

response along the direction of motion in the conventional reconstruction, as shown in Fig. 1(b). Fig. 1(c) demonstrates the output of the sparsity-driven focusing [4] without LRSD. There were eight spurious points in addition to the three original moving targets. It follows that the direct application of sparsity-driven focusing could not perform well under very low SCR conditions and the LRSD step becomes imperative.

Two subapertures were constructed using square window functions for the frequency domain filtering to mask half of the azimuth spectrum at a time followed by a zero-padded IDFT. Fig. 1(d, e) depict the subaperture images formed this way. The resolution along the vertical axis (azimuth) was lowered due to the masked frequency samples. This loss of resolution is unavoidable at this stage since subaperture images are required for the LRSD step, but it is recovered later.

A slight shift of the mean position of the moving targets may be noticed along the vertical axis which is necessary for the LRSD to detect the moving targets as sparse components. Our formulation in (12) leads to LRSD of the scene as well as the corresponding data through subaperture-based processing. Then one can process the full aperture data for the sparse and low-rank components separately in various ways. Just as an

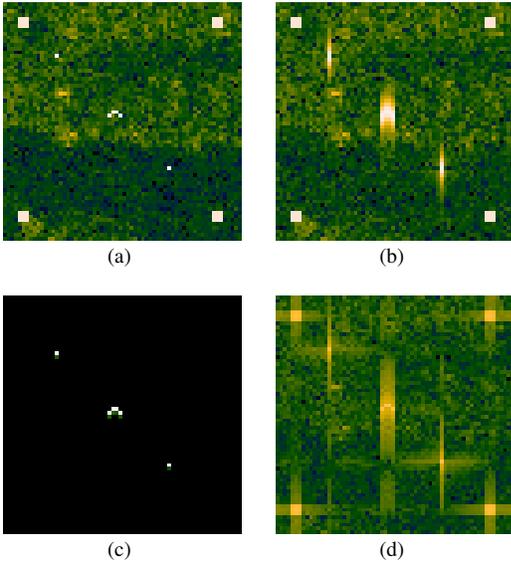


Fig. 2. Results of the real SAR scene experiment. (a) Ground truth scene indicating the locations of three moving targets in a real SAR background. Four strong static points were also added near the corners of the scene. (b) Conventionally reconstructed scene with quadratic phase errors due to the targets' motions. (c) Focused moving target image after processing by the proposed framework and subsequent sparsity-driven focusing. (d) Background image after LRSO and full resolution conventional reconstruction.

example, here we demonstrate the use of [4] to process the data corresponding to the sparse component which contains the moving targets. As shown in Fig. 1(h), the image is well focused without any ghost targets. Two subaperture low-rank images were also obtained from the LRSO (not shown). These images were conventionally reconstructed to get the full resolution background image as shown in Fig. 1(i).

### B. Real SAR Scene Experiments

Real SAR scene experiments were performed with the MiniSAR data [16] where we have introduced synthetic moving targets for performance evaluation. Fig. 2 depicts the results of our real SAR scene experiments. Three moving targets were added to a  $64 \times 64$  real SAR background where one of the targets has a bigger signature as compared to the other two. Furthermore, four static targets were added in the scene to evaluate the LRSO performance for the strong static components. Fig. 2(a, b) present the ground truth and the conventional reconstructions respectively.

Fig. 2(c) shows the moving target image obtained after processing by our proposed framework and subsequent sparsity-driven focusing. It contains the focused moving targets without any significant background or false targets.

The full resolution background image is shown in Fig. 2(d). It is worth noting that most of the MiniSAR scene is included in the low-rank (background) image since its components are mostly consistent across the subapertures. The four synthetic static targets also appear in the low-rank image for the same reason. Another important effect which requires attention is the leakage of the residual from the moving targets response.

There are two main factors that cause this leakage. The first factor is that the moving target response (spread function) has some overlapping components across the subapertures which do not change significantly. The second factor is the regularization parameter values used for the LRSO. If we optimize for a sparser solution, some parts of the moving target response would be forced to the low-rank part.

### V. CONCLUSION

In this paper, a subaperture based approach for moving target radar imaging was presented which utilizes the LRSO framework. Moving targets change their position from one subaperture to the other and consequently they are decomposed as the sparse components. Static background on the other hand is mostly fixed in its location, hence it separates out as the low-rank part. This work extends the applicability of sparsity-driven focusing to very low SCR scenarios. Another contribution is the ability to reconstruct full resolution SAR images after the LRSO stage. The anisotropic behaviour of background reflectivity in real SAR scenes can lead to a leakage of the background components into the sparse image which motivates future work.

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