

Distribution System State Estimation-A step towards Smart Grid

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Abstract: State estimation (SE) is well-established at the transmission system level of the electricity grid, where it has been in use for the last few decades and is a most vital component of energy management systems employed in the monitoring and control centers of electric transmission systems. However, its use for the monitoring and control of power distribution systems (DSs) has not yet been widely implemented because DSs have been majorly passive with uni-directional power flows. This scenario is now changing with the advent of smart grid, which is changing the nature of electric distribution networks by embracing more dispersed generation, demand responsive loads, and measurements devices with different data rates. Thus, the development of distribution system state estimation (DSSE) tool is inevitable for the implementation of protection, optimization, and control techniques, and various other features envisioned by the smart grid concept. Due to the inherent characteristics of DS different from those of transmission systems, transmission system state estimation (TSSE) is not applicable directly to distribution systems. This paper is an attempt to present the state-of-the-art on distribution system state estimation as an enabler function for smart grid features. It broadly reviews the development of DSSE, and challenges faced by its development, and

1 various DSSE algorithms, as well as identifies some future research lines for DSSE.

2 **Keywords.** Distribution system state estimation; DSSE; Smart grid; Microgrid; Distributed
3 energy sources (DERs); Energy management system; Distribution management system.

4 **NOMENCLATURE**

5	SE	State estimation
6	DSSE	Distribution system state estimation
7	TSSE	Transmission system state estimation
8	DS	Distribution system
9	DN	Distribution network
10	TS	Transmission system
11	DER	Distributed Energy resource
12	SCADA	Supervisory Control and Data Acquisition
13	EMS	Energy management system
14	PMU	Phasor measurement unit
15	μ PMU	Micro-phasor measurement unit
16	DG	Distributed generator
17	DR	Demand response
18	DA	Distribution automation
19	R/X	Resistance-to-reactance
20	BC-DSSE	Branch-current-based DSSE
21	NV-DSSE	Node-voltage-based DSSE
22	RTU	Remote terminal unit
23	PDC	Phasor data concentrator
24	WLS	Weighted least squares
25	DMS	Distribution management system
26	DC	Direct current
27	MSE	Microgrid state estimator
28	ACSR	Aluminum Conductor Steel-Reinforced
29	Y-type	Star-connected
30	Δ -type	Delta-connected
31	FA	Firefly
32	PCC	Point-of-common-coupling
33	%RMSE	Percent-Root means square error
34	WLAV	Weighted-Least-Absolute-Value
35	SHGM	Schweppes-Huber-generalized M-estimator
36	IRLS	Iterative reweighted least squares
37	DSE	Dynamic SE
38	FASE	Forecast-aided state estimation
39	ANN	Artificial Neural Network
40	EKF	Extended Kalman filter
41	UKF	Unscented Kalman filter
42	LSE	Local state estimator
43	MASE	Multi-area state estimation
44	BSE	Bi-linear state estimation
45	ML	Machine learning
46	EM	Expectation maximization

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RBA	Recursive bayesian approach
SOR	Successive-over-relaxation
MV	Medium voltage
GPS	Global positioning system
AMI	Advanced metering infrastructure
ADMS	Advanced distribution management system

1. Introduction

SE, after it was first introduced to power systems by Fred Schweppes in 1970 [1], is nowadays an important function in the management and control of the operations of electric transmission networks all over the world. It has strengthened the SCADA systems and eventually led to the development of the EMS [2]. The state estimator obtains the system state using the SCADA measurements, measurements from PMUs [3,4], pseudo-measurements and the topology information [2,3]. After the state is known, various functions of EMS like contingency analysis, security analysis, optimal power flow and other functions can be carried out as shown in Figure 1. Therefore, SE is the backbone function of TS EMS [5], however, its application to DS was not required. This was due to its passive nature with uni-directional power flows since there was no active generation at this level. However, due to shift towards the smart grid encompassing DG inputs and other features such as DR and DA functions, the shape of the power DS is changing and it is no longer passive due to bi-directional power flows (see, Figure 2). This establishes need for bringing DS into the operation circle of monitoring and control, which makes the role of DSSE more significant.

Figure 1. Role of SE in EMS/SCADA

DS and TS differ from one another in many ways, such as DS have high R/X, imbalances among phases and low availability of real-time measurements. This makes the use of TSSE techniques unsuitable for application to DS. This paper is an attempt to encompass various SE techniques applied to DS by reviewing the relevant literature. Many review papers on the subject can be found with the deficiency of putting various techniques together but not mentioning the adequacy of those techniques for DS. This paper attempts to address this deficiency by mentioning the

adequate estimation techniques for DS. It further provides future research directions for DSSE, including intelligent load modeling techniques [6] for pseudo measurement generation [7], event-triggered SE techniques [8], incorporation of smart meter [9] data and micro-synchrophasors (μ PMU) [10] data in DSSE, and finally development of advanced energy management systems [6].

Figure 2. The smart grid and active DS

This paper is divided into the following sections: Section 2 presents SE and its mathematical formulation. Section 3 discusses the need for DSSE; modification on conventional SE for DSSE; NV- DSSE; BC- DSSE; and comparison of the voltage and branch current based DSSE. Section 4 discusses the classification of DSSE techniques, Section 5 presents multi-area or distributed DSSE techniques, future research directions in DSSE outlining five areas of active research on DSSE are briefly discussed in Section 6, and finally Section 7 concludes the paper.

2. State Estimation in Power Systems

System state is the minimum set of variables that can be used to completely define the power system using network topology and impedance parameters e.g. complex node voltages or branch currents [3]. SE is the process of determining the system state using system measurements based on minimization of certain statistical criteria (e.g. Least Squares) [1]. The major objectives of SE are the following [11,12]:

1. Bad measurement data detection;
2. Smoothing out of small errors;
3. Detection of topology errors i.e. wrong switch statuses;
4. Provision of estimates for unmonitored parts of the system .i.e. filling in meter measurements for missing or delayed measurements;
5. Estimation of network parameters based on redundancy in measurements.

Four main processes are carried out by the TSSE present in the EMS [2,3] as depicted in Figure

1 3. Topology processing uses network parameters such as circuit breaker and switch status
2 information and updates the network topology. It makes sure that the correct topology information
3 is used in the SE process [13,14]. Other works on topology processor can be found in [15,16].
4 Observability analysis determines whether the measurements are sufficient to carry out the SE. To
5 ensure the observability, measurements based on historical load data, called pseudo
6 measurements, and zero injection measurements, known as virtual measurements, are used. A
7 Null space method for observability analysis can be found in [17]. The bad data processor is
8 another important function of SE, which processes the measurements and detects the erroneous
9 measurements which get corrupted due to reasons such as communication network failures or
10 dropped measurement packets. It detects and eliminates the gross measurement errors subject to
11 the presence of sufficient measurement redundancy. Bad data processing and elimination can be
12 found in [18–23]. Finally, the system state is obtained by the state estimator using the processed
13 measurements and results from observability analyzer and topology processor [3,24]. The layout
14 of the EMS/SCADA system is shown in Figure (2). EMS is used to monitor and control the
15 operation of a power system, where SE plays important role. The measurement data is received
16 from devices such as RTUs, and more recently, PDCs [3]. These measurements, along with other
17 measurements (pseudo measurements and virtual measurements) and information from the
18 observability analysis and the topology processor, are used to estimate the system state. This
19 state is also used by the supervisory control system, which generates the control sequence for the
20 switchgear (circuit breakers).

21 ***Figure 3.*** SE Process

22
23 ***2.1. Conventional SE problem formulation***

24
25 SE has widely been adopted in industry and has attained much research attention over the last few
26 decades [25,26].

27 The measurement model \mathbf{z} is given in equation (1) as;

28
29
$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{e} \tag{1}$$

1 Where $\mathbf{z} \in \mathbb{R}^{m \times 1}$ is the measurements vector having ‘ m ’ measurements (actual, pseudo and
2 virtual) and $\mathbf{x} \in \mathbb{R}^{N \times 1}$ (where N being the number of network buses) is the vector of state variables
3 consisting of node voltage magnitudes and phase angles, and it may include tap positions,
4 $\mathbf{e} \sim \mathcal{N}(\mathbf{0}, \mathbf{R})$ is the observation noise, with Gaussian distribution of zero mean and covariance
5 matrix \mathbf{R} , and finally, $\mathbf{h}(\cdot)$ is a non- linear function vector relating the measurements to the state
6 variables, for instance, power flow equations [2]. These are given in equations (2-5):

$$7 \quad P_i = V_i \sum_{j=0}^N V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \quad (2)$$

$$8 \quad Q_i = V_i \sum_{j=0}^N V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \quad (3)$$

$$9 \quad P_{ij} = V_i V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) - G_i V_i^2 \quad (4)$$

$$10 \quad Q_{ij} = V_i V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) + B_i V_i^2 \quad (5)$$

11 The measurement is linearized about an operating point using Taylor’s series expansion.

$$12 \quad \mathbf{z} = \mathbf{h}(\mathbf{x}_0) + (\mathbf{x} - \mathbf{x}_0) \left(\frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}} \right) + \mathbf{e}(\mathbf{x}) + h.o.t \quad (6)$$

13 Where;

$$14 \quad \Delta \mathbf{z} = \mathbf{z} - \mathbf{h}(\mathbf{x}_0) \quad (7)$$

15 Hence, after ignoring higher order terms (*h.o.t*) in (6)

$$16 \quad \Delta \mathbf{z} = \mathbf{H} \Delta \mathbf{x} + \mathbf{e}(\mathbf{x}) \quad (8)$$

17 Where \mathbf{H} is the Jacobean matrix and is given by

$$18 \quad \mathbf{H} = \frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}} \quad (9)$$

19 The measurement covariance matrix \mathbf{R} is defined based on the variances of various measurements

20 as;

$$21 \quad \mathbf{R} = \text{diag}(\sigma_1^2, \sigma_2^2, \dots, \sigma_m^2) \quad (10)$$

22 Also, the gain matrix is obtained as:

$$23 \quad \mathbf{G} = \mathbf{H}^T \mathbf{W} \mathbf{H} \quad (11)$$

1 Where $\mathbf{W} = \mathbf{R}^{-1}$ is the measurement weight matrix. If the system is fully observable the gain
 2 matrix \mathbf{G} is positive definite and non-singular. This is ensured by including independent
 3 measurements in the measurement set with size greater than the size of state vector.

4 The WLS formulation for the SE is done as the minimization problem as;

$$5 \quad \min \mathbf{f} = [\mathbf{z} - \mathbf{h}(\mathbf{x})] \quad (12)$$

6 Subject to:

$$7 \quad \mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{e}(\mathbf{x}) \quad (13)$$

8 This can be written in the famous WLS objective function form as;

$$9 \quad \min \mathbf{f} = \sum_{n=1}^m \mathbf{W}[\mathbf{z} - \mathbf{h}(\mathbf{x})]^2 \quad \text{Or} \quad \min [\mathbf{z} - \mathbf{h}(\mathbf{x})]^T \mathbf{W} [\mathbf{z} - \mathbf{h}(\mathbf{x})] \quad (14)$$

10 Due to a non-linear measurement model, iterative techniques such as the Newton-Raphson method
 11 [27] are used to obtain the state estimate.

$$12 \quad \Delta \mathbf{x} = \mathbf{G}^{-1} \mathbf{H}^T \mathbf{W} [\mathbf{z} - \mathbf{h}(\mathbf{x})] \quad (15)$$

$$13 \quad \mathbf{x}^{k+1} = \mathbf{x}^k + \Delta \mathbf{x}^{k+1} \quad (16)$$

14 Where k is the iteration count for SE process. Once the state is determined, bad data analysis can
 15 be done using some statistical measures such as largest normalized residual test [28]. This method
 16 is mostly applied in TSSE. Equation (15) is known as Normal Equation in the literature.

17 **3. Distribution system state estimation (DSSE)**

18
 19 Research on DSSE began near 1990 [29–32]. The research motivation for DSSE came due to
 20 various reasons. The following sections describe the need for DSSE and its various formulations
 21 available in the literature.

22 **3.1. Need for DSSE**

23
 24 DSSE will play a central role in the implementation of the smart grid features such as DA, DR
 25 and increased involvement of renewable energy sources and hybrid electric vehicles. Thus,
 26 distribution grid will become an active network that will be more dynamic compared to the
 27

1 current passive DS. Due to fast changing dynamics, an efficient monitoring and control has to be
2 developed incorporating smart grid features such as DA, situational-awareness and DR. The
3 conventional SE techniques applied to TSs, are not directly applicable to the DNs because they
4 differ in the following ways [3,24,33];

5
6 **3.1.1. High R/X ratios:** Electric DNs, due to low voltage levels and comparatively shorter
7 lengths, have higher R/X ratios than the TSs. Therefore, DSs cannot be modeled and analyzed as
8 TSs due to the fact that the assumptions made for these networks are not true for DNs. Iterative
9 algorithms that use the Newton-Raphson simply do not converge for networks with higher R/X
10 ratios. Moreover, for such networks, DC approximation and de-coupled power-flow solutions
11 also becomes invalid [11,34–36]. A line-plot for different cross-sectional areas of ACSR cables,
12 for both TSs and DSs data given in [37,38], is shown in Figure 4. A separator is used to
13 distinguish the two systems using R/X ratios. This characteristic shows why state estimators
14 developed for transmission networks fail to work for DSs.

15 **Figure 4.** R/X ratios for TSs and DSs

16
17 **3.1.2. Low real-measurements availability:** Unlike TS, real-time measurements are very
18 limited in DS and are not enough for the observability analysis required in the estimation
19 process. The conventional SE assumes the system to be over-determined by having redundant
20 measurements but DSs are under-determined. Various attempts have been made to solve this
21 measurement scarcity problem in DSs by generating pseudo measurements using load data.
22 Greater proportion of pseudo measurements compared to the real-time measurements can
23 compromise the accuracy of DSSE. To improve the accuracy of DSSE algorithm, many
24 intelligent load estimation techniques have been proposed in the literature (see, section 6.1).
25 However, data from recent vast deployment of smart meters have made it possible to develop
26 accurate DSSE algorithms (see, section 6.4).

27
28 **3.1.3. Scalability and complexity:** The DS are complex as they depend on the area. Rural area
29 DNs are less dense as compared to those in urban areas. The more the density of the DN, the
30 more its complexity. Thus, distributed or multi-area DSSE techniques should be developed that are

1 efficient and scalable to achieve all sort of complexities (see, section 5).

2
3 **3.1.4. Complex measurement functions:** The measurements available at feeders are current
4 and power injections. Direct voltage and power measurements are rarely available, which
5 complicates the measurements functions. Recently PMUs or μ PMUs have been researched for DSs
6 which provide direct measurements of voltage and current phasors thereby eliminating the non-linearity
7 of measurement functions (see, section 6.3).

8
9 **3.1.5. Unbalanced phases:** In DS, it is very common to have three phase imbalances. The
10 conventional SE works on the assumption of positive sequence or three phase balanced network
11 where three phase models are not needed. However, these methods cannot be applied if there are
12 phase imbalances which is a common scenario in DS.

13 **3.2. Modification on conventional SE for DSSE**

14
15 The distinctive characteristics of DSs are making them different from the TSs. Therefore, the SE
16 techniques applicable to TSs are not applicable to DSs in their original form and requires
17 modification. In the literature, many papers have built upon by making the conventional
18 techniques work for DS. Papers in the literature can be categorized in the following four classes
19 based on the nature of such modification.

20
21 **3.2.1. Adapting WLS based TSSE to DSSE:** In the literature, some modifications on WLS
22 TSSE are given. A SE technique, which uses available set of remote measurements (voltages,
23 real and reactive power and substations currents) along with statistical load data of distribution
24 transformers, is proposed in [29]. Similar SE techniques with three phase details are presented in
25 [30] and [31]. The work in [31] used synchronized measurements along with asymmetric model
26 of DS. A 3-phase fast decoupled state estimator is proposed in [39]. The advantage of this
27 approach is that the gain matrix stays constant and symmetric which reduces the computational
28 burden. The disadvantage of all these methods is that they lack robustness and will not converge
29 to a unique solution in the presence of bad measurements (e.g. measurements majorly corrupted
30 by noise) [40].

1
2 **3.2.2. Load Estimation:** A few authors have discussed load estimation for DSSE due to the
3 fact that metered measurements are very limited in DS which are not sufficient to ensure
4 observability. Therefore, pseudo measurements are used to solve the issue of observability. Since
5 DSSE algorithms have to rely more on pseudo-measurements, the authors in [41] propose DSSE
6 which increases the accuracy of these measurements by taking into account the three-phase
7 details and limited availability of real-time measurements. Authors in [42], used WLS-based
8 DSSE algorithm to estimate both star-connected (or Y-type) and delta-type loads in a real-life
9 radial DS. Although this algorithm works better for the radial system, but it doesn't take into
10 account the DG penetration, which is changing the shape of the DN from radial to meshed
11 configuration. To account for this, a modification on [42], is proposed in [43], in which the
12 voltages measurements and meshed network topologies (due to enhanced DG penetration) are
13 incorporated.

14
15 **3.2.3. Phase imbalance problem:** Another issue with DS is phase imbalances that exists in
16 practice. This has to be considered in order to perform accurate SE. A few papers have been built
17 on this problem such as [32,44,45]. These SE methods work with phase imbalances and with
18 high R/X ratios, where the conventional WLS approaches fail to provide a solution. To apply
19 conventional WLS methods, a current-based SE is proposed in [32]. Branch current formulation is
20 considered instead of node voltage based formulation because the per-phase decoupling of the
21 Jacobian matrix \mathbf{H} is possible. This makes it possible to treat each phase as an independent SE
22 problem and thus helps in the application of conventional WLS method to unbalanced DNs.

23
24 **3.2.4. Incorporation of DERs:** With shift towards smart grid the DS is changing due to
25 the integration of DERs, and energy storages in the form of flexible structures such as
26 microgrids. To enhance observability of DS, the effect of DERs or microgrid must be
27 incorporated in the SE problem. A microgrid state estimator is proposed in [33], which is
28 based on conventional WLS and incorporates the additional dynamics introduced by DER.
29 it works better with the topology errors but bad data may affect the state estimate. In [46],
30 the autonomous SE method is proposed which takes into account the fast changing

1 topology information due to the presence of DERs. Whenever a DER connects to or
 2 disconnects from the DS, it can be detected automatically and the system model is updated.
 3 Another WLS and FA-based hybrid DSSE algorithm, that considers substantial penetration
 4 of DER, is proposed in [47]. The FA algorithm is a heuristic method which is employed to
 5 increase the estimation accuracy. Another similar study can be found in [48], in which
 6 authors extended the DSSE algorithm for identifying unexpected power-injections (both
 7 active and reactive) at PCCs of DERs and/or Microgrids. The estimates helps the system
 8 operator in taking proper actions by comparing the estimated injections with real values.

9 10 *3.3. Node-Voltage-based state estimation*

11 In the NV-DSSE, complex node-voltages are considered as state variables. The state
 12 variables can be either expressed in polar-coordinates such as $\mathbf{x} =$
 13 $[\theta_2, \theta_3, \dots, \theta_N, V_1, V_2, \dots, V_N]$, or in rectangular-coordinates containing the real and imaginary
 14 parts of node-voltages such as $\mathbf{x} = [V_1r, V_2r, \dots, V_Nr, V_1x, V_2x, \dots, V_Nx]$, where N represents the
 15 number of system nodes or buses. The measurement function is given in (1) where \mathbf{z} is the
 16 measurement or observation vector containing measurements of all types that is:

- 18 • Real-time non-synchronized measurements such as line power flows, bus power
 19 injections, and voltage, and current magnitudes.
- 20 • Real-time Synchronized measurements from PMUs such as Voltage and current
 21 measurements along with phase angles.
- 22 • Pseudo-measurements obtained using statistical load profiles.

23 In the polar formulation, bus-1 is normally treated as reference bus and its angle is considered
 24 zero (i.e. $\theta_1 = 0$). The phase angles of all other buses are measured with respect to this angle
 25 therefore ' θ_1 ' is excluded from the state-vector. However, if the PMU measurements are present,
 26 ' θ_1 ' may be included as one of the state variable since the reference is not required [49,50].
 27 Later, the system state can be determined by using the WLS approach. In [41], a three-phase
 28

1 state estimator is developed using the NV-DSSE formulation. After the state is determined, the
2 branch currents can easily be calculated using the voltage drops at every node. Other similar
3 readings can be found in [30,51].

4 5 **3.4. Branch-Current-based state estimation**

6
7 In the BC-DSSE method, state variables are complex branch-currents. In this formulation the
8 rectangular coordinates of state variables are used. In the system where there is no PMU
9 available, the state vector solely consists of branch currents (real and imaginary
10 components); $\mathbf{x} = [I_1r, I_2r, \dots, I_Nr, I_1x, I_2x, \dots, I_Nx]$, where N represents the number of
11 branches. However, if PMUs are installed then the state vector will contain the slack bus
12 voltage [52]. In [53], authors propose a BC-DSSE algorithm that considers both traditional
13 SCADA measurements and synchronized measurements from PMUs.

14 BC-DSSE algorithm consists of the following steps to be carried out at each iteration update [52].

- 15
16 • Conversion of power measurements to equivalent current measurements [32].
- 17 • Estimate of branch currents by solving (15).
- 18 • Update of state vector using equation (16).
- 19 • Compute the network node voltages using forward-sweep starting from the slack bus and
20 tracing down the network graph.

21 The details of forward-sweep algorithm can be found in [44,54].

22 The BC-DSSE is presented by Mesut Baran in his paper [32] using the WLS-based approach. A
23 few other readings and implementations of this algorithm with slight variations can be found in
24 [55–57].

25 **3.5. NV-DSSE and BC-DSSE-Comparison**

26
27 In [58] and [59] the two SE formulations, i.e. NV-DSSE and BC-DSSE, are compared. In [58],
28 the authors present an extensive comparison of the two formulations regarding complexity,
29 numerical stability, convergence, computational expense and sensitivity to measurement weights.

The results are tabulated in Table 1, which justify and endorse the BC-DSSE method as a promising one and emphasize its use in DS as compared to NV-DSSE.

Table 1: NV-DSSE Vs BC-DSSE [32,58]

Similar comparison is made in [59] with synchronized measurements from PMUs and traditional (non-synchronized) SCADA measurements. The two SE formulations are compared, based on RMSE%, convergence and computational time, with and without PMU data. BC-DSSE and NV-DSSE with traditional measurements are named original-traditional and original-synchronized when PMU measurements are included. Similarly, the extended BC-DSSE and NV-DSSE algorithms, given in [53] and [50], respectively, are named extended-traditional and extended-synchronized for traditional and PMU measurements, respectively. Comparison results are given in Figures 5, 6 and 7. Figures 5 and 6 show the RMSE% in Voltage and Current respectively, for BC-DSSE and NV-DSSE methods. In Figure 5, it can be noted that the performance of NV-DSSE (both original and extended) is better than that of the original BC-DSSE algorithm and similar to that of the extended BC-DSSE algorithm in terms of RMSE% of voltage.

Figure 5. Percent-RMSE in Voltage

However, in terms of RMSE% current (Figure 6), the performance of both NV-DSSE and BC-DSSE is deemed neutral.

Figure 6. Percent-RMSE in Current

In Figure 7, the computational comparison of both algorithms shows that BC-DSSE algorithms are faster than those of NV-DSSE.

Figure 7. Average iterations and computational time

4. Classification of DSSE techniques

Based on the time evolution of state vector and measurement model, the DSSE techniques are broadly categorized into two categories, namely Static DSSE, and dynamic (or forecast aided) DSSE.

4.1. Static DSSE techniques

In static SE it is assumed that the state of power system is not changing much between two consecutive state updates. This is called quasi-steady state condition of power system. Static SE techniques have been researched a lot and many techniques can be found in the literature.

Using the measurement in (1) and the normalized residual vector for k^{th} measurement can be defined as:

$$\mathbf{r}_k = \frac{z_k - \mathbf{h}(x_k)}{\sigma_k} \quad (17)$$

The objective is to minimize the residual given in (17). Generally this is expressed as [60]:

$$J = \sum_{k=1}^M \zeta_k(r) \quad (18)$$

The function $\zeta_k(r)$ can be evaluated differently thus producing different estimators such as WLS, WLAV and SHGM. These estimators will be reviewed briefly.

4.1.1. WLS: For WLS algorithm the function $\zeta_k(r)$ takes the following form:

$$\zeta_k(r) = \frac{1}{2} r_k^2 \quad (19)$$

The objective function to be minimized is given in (14).

4.1.2. WLAV: In this algorithm the function $\zeta_k(r)$ has the following form:

$$\zeta_k(r) = |r_k| \quad (20)$$

In WLAV, the minimization of the following objective function is required

$$\text{Min } (J(\mathbf{x})) = \left\| R^{-\frac{1}{2}} [\mathbf{z} - \mathbf{h}(\mathbf{x})] \right\|_1 \quad (21)$$

Further details of this algorithm can be found in [61].

4.1.3. SHGM: This estimator is based on Huber function and represents a good compromise of WLS and WLAV.

$$\zeta_k = \begin{cases} \frac{1}{2} \mathbf{r}_k^2, & \text{if } |\mathbf{r}_k| \leq a\omega_k \\ a\omega_k - \frac{1}{2} a^2 \omega_k^2, & \text{otherwise} \end{cases} \quad (22)$$

This estimator is sensitive to weight parameter ‘ ω_k ’ and tuning factor ‘ a ’. The solution of this estimator is obtained through IRLS algorithm [2]. In [60], the three algorithms, i.e. WLS, WLAV and SHGM, are compared based on three statistical measures, namely bias, consistency and quality. Based on these measures results are produced and it is shown that WLS gives the best performance and is a preferred choice for DSSE solver. Detailed analysis and simulation results can be found in [60].

4.2. Dynamic DSSE techniques

The previous section discussed SE methods that are static. These methods takes into account single snapshot of measurement data for the state estimate and its evolution over successive measurement instants is disregarded [62,63]. On the other hand, the DSE techniques, or more appropriately the FASE techniques, consider the time evolution of state over time and can track system changes during its normal operation. FASE techniques inherently consists of a forecasting feature which can help provide near real-time monitoring of the system [62,64]. Generally FASE process involves the following four steps as shown in Figure 8.

Figure 8. Steps involved in the DSE process

4.2.1. Mathematical model: The DSE process considers the following mathematical model [65,66].

$$\mathbf{x}_{k+1} = \mathbf{g}(\mathbf{x}(k), \mathbf{u}(k), \boldsymbol{\omega}(k), k) \quad (23)$$

Here, k represents the time instant, and $\mathbf{x}(k)$, $\mathbf{u}(k)$ and $\boldsymbol{\omega}(k)$ are the values of state vector, input vector and process noise at time instant k . These values are related to the future state vector, \mathbf{x}_{k+1} , through a non-linear vector function $\mathbf{g}(\cdot)$. Model in (23) is far more complex, therefore following assumptions are made to simplify it for easy implementation [65,66].

- Power system is assumed to be operating in quasi-steady state. In quasi-steady state,

the state transition can be considered linear.

- The process noise is modelled as a zero-mean Gaussian distribution with constant covariance \mathbf{P} .

Equation (23) can be re-written in a simpler form using the above assumptions as:

$$\mathbf{x}_{k+1} = \mathbf{F}_k \mathbf{x}_k + \mathbf{g}_k + \mathbf{v}_k \quad (24)$$

The system observation model is given by equation (25).

$$\mathbf{z}_k = \mathbf{h}_k(\mathbf{x}_k) + \mathbf{w}_k \quad (25)$$

Where $\mathbf{F}_k \in \mathbb{R}^{L \times L}$ is state transition matrix, where $L = 2N - 1$ is the number of state variables with N being the number of buses; \mathbf{g}_k is related to trend behavior and state trajectory; \mathbf{z}_k is the measurement vector; \mathbf{v}_k and \mathbf{w}_k are process noise and observation noise respectively, both having a zero-mean Gaussian distribution with covariance matrices \mathbf{P} and \mathbf{Q} respectively. The covariance matrix \mathbf{P} is normally considered to be constant (e.g. 10^{-6}).

The observation model is non-linear and is linearized using Taylor's series as:

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{w}_k \quad (26)$$

Where \mathbf{H}_k is the Jacobian matrix previously defined in equation (9).

4.2.2. Parameter identification: This is the second step of DSE process. In the literature, many authors [64–71] have adopted the Holt-Winters exponential smoothing technique for the identification of \mathbf{F}_k and \mathbf{g}_k . Debs and Larson, in [66], assume a simple state transition model by considering \mathbf{F}_k as an identity matrix and a zero \mathbf{g}_k . This reduces equation (24) to a simpler form given by (27). Similar assumption is also applied by the authors in [67,72,73].

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{v}_k \quad (27)$$

4.2.3. State prediction: State prediction utilizes the assumed system model and predicts

the future values of the state variables. In the literature, there are many algorithms that perform this prediction step, e.g. ANNs [74] and algorithms based on Fuzzy logic [75]. Some authors used auto-regression-based models for state prediction [65].

4.2.4. State filtering: Filtering is the final step involved in the DSE process. In this step, bad data are filtered out from the measurement set using the measurements that arrive at time instant ' $k + 1$ ' and the predicted information obtained in the prediction step of the dynamic estimation process. The EKF is widely used method to do the filtering step [76]. The recursions of EKF based methods, using the measurements coming at time instant ' $k + 1$ ' i.e. $\mathbf{z}_{k+1} = \mathbf{h}(\mathbf{x}_{k+1}) + \boldsymbol{\omega}_{k+1}$ are given by [3].

$$\hat{\mathbf{x}}_{k+1} = \tilde{\mathbf{x}}_{k+1} + \mathbf{K}_{k+1}[\mathbf{z}_{k+1} - \mathbf{h}(\tilde{\mathbf{x}}_{k+1})] \quad (28)$$

Where;

$$\tilde{\mathbf{x}}_{k+1} = \mathbf{F}_k \hat{\mathbf{x}}_k + \mathbf{g}_k \quad (29)$$

$$\mathbf{K}_{k+1} = \boldsymbol{\Sigma}_{k+1} \mathbf{H}_{k+1}^T \mathbf{Q}^{-1} \quad (30)$$

$$\boldsymbol{\Sigma}_{k+1} = [\mathbf{H}_{k+1}^T \mathbf{Q}^{-1} \mathbf{H}_{k+1} + \mathbf{M}_{k+1}^{-1}]^{-1} \quad (31)$$

$$\mathbf{M}_{k+1} = \mathbf{F}_k \boldsymbol{\Sigma}_k \mathbf{F}_k^T + \mathbf{P} \quad (32)$$

Other formulations of DSE can be found in [25], and more detailed survey on DSE techniques can be found in [77].

From the literature, it can be observed that the DSE or FASE techniques are more focused on TSSs, such as the studies done in [67,71] and [78]. In [67], an exponential-weight function is used to increase the robustness of the EKF-based estimator. Another modification to this algorithm is given in [69], in which the Taylor's series of the non-linear measurement function is expanded to include the second order term, which enhances the accuracy of the estimation process. EKF utilizes the linearization of the dynamics of the system involved, which has inherent flaws. In [79], a new variant of Kalman filter, namely

UKF, is proposed which has the same computational expense as EKF. The authors in [80], discuss various flaws of EKF and extend the use of UKF to other diverse applications such as system-identification, ANN-training and problems of dual-estimation. Although UKF has been applied to non-linear systems with specific models, it was first applied to power systems, in which a specific system model does not exist, in [81], in which the authors have re-formulated UKF for power systems. Here, UKF is compared to EKF and to WLS for diverse simulation scenarios such as normal operating conditions, bad data, and sudden load variation. The authors concluded that comparatively UKF performs better in all simulated scenarios. In [82], a UKF-based hybrid-dynamic estimator is proposed which incorporates measurements from both the commonly available SCADA system and PMUs. The algorithm was validated for IEEE 14-bus and New England 39-bus networks and compared with conventional UKF, and it was shown that the proposed algorithm outperforms the conventional UKF. Another UKF-based estimator that accounts for randomly delayed measurements, namely UKF-RD, is presented in [83], in which simulation results demonstrated the accuracy of UKF-RD compared to conventional UKF.

Although dynamic estimators have been proposed for TSs, only a few studies have discussed the DSE solution for DSs. In [84], a UKF-based estimator is proposed for DNs considering renewable energy integration. Since DNs lack real-time measurements, network observability is achieved using pseudo measurements, which are generated using historical load forecasting. In this regard, the authors in [85], propose a UKF-based dynamic estimator, which utilizes short-term load and DG forecasting for generating pseudo measurements. This algorithm was validated using a 123-bus DN to demonstrate its effectiveness. Various dynamic estimators have been applied to power TSs, but from the

DS perspective, DSE techniques are not very prevalent, possibly due to the following reasons.

1. FASE techniques require measurements with high resolution (e.g. from PMUs), which is so far not possible in DS due to a lack of communication infrastructure [86,87].
2. The large problem size of DS (due to its dense nature) can lead to a huge computational burden [8].

5. Multi-area DSSE

The DSs are denser as compared to TSs due to the increased number of nodes per unit area. This predicts that SE is likely to face large computational challenges, creating the need for more computational resources. To remedy this problem, the large network is divided into smaller networks, each consisting of an LSE. The LSE of each network area estimates the state of its concerned area network using the measurements from that particular area.

$$\mathbf{z}_l = \mathbf{h}_l(\mathbf{x}_l) + \boldsymbol{\omega}_l, l = 1 \dots L \quad (33)$$

Where $\mathbf{x}_l = [\mathbf{x}_{il}^T \ \mathbf{x}_{bl}^T]^T$ is the state estimate of local area network ‘ l ’. The state vector comprises the internal state variables (i.e. \mathbf{x}_{il}) of any particular area and border or tie-lines state variables (i.e. \mathbf{x}_{bl}) between two neighboring network-areas. A central estimator coordinates all the network-areas and processes and augments the states of the individual network-areas into a single state vector that represents the whole system (see Figure 9).

Figure 9. Multi-area SE

MASE may or may not contain a central coordinator. MASE with central coordinators are called hierarchical-MASE in the literature, whereas those without it are decentralized-MASE [27]. Figure 9 shows an example of hierarchical-MASE, but if the central estimator

is ignored it will become decentralized-MASE. In [88], authors propose a hierarchical-MASE with an alternative approach through which sensitivity-functions are exchanged instead of system-states between the neighboring areas. This improves convergence speed and reduces the data exchanges between the neighboring areas [88]. A decentralized-MASE is proposed in [89], which undertakes a two-step estimation process to determine the state of a large DS. A large DN is first divided into manageable local-area networks based on geography, various topological-constraints and available metering infrastructure. Later, local estimates for all the network-areas are obtained, which are utilized by the second estimation step to determine the updated state of the whole DS. In [90], a distributed-DSSE is presented which can take into account different types of measurement data from PMUs, smart meters, and SCADA to estimate the state of the DN. The main advantage of this method is its applicability to both radial and meshed networks with frequently varying system configuration. Another robust and fully decentralized-MASE based on BSE is proposed in [91], which takes into account non-linear measurements. Although MASE is an attractive paradigm, it has certain inherent drawbacks such as heavy dependency on the communication network between the neighboring areas. To cope with the challenge of this computational burden and to relieve the communication infrastructure, a decentralized UKF-based MASE is proposed in [92] for the power system SE along with a consensus-algorithm. The authors propose a multi-area dynamic state estimator which splits the network into non-overlapping areas and carries out estimation for each area locally. Later, the consensus-algorithm initiates local communication among the neighboring network-areas to exchange state information. Another work on MASE technique can be found in [93], in which event-driven sensing, estimation and communication is implemented to minimize the data exchange and thus, reduce the

dependency on the communication network.

Multi-area DSSEs are robust and computationally efficient, but they come with problems of time-skewness due to non-synchronized measurements obtained in different network-areas [3]. A more detailed survey on MASE and associated challenges can be found in [27].

6. Future Research directions for DSSE

With the grid becoming more and more intelligent, it is getting more dynamic and complex. The events occurring in smart grids will be difficult to control manually. This in turn would require the extension of monitoring and control to the distribution level. Hence, DSSE will play a vital role in future smart DS. In the following sections, several new research areas for DSSE are discussed.

6.1. Intelligent load forecast techniques for DSSE.

DSs have fewer available real measurements than TSs do. These are not sufficient for ensuring system observability, which is crucial for the state estimator to work. In the literature, the remedy to this problem is performed through load forecasting. In this regard, ML and ANN-based methods present a viable solution. A detailed review on the application of ANNs for load forecasting can be found in [94]. In [95], an ANN-based load forecasting model is presented in which pseudo measurements are generated for DSSE. Another load estimator based on a ML technique is proposed in [96], in which the load model developed works in a closed loop and has the capability of training itself as new measurement data comes in and thus, enhances the performance of DSSE. Closed loop models are developed for the load forecast in [97,98]. These have the advantage of increased accuracy resulting in improved the performance of DSSE. Similar approaches are used by [99,100] to accurately estimate the load, thus enhancing the accuracy of DSSE. A real-time load modeling technique is presented in [99], in which the customer

load curves data and measurements of line flows have been utilized to approximate the uncertainty in the load estimates, which are used by DSSE. Another technique for generating pseudo measurements for DSSE is proposed in [100], in which the authors use a Gaussian mixture model to represent the load probability density function in DSs, where the mixture parameters are attained through a EM algorithm. These load models can be used by DSSE as pseudo-measurements.

6.2. *Event-triggered DSSE techniques.*

One of the challenges involved in extending the TSSE to the DS is increased computational burden due to the big problem size of DNs. In [8] and [101,102] an event-triggered approach is applied to the existing WLS SE to improve its computational efficiency and estimation accuracy in the presence of variable energy sources. The results obtained with the developed SE technique are more promising than the existing WLS SE. In [93] an event-triggered MASE is developed which is able to perform event-based sensing, estimation and communication. The occurrence of an event happens if there is sufficient novelty in the measurements above a certain threshold value. The advantage of this method is that it makes efficient use of communication and computational resources. In the literature, event-triggered approaches have also been adopted for model identification, especially in DSs where topology errors are more common because not all the breakers are monitored. The detection and elimination of these errors are necessary for accurate estimation. In [103], the authors present a topology identification algorithm based on RBA. Various critical power system configurations are defined as different topologies in a model bank. The SE algorithm is run for all topologies in parallel, and Bayesian-based probabilities are calculated for all of the models. The probability of the correct topology model reaches '1', whereas those of

others converge to zero. The a-posteriori probability of correct model ψ_i is given by (35);

$$p(\psi_i|\boldsymbol{\varepsilon}^j) = \frac{p(\mathbf{e}^j|\psi_i)p(\psi_i|\boldsymbol{\varepsilon}^{j-1})}{\sum_{k=1}^{N_m} p(\mathbf{e}^j|\psi_k)p(\psi_k|\boldsymbol{\varepsilon}^{j-1})} \quad (34)$$

Where $\boldsymbol{\varepsilon} = \{\mathbf{e}_1^j, \mathbf{e}_2^j, \mathbf{e}_3^j, \dots, \mathbf{e}_{N_m}^j\}$, are the corresponding error vectors for models $\{\psi_1, \psi_2, \psi_3, \dots, \psi_{N_m}\}$ respectively; $p(\psi_i|\boldsymbol{\varepsilon})$ is the a-priori probability, and $p(\mathbf{e}^j|\psi_i)$ is the probability of i-th model error. Although this approach is effective in identifying the correct topology, it may converge slowly in the presence of noise. This issue is addressed by [104], in which the authors propose a Seidel-type recursive Bayesian approach, in which it is shown that the convergence speed is improved even in the presence of noise. Very recently, an SOR-based RBA is proposed in [105], which has further increased the convergence speed of both basic-RBA and Seidel-type RBA. Topology identification results for all three algorithms for the IEEE 6-bus system [11] are shown in Figure 10, for a case of 10% noise in the measurements. It can be observed that all the algorithms end up selecting the correct system configuration but with different convergence speed, and SOR-RBA converges quickly as compared to the Seidel-type RBA and the basic-RBA in the presence of noise.

Figure 10. Three RBA approaches: a) Basic-RBA [103] b) Seidel-type RBA [104] c) SOR-RBA [105].

A similar approach can be found in [106], where three estimators, namely WLS, EKF and UKF, are used, and it is shown that when the topology is known a-priori, UKF performs better than WLS and EKF. When the topology is not known a-priori, a configuration change is detected using a forecast-aided technique and later, the correct topology is recognized from a bank of available options using an event-triggered based recursive Bayesian filter. Although Bayesian based topology identification methods perform well for smaller and

medium sized networks, their performance may degrade for larger networks due to an increased number of possible topologies in the model bank [107]. In [107], the authors implemented event-driven RBA algorithm and generalized-SE for configuration identification of 48-bus MV DN with DG and microgrid integration. The performance of both algorithms is evaluated in the presence of an increased number of system topologies and in the presence of noise. It is shown that the computational performance of the RBA-based approach deteriorates compared to that of the generalized-SE when the number of configurations in the model bank are increased. This in turn motivates seeking accurate and computationally efficient power system topology identification algorithms in the future.

6.3. Incorporation of PMU measurements in DSSE.

Power systems are becoming more dynamic with the added role of DERs. These resources, being stochastic in nature, add uncertainties to power system dynamics. These fast changing dynamics may not fully be captured by traditional SCADA sensors. Therefore, PMUs came into picture in the year 1980 [3] which are capable of providing synchronized measurements of voltage and current phasors with a time stamp from a GPS-based universal time clock. These synchronized measurements can help avoid iterative SE techniques by providing a linear relation between measurements and states, ultimately reducing the computational complexity of these algorithms [3]. They could prove to be more useful in DSs, where DSSE would likely face more challenges like computational complexity and estimation accuracy. In [108,109] and [110,111], the authors have worked on the incorporation of PMU in the DSSE algorithm. Beside all these studies, the deployment of PMUs in DNs is not economical. Hence, in [10], the authors developed μ PMU to offset the installation costs of these units in DS. In [112], a

linear DSSE algorithm is formulated assuming that the DN is completely observable with μ PMUs. The disadvantage of [112] is that it assumes the full scale installation of μ PMU, which is not yet possible due to economic constraints. In [113], the authors propose a compressive sensing based DSSE algorithm that makes use of a lower μ PMU and utilizes l^1 -norm to solve the underdetermined system. The algorithm was validated for weakly-meshed 123-bus and 134-bus networks with different levels of DER penetration, and a performance comparison of the proposed algorithm and conventional WLS-based DSSE algorithm. Apart from all these studies, μ PMUs are still expensive and their massive deployment in DS is not possible; today's need is to develop DSSE that can use both synchronized data (e.g. PMU or μ PMU data) and non- synchronized data (e.g. smart meters, SCADA sensors and pseudo measurements). In [114], two different ways of fusing PMU data and conventional SCADA measurement data were found for static SE. These are:

- A single stage state estimator in which both conventional SCADA and PMU or μ PMU data can be combined to reach an optimal state estimate.
- A hierarchical double stage state estimator in which a state estimate is obtained by using only conventional SCADA measurement data. This estimated state is then mixed with the measurement data from PMU and similar units to get the optimal state estimate.

Such methods can be extended to DSSE in future.

6.4. Inclusion of smart meter measurement data in DSSE.

The availability of more DS loads data from smart meters can help better estimate and model the load behavior and as a result can increase the accuracy of DSSE. In the

literature, inclusion of smart meter data from AMI is also exploited to increase the accuracy of DSSE algorithms [115,116]. In [117], measurement data from smart meters is used for estimating various network variables such as voltages and line flows etc. Another method that uses compressed smart meter data and data from DERs, is proposed in [118]. In [119], an energy forecasting methodology is developed based on smart meter data for the operation of DS having substantial presence of DERs. Voltage and power or equivalent current measurements from AMI are used to estimate the 1-phase or 3-phase DN models [120]. Incorporation of data from smart meters is still challenging because of its non-synchronized and low data rate. The data reporting rate of smart meter is about 15 minutes, which may not capture the snapshot of system more effectively. In [121], authors propose a DSSE algorithm that utilizes non-synchronized smart meter data by proper adjustment of variances for these measurement. Using data from smart meters can help in providing system observability for certain unmonitored network-areas. In this regard, the hierarchal estimation techniques that make use of non-synchronized heterogeneous measurements (e.g. PMU data, smart meter data, and SCADA measurements) would be a better solution.

6.5. *Advanced energy management systems for DS.*

ADMS is another good research area where DSSE has a fundamental role. The relationship of DMS with its TS counterpart, i.e. EMS, is depicted in Figure 11. Earlier, DSs were passive with uni-directional power flows, which made their management and control easy. However, the future smart grid is transforming the existing power distribution grid in terms of 1) communication infrastructure, 2) integration of sources of different nature, 3) involvement of different types of loads and equipments, 4) data accumulation, 5) data security and sharing, and 6) deregulation of electricity grid which brings in many business

players [122]. Thus, the future grid would be an extra ordinary complex grid, whose operations would require certain common platform to increase its operational flexibility by facilitating flexible data exchange and system interoperability [122,123]. This would in turn require a fully functional DMS, which integrate sources and loads of different nature, and provide a platform for different utilities to cooperate in data sharing. In this regard, many researchers have tried to develop management and control functions to enhance system monitoring at the distribution level [124,125]. Algorithms for three important functions of DMS, namely load estimation, ac power flow, and optimal system re-configuration, are presented in [126]. Another similar study is performed in [127], in which the authors demonstrate the development of standard measurement-acquisition system and a real-time situational-awareness function for the Korean Smart grid initiative project. In [128], the authors develop an application software for DMS, which was used to investigate the effect of missing or delayed measurements on DSSE. In [129], a two-level DSSE algorithm is proposed for DMS of low-voltage (LV) DN. This algorithm was tested on a LV-network which has a mixture of conventional-generation sources and DGs, smart-loads, and storage-facility. The authors in [130], develop a DMS framework integrating network modeling, SE and control for the implementation of Volt/Var support service. However, these algorithms are proposed mainly for radial DNs and doesn't take into account meshed network topology. In this regard, a possible future work may consider the modification of [130] for meshed network topology with enhanced DG integration. Efficient and quickly convergent power flow algorithms, for instance [131–133], that considers both radial and meshed network models and integration of multi-DER, may be adopted.

Figure 11. Relationship between EMS and DMS

7. Conclusion

TSSE is well established and is present as a critical component of EMS because of the well-developed communication infrastructure. However, it is not implemented in the DSs firstly, due to passive nature of DS in which power flows are uni-directional and are easily manageable and secondly, due to the absence of communication infrastructure at this level. With the development of smart grid, which promises many features such as DA, demand responsive loads and increased integration of DERs, the distribution grid is evolving and is turning into active network. In active DS, power flows are bi-directional due to penetration of renewable energy sources such as wind energy, solar energy etc. Therefore, DSSE will have an essential role in such future active networks. This paper gives an overview of DSSE methods, its formulations and types present in the literature. Furthermore it provides brief possible future research directions for DSSE, including load forecasting for pseudo measurement generation, event-triggered SE, incorporation of PMU or μ PMU and smart meter data and finally development of ADMS.

8. Conflict of interest

The author(s) declare(s) that there is no conflict of interest regarding the publication of this paper. Also the financial support did not lead to any conflict of interests regarding the publication of this manuscript.

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