# Stochastic Last Mile Relief Network Design with Resource Reallocation

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STOCHASTIC LAST MILE RELIEF NETWORK DESIGN WITH RESOURCE REALLOCATION

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## Özet

Çalışmamız kapsamında, yardım malzemelerinin afet öncesinde depolandığı yerel dağıtım merkezlerinin mevcut olduğu durumlar için afet sonrası müdahale ağı tasarımı problemi üzerinde durulmaktadır. Afet öncesi mevcut ağ ile bütünleşik olarak, yerel dağıtım merkezlerinin ve son dağıtım noktalarının yerleri ve kapasiteleri afet sonrası duruma ilişkin belirsizlikler de göz önüne alınarak belirlenmektedir. Yeni erişilebilirlik ölçütleri tanımlanmış ve de daha erişilebilir ve daha adil bir şekilde yardım malzemelerinin dağıtılmasını sağlayacak iki alternatif iki aşamalı rassal programlama modelleri geliştirilmiştir. Ortaya konulan eniyileme modellerinin çözülmesi bilgisayısal açıdan güç olduğundan ayrışım yaklaşımna dayalı dal-ve-kesi algoritmaları geliştirilmiştir. Önerilen modellerle ilgili çıkarımlar sağlamak adına 2011 tarihinde Van ilimizde gerçekleşmiş olan depreme ilişkin verilere dayalı sayısal bir analiz yapılmıştır. Ayrıca, çözüm yöntemlerimizin etkinliğini gösteren sayısal bir çalışma da ortaya konmuştur.

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## Abstract

We study a last mile distribution network design problem for situations where there exist local distribution centers (LDCs) with pre-positioned supplies. Given the information on the existing pre-disaster relief network, the problem determines the locations and capacities of LDCs and points of distribution in the relief network, while capturing the uncertain aspects of the post-disaster environment. We introduce new accessibility metrics, and develop two alternate two-stage stochastic optimization models that would allow more accessible and equitable distribution of relief supplies. Since solving the proposed optimization models is computationally challenging, we employ decomposition-based branch-and-cut algorithms. We perform numerical analysis based on the real-world data from the 2011 Van earthquake in Turkey to provide insights about the proposed models, and also conduct a computational study that demonstrates the effectiveness of the solution method.

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# Chapter 1

# Introduction

As observed during recent disasters (e.g., 2005 Pakistan earthquake, 2010 Haiti earthquake, 2011 Turkey earthquake), relief organizations may face serious logistical challenges in the final stage of relief operations, in which aid is delivered to the people in affected areas. This final stage is known as the "*last mile*", and it is indeed usually the toughest stage of relief operations and can affect the overall effectiveness of response. The main challenges involve a high level of uncertainty in demand and transportation infrastructure, and high stakes associated with quick demand satisfaction (Kovacs and Spens, 2007; Balcik et al., 2008). Considering these challenges and the need of allocating scarce resources in an effective way, last mile logistics could greatly benefit from operations research methods (Altay and Green, 2006; Van Wassenhove and Pedraza Martinez, 2012). Along these lines, our study focuses on developing optimization models for the last mile network design problem, which incorporate the inherent uncertainty and the critical concerns in providing effective response service.

In disaster-prone regions (e.g., Turkey), local relief organizations pre-position relief supplies at several logistical facilities (local distribution centers) to decrease the response times, and consequently, to alleviate the suffering of the people in need, in case of a disaster. For instance, the Istanbul Metropolitan Municipality stores several types of relief items (e.g., water, blankets, tents, portable kitchen) at the distribution center located in Halkalı, which is constructed in 2006 with a total area of 38,000 square meters (Istanbul Metropolitan Municipality, 2006). The importance of establishing such pre-disaster relief networks has been emphasized in the recent literature (see e.g., Balcik et al., 2008; Salmerón and Apte, 2010; Hong et al., 2014). When a disaster occurs, it is crucial especially for the local relief organizations to evaluate the existing distribution network, and effectively utilize it in a post-disaster setting. In particular, post-disaster network conditions such as damaged transportation infrastructure can strongly affect access to the pre-positioned relief supplies. In such situations, relief organizations can set up new distribution centers in order to quickly deliver the pre-stocked supplies -as well as the relief supplies rushing into the country- to the affected areas. Thus, decisions on how to distribute the pre-stocked supplies include also decisions on whether or not to transfer them to the newly established distribution centers. Therefore, it is essential to consider the integrated structure of the overall network while making the last mile network design decisions, when the post-disaster environment entails setting up new distribution centers in addition to the existing ones.

The critical considerations in last mile relief network design include ensuring a high level of access to the supplies and ensuring equitable services (Sphere Project, 2011; Noyan et al., 2013). As emphasized in Noyan et al. (2013), post-disaster network conditions (such as damaged roads, topographical barriers, etc.), and demographical (such as gender and age) and socio-economical (such as vehicle ownership) characteristics of the affected population groups can strongly affect people's access to relief supplies. The definition of equity highly depends on the context (Marsh and Schilling, 1994). As in Noyan et al. (2013), we consider equity both in accessibility and supply allocation; equity in supply allocation is defined based on proportion of unsatisfied demand. In addition, it is also important to consider the logistics costs due to the decisions on locating new LDCs and distributing the relief supplies between the LDCs. In this spirit, we develop mathematical programming models for last mile relief network design, given a pre-disaster network that incorporates accessibility, equity and logistics costs.

In this research, we study the Stochastic Last Mile Relief Network Design Problem with Resource Reallocation (SLMRNDR), which assumes that there already exist some resources located before a disaster occurs, and integrates the decisions on the reallocation of pre-stocked relief supplies. In particular, we *mainly* determine the locations and capacities of LDCs and PODs, how to reallocate the pre-stocked relief supplies between the LDCs, and how to allocate the total available supply among the PODs. SLMRNDR incorporates the accessibility and equity concerns, and the uncertainty in post-disaster relief demands and transportation network conditions. To the best of our knowledge, this is a first in the humanitarian logistics literature. A closely related study Noyan et al. (2013) consider a similar last mile network design problem; however, it does not integrate any existing relief network into last mile network design, and assumes that there is a single LDC, whose location is fixed and known. More specifically, we extend the study -on a two-echelon relief system- by Noyan et al. (2013) to a more elaborate integrated last mile network design problem -for a three-echelon relief system.

Following the characterization of equity in supply allocation proposed by Noyan et al. (2013), we consider two policies for allocating supplies equitably among the selected PODs. The first supply allocation policy is proportional allocation (referred to as the PD Policy); it allocates the available relief supplies among the PODs in proportion to the total demand assigned to the PODs. The second policy (referred to as the TD Policy) allocates supplies by limiting the shortage amount at each POD by a specific proportion of the corresponding total demand; setting the proportion parameters equal for all PODs ensures equitable allocation. Noyan et al. (2013) show that enforcing the PD policy is better than enforcing the TD policy in terms of the equity performance. However, they also show that enforcing a strict proportional allocation can significantly compromise accessibility compared to enforcing the TD policy; this tradeoff between equity and accessibility becomes even more evident when capacity restrictions of PODs are more severe. In order to balance this tradeoff, as proposed by Noyan et al. (2013), we consider a hybrid allocation policy that combines the two equitable supply allocation policies. Different from Noyan et al. (2013), considering additional decisions regarding the multiple LDCs creates challenges in defining the accessibility metrics for the three-echelon relief network. Taking this challenge into account, we propose two new approaches to incorporate accessibility into optimization models. Consequently, we develop two alternative two-stage stochastic programming models incorporating the hybrid allocation policy that can achieve high levels of equity and accessibility simultaneously.

We characterize the inherent randomness by a finite set of scenarios, where a scenario represents a joint realization of all random parameters. It is well known that stochastic programming models are computationally challenging due to the potentially large number of scenario-dependent variables and constraints. Introducing integer variables into stochastic programs, as in the proposed ones, further complicates solving these models. To be able to solve large problem instances, we develop an effective branch-and-cut algorithm based on Benders decomposition.

The rest of the study is organized as follows. In Chapter 2 we review the relevant literature. In Chapter 3, we describe the problem SLMRNDR in detail and present the corresponding mathematical programming formulations. Chapter 4 is dedicated to the solution method and computational enhancements, while Chapter 5 presents a case study and an extensive computational study. We conclude in Chapter 6 with further research directions.

# Chapter 2

# Literature

There is a growing Operations Research literature that addresses problems related to the different phases of the disaster management cycle. The existing studies that focus on facility location and network design problems in humanitarian relief mostly focus on the disaster preparedness phase, and propose models for selecting locations to pre-position relief supplies for responding to future disasters (e.g., Balcik and Beamon, 2008; Ukkusuri and Yushimoto, 2008; Rawls and Turnquist, 2010a; Mete and Zabinsky, 2010; Döyen et al., 2011; Duran et al., 2011; Gormez et al., 2011; Noyan, 2012). The majority of these studies incorporate uncertainty related to the occurrence of disasters through scenarios, and uses stochastic programming models where the objective is to minimize the expectation of one or more of the following: logistics costs, traveling distances, and unmet demand. Some of these studies use commercial software to solve the proposed models (e.g., Balcik and Beamon, 2008; Duran et al., 2011), while others develop specialized solution methods (e.g., Rawls and Turnquist, 2010a; Noyan, 2012). Moreover, some studies also feature case studies and/or disaster scenarios based on real-world data (e.g., Rawls and Turnquist, 2010a; Duran et al., 2011). Despite the richness of the literature on network design problems that address pre-disaster decisions, post-disaster network design problems have not received much attention.

In the fairly developed post-disaster literature, the main focus is on vehicle routing and network flow type models (e.g., Barbarosoglu and Arda, 2004; Tzeng et al., 2007; Balcik et al., 2008; Huang et al., 2012). However, there are only a few studies that address locating last mile facilities. Horner and Downs (2008) consider the problem of locating distribution centers in a hurricane region for providing relief supplies after a disaster occurs, while considering the socio-economic characteristics (e.g., household income levels) of the affected population. Horner and Downs (2010) extend this study by incorporating the decisions on the flow of relief goods shipments between facilities, while Widener and Horner (2011) extend it by considering the capacitated last mile facilities. There are also some studies that address location and routing decisions simultaneously in a post-disaster setting (e.g., Yi and Ozdamar, 2007; Rath and Gutjahr, 2011; Afshar and Haghani, 2012; Lin et al., 2012; Tricoire et al., 2012). The majority of the studies in the post-disaster literature either assume a deterministic setting; although in a practical setting, the exact values of various factors are not known with certainty at the time of decision making, and/or do not address the critical concerns in last mile relief network design, namely accessibility and equity. To the best of our knowledge, Noyan et al. (2013) is the only study, which considers accessibility and equity in last mile relief network design with a stochastic setting.

Several factors that affect accessibility in a post-disaster setting have been discussed in the literature (e.g., Morrow, 1999; Zakour and Harrell, 2004; Kovacs and Tatham, 2009) and also in the real-life practice (e.g., IFRC (International Federation of Red Cross and Red Crescent Societies), 2013; OCHA (Office for the Coordination of Humanitarian Affairs), 2012). The mainstream approach is to define accessibility only based on the travel times, yet various demographical/socio-economical factors (such as age, gender, being in a female-headed household with young children) can also drastically affect access to aid. Noyan et al. (2013) emphasize the importance of such factors and incorporate these factors into the characterization of accessibility within the context of last mile distribution network design. In particular, they develop accessibility metrics by focusing on two echelons of the last mile relief network separately. They take into consideration the physical factors (e.g., geographical, topographical, infrastructural) along the first echelon, whereas they consider both physical and demographical/socio-economical factors (e.g., age, gender, economical status) along the second echelon.

The concept of equity has been receiving increasing attention in the context of humanitarian logistics (e.g., Tzeng et al., 2007; Balcik et al., 2008; Campbell and Jones, 2011; Vitoriano et al., 2011; Huang et al., 2012). The equity concern in the last mile relief network was initially motivated by studies which incorporate equity into facility location problems in other settings (e.g., locating public facilities such as fire and ambulance stations; see e.g., Mulligan (1991); Current and Ratick (1995); Felder and Brinkmann (2002); Noyan (2010)). However, there is no consensus on how to characterize and measure equity; its definition highly depends on the context as discussed in Marsh and Schilling (1994). We can say that there are only a few studies incorporating equity into last mile relief network design. The most relevant one is by Noyan et al. (2013), which proposes methods to model equity in supply allocation and equity in accessibility.

In this study, we consider a stochastic last mile relief network design problem, which is an extension of the one introduced in Noyan et al. (2013). Our problem is a significant and non-trivial extension, which considers a three-echelon-relief system with pre-stocked relief supplies at the first-echelon. In practice, relief supplies are not merely reallocated among existing distribution centers, relief organizations can also reallocate all/some of the supplies available at existing distribution centers to new facilities that are like to be more accessible. For instance, in the 2011 Van earthquake, the Van Airport was used to store relief items, some of which were sent from existing distribution centers due to its ease of access (Turkish Red Crescent Disaster Management, 2012). In this spirit, we allow opening new LDCs at selected set of locations as well as closing the existing ones or reallocating the existing resources within the network, if necessary. In contrast to extensive literature on resource reallocation in inventory planning, a limited number of studies consider the reallocation of relief goods in a post-disaster setting.

Rottkemper et al. (2011) address the importance of relocation of relief supplies for post-disaster situations, where a sudden change in demand or supply occurs and triggers the reallocation of a relief item during an ongoing humanitarian operation. Considering a post-disaster setting where such overlapping disasters can occur, they introduce a distribution network design problem, which minimizes the total cost (transportation, inventory holding, and unsatisfied demand costs) for a particular relief item over a specified planning horizon. Their model is based on an existing distribution network which comprises a global, a central and a number of regional depots. We note that global, central, and regional depots correspond to central depot, LDC, and POD in our setting, and different from their study, we do not assume that the locations of the central and regional depots are given. They also assume that all parameters (e.g., distances between depots, average travel times, the demand during the ongoing humanitarian operation) excluding the demand triggered by sudden changes are known. Although the authors assume that a known and limited amount of a particular relief item is pre-positioned at the regional depots as well as the central depot, the global depot is assumed to have unlimited inventory. In case of an overlapping disaster, their model determines a reallocation plan of relief items – determines the inventory levels and transhipment decisions– while considering the possible future disruptions. In order to deal with the new information arising during the ongoing relief operation, they use a rolling horizon solution approach. Rottkemper et al. (2012) extend the study of Rottkemper et al. (2011) by incorporating a time span dimension; in particular, the longer the demand has remained unsatisfied, the higher are the penalty costs. Focusing on unsatisfied demand and logistics costs, they introduce a multi-objective problem based on a weighting method, and conduct a sensitivity analysis on the choice of the weights to investigate the trade-off between the unsatisfied demand and logistics costs. Although the main goal of humanitarian operations is to effectively provide help to the affected people, in practice, the humanitarian organizations face limited financial resources (Tomasini and Wassenhove, 2009). Such financial concerns have been addressed in the post-disaster literature (e.g., Li et al., 2011; Rottkemper et al., 2011, 2012). However, while incorporating cost into humanitarian operations, the key issue is the relative importance of logistics (e.g., transportation, inventory) costs versus social (related to equity, accessibility, unsatisfied demand, etc.) costs (Holguín-Veras et al., 2012). In last mile, the humanitarian relief organizations are primarily concerned with the well-being of the people in the affected area, however, they shall take actions in accordance with their scarce financial resources. Therefore, we give more importance to accessibility and equity issues, while we still intend to keep the logistics costs associated with locating additional LDCs and reallocation operations within a specified limited budget.

Our study contributes to the humanitarian logistics literature by introducing new accessibility metrics, and developing alternate optimization models that address accessibility, equity, and budget issues for the last mile relief network design problem with resource reallocation in a stochastic setting. We assert that our accessibility metrics are suitable in designing such integrated relief networks. To the best of our knowledge, our study is a first in developing such stochastic optimization models that incorporate the decisions on the reallocation of relief goods while allowing more accessible and equitable services to the beneficiaries.

# Chapter 3

# Stochastic Optimization Models

In this chapter, we first describe the main characteristics of the stochastic last mile relief network design problem with resource reallocation – SLMRNDR –, and then develop the corresponding stochastic optimization models.

## 3.1 Problem Setting

We assume that there already exist some recourses allocated – to serve the affected region in case of a disaster– before the disaster occurs. More specifically, we are given a set of existing LDCs and the amounts of relief supplies pre-positioned at those LDCs. We focus on designing a relief distribution network which is integrated with the given pre-disaster relief network. The last-mile network structure of interest is illustrated in Figure 3.1. As shown in the figure, relief supplies arriving at a central depot are sent to LDCs, and pre-stocked supplies can be reallocated. A candidate LDC can receive delivery from existing LDCs and the central depot, while an existing LDC can receive delivery also from other existing LDCs. Then, the relief supplies are distributed to PODs, where aid recipients are delivered the supplies. Each demand location is assumed to be served by a single POD as it makes it easier to track whether the aid reaches those intended effectively. We ignore any potential behavioral elements that may cause some beneficiaries to travel to PODs other than their assigned POD to receive aid.

Given a network that involves a set of demand locations, a set of existing LDCs, a set of candidate additional LDCs, a set of candidate PODs, the SLMRNDR determines i) the locations and capacities of the LDCs, i) the locations and capacities of the PODs, ii) the amounts of supplies to be delivered from the central depot to LDCs, iii) the amounts of

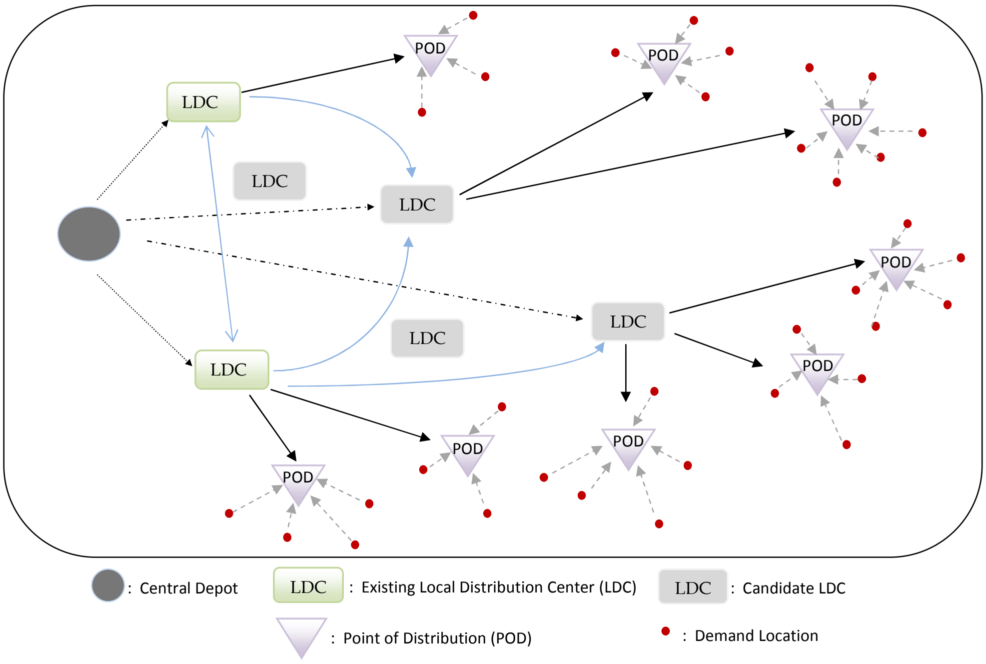


Figure 3.1: Example last mile relief network (Modified from Horner and Downs (2007); Noyan et al. (2013))

supplies to be delivered between LDCs, iv) the amounts of supplies to be delivered from LDCs to PODs, and vi) the assignments of the demand points to the PODs, while considering accessibility and equity issues, and incorporating the uncertainties in the demand and transportation network conditions.

We follow the characterization of the accessibility and equity presented in Noyan et al. (2013). Following their approach, we obtain accessibility scores from the weighted travel times, where the weights are based on a mobility score and a risk score. The mobility score reflects the proportions of socially-disadvantaged populations (disabled people, elderly, and women with children), whereas the risk score is related to topographical barriers (e.g., flooding risks). Different from Noyan et al. (2013), in our setting the lower the accessibility score the higher the accessibility (for a detailed discussion see Remark 2). Along these lines, we aim to minimize the specified aggregated accessibility metrics, which are defined based on accessibility scores, in order to improve the performance of the system in terms of accessibility. In terms of equity in supply distribution, we enforce the hybrid allocation policy, and in terms of equity in accessibility, we define the coverage sets for PODs and demand locations by enforcing an upper bound on each accessibility score associated with candidate LDCs and PODs, respectively.

As mentioned in Chapter 1, we extend the study by Noyan et al. (2013) to a more elaborate integrated last mile network design problem. The main difference in our setting compared to the one described in Noyan et al. (2013) is the additional echelon involving multiple LDCs, and the way we define the aggregated accessibility metrics. A detailed discussion on our aggregated accessibility metrics is provided in Section 3.3. In fact, Noyan et al. (2013) introduce alternate supply allocation policies, while we introduce alternate accessibility metrics. Considering all the critical issues, it can be quite costly to establish a last mile relief distribution network, yet it should be done with a limited budget. We admit that, in post-disaster stage, cost issues are secondary compared to the accessibility and equity issues. However, in our setting, it would be reasonable to consider at least the logistics costs associated with the first-echelon while determining the decisions on locating new LDCs and distributing the relief supplies between the LDCs. In particular, we consider two types of cost: fixed cost of opening a LDC or extending the capacity of an existing LDC, and cost of delivering the relief supplies to open LDCs. We still give more importance to access to relief supplies by incorporating the accessibility metrics into the objective function, and controlling the logistics costs only via a budget constraint.

## 3.2 Mathematical Models

We consider a network where each node represents a geographical area (a village or a number of settlements, etc.) according to the size of the affected region. Considering the three echelons of the last mile relief network, we introduce the following notation: the sets of nodes representing the candidate facilities in the second and the third echelon are denoted by $J^{(1)}$ and $J^{(2)}$, respectively. That is, $J^{(1)}$ denotes the set of candidate additional LDCs, while $J^{(2)}$ denotes the set of candidate PODs. We denote the sets of demand nodes by $I$, and assume that $J^{(2)} \subseteq I$. In addition, the set of nodes in the network that represent the existing LDCs are denoted by $H$, and the set of nodes representing all the LDCs is denoted by $\bar{J}^{(1)}$, i.e., $\bar{J}^{(1)} = J^{(1)} \cup H$. The LDCs can have different storage capacities; the set of size categories of an LDC at node $k$ is denoted by $L_k,\ k \in \bar{J}^{(1)}$. In order to ensure equitable accessibility of PODs from the LDCs and demand locations, the coverage sets are defined based on the accessibility scores. In particular, we enforce a maximum threshold requirement on each weighted travel time (accessibility score), where $\tau^{(1)}$ and $\tau^{(2)}$ denote the specified common threshold value for the second and the third echelons, respectively.

We capture the inherent uncertainty through a finite set of scenarios denoted by $S$, where each scenario represents the joint realization of the demands at all nodes and the accessibility scores associated with the links of the network. We follow the scenario generation methods presented in Noyan et al. (2013) and Hong et al. (2014), which characterize the dependency structure in relief networks by taking into account the geographical distribution of the affected locations. Since each accessibility score is defined as a weighted travel time, we aim to minimize the accessibility metrics that are defined as a linear combination of accessibility scores in order to improve the performance of the system in terms of accessibility.

Decision-makers (such as relief organizations, governments) usually have to make the last mile distribution network design decisions before the inherent uncertainties related to the post-disaster conditions are resolved. Due to this special structure, we propose a mathematical programming approach in the presence of uncertainty for the SLMRNDR. In our two-stage framework, the decisions on the locations and capacities of the LDCs and PODs are made at the first stage. Then, once the uncertainty in accessibility scores and demand are resolved, given the predetermined first-stage decision variables the second-second problem is solved to determine the decisions on the assignment of demand points to PODs, the amount of supplies delivered to LDCs (from the central depot and the existing LDCs) and PODs (from the open LDCs).

We introduce the following notation for our models:

Scenario-dependent input parameters

$p^s$: probability of scenario $s \in S$,

$d_i^s$: demand at node $i$ under scenario $s$, $i \in I,\ s \in S$,

$\nu_{kjs}^{(1)}$: score for accessibility to POD $j$ from LDC $k$ under scenario $s$, $k \in \bar{J}^{(1)},\ j \in J^{(2)},\ s \in S$,

$\nu_{ijs}^{(2)}$: score for accessibility to POD $j$ from demand node $i$ under scenario $s$, $i \in I,\ j \in J^{(2)},\ s \in S$,

$N_{js}^{(1)} = \{k \in \bar{J}^{(1)} \mid \nu_{kjs}^{(1)} \leq \tau^{(1)}\}$: coverage set of candidate LDCs that can serve POD $j$ under scenario $s$, $j \in J^{(2)},\ s \in S$,

$M_{ks}^{(1)} = \{j \in J^{(2)} \mid \nu_{kjs}^{(1)} \leq \tau^{(1)}\}$: coverage set of candidate PODs that can be served by the LDC $k$, $k \in \bar{J}^{(1)},\ s \in S$,

$N_{is}^{(2)} = \{j \in J^{(2)} \mid \nu_{ijs}^{(2)} \leq \tau^{(2)}\}$: coverage set of candidate PODs that can serve demand node $i$ under scenario $s$, $i \in I,\ s \in S$,

$M_{js}^{(2)} = \{i \in I \mid \nu_{ijs}^{(2)} \leq \tau^{(2)}\}$: coverage set of demand nodes that can be served by the candidate POD $j$ under scenario $s$, $j \in J^{(2)},\ s \in S$.

Scenario-independent input parameters

$I$: set of nodes in the network,

$J^{(1)}$: set the candidate additional LDCs,

$J^{(2)}$: set of the candidate PODs,

$H$: set of the existing LDCs,

$\bar{J}^{(1)}$: set of all the LDCs,

$L_k$: set of size categories of an LDC at node $k$, $k \in \bar{J}^{(1)}$,

$\delta_{kl}$: capacity of LDC $k$ of size category $l$, $k \in \bar{J}^{(1)},\ l \in L_k$,

$\theta_k$: amount of pre-stocked supply at the existing LDC $k,\ k \in H$,

$\bar{\theta}$: amount of additional available supplies,

$K_j$: upper bound on the capacity of POD $j,\ j \in J^{(2)}$,

$\kappa^{(1)}$: upper bound on the number of LDCs (including opened and existing ones),

$\kappa^{(2)}$: upper bound on the number of PODs to be opened,

$B$: available budget,

$f_{kl}$: fixed cost of opening an LDC of size category $l$ at node $k$, $k \in \bar{J}^{(1)},\ l \in L_k$; for an existing LDC it represents the cost of extending the capacity.

$c_{hk}^{s}$: unit shipping cost to LDC $k$ from LDC $h$ under scenario s, $h \in H,\ k \in \bar{J}^{(1)},\ s \in S$,

$c_{0k}^{s}$: unit shipping cost to LDC $k$ from the central depot, $k \in \bar{J}^{(1)}$.

First-stage decision variables

$z_{kl} = 1$ if an LDC of size category $l \in L_k$ is located at node $k,\ k \in \bar{J}^{(1)}$, and $z_{kl} = 0$ otherwise. For an existing LDC $k \in H$, the value of 0 indicates that it is closed/not used in the post-disaster stage.

$y_j = 1$ if a POD is located at node $j \in J^{(2)}$, and $y_j = 0$ otherwise,

$R_j$: the capacity of POD $j \in J^{(2)}$.

Second-stage decision variables

$x_{ij}^s = 1$ if demand node $i \in I$ is served by POD $j \in N_{is}^{(2)}$ under scenario $s \in S$, $x_{ij}^s = 0$ otherwise.

$r_{0k}^s$: amount of supply delivered from the central depot to LDC $k \in \bar{J}^{(1)}$,

$r_{hk}^s$: amount of supply delivered from the existing LDC $h \in H$ to LDC $k \in \bar{J}^{(1)} \setminus h$,

$w_{kj}^s$: amount of supply delivered to POD $j \in J^{(2)}$ from LDC $k \in N_{js}^{(1)}$ under scenario $s \in S$.

Auxiliary decision variables (introduced mostly for ease of exposition)

$\mathrm{NS}_k^s$: amount of total net supply at LDC $k \in \bar{J}^{(1)}$,

$\mathrm{TD}_j^s$: total demand assigned to the PODs; expressed in terms of the assignment decisions as $\mathrm{TD}_j^s = \sum_{i \in M_{js}^{(2)}} x_{ij}^s d_i^s, \quad j \in J^{(2)},\ s \in S.$

We next present the mathematical formulation of the first-stage problem

$$
\begin{aligned}
\min \quad & E[Q(\mathbf{z}, \mathbf{y}, \mathbf{R}, \boldsymbol{\xi})] && (3.1)\\
\text{subject to:} \quad & \sum_{k \in \bar{J}^{(1)}} \sum_{l \in L_k} z_{kl} \leq \kappa^{(1)}, && (3.2)\\
& \sum_{j \in J^{(2)}} y_j \leq \kappa^{(2)}, && (3.3)\\
& \sum_{l \in L_k} z_{kl} \leq 1, \qquad k \in \bar{J}^{(1)}, && (3.4)\\
& R_j \leq K_j y_j, \qquad j \in J^{(2)}, && (3.5)\\
& z_{kl} \in \{0,1\}, \qquad k \in \bar{J}^{(1)},\ l \in L_k, && (3.6)\\
& y_j \in \{0,1\}, \qquad j \in J^{(2)}, && (3.7)\\
& R_j \geq 0, \qquad j \in J^{(2)}. && (3.8)
\end{aligned}
$$

Here $\boldsymbol{\xi}$ denotes the random data and $Q(\mathbf{z}, \mathbf{y}, \mathbf{R}, \boldsymbol{\xi})$ is the objective function of the second-stage problem for a given set of first-stage decisions. For ease of exposition, we use the notation $Q(\boldsymbol{\eta}, \boldsymbol{\xi})$ with $\boldsymbol{\eta} = (\mathbf{z}, \mathbf{y}, \mathbf{R})$.

It is well-known that expressing the exact expected value of $Q(\boldsymbol{\eta}, \boldsymbol{\xi})$ is in general not possible, since it requires solving an optimization model in the second-stage. To alleviate this issue, as in most of the applied stochastic programming models, a point estimation of the expected second-stage objective function value is obtained via sample averaging. More specifically, we consider $|S|$ samples of the random data $\boldsymbol{\xi}$, then obtain corresponding values of $Q(\boldsymbol{\eta}, \boldsymbol{\xi})$ by solving the second-stage problem, and finally take the average of these values across all $|S|$ samples (scenarios). For $\boldsymbol{\xi}^s = (\mathbf{d}^s, \boldsymbol{\nu}_s^{(1)}, \boldsymbol{\nu}_s^{(2)})$ denoting the realization of the random data under scenario $s \in S$, the general form of the second-stage problem is given by

$$Q(\boldsymbol{\eta}, \boldsymbol{\xi}^s) = \min \quad f(\mathbf{x}^s, \mathbf{w}^s, \mathbf{r}^s) + \epsilon \sum_{j \in J} \beta_j^s \tag{3.9}$$

$$\text{s. t.} \quad \mathrm{NS}_k^s \leq \sum_{l \in L_k} z_{kl} \delta_{kl}, \qquad k \in \bar{J}^{(1)}, \tag{3.10}$$

$$\mathrm{NS}_k^s = \begin{cases} \sum\limits_{h \in H \setminus k} r_{hk}^s + r_{0k}^s, & k \in J^{(1)}, \\ \theta_k + \sum\limits_{h \in H \setminus k} r_{hk}^s - \sum\limits_{h \in \bar{J}^{(1)} \setminus k} r_{kh}^s + r_{0k}^s, & k \in H, \end{cases} \tag{3.11}$$

$$\sum_{k \in \bar{J}^{(1)}} r_{0k}^s = \bar{\theta}, \tag{3.12}$$

$$\sum_{j \in J^{(2)}} \sum_{k \in N_{js}^{(1)}} w_{kj}^s = \min\Big(\sum_{k \in H} \theta_k + \bar{\theta}, \sum_{i \in I} d_i^s\Big), \tag{3.13}$$

$$\sum_{j \in M_{ks}^{(1)}} w_{kj}^s \leq \mathrm{NS}_k^s, \qquad k \in \bar{J}^{(1)}, \tag{3.14}$$

$$\sum_{k \in N_{js}^{(1)}} w_{kj}^s \leq R_j, \qquad j \in J^{(2)}, \tag{3.15}$$

$$\sum_{j \in N_{is}^{(2)}} x_{ij}^s = 1, \qquad i \in I, \tag{3.16}$$

$$x_{ij}^s \leq y_j, \qquad i \in I, \ j \in N_{is}^{(2)}, \tag{3.17}$$

$$x_{jj}^s \geq y_j, \qquad j \in J^{(2)}, \tag{3.18}$$

$$\sum_{k \in N_{js}^{(1)}} w_{kj}^s \leq \mathrm{PD}_j^s + \beta_j^s \leq \mathrm{TD}_j^s, \qquad j \in J^{(2)}, \tag{3.19}$$

$$\mathrm{TD}_j^s - \sum_{k \in N_{js}^{(1)}} w_{kj}^s \leq \rho\, \mathrm{TD}_j^s, \qquad j \in J^{(2)}, \tag{3.20}$$

$$\sum_{l\in L_k}\sum_{k\in \bar{J}^{(1)}} z_{kl}f_{kl} + \sum_{h\in H}\sum_{k\in \bar{J}^{(1)}\setminus h} r^s_{hk}c^s_{hk} + \sum_{h\in \bar{J}^{(1)}} r^s_{0h}c^s_{0h} \le B, \tag{3.21}$$

$$w^s_{kj} \ge 0, \qquad j \in J^{(2)},\ k \in N^{(1)}_{js}, \tag{3.22}$$

$$\beta^s_j \ge 0, \qquad j \in J^{(2)}, \tag{3.23}$$

$$x^s_{ij} \in \{0,1\}, \qquad i \in I,\ j \in N^{(2)}_{is}, \tag{3.24}$$

$$r^s_{hk} \ge 0, \qquad h \in \{H \cup 0\},\ k \in \bar{J}^{(1)} \setminus h, \tag{3.25}$$

$$\mathrm{NS}^s_k \ge 0, \qquad k \in \bar{J}^{(1)}. \tag{3.26}$$

Here $\mathrm{PD}^s_j$ denotes a specific proportion (based on the total assigned to POD $j$ under scenario $s$) of the total delivery amount $\Theta^s \stackrel{\text{def}}{=} \min(\sum_{k\in H}\theta_k + \bar{\theta}, \sum_{i\in I} d^s_i)$; in particular,

$$\mathrm{PD}^s_j = \Theta^s(\mathrm{TD}^s_j \,/ \sum_{i\in I} d^s_i) \quad j \in J,\ s \in S.$$

The objective function (3.1) minimizes the expected value of a specified aggregated metric, which quantifies people's access to relief supplies. Specifically, we propose two types of $f(\cdot)$ function to express the aggregated accessibility metric, which will be explained in the next section. By constraints (3.2) and (3.3), the numbers of LDCs and PODs do not exceed the specified limits, whereas (3.4) ensures that at most one and a single type of LDC can be located at any node representing a candidate or existing LDC. Constraints (3.5) impose a maximum capacity limit for PODs to prevent oversized facilities.

In the second-stage problem, constraints (3.10) guarantee that relief supplies are stored at open LDCs and the amount of supplies allocated at an LDC does not exceed its capacity. The conservation of flow at each node associated with an LDC is represented by constraints (3.11); they determine the net supply amount at each LDC and imply that $\sum_{k\in \bar{J}^{(1)}} \mathrm{NS}^s_k = \sum_{k\in H}\theta_k + \bar{\theta}$ for all $s \in S$. By constraint (3.12), all the additional available supplies are delivered to LDCs, and then constraint (3.13) ensures that the total amount of delivery to the PODs is equal to the total available supplies, unless the total demand is less than this value. Additionally, by (3.14) and (3.15), the amount of total supply delivered from an LDC is limited by its total net supply, and the total amount of delivery to any POD is limited by its capacity, respectively. Observe also that constraints (3.15) together with constraints (3.5) ensure that there has to be a POD located at node $j$ if there is any delivery to that node (i.e., if $\sum_{k\in N^{(1)}_{js}} w^s_{kj} > 0$ for at least one scenario). Constraints

(3.16)-(3.18) guarantee the connectivity of the network in such a way that each demand node is assigned to a single open POD, and a POD located at a demand node serves its own location. In accordance with the latter assignment condition, which would be reasonable in practical settings, we assume that the values of the capacity parameters $K_j$ are sufficiently large to allow an open POD to serve at least its own location under any scenario; $K_j \geq \max_{s\in S} d_j^s$ for $j \in J^{(2)}$. As in Noyan et al. (2013), constraints (3.19) and (3.20) are enforced to ensure an equitable supply distribution -at the POD level- based on the hybrid allocation policy. According to this policy, we aim to distribute supplies as proportionally as possible without comprising accessibility; this is achieved by (3.19) and minimizing the term involving the $\beta$ variables and a very small penalty coefficient $\epsilon$. By changing the value of the parameter $\epsilon$, the decision makers can control the balance between accessibility and equity. In fact, for extremely large and small values of this parameter, the hybrid policy behaves like the PD policy and the TD policy, respectively. In addition, we limit the demand shortages via constraint (3.20); the upper bound on shortages is specified as a common proportion of the total demand assigned to PODs. On the other hand, constraint (3.21) will not allow the total logistics cost associated with the first echelon – total cost of opening LDCs and extending the capacities of existing LDCs, and delivering the relief supplies to open LDCs– to exceed the available budget. The rest of the constraints are for non-negativity and binary restrictions.

According to this formulation, as desired, it is not possible to obtain a solution where $y_j = 1$ if there will be no delivery to POD $j$, i.e, if $\sum_{k\in N_{js}^{(1)}} w_{kj}^s = 0$ for all $s \in S$. Suppose that $y_j = 1$ for an index $j \in J^{(2)}$. Then, by (3.18) and (3.20), we have $x_{jj}^s > 0$ (implying $\mathrm{TD}_j^s > 0$), and $\sum\limits_{k\in N_{js}^{(1)}} w_{kj}^s > 0$ for all $s \in S$, respectively. In another words, any open POD should satisfy at least a certain amount of the total demand based on the value of parameter $\rho$ ($\rho < 1$) under each scenario. Therefore, we have $w_{kj}^s > 0$ for at least one index $k \in N_{js}^{(1)}$ for all $s \in S$, which contradicts with the assumption that $\sum_{k\in N_{js}^{(1)}} w_{kj}^s = 0$ for all $s \in S$. However, at an optimal solution we can observe that $\sum_{l\in L_k} z_{kl} = 1$ even if there is no need to keep/open LDC $k$, i.e., $\max_{s\in S} \sum_{j\in M_{ks}^{(1)}} w_{kj}^s = 0$ (since we do not minimize the cost of opening facilities). We can deal with this issue by constructing an alternative optimal solution: set $z_{kl} = 0$ for all $l \in L_k$ if $\max_{s\in S} \sum_{j\in M_{ks}^{(1)}} w_{kj}^s = 0$ at any optimal solution. It is easy to see that this new solution is feasible with the same objective function value.

## 3.3 Alternative Accessibility Metrics

We aim to define an aggregated accessibility metric, which quantifies the total accessibility of the supplies (at the open LDCs) all the way from the demand locations. An alternative approach can be found in Noyan et al. (2013); in particular, they define an accessibility metric as the summation of the total accessibility of the PODs from the LDC and the total accessibility of the PODs from the demand locations. Different from their approach, we opt for defining a metric that would consider the structure of a multi-echelon network in a more integrated way. To be able to discuss the differences in these alternative approaches and provide motivation for our approach, we first present the objective functions proposed in Noyan et al. (2013).

In the study of Noyan et al. (2013), it is assumed that there is a single fixed LDC, and therefore, it is sufficient to consider a single type of facilities (i.e., consider $J$ instead of $J^{(1)}$ and $J^{(2)}$). Denoting the expected accessibility to POD $j$ from the single LDC by $\bar{\nu}_{0j} = \sum_{s\in S} p^s \nu^s_{0j},\ j \in J$, their first-stage objective function is given by

$$f(\mathbf{y}, \mathbf{R}) = \sum_{j\in J} \bar{\nu}_{0j} y_j + E[Q(\mathbf{y}, \mathbf{R}, \boldsymbol{\xi})], \tag{3.27}$$

with

$$E[Q(\mathbf{y}, \mathbf{R}, \boldsymbol{\xi})] = \sum_{s\in S} p^s Q(\mathbf{y}, \mathbf{R}, \boldsymbol{\xi}^s) = \sum_{s\in S} p^s \sum_{i\in I} \sum_{j\in N_i^s} \nu^s_{ij} x^s_{ij}.$$

In general, the above expected total accessibility is an effective metric in establishing last mile relief networks. However, there is a slight concern arising from completely separating the quantification of the accessibility in two echelons (from LDCs to PODs and from PODs to demand points). For now, consider the case where larger accessibility score means higher accessibility, i.e., the objective is to maximize the function (3.27). This approach can reward locating additional PODs in a way that results in higher total accessibility but lower accessibility (considering the same demand locations) in both echelons. We illustrate this claim by using the following small example in Figure 3.2, where the scenario dependence is ignored, and only a very small portion of a network is considered. The objective function (3.27) is not consistent with the rational choice of setup 1 over setup 2. As a remedy, we can quantify the accessibility to the LDC all the way from demand locations. More precisely, the corresponding first-stage objective function becomes

$$f(\mathbf{y}, \mathbf{R}) = \mathbb{E}[Q(\mathbf{y}, \mathbf{R}, \boldsymbol{\xi})], \tag{3.28}$$

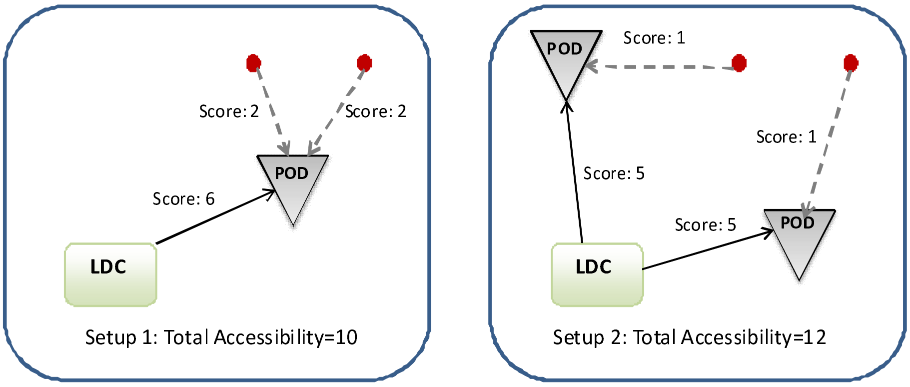


Figure 3.2: Illustrative example for the existing accessibility metric

with

$$E[Q(\mathbf{y}, \mathbf{R}, \boldsymbol{\xi})] = \sum_{s \in S} p^s Q(\mathbf{y}, \mathbf{R}, \boldsymbol{\xi}^s) = \sum_{s \in S} p^s \sum_{i \in I} \sum_{j \in N_i^s} (\nu_{0j}^s + \nu_{ij}^s) x_{ij}^s. \tag{3.29}$$

This integrated version ensures accounting the accessibility to the LDC from the single POD in setup 1 twice (since it serves two demand points, implying that both $x$ assignment decisions are 1), and the resulting total accessibility would be 16 instead of 10. Alternatively, one can give more weight to the second term in (3.27) to avoid any potential issues illustrated in the above example. We would like to note that the raised issue is not a practical concern in situations where there is a limited fixed number of PODs to be opened (as in Noyan et al., 2013). As illustrated in Figure 3.2, the main problem arises from comparing the options (setups) with different number of PODs. If we would compare the setups with a fixed number of PODs (such as 1 or 2 in our small example), the objective function (3.27) would also favor the setup with higher accessibility in both echelons. Thus, even if the way the aggregated accessibility metric is reasonable for certain settings, we shall still develop a more elaborate version of it for more general settings. In fact, such an elaborate metric is essential for our setup, which has a more complicated structure due to the additional echelon involving multiple LDCs. In addition, we also consider a demand-weighted version of (3.29), which is a very natural extension to assess the effectiveness of the response in terms of equitable accessibility. We next formally define our new aggregated accessibility metrics.

In line with the above discussions, according to our notation, an ideal demand-weighted objective function should have a representation of the form

$$\sum_{i \in I} \sum_{j \in N_{is}^{(2)}} (\hat{\nu}_{js} + \nu_{ijs}^{(2)}) x_{ij}^s d_i^s, \tag{3.30}$$

where $\hat{\nu}_{js}$ designates an estimated score for accessibility of POD $j$ from the open LDCs. It is important to observe that this parameter is trivially given by $\hat{\nu}_{0j}$ in (3.29), since there is a single LDC in Noyan et al. (2013). The challenge in estimating the parameters $\hat{\nu}_{js}$ stems form the fact that the open LDCs and the delivery amounts to PODs from the open LDCs are not known a priori. An ideal way of estimating these scores is given by

$$\hat{\nu}_{js} = \frac{\sum\limits_{k \in N_{js}^{(1)}} w_{kj}^s \nu_{kjs}^{(1)}}{\sum\limits_{k \in N_{js}^{(1)}} w_{kj}^s}, \quad j \in J^{(2)}. \tag{3.31}$$

It is clear that a proper approach, which quantifies the access to the supplies, should give more importance to the accessibility scores associated with the links between POD $j$ and the LDCs that deliver more supplies to that POD. In (3.31) we also need scaling to obtain a single score which is appropriate in terms of unit dimension. Since $\hat{\nu}_{js}$ depends on the $\mathbf{w}$ decisions on the delivery amounts, it is also a decision variable. Unfortunately, incorporating these decisions into (3.29) would lead to fractional and quadratic terms ($w_{kj}^s x_{ij}^s$) in the objective function, and consequently, we have to deal with non-convex global optimization problems. It is evident that we need to develop computationally tractable ways of estimating the parameters $\hat{\nu}_{js}$. In this regard, taking into the delivery amounts into account, we propose two approaches to approximate the desired objective function (3.30).

- **Alternative 1:** Taking a closer look into (3.30) of the form

$$\sum_{i \in I} \sum_{j \in N_{is}^{(2)}} \hat{\nu}_{js} x_{ij}^s d_i^s + \sum_{i \in I} \sum_{j \in N_{is}^{(2)}} \nu_{ijs}^{(2)} x_{ij}^s d_i^s, \tag{3.32}$$

  we can say that the coefficients of the accessibility scores in the first term is related to the amount of the delivered supply, as the accessibility scores themselves. In an intuitive way, considering the scalarization appearing in (3.31), we propose the following approximation:

$$\sum_{k \in \bar{J}^{(1)}} \sum_{j \in M_{ks}^{(1)}} w_{kj}^{s} \nu_{kjs}^{(1)} + \sum_{i \in I} \sum_{j \in N_{is}^{(2)}} \nu_{ijs}^{(2)} x_{ij}^{s} d_{i}^{s}. \tag{3.33}$$

To provide additional insights, we note that $\sum_{i \in I} \sum_{j \in N_{is}^{(2)}} x_{ij}^{s} d_{i}^{s} = \sum_{i \in I} d_{i}^{s}$ and the scalarization factor $\sum_{k \in N_{js}^{(1)}} w_{kj}^{s}$ (appearing in (3.31)) is related to $\sum_{i \in I} d_{i}^{s}$ via the constraint (3.13).

- **Alternative 2:** In this approach, we propose to consider a selected subset of LDCs in (3.31), i.e., consider a subset of the coverage set $N_{js}^{(1)}$, denoted by $\hat{N}_{js}^{(1)}$. In consistent with the significance of the delivery amounts, we particularly select the LDCs that correspond to the $\alpha_j$ largest of the delivery amounts $w_{kj}^{s},\ k \in N_{js}^{(1)}$, where $\alpha_j$ is a specified constant ($1 \leq \alpha_j \leq \min_{s \in S} |N_{js}^{(1)}|$). Giving equal importance to those selected LDCs, we calculate the associated average score for the accessibility of POD $j$ through the $\alpha_j$ most influential LDCs as:

$$\sum_{k \in \hat{N}_{js}^{(1)}} \nu_{kjs}^{(1)} / \alpha_j. \tag{3.34}$$

  Thus, we propose to use (3.34) to approximate (3.31), which results in the following approximation of the ideal objective function (3.30):

$$\sum_{i \in I} \sum_{j \in N_{is}^{(2)}} \sum_{k \in \hat{N}_{js}^{(1)}} \frac{1}{\alpha_j} \nu_{kjs}^{(1)} x_{ij}^{s} d_{i}^{s} + \sum_{i \in I} \sum_{j \in N_{is}^{(2)}} \nu_{ijs}^{(2)} x_{ij}^{s} d_{i}^{s}. \tag{3.35}$$

  The challenge in incorporating the above objective function into our optimization model stems from the fact that the selection of the LDCs (the identification of the subset $\hat{N}_{js}^{(1)}$) depends on the sorting of the $w_{kj}^{s}$ decisions, which cannot be known a priori. To this end, we obtain the following analytical result.

**Proposition 1** *The function* (3.35) *can be equivalently expressed as*

$$\sum_{i \in I} \sum_{j \in N_{is}^{(2)}} \sum_{k \in N_{js}^{(1)}} \frac{1}{\alpha_j} \nu_{kjs}^{(1)} q_{kji}^{s} d_{i}^{s} + \sum_{i \in I} \sum_{j \in N_{is}^{(2)}} \nu_{ijs}^{(2)} x_{ij}^{s} d_{i}^{s} \tag{3.36}$$

*if the non-negative variables* $q^s_{kji},\ i \in I,\ j \in N^{(2)}_{js},\ k \in N^{(1)}_{js}$, *satisfy the following constraints:*

$$\vartheta^s_j \geq w^s_{kj} - \zeta^s_{kj}\mathcal{M}^s_{jk}, \qquad j \in J^{(2)},\ k \in N^{(1)}_{js}, \tag{3.37}$$

$$\vartheta^s_j \leq w^s_{kj} + (1-\zeta^s_{kj})\mathcal{M}^s_{jk}, \qquad j \in J^{(2)},\ k \in N^{(1)}_{js}, \tag{3.38}$$

$$\sum_{k \in N^{(1)}_{js}} \zeta^s_{kj} = \alpha_j, \qquad j \in J^{(2)}, \tag{3.39}$$

$$\zeta^s_{kj} \in \{0,1\}, \qquad j \in J^{(2)},\ k \in N^{(1)}_{js}, \tag{3.40}$$

$$q^s_{kji} \leq \zeta^s_{kj}, \qquad i \in I,\ j \in N^{(2)}_{is},\ k \in N^{(1)}_{js}, \tag{3.41}$$

$$q^s_{kji} \leq x^s_{ij}, \qquad i \in I,\ j \in N^{(2)}_{is},\ k \in N^{(1)}_{js}, \tag{3.42}$$

$$q^s_{kji} \geq \zeta^s_{kj} + x^s_{ij} - 1, \qquad i \in I,\ j \in N^{(2)}_{is},\ k \in N^{(1)}_{js}, \tag{3.43}$$

*where* $\mathcal{M}^s_{jk}$ *are sufficiently large constants to make the constraints* (3.37) *and* (3.38) *redundant whenever* $\zeta^s_{kj} = 1$ *and* $\zeta^s_{kj} = 0$, *respectively.*

**Proof.** Let $B^s_j = \{k \in N^{(1)}_{js} \ :\ \zeta^s_{kj} = 1\}$ and $\bar{B}^s_j$ be its complement set for all $j \in J^{(2)},\ s \in S$. Then, constraints (3.37) and (3.38) ensure that $w^s_{kj} \leq \vartheta^s_j$ for all $k \in \bar{B}^s_j$, and $\vartheta^s_j \leq w^s_{kj}$ for all $k \in B^s_j$. These orderings imply that $w^s_{kj} \geq w^s_{k'j}$ for any pair of $k \in B^s_j$ and $k' \in \bar{B}^s_j$. In addition, by (3.39) and (3.40), we have $|B^s_j| = \alpha_j$. Therefore, $\zeta^s_{kj}$ takes the value of 1 if $w^s_{kj}$ is among the $\alpha_j$ largest values, and so the set $B^s_j$ corresponds to the $\alpha_j$ largest of the delivery amounts $w^s_{kj},\ k \in N^{(1)}_{js}$, i.e., $B^s_j = \hat{N}^{(1)}_{js}$ for all $j \in J^{(2)},\ s \in S$. Consequently, $\sum_{k \in N^{(1)}_{js}} \nu^{(1)}_{kjs}\zeta^s_{kj}/\alpha_j$ is equivalent to (3.34), and (3.35) can be rewritten as

$$\sum_{i \in I}\sum_{j \in N^{(2)}_{is}}\sum_{k \in N^{(1)}_{js}} \frac{1}{\alpha_j}\nu^{(1)}_{kjs}\zeta^s_{kj}x^s_{ij}d^s_i + \sum_{i \in I}\sum_{j \in N^{(2)}_{is}} \nu^{(2)}_{ijs}x^s_{ij}d^s_i.$$

We can linearize the quadratic terms $\zeta^s_{kj}x^s_{ij}$ by introducing the non-negative variables $q^s_{kji},\ i \in I,\ j \in N^{(2)}_{is},\ k \in N^{(1)}_{js}$, and the additional constraints (3.41)-(3.43). It is easy to see that constraints (3.41)-(3.43) guarantee that $q^s_{kji} = \zeta^s_{kj}x^s_{ij}$ for all $i \in I,\ j \in N^{(2)}_{is},\ k \in N^{(1)}_{js}$, which proves our claim. ■

It is well-known that the choice of the Big-M coefficients is crucial in obtaining stronger formulations. In our implementations, observing that for any feasible solution $w^s_{kj} \leq \sum_{i \in M^{(2)}_{js}} d^s_i(1-\rho)$ and $w^s_{kj} \leq K_j$, we set $\mathcal{M}^s_{jk} = \mathcal{M}^s_j \stackrel{\text{def}}{=} \min\ \{\sum_{i \in M^{(2)}_{js}} d^s_i(1-$

$\rho),\ K_j\}$ for all $j \in J^{(2)},\ k \in N^{(1)}_{js},\ s \in S$. One can also consider the capacity of the LDCs while setting these parameters: $\mathcal{M}^s_{jk} = \min\ \{ \sum\limits_{i \in M^{(2)}_{js}} d^s_i(1-\rho),\ K_j,\ \sum_{l \in L_k} z_{kl}\delta_{kl}\}$ (observe that for a given first-stage decision vector $\sum_{l \in L_k} z_{kl}\delta_{kl}$ is a well-defined parameter).

**Corollary 2** *According to approach based on the second alternative way of approximating* (3.30)*, the second-stage problem is formulated as*

$$\min \quad \sum_{i \in I} \sum_{j \in N^{(2)}_{is}} \sum_{k \in N^{(1)}_{js}} \frac{1}{\alpha_j} \nu^{(1)}_{kjs} q^s_{kji} d^s_i + \sum_{i \in I} \sum_{j \in N^{(2)}_{is}} \nu^{(2)}_{ijs} d^s_i x^s_{ij} + \epsilon \sum_{j \in J} \beta^s_j \tag{3.44}$$

*s.t.* $\quad$ (3.10) – (3.26)

$$\vartheta^s_j \geq w^s_{kj} - \zeta^s_{kj}\mathcal{M}^s_{jk}, \qquad j \in J^{(2)},\ k \in N^{(1)}_{js}, \tag{3.45}$$

$$\vartheta^s_j \leq w^s_{kj} + (1-\zeta^s_{kj})\mathcal{M}^s_{jk}, \qquad j \in J^{(2)},\ k \in N^{(1)}_{js}, \tag{3.46}$$

$$\sum_{k \in N^{(1)}_{js}} \zeta^s_{kj} = \alpha_j, \qquad j \in J^{(2)}, \tag{3.47}$$

$$q^s_{kji} \leq \zeta^s_{kj}, \qquad i \in I,\ j \in N^{(2)}_{is},\ k \in N^{(1)}_{js}, \tag{3.48}$$

$$q^s_{kji} \leq x^s_{ij}, \qquad i \in I,\ j \in N^{(2)}_{is},\ k \in N^{(1)}_{js}, \tag{3.49}$$

$$q^s_{kji} \geq \zeta^s_{kj} + x^s_{ij} - 1, \qquad i \in I,\ j \in N^{(2)}_{js},\ k \in N^{(1)}_{js}, \tag{3.50}$$

$$q^s_{kji} \geq 0, \qquad i \in I,\ j \in N^{(2)}_{is},\ k \in N^{(1)}_{js} \tag{3.51}$$

$$\vartheta^s_j \geq 0, \qquad j \in J^{(2)}, \tag{3.52}$$

$$\zeta^s_{kj} \in \{0,1\}, \qquad j \in J^{(2)},\ k \in N^{(1)}_{js}. \tag{3.53}$$

**Remark 1** ***(Related to the Alternative 2)*** *Here we discuss two weaknesses of the second modeling approach.*

- *When $\alpha_j > 1$, we cannot guarantee that all of the $\alpha_j$ largest delivery amounts are positive. This implies that, we can incorporate the accessibility score of an unused links (with zero delivery) into the estimation of the accessibility score associated with the links between POD $j$ and the LDCs that deliver supplies to that POD. To deal with this issue, one can update $\alpha_j$ as $\alpha_j := \min(\alpha_j, \min_{s \in S} |\{k \in N^{(1)}_{js}\ :\ w^s_{kj} > 0\}|)$ (or, even can consider a scenario-dependent version $\alpha^s_j = \min(\alpha_j, |\{k \in N^{(1)}_{js}\ :\ w^s_{kj} > 0\}|)$. However, in such an approach, $\alpha_j$ would be a decision variable, and we would end up with fractional terms in the objective function, as the ideal one based on the desired form* (3.31)*. Observe that, there is no* *such a concern when $\alpha_j = 1$, since $|\{k \in N_{js}^{(1)} \ : \ w_{kj}^s > 0\}| \geq 1$ for an open POD $j$. Therefore, in our implementation, we generally use $\alpha_j = 1$ for all $j \in J$. On the other hand, one can also use the following ad-hoc approach. Solve the model with $\alpha_j = 1, \ j \in J$, and according to the obtained results, update $\alpha_j$ parameters and resolve the model. More specifically, the $\alpha_j$ parameter can take larger values for POD $j$ that are more likely to be assigned to multiple LDCs.*

- *The second approach gives more importance to the accessibility scores associated with the links between the PODs and LDCs that deliver more supplies to the open PODs. However, it gives equal importance to the selected links by using the average of their associated accessibility scores. Thus, it intends to take into consideration the delivery amounts but still does not incorporate their exact values into the estimation of the accessibility of PODs from LDCs.*

**Remark 2** ***(Related to Accessibility Metrics)*** *We follow the characterization of the accessibility proposed in Noyan et al. (2013) to obtain the accessibility scores. In particular, they describe methods to estimate the weighted travel times associated with each link, and then, they use a decreasing function of the weighted travel times to obtain the accessibility scores. Thus, in their setting, higher the accessibility score higher the accessibility. In our study, we directly use the weighted travel times as the accessibility scores, since it seems more natural for our metrics defined in accordance with* (3.30).

*Observe that the summation of the weighted travel times in* (3.30) *would still correspond to a weighted travel time. Then, one can use a decreasing function of the resulting weighted travel times to assess the accessibility to supplies all the way from demand points. It seems more natural than taking a decreasing function of each weighted travel time separately and then sum them. According to this approach, we do not even need to specify a decreasing function, we just focus on minimizing aggregated accessibility metrics based on the weighted travel terms.*

**Remark 3** ***(Related to an Alternative Model)****. If we assume that each POD is served by exactly one open LDC, then we can express the desired objective function* (3.30) *without using an approximation. More precisely, we would introduce the binary variables $\gamma_{jk}^s$ to identify the assignments of PODs to the LDCs: $\gamma_{kj}^s = 1$ if POD $j \in J^{(2)}$ is served by LDC $k \in N_{js}^{(1)}$ under scenario $s \in S$, $\gamma_{kj}^s = 0$ otherwise. Then, the ideal expression* (3.31) *takes the form*

$$\hat{\nu}_{js} = \sum_{k \in N_{js}^{(1)}} \gamma_{kj}^s \nu_{kjs}^{(1)},$$

*which can be easily incorporated into our optimization model. In our study, we do not make this simplifying assumption because it is rather restrictive for practical settings.*

Here we describe some simplifying modifications made in the mathematical formulations of the two-stage models to be able to develop a computationally effective solution algorithm, which is applicable only to the models with pure binary decision variables in the first-stage. More specifically, we drop the continuous decision variables $R_j$ from the first-stage problem, and replace it by $K_j y_j$ in the second-stage problem. Then, constraint (3.15) becomes

$$\sum_{k \in N_{js}^{(1)}} w_{kj}^s \leq K_j y_j, \quad j \in J^{(2)}. \tag{3.54}$$

It is easy to see that if any of our proposed models are feasible, then there exist an optimal solution with

$$R_j = \max_{s \in S} \sum_{k \in N_{js}^{(1)}} w_{kj}^s, \quad j \in J^{(2)}. \tag{3.55}$$

In fact, from the relief organization's point of view, a rational decision-maker would set the POD capacity values as in (3.55). Then, the equivalence of the modified models and the original ones trivially follows. For the modified models, with some abuse of notation, the first-stage decision vector $\boldsymbol{\eta}$ represents $(\mathbf{z}, \mathbf{y})$ instead of $(\mathbf{z}, \mathbf{y}, \mathbf{R})$. In the rest of the study, we regard modified versions of the proposed models.

We can combine our first-stage problem and our second-stage problem into a single large-scale mixed-binary linear program, which is known as the deterministic equivalent formulation (DEF). For ease of reference, we provide the compact DEFs of the proposed stochastic optimization models in Table 3.1.

| | Large-scale Deterministic Equivalent Formulations |
|---|---|
| Model_1 | minimize$\{\sum_{s \in S} p^s f(\mathbf{x}^s, \mathbf{w}^s, \mathbf{r}^s)$ : (3.2) – (3.4), (3.6) – (3.7), (3.10) – (3.14), (3.16) – (3.26), (3.54) for all $s \in S\}$, where $f(\mathbf{x}^s, \mathbf{w}^s, \mathbf{r}^s)$ given by (3.33) |
| Model_2 | minimize$\{\sum_{s \in S} p^s f(\mathbf{x}^s, \mathbf{w}^s, \mathbf{r}^s)$ : (3.2) – (3.4), (3.6) – (3.7), (3.10) – (3.14), (3.16) – (3.26), (3.45) – (3.54) for all $s \in S\}$ where $f(\mathbf{x}^s, \mathbf{w}^s, \mathbf{r}^s)$ given by (3.36) |

Table 3.1: Compact DEFs of our proposed stochastic programming models

# Chapter 4

# Solution Methods

Stochastic mixed-integer programming models are generally known to be computationally challenging, which can partially be attributed to the potentially large number of scenario-dependent variables and constraints (see, e.g., Birge and Louveaux, 1997; Sen, 2005). The studies that focus on developing solution methods for stochastic integer programs mainly consider the two-stage framework, and propose decomposition-based algorithms. The integer L-shaped algorithm proposed by Laporte and Louveaux (1993) is the first decomposition method for stochastic programs with integer decisions in the second-stage. It is a Benders decomposition-based branch-and-cut algorithm, and relies on the assumption that there exist only binary decision variables in the first-stage. For other types of decomposition-based algorithms developed to solve two-stage stochastic mixed-integer programming models, we refer to the overviews by Birge and Louveaux (1997), Klein Haneveld and van der Vlerk (1999), Louveaux and Schultz (2003), and Sen (2005), and a comprehensive bibliography (van der Vlerk, 2007).

Among the existing solution methods, we mention here the disjunctive decomposition-based branch-and-cut algorithms which rely on value function convexification and set convexification of the second-stage (Sen and Higle, 2005; Yuan and Sen, 2009). These studies consider a special setting with binary variables in the first-stage and mixed-binary variables in the second-stage, and therefore, they can benefit from the theory of disjunctive programming. In fact, these disjunctive decomposition-based branch-and-cut algorithms are in general very effective (for an illustrative computational study we refer the reader to Ntaimo and Sen (2008)). However, they assume to have a *deterministic recourse matrix* (representing the coefficients associated with the second-stage decisions in the constraints of the second-stage problem). In addition, for ease of implementation, they assume *rel-*

*atively complete recourse*; a two-stage stochastic program has relative complete recourse when for each first-stage feasible solution the second-stage problem is feasible. In contrast, our models do not have relative complete recourse, and all the parameters including the recourse matrix are random in the second-stage problems; specifically, the recourse matrix is random due to scenario-dependent coverage sets. We observe that we can reformulate our models to guarantee the relative complete recourse and avoid randomness in the recourse matrix, but it requires introducing additional variables and a large number of Big-M type constraints and Big-M type penalty terms in the objective function. These modifications would result in computationally even more challenging formulations. In addition, the integer L-shaped method appears to be more easy to implement. Along these lines, we opt to develop an algorithm based on the integer L-shaped method, which is applicable to our models due to their equivalent reformulations involving pure binary variables in the first-stage (for details, see the end of Chapter 3).

Following the line of research of Noyan et al. (2013), we implement an enhanced version of the classical integer L-shaped method by employing state-of-the-art computational features, such as the lazy constraint callback of `IBM ILOG CPLEX` and a parallelization of the Benders subproblems via the Boost C++ Libraries. The lazy constraint callback feature is the key to remove the burden of implementing a full-fledged branch-and-cut algorithm procedure (Rubin, 2011). Exploiting the special structure of their model Noyan et al. (2013) prove that the number of opened PODs is always equal to the corresponding upper bound value, and consequently, they obtain slightly stronger Benders (feasibility and optimality) cuts (compared to those presented in Laporte and Louveaux (1993)). These stronger cuts are not valid in our setting, since similar claims – related to the number of open facilities – do not hold in general due to the additional logistics cost issues. Furthermore, our models are harder to solve than those presented in Noyan et al. (2013). Considering these challenges, we incorporate three additional features into the enhanced integer L-shaped algorithm described in Noyan et al. (2013): (i) starting solutions, (ii) alternating cuts, and (iii) scenario prioritization (for second-stage infeasibility detection). We next explain the proposed Benders decomposition-based branch-and-cut algorithm in detail.

In both proposed models the first-stage problems are identical and the structures of the second-stage problems are similar from an algorithmic point of view. Therefore, we consider the following general representation of the second-stage problem while explaining the details of the solution algorithm:

$$Q(\boldsymbol{\eta}, \boldsymbol{\xi}^s) = \max_{\mathbf{u}} \ \{(\mathbf{q}^s)^T\mathbf{u} \ : W^s\mathbf{u} = \mathbf{h}^s - T^s\boldsymbol{\eta}, \ \mathbf{u} \in \mathbb{R}^{n_1} \times \{0,1\}^{n_2}\}. \tag{4.1}$$

At a given iteration of the multi-cut integer L-shaped algorithm, we consider the following *relaxed master problem* (RMP):

$$\begin{aligned}
&\textbf{(RMP)} && \max \ \sum_{s\in S} p_s \vartheta^s && && (4.2)\\
&\text{subject to:} && (3.2)-(3.4), (3.6)-(3.7), && && (4.3)\\
& && D_l\boldsymbol{\eta} \le d_l, && l = 1,\dots,r, && (4.4)\\
& && E_l^s\boldsymbol{\eta} + \vartheta^s \le e_l^s, && l = 1,\dots,t, \ s \in S, && (4.5)\\
& && \boldsymbol{\eta} \in [0,1]^n, \quad \vartheta \in \mathbb{R}^{|S|}, && && (4.6)
\end{aligned}$$

where $n$ denotes the size of the binary first-stage decision vector $\boldsymbol{\eta}$, i.e., $n = \sum_{k\in \bar{J}^{(1)}} |L_k| + |J^{(2)}|$.

In this RMP, the second-stage feasibility requirements are relaxed, and the so-called *feasibility cuts* (4.4) are used to represent the conditions that ensure the second-stage feasibility given the first-stage decisions. In addition, the exact calculation of the second-stage objective values is relaxed, and the auxiliary $\vartheta^s$ variables, along with *optimality cuts* (4.5), are employed to obtain appropriate approximations of the scenario-dependent second-stage objective values. Valid feasibility and optimality cuts are added to the RMP if necessary during the course of the algorithm, and $r$ and $t$ denote the number of feasibility and optimality cuts generated so far, respectively. The final relaxation concerns the integrality restrictions as in any branch-and-cut algorithm. We also note that relaxation of the exact calculation of $Q(\boldsymbol{\eta}, \boldsymbol{\xi}^s), \ s \in S$, using a polyhedral representation is the key point of a Benders decomposition based L-shaped method. We next present the feasibility and optimality cuts that are valid for our proposed two-stage models.

In order to cut a candidate incumbent (best available feasible) solution, for which there exists an infeasible second-stage subproblem, we use the combinatorial benders (CB) cuts of the form:

$$\sum_{j\in G_r} \eta_j - \sum_{j\notin G_r} \eta_j \le |G_r| - 1. \tag{4.7}$$

Here $G_r = \{j \in \{1,\dots,n\} : \eta_j = 1\}$ corresponds to the $r$-th first-stage feasible solution. Observe that the left-hand side of (4.7) is always less than or equal to the cardinality of the set $G_r$, and it is equal to its upper bound of $|G_r|$ only at the $r$-th first-stage feasible solution. This implies that (4.7) solely eliminates further consideration of the $r$-th first-stage feasible solution, which leads to an infeasible second-stage problem. It is common to use such CB cuts for two-stage models with pure binary decisions in the first-stage (see, e.g., Sen, 2005; Codato and Fischetti, 2006). However, such valid inequalities tighten the set of first-stage feasible solutions by cutting only a particular candidate solution. Thus, they do not provide much information about the conditions for the second-stage feasibility, and consequently, a large number of candidate solutions can be enumerated by the branch-and-cut algorithm. In order to mitigate the inefficiency caused by the CB cuts, we also employ the continuous L-shaped feasibility cuts, which will be presented later in this chapter.

We adapt the optimality cuts proposed in Laporte and Louveaux (1993), and use their multi-cut versions while approximating the expected second-stage objective values. Let $\varrho_r^s$ denote the optimal objective function value of the second-stage problem for the $r$-th first-stage feasible solution and scenario $s \in S$. Then, the following set of optimality cuts is valid for our models:

$$\vartheta^s \geq (\varrho_r^s - L^s)(\sum_{j \in G_r} \eta_j - \sum_{j \notin G_r} \eta_j) - (\varrho_r^s - L^s)(|G_r| - 1) + L^s, \quad s \in S, \tag{4.8}$$

where $L^s$ denotes a valid lower bound on the second-stage objective value for scenario $s \in S$. Note that there always exist finite lower bounds on $Q(\boldsymbol{\eta}, \boldsymbol{\xi}^s),\ s \in S$, due to the boundedness of our second-stage problems. In Section 4.1, we present optimization models to obtain the lower bound values used in (4.8).

Let us now consider the (LP) relaxation of a two-stage stochastic integer program, which is obtained by dropping the integrality restrictions in the second-stage problem. It is well-known that any feasibility or optimality cut that is valid for such a relaxed two-stage model is also valid for the original model with integer variables in the second-stage (see, e.g., Laporte and Louveaux, 1993). For the relaxed two-stage model, we can derive the Benders cuts using the duality theory; this Bender decomposition-based approach is known as the *continuous L-Shaped method* (Van Slyke and Wets, 1969). In line with these discussions, in our implementation we use both the continuous feasibility and optimality cuts (as in the continuous L-shaped method) to obtain stronger formulations of the RMP. For details of the continuous L-shaped cuts, we refer to Prékopa (1995) and Birge and Louveaux (1997). According to the representation in (4.1), the continuous feasibility cuts are of the form

$$(\boldsymbol{\upsilon}^{rs})^T(\mathbf{h}^s - T^s\boldsymbol{\eta}) \leq 0, \tag{4.9}$$

and the continuous optimality cuts are given by

$$\vartheta^s \geq (\boldsymbol{\pi}^{rs})^T(\mathbf{h}^s - T^s\boldsymbol{\eta}), \quad s \in S. \tag{4.10}$$

Given the $r$-th first-stage feasible solution, $\boldsymbol{\upsilon}^{rs}$ in (4.9) denotes an extreme ray of the dual feasible region of the continuous relaxation of the second-stage problem under scenario $s$, whereas $\boldsymbol{\pi}^{ls}$ in (4.10) denotes an optimal solution of the corresponding dual problem. Note that these continuous cuts are not sufficient for the convergence of our Benders decomposition-based algorithm to the correct optimal solution, but we observe that they notably improve the performance of the algorithm.

We note that in the classical implementation of the Benders decomposition with multi-cuts, adding a large number of cuts at each iteration may degrade the computational performance (even if such disaggregated cuts in general provide better approximations by capturing more information). Fortunately, the lazy constraint callback feature in our enhanced L-shaped method addresses this only obstacle of the multi-cut approach.

## 4.1 Computing Lower Bounds on the Second-Stage Objective Values

It is clear that the choice of the lower bounds in (4.8) is crucial in obtaining stronger formulations of the RMP, and consequently, in enhancing the computational performance of the integer L-shaped algorithm. However, computing tighter lower bounds becomes expensive as the problem size gets larger. To this end, we solve the following two types of initialization problems (one for each alternative model) under each scenario:

| | Initialization Problem (InitP$_s$) |
|---|---|
| Model_1 | $\underset{\mathbf{z},\mathbf{y},\mathbf{x}^s,\mathbf{w}^s,\boldsymbol{\beta}^s}{\text{minimize}} \; \{(3.33) \;:\; (3.2)-(3.4),(3.6),(3.7),(3.13),(3.14)^*,(3.16)-(3.20),(3.22)-(3.24),(3.54)\}$ |
| Model_2 | $\underset{\mathbf{z},\mathbf{y},\mathbf{x}^s,\mathbf{w}^s,\boldsymbol{\beta}^s,\mathbf{q}^s,\boldsymbol{\vartheta}^s,\boldsymbol{\zeta}^s}{\text{minimize}} \; \{(3.36) \;:\; (3.2)-(3.4),(3.6),(3.7),(3.13),(3.14)^*,(3.16)-(3.20),(3.22)-(3.24),(3.45)-(3.52),(3.53)^\dagger,(3.54)\}$ |

Table 4.1: Compact formulations of the initialization problems (for obtaining the initial lower bound values)

$^\dagger$: Binary restriction is relaxed.
$^*$: $\text{NS}_k^s$ is replaced by $\sum_{l \in L_k} z_{kl}\delta_{kl}$.

It is easy to see that the optimal objective function values of the above optimization problems provide valid lower bounds for each model, since we consider a relaxed version of the corresponding second-stage problem by excluding the decisions and constraints related to the amounts of supplies delivered to the LDCs. Moreover, for further computational enhancement, we update these lower bounds during the course of algorithm according to the bounding scheme presented in Laporte and Louveaux (1993). In particular, we determine new bounds for any finite value $t \geq 1$ as follows:

$$L^s = \underset{\vartheta^s,\mathbf{y},\mathbf{z}}{\text{minimize}} \; \{\vartheta^s \;:\; (3.2)-(3.4),(3.6)^\dagger,(3.7)^\dagger, \text{ and } (\vartheta^s, \boldsymbol{\eta}) \text{ satisfies the continuous optimality cuts (4.10) for } l = 1,\ldots,t\}. \quad (4.11)$$

We update the current lower bounds if they are smaller than the corresponding values in (4.11). Note that this bound updating scheme captures additional information about the second-stage optimality without spending too much computational time. In fact, it can provide tighter lower bounds than the initial ones, although the binary restrictions on the first-stage decisions are relaxed.

## 4.2 Outline of the Algorithm

The outline of the solution algorithm is similar to the one presented in Noyan et al. (2013). However, our models are computationally more challenging, and therefore, we employ three additional methods that significantly impact the computational performance. To keep our presentation self-contained, we provide the description of the whole algorithm but elaborate only on the new enhancements, which prove to be very essential for solving the proposed models.

1. **Starting solutions.** One can use a heuristic approach to obtain an initial first-stage solution, which is not necessarily feasible for the original second-stage model. We note that finding a first-stage solution for which all the second-stage subproblems are feasible may require more elaborative approaches. Since our main goal is to provide a better start, we only focus on obtaining a reasonably good candidate first-stage solution. Along these lines, we propose the heuristic approach described below.

   We solve InitP$_s$ to retrieve the optimal LDC and POD location decisions under each scenario $s$, denoted by $\mathbf{z}^s$ and $\mathbf{y}^s$, respectively. Then, we assign a frequency score to each POD and each LDC based on their total occurrence as an open facility over all scenarios. More specifically, the frequency scores of the PODs are given by

   $$c_j = \left|\{s \in S \ : \ y_j^s = 1\}\right|, \quad j \in J^{(2)},$$

   while the frequency scores of LDCs are obtained as follows:

   $$c_{kl} = \left|\{s \in S \ : \ z_{kl}^s = 1\}\right|, \quad k \in \bar{J}^{(1)}, \ l \in L_k.$$

   Let us first focus on the $y$ decisions and consider a permutation $\boldsymbol{\sigma}$ describing a non-increasing order of $c_1, c_2, \cdots, c_{|J^{(2)}|}$, i.e.,

   $$c_{\boldsymbol{\sigma}(1)} \geq c_{\boldsymbol{\sigma}(2)} \geq \cdots \geq c_{\boldsymbol{\sigma}(|J^{(2)}|)}. \tag{4.12}$$

   Given these notation, our heuristic sets the initial values of $y_j, \ j \in \bar{J}^{(2)}$, denoted by $\bar{y}_j, \ j \in \bar{J}^{(2)}$, as follows: $\bar{y}_j = 1$ for $j \in \{\boldsymbol{\sigma}(1), .., \boldsymbol{\sigma}(\kappa^{(2)})\}$, and $\bar{y}_j = 0$ otherwise. In other words, we select the most frequently opened $\kappa^{(2)}$ of the PODs.

   A very similar approach is applied for the decision vector $\mathbf{z}$. Recall that we consider random coverage sets, and incorporating the coverage sets in the second echelon ensures to open at least one LDC belonging to the coverage set of each open POD. In order to obtain scenario-independent $z$ decisions – given the open PODs (initial values of $y$ decisions) – we solve a set covering type problem, which ignores the randomness of the coverage sets in the second echelon. To this end, we consider fixed coverage sets, denoted by $\hat{N}_j^{(1)}, \ j \in J^{(2)}$; these sets are defined based on the worst-case scenario, i.e., $\hat{N}_j^{(1)} = \bigcap\limits_{s \in S} N_{js}^{(1)}$ for all $j \in J^{(2)}$. We aim to maximize the sum of the frequency scores while selecting the LDCs to be opened, and solve the following set covering (SC) type problem:

$$\text{maximize} \quad \sum_{k \in N_{js}^{(1)}} \sum_{l \in L_k} c_{kl} z_{kl} - \epsilon \sum_{j \in J^{(2)}} \beta_j \tag{4.13}$$

$$\text{subject to:} \quad \sum_{l \in L_k} z_{kl} \leq 1, \quad k \in \bar{J}^{(1)}, \tag{4.14}$$

$$\sum_{k \in \bar{J}^{(1)}} \sum_{l \in L_k} z_{kl} \leq \kappa^{(1)}, \tag{4.15}$$

$$\beta_j + \sum_{k \in \hat{N}_j^{(1)}} \sum_{l \in L_k} z_{kl} \geq \bar{y}_j, \quad j \in J^{(2)}, \tag{4.16}$$

$$0 \leq \beta_j \leq 1, \quad j \in J^{(2)}. \tag{4.17}$$

As in SLMRNDR, constraints (4.14) and (4.15) enforce at most one and single type of LDC at any node and limit the number of LDCs to be opened, respectively. If POD $j$ is decided to be open, constraint (4.16) aims to open at least one LDC which can serve POD $j$ in the worst-case scenario. If this is not possible, $\beta_j$ would take value $1$, and such occurrences are penalized in the objective function. The existence of $\beta_j$ variables guarantees the feasibility of (4.13)-(4.17). Denoting the optimal solution of this problem by $\bar{z}$, we feed $(\bar{\mathbf{y}}, \bar{\mathbf{z}})$ as an initial solution to the branch-and-cut algorithm.

2. **Alternating cuts.** In the enhanced integer L-shaped algorithm, whenever the second-stage feasibility of a candidate incumbent solution is certified, the lazy constraint callback routine checks whether there is a missing optimality cut. Identifying an optimality cut is computationally very expensive, since it requires solving all the mixed-integer second-stage problems to optimality. A natural approach is to avoid or postpone solving these problems to optimality, if possible. Angulo et al. (2014) have recently proposed a simple cut strategy, which postpones generating the regular optimality cuts of the form (4.8) as long as there are missing continuous optimality cuts. Thus, instead of regular optimality cuts, continuous optimality cuts are introduced as lazy constraints if possible, and this strategy is referred to as *alternating cuts*.

   Recall that the lazy constraint callback routine checks whether there is any missing optimality cut only after it ensures that all the second-stage problems are feasible for the candidate incumbent solution. In this case, we have to solve the LP relaxations of all the second-stage problems to optimality, and therefore, there is no additional computational burden while we append the continuous optimality cuts, if necessary. Furthermore, we avoid solving our computationally challenging mixed-integer second-stage problems as long as there is a violated continuous optimality cut. Due to these benefits, the alternating cut strategy is effective in significantly improving the computational performance of the algorithm.

3. **Scenario prioritization**. In the feasibility check procedure of an integer L-shaped algorithm, the second-stage problems are not solved in a particular order to detect the first infeasible scenario. That may lead to solving a large number of second-stage problems until an infeasible scenario is detected. Therefore, a fast discovery of the first infeasible second-stage problem can save computational effort. In this spirit, we propose a scoring scheme for prioritizing the scenarios while checking the feasibility of the second-stage problems. Given a candidate solution, we assign an infeasibility score for each second-stage problem and solve the feasibility problems in a specific order based on these scores. We next describe the steps we follow to calculate the infeasibility score. We keep a counter, denoted by $o_s$, throughout the algorithm to record the total number of occurrences where a missing feasibility cut is identified for $s \in S$. We also calculate an additional score, denoted by $\hat{c}_s$, considering the impact of the coverage sets on the second-stage infeasibility. In particular, we define $\hat{c}_s$ as follows:

$$\hat{c}_s = \sum_{i \in I} \left| N_{is}^{(2)} \cap \bar{J} \right|, \tag{4.18}$$

where $\bar{J}$ is the set of open PODs at the given candidate solution. It is evident that for smaller values of $\hat{c}_s$ the second-stage problem under scenario $s$ is more likely to be infeasible. In this spirit, given a candidate solution the infeasibility score, indicated by $\text{infeas}_s$, is calculated as follows:

$$\text{infeas}_s = -\hat{c}_s o_s, \quad s \in S. \tag{4.19}$$

Intuitively, the second-stage problems with higher infeasibility scores are more likely to turn out to be infeasible. Hence, we sort the scenarios in descending order with respect to their infeasibility scores at the beginning of the feasibility check procedure, and evaluate the feasibility of the corresponding second-stage problems in this order during the feasibility check procedure.

We next give a brief description of our algorithm in order to keep it self-contained. Before invoking the branch-and-cut algorithm, we solve $\text{InitP}_s$ for all $s \in S$ in order to provide appropriate lower bounds on the second-stage objective values, and to obtain an initial solution (as outlined in Algorithm 1). We next initiate the branch-and cut algorithm by feeding the obtained initial solution to `CPLEX`. Whenever a candidate incumbent solution is found in the search tree, the callback routine checks whether there is a missing feasibility cut. We utilize the proposed scenario prioritization feature, and the 3-phase feasibility procedure presented in Noyan et al. (2013) for early detection of a violated feasibility cut. If second-stage infeasibility for a scenario is certified at one of the phases, we add the corresponding feasibility cuts of types (4.9) and (4.7) as lazy constraints. Note that identifying the first infeasible scenario is sufficient to generate (4.9). If all the second-stage problems turn out to be feasible, the lazy callback callback routine inquires about any missing continuous optimality cuts. If any detected, the corresponding continuous optimality cuts (4.10) are generated. Note that the continuous second-stage problems are already solved to optimality in the 3-phase feasibility procedure, hence generating (4.9) becomes a quite fast procedure in the overall algorithm. Unless a missing continuous optimality cut is identified, the second-stage problems are solved to optimality, and for any second-stage problem with an optimal objective value larger than the approximated value $\vartheta^s$ we append the optimality cuts (4.8). Finally, at each candidate incumbent solution that leads to feasible second-stage problems, we solve the problem (4.11) in order to update the lower bounds $L^s, s \in S$. The callback routine certifies the candidate solution as the incumbent once all types of appended lazy constraints are satisfied in the RMP. The algorithm terminates when the incumbent is proved to be optimal. The pseudocodes of our heuristic method to obtain starting solutions, and our exact branch-and-cut algorithm are presented in Algorithms 1 and 2, respectively.

## 4.3 Parallel Computing

We utilize parallel computing techniques similar to Noyan et al. (2013) in order to improve the computational performance of the algorithm. In order to benefit from our decomposition approach, our strategy is to distribute independent operations (such as the initialization problems and solving the second-stage problems) over a fixed number of available threads. To this end, we utilize the Boost C++ Libraries. Note that one can also

**Algorithm 1:** Procedure $starting\_solution$

**for** $j \in J^{(2)}$ **do**
  Given the optimal $y_j^s,\ s \in S$, calculate the frequency score $c_j$;
**end**
Sort $c_j$ in a non-increasing order and let $\boldsymbol{\sigma}$ be the permutation that satisfies (4.12);
Set $\bar{y}_j = 1$ for all $j \in \{\boldsymbol{\sigma}(1), ..\boldsymbol{\sigma}(\kappa^{(2)})\}$, and 0 otherwise;
**for** $k \in \bar{J}^{(1)},\ l \in L_k$ **do**
  Calculate the frequency score $c_{kl}$;
**end**
Given $\bar{\mathbf{y}}$ and the $c_{kl}$ values, solve the set covering problem (4.13)-(4.17);
Retrieve $\bar{z}$ values;
Set the initial solution to $(\bar{\mathbf{y}}, \bar{\mathbf{z}})$.

use built-in parallel programming features of CPLEX to solve a single problem by tuning the *threads* parameter of CPLEX. We next give the details of allocation of threads to each operation in our implementation.

The maximum number of threads that can be utilized simultaneously is $4$, whereas we allocate $1$ thread to CPLEX. Furthermore, we set the parallelization mode as *opportunistic*, in which CPLEX utilize all opportunities for parallelism in order to provide better performance at the expense of non-deterministic solution times and solution vectors (IBM ILOG CPLEX (2010)). Since the initialization and second-stage problems are not computationally demanding compared to RMP, we distribute these problems among the multiple threads and disable the parallelization features of CPLEX. To this end, we allow concurrent usage of maximum $4$ threads while solving the second-stage problems as well as the initialization problems.

**Algorithm 2:** Enhanced Integer L-shaped Algorithm

`// Initialization`

Set $r = t = 0$. Solve $\text{InitP}_s$ and set an appropriate lower bound on $\vartheta^s$ for all $s \in S$.

Use $starting\_solutions$ procedure to retrieve an initial solution;

`// Main loop`

Provide the initial solution to `CPLEX`. Invoke `CPLEX` to solve the RMP and initiate branch-and-cut procedure;

**while** *`CPLEX` determines that both the relative and absolute optimality gaps of the current incumbent are greater than the specified thresholds* **do**

  Identify a new candidate incumbent solution $(\hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\vartheta}})$;

  Check the second-stage feasibility for $\hat{\boldsymbol{\eta}}$;

  **if** *the second-stage problem is feasible for all the scenarios* **then**

    $violation = false$;

    **for** $s \in S$ **do**

      **if** $(\hat{\varrho}^s - \hat{\vartheta}^s)/\hat{\vartheta}^s \geq \epsilon$ **then** `//` $\hat{\boldsymbol{\eta}}$ `violates some of the missing continuous optimality cuts`

        Add the corresponding continuous optimality cuts of the form (4.10) to the RMP as lazy constraints, $violation = true$;

      **end**

    **end**

    **if** $violation = false$ **then** `//If` $\hat{\boldsymbol{\eta}}$ `does not violate any continuous cuts`

      **for** $s \in S$ **do**

        Solve the second-stage problem under scenario $s$ to optimality and let $\hat{\vartheta}^s,\ s \in S$, denote the optimal second-stage objective values;

        **if** $(\hat{\varrho}^s - \hat{\vartheta}^s)/\hat{\vartheta}^s \geq \epsilon$ **then** `//` $\hat{\boldsymbol{\eta}}$ `violates some of the missing optimality cuts`

          Add the corresponding optimality cuts of the form (4.8) to the RMP as lazy constraints;

        **end**

      **end**

    **end**

    Solve the problem (4.11) for all $s \in S$ to calculate the new bounds, and update the current bounds if possible;

  **else** `//` $\hat{\boldsymbol{\eta}}$ `violates some of the missing feasibility cuts`

    Solve the continuous relaxation of the first infeasible second-stage problem without *presolve*, retrieve $\boldsymbol{v}^{ls}$ utilized in (4.9);

    Add the corresponding feasibility cuts of the form (4.7) and (4.9) to the RMP as lazy constraints;

  **end**

**end**

# Chapter 5

# Computational Analysis

We apply our models on a case study developed based on real-world data from the 2011 Van earthquake, which is an extension of the case study presented in Noyan et al. (2013). First, we provide the details of the data set which we utilize in this study. We next present our extensive numerical analysis in order to compare the performances of the proposed models in terms of the desired accessibility metrics. Additionally, we evaluate the trade-off between accessibility and equity metrics by analysing alternate supply allocation policies (PD, TD, and hybrid policies) given in Noyan et al. (2013). In section 5.3, we demonstrate the computational performance of our solution approach.

## 5.1 Data Set

We mainly use the data structure of the case study in Noyan et al. (2013), however, we incorporate additional parameters relevant to our proposed models into the case study. Furthermore, as stressed in Remark 2, we define the accessibility scores as the weighted travel times. Therefore, in this section, we emphasize more on the additional parameters and accessibility metrics.

Here we briefly indicate the key points of the case study in Noyan et al. (2013) and refer the reader to this study for further details. The authors consider a network consists of $94$-settlements affected by the 2011 Van Earthquake. They cluster this network with 30 and 60 nodes through a p-median model which minimizes the demand-weighted travel times. Based on the intensity of the earthquake, one of three possible damage states is associated with each cluster. According to the damage state of a cluster, the number of affected people are estimated. In order to represent the post-disaster relief network conditions, they construct the random realizations of demand and accessibility for a set of scenarios ($|S| = 50, 100, 200, 500$) given the base values for demands and accessibility metrics. Specifically, the realizations of the demands and accessibility metrics for each scenario are obtained by multiplying their corresponding estimated base values by specified deviation factors. For each node and link, a separate deviation factor is independently sampled from particular uniform distributions depending on the damage level of the area in consideration, which is determined based on the proximity to the epicenter of the earthquake. Note that a link may span multiple damage areas, then the proportion of the link length in different damage areas are considered while sampling the deviation factor.

We next elaborate on the additional and modified model parameters. Different from the network structure in Noyan et al. (2013), we allow multiple LDC locations in the network where either existing LDCs or additional LDCs can be set up. In total, there are $9$ LDC locations in the network, consisting of $5$ pre-position LDC locations and $4$ candidate LDC locations. Below we describe how we choose both existing and additional candidate LDC locations.

## I. Existing LDC locations.

In the report of the Van Earthquake 2011, Kizilay lists the particular disaster management centers which sent relief supplies to the affected region (Turkish Red Crescent Disaster Management (2012)). Indeed, the action plan of Disaster and Emergency Management Presidency of Turkey proclaims these disaster management centers as the liable group of disaster management centers particularly for the Van city in case of a disaster ( Disaster and Emergency Management Presidency (AFAD) (2013)). According to this action plan, each city is assigned to a specified group of disaster management centers in Turkey for the purpose of disaster preparedness. These disaster management centers preposition a specific amount of relief supply. In case of a disaster, they store the additional supplies rushing in the country, and transfer both additional and existing relief supplies to the affected region that they are in charge of. In this spirit, we choose the liable disaster management centers of the Van region located at Muş, Van, Ağrı and Diyarbakır as the existing LDCs in the network.

## II. Additional LDC locations.

In practice, relief organizations set up additional LDCs close to the locations wither high demand values, if necessary. In this spirit, we select the demand nodes with higher base demand values as the candidate LDC locations, therefore we identify the most populated 4 demand nodes also as the candidate LDC locations, namely Bostaniçi, Erçek, Bardakçı and Güvençli. Furthermore, relief supplies were also received by a center close to the Van airport in the Van 2011 Earthquake. Hence, we add the Van airport to the last mile network and allow it to be a candidate LDC location.

## III. The central depot.

The relief organizations usually stage the relief items which are supplied by international and national organizations at a substantially large depot. It is located at a convenient place in terms of accessibility particularly for the international relief organizations. Therefore, an airport near the affected region is generally preferred as the central depot. In the Van Earthquake 2011, the Erzurum Airport is preferred as the central depot over the Van Airport. That is because, the operational capacity of the Erzurum airport is larger than the one in Van. Furthermore, the Erzurum airport is much safer from aftershocks. In this spirit, we choose the central depot as the Erzurum Airport.

## IV. Computing the accessibility metric.

We define the accessibility metric in a similar fashion to Noyan et al. (2013). However, while obtaining the accessibility scores, we directly use the weighted travel times instead of using an inverse function of it, since this approach is more natural while characterizing the proposed aggregated accessibility metrics in our context. Thus, a link with a lower accessibility score indicates higher accessibility. In order to stress the difference between our accessibility metric and the one in Noyan et al. (2013), we give a complete list of steps to calculate the values of accessibility metrics both at the first and second echelon for each test instance:

1. The travel time in hours between each pair of nodes in the network is obtained from Google Maps.

2. Each node in the graph is associated with a risk score based on the distance from the Lake of Van. The flooding risks are attached to all links in the network via this metric, and the risk score is higher in some areas of the affected regions, which are closer to the Lake of Van.

3. To calculate the accessibility metric between LDCs and PODs, we simply multiply the travel time by the risk score of the link between the pairs.

4. To calculate the accessibility metric between PODs and demand nodes , we additionally multiply the metric described in item 2 with a mobility score $\beta_i$ of each demand node $i$. Particularly, the mobility score of a demand node is calculated by taking the weighted sum of proportions of elderly, disabled and women with child at that node.

We remark that the mobility score is not considered while calculating the accessibility scores for the first echelon. That is because, the mobility of individuals between LDCs and PODs is inapplicable. We also note that the the travel times between each pair of nodes has been slightly changed since 2013 due to new road constructions in the region. Therefore, while calculating the accessibility metrics, we utilize the travel time data retrieved from Google Maps in 2014 rather than the given travel times in the case study of Noyan et al. (2013).

## V. Other input parameters

In order to implement the proposed models, we need to specify the values of other input parameters as well. We describe how we set the values for these parameters (particularly, $\bar{\theta}$, $\theta_k$, $\delta_{kl}$, $K_j$, $\tau^{(1)}$, $\tau^{(2)}$, $\kappa^{(1)}$, $\kappa^{(2)}$, $\rho$, $c^s_{hk}$, $f_{kl}$, $B$, $\alpha_j$) below:

- The amount of total available supply in the network is denoted by $\tilde{\theta} = \bar{\theta} + \sum\limits_{k \in H} \theta_k$, which corresponds 90% of the expected total demand. We calculate the expected total demand as in the study of Noyan et al. (2013). We set $\bar{\theta} = 0.6\tilde{\theta}$ and $\sum\limits_{k \in H} \theta_k = 0.4\tilde{\theta}$.
- The relief organizations generally establish LDCs with different capacities considering the needs and available sources in the post-disaster environment. We assume that three size categories for LDCs, specifically small, medium, and large, are adequate to capture the real life practices of relief organizations. Along these lines, we determine the capacities as 20%, 40% and 60% of $\tilde{\theta}$ for the LDCs with small, medium and large sizes, respectively. Recall that each existing LDC has a prespecified capacity, thus it is associated with a particular size category. We identify the size categories of the existing LDCs by considering their areas and the ratio between the specified size categories. (we retrieve the areas of the existing LDCs via a personnel contact at Kizilay). According to this approach, the size category of the LDC in Mus is identified as large, whereas the others are identified as small.

- We set the pre-stocked supply amount at each existing LDC to a certain proportion of $\sum_{k \in H} \theta_k$ with respect to its capacity. More specifically, we distribute $\sum_{k \in H} \theta_k$ among the existing LDCs proportional to the storage capacity ratios between size categories.

- The upper bounds on the capacities of the PODs denoted by $K_j$ impact the decisions of both models significantly. Following the study of Noyan et al. (2013), we set $K_j = c\hat{d}_j$, where $\hat{d}_j$ indicates the estimated base value of the demand node $j$. We select $c$ from the following set $\{2, 2.5, 3\}$.

- Recall that $\tau^{(1)}$ and $\tau^{(2)}$ are the upper bounds on the accessibility thresholds used for defining the coverage sets at the first and second echelons of the last-mile relief network, whereas $\kappa^{(1)}$ and $\kappa^{(2)}$ denote the upper bounds on the number of LDCs and PODs to be opened, respectively. If both upper bounds (specifically, on the accessibility threshold and the number of open facilities) at any of the echelon are not sufficiently large enough, then it might not be possible to identify feasible assignment decisions while meeting the accessibility threshold. Particularly, assigning a POD to a LDC and/or each demand point to a POD might not be achieved. Therefore, we consider the number of demand points (i.e., the demand nodes with small $|N_{is}^{(2)}|$) and PODs (i.e., the PODs with small $|N_{js}^{(1)}|$) that are challenging to be covered at the first and second echelon, respectively. For instance, in order to determine the value of parameter $\tau^{(1)}$, we restrict the number of candidate PODs for which the coverage set includes less than a certain number of candidate LDCs. Thus, we consider the worst (largest) possible values of the realizations of the accessibility metric (denoted by $\hat{v}_{ij}^{(2)}$ at the second echelon and $\hat{v}_{kj}^{(1)}$ at the first echelon). Consequently, we set the values of $\tau^{(1)}$ and $\tau^{(2)}$ based on the following conditions:

$$\left|\{i \in I : \left|N_i^{(2)}\right| = \left|\{j \in J^{(2)} \mid \hat{v}_{ij}^{(2)} \leq \tau^{(2)}\}\right| \leq \lceil \gamma_1 \ |I| \rceil \}\right| < \lceil \gamma_2 \ |I| \rceil, \quad (5.1)$$

$$\left|\{j \in J^{(2)} : \left|N_j^{(1)}\right| = \left|\{k \in \bar{J}^{(1)} \mid \hat{v}_{kj}^{(1)} \leq \tau^{(1)}\}\right| \leq \lceil \gamma_1 \ |J^{(2)}| \rceil \}\right| < \lceil \gamma_2 \ |J^{(2)}| \rceil. \quad (5.2)$$

For our 30-node case study network, we set $\gamma_1 = 0.05$ and $\gamma_2 = 0.15$, and observe that the above condition is satisfied for $\tau^{(1)} \in [1.96, 2.26]$, whereas for $\tau^{(2)} \in [14.49, 1076]$.

- In real-world, the decision makers can limit the $\kappa^{(1)}$ and $\kappa^{(2)}$ parameters based on available resources (i.e., budget, personnel, etc.). In this case study, we set $\kappa^{(1)} = 6$, $\kappa^{(2)} = 8$.

- Recall that $\rho$ is the proportion of unsatisfied demand at POD level and we enforce the same proportion for all PODs in our models. For our case study, we set $\rho = 0.30$.

- We consider the unit shipment cost from the existing LDCs/central depot to all LDCs. In order to calculate the unit shipment costs between two nodes, we first calculate the distance between them by using the Vincent algorithm, then we scale it by a constant ($0.001$). We generate the realizations of the random cost parameter through following the same approach for accessibility metric described in Noyan et al. (2013), since the unit shipment cost between two nodes also depends on the damage level of the link between them. Specifically, we obtain the realizations of the random cost parameter for a particular link through multiplying its base unit shipment cost by a particular deviation factor (see section $6$ in Noyan et al. (2013) for a more detailed explanation).

- We assume that the fixed cost of opening a LDC increases proportional to its size. A study in the pre-disaster literature by Rawls and Turnquist (2010b) defines the size categories (small, medium and large as in our case study) of facilities that are used for pre-positioning relief supplies and exhibits their respective storage capacities and fixed costs. By using the given storage capacities and fixed costs of facilities in this study, we deduce the corresponding proportional increase in fixed cost against the proportional increase in storage capacity between different facility sizes. In this spirit, we assume the fixed cost of opening a small LDC to $50000$ and considering both the ratios drawn from Rawls and Turnquist (2010b) and storage capacity ratios between different size categories in our setting, we set the fixed costs as $85000, 110000$ for medium and large size LDCs, respectively. Recall that the existing facilities can also be expanded at a certain cost into upper size categories. We set the expansion cost of an existing LDC into a larger size category by simply taking the difference between the fixed costs of opening a LDC at its existing size and enlarged size. For instance, a small size existing LDC can be upgraded to a large size category at a cost of $60000$.

- We set $B = a\underline{B}$, where $\underline{B}$ denotes the minimum required budget in the best-case scenario where $a \in \{1.3, 1.5, 1.7\}$. Here the best-case scenario indicates the least demand values for each demand node, and the least accessibility scores and cost parameters between each pair of nodes in a test network. By using the best-case scenario parameters, we replace the objective function (3.9) by the left hand side of constraint (3.21), and solve the modified two-stage problem in order to obtain $\underline{B}$. We observe that the budget slightly impact the location decisions where $a = 1.7$. Hence, it is appropriate to categorize the budget as low, medium, and high where $a = 1.3, 1.5, 1.7$, respectively.

- Recall that the $\alpha_j$ parameter is used to estimate the average accessibility between LDCs and PODs in (3.36). $\alpha_j$ determines to which extent the links with higher delivery amounts should be considered in the estimation of average accessibility of a POD. For instance, given $\alpha_j = 1$, the link which has the highest delivery amount to POD $j$ is solely considered while calculating $\nu_j^s$ for all $s \in S$. Note that the smallest cardinality of $\hat{N}_j^{(1)}$ among all PODs is the upper bound on the value of $\alpha_j$. In this case study, we set the value of parameter $\alpha_j = 1$ due to the drawback of the second model as addressed in Remark 1.

## 5.2 Comparison of Models

We conduct numerical experiments to compare the performances of the proposed models with respect to accessibility metrics. Observe that, the proposed models employ different approaches to approximate the ideal objective function (3.30). It follows that the comparison of the models based on their optimal function values would not be fair, thus this may arise misleading conclusions. However, we can fairly compare the two models based on their corresponding ideal objective function values given their optimal solutions. In particular, we can simply calculate the ideal aggregated accessibility metrics that both models achieve with their optimal decisions.

The results for the ideal aggregated accessibility metrics for different instances of Model_1 and Model_2 appear in Table 5.1. (i.e, metric IOFV (the sum of IOFV-I+IOFV-II) in the table), where the metrics (IOFV-I) and (IOFV-II) stand for the ideal aggregated accessibility in the first and second echelon, respectively. Recall that both models minimize the expected value of a particular aggregated accessibility (see (3.33) and (3.36)). We also present the model-specific accessibility metrics in the first and second echelon in this table, denoted by (I) and (II), respectively. Particularly, the metric (I) stands for the expected total weighted accessibility of the PODs from the LDCs, whereas the metric (II) denotes the expected total weighted accessibility of the PODs from the demand locations as given in the objective functions of Model_1 (3.33) and Model_2 (3.33).

In the presented instances, we alter the values of the parameters $\tau^{(1)}$, $\tau^{(2)}$, $c$, and $B$, while we set $\rho = 0.3$, $\kappa^{(1)} = 6$, $\kappa^{(2)} = 8, \alpha_j = 1$ for all $j \in J^{(2)}$.

**Observation 1** *Model_1 performs better than Model_2 in terms of IOFV. In particular, IOFV-I notably contributes to the gap between the two models in terms of IOFV, whereas IOFV-II are quite similar in both models.*

We can see from Table 5.1 that the gap with respect to IOFV is always larger than $6\%$, which leads to the fact that the two models are not comparable. The numerical results for Model_2 in Table 5.1 support the weaknesses addressed in Remark 1. We propose an elaborate approach to approximate the ideal objective function in Model_2. Unfortunately, we observe that a simpler modeling approach (presented in Model_1) performs better than the elaborate modelling approach in terms of accessibility metrics; particularly the accessibility of the PODs from the LDCs. Although Model_2 focuses more on the links with higher delivery amounts, it fails to differentiate the selected links with respect to their delivery amounts by simply taking the average of the corresponding accessibility scores. That is to say, Model_2 is indifferent between the delivery supply amounts among the selected links. As stressed in Remark 1, when $\alpha_j > 1$, it is possible to incorporate the accessibility scores of the links with zero delivery amounts. Model_2 with the ad-hoc approach usually achieves better IOFV than Model_2; however, it does not obtain better IOFV than Model_1. The superiority of Model_1 stems from the fact that it incorporates the $\mathbf{w}$ decisions weighted by the accessibility scores of the selected links; it is a simple but effective approach to approximate the ideal objective function. As opposed to the simple version, Model_2 gives more importance to the accessibility scores associated with higher amounts of delivery, but neglects the significance of delivery amounts among the selected links. Furthermore, Model_2 is computationally hard to solve due to its complex formulation as shown in Table 5.4. Considering these results, we emphasize more on the computational analysis of Model_2, yet we address the settings where the difference between two models is relatively high.

**Observation 2** *When we focus more on ensuring equitable accessibility in the second echelon, the relative gap between Model_1 and Model_2 decreases in terms of IOFV. In particular, for Model_2, the impacts of higher levels of equitable accessibility in the second echelon are more prominent in the first echelon (IOFV-I) compared to the second-echelon (IOFV-II).*

| Parameters | Model | IOFV-I | IOFV-II | IOFV | Relative Increase | (I) | (II) | (I +II) |
|---|---|---|---|---|---|---|---|---|
| ($\tau^{(1)}$=2.18,$\tau^{(2)}$=44,c=2.5,$B$=Medium) | Model_1 | 91716 | 143662 | 235378 | 9% | 78722 | 143662 | 222384 |
| | Model_2 | 112264 | 143947 | 256211 | | 88001.4 | 143947 | 231948.4 |
| ($\tau^{(1)}$=2.18,$\tau^{(2)}$=22,$c$=2.5,$B$=Medium) | Model_1 | 91712 | 143666 | 235378 | 6% | 78722 | 143666 | 222388 |
| | Model_2 | 105999 | 143958 | 249957 | | 87991.4 | 143958 | 231949.4 |
| ($\tau^{(1)}$=2.1,$\tau^{(2)}$=44,$c$=2.5,$B$=Medium) | Model_1 | 91712 | 143666 | 235378 | 11% | 78722 | 143666 | 222388 |
| | Model_2 | 116890 | 143638 | 260528 | | 88344 | 143638 | 231982 |
| ($\tau^{(1)}$=2.18,$\tau^{(2)}$=44,$c$=2,$B$=Medium) | Model_1 | 91818 | 148366 | 240184 | 9% | 79272.3 | 148366 | 227638.3 |
| | Model_2 | 113891 | 148377 | 262268 | | 88357.3 | 148377 | 236734.3 |
| ($\tau^{(1)}$=2.18,$\tau^{(2)}$=44,$c$=2,$B$=Low) | Model_1 | 91806 | 148380 | 240186 | 16% | 79259.6 | 148380 | 227639.6 |
| | Model_2 | 130832 | 148423 | 279255 | | 91986.2 | 148423 | 240409.2 |
| ($\tau^{(1)}$=2.18,$\tau^{(2)}$=44,$c$=3,$B$=Medium) | Model_1 | 91458 | 142573 | 234031 | 6% | 78012.9 | 142573 | 220585.9 |
| | Model_2 | 105637 | 142580 | 248217 | | 88228.9 | 142580 | 230808.9 |
| ($\tau^{(1)}$=2.20, $\tau^{(2)}$=60,$c$=2.5,$B$=Medium) | Model_1 | 91712 | 143666 | 235378 | 12% | 78722 | 143666 | 222388 |
| | Model_2 | 119380 | 143631 | 263011 | | 88343.4 | 143631 | 231974.4 |
| ($\tau^{(1)}$=2.20, $\tau^{(2)}$=44,$c$=2.5,$B$=Medium) | Model_1 | 91711 | 143662 | 235373 | 10% | 78722 | 143662 | 222384 |
| | Model_2 | 116359 | 143641 | 260000 | | 88343.7 | 143641 | 231984.7 |
| ($\tau^{(1)}$=2.20, $\tau^{(2)}$=44,$c$=3,$B$=Medium) | Model_1 | 91462 | 142573 | 234035 | 9% | 78012.9 | 142573 | 220585.9 |
| | Model_2 | 112756 | 142575 | 255331 | | 88225.2 | 142575 | 230800.2 |
| ($\tau^{(1)}$=2.40, $\tau^{(2)}$=44,$c$=2.5,$B$=Medium) | Model_1 | 91712 | 143662 | 235374 | 8% | 78722 | 143662 | 222384 |
| | Model_2 | 110786 | 143958 | 254744 | | 87979 | 143958 | 231937 |
| ($\tau^{(1)}$=2.20, $\tau^{(2)}$=60,$c$=3,$B$=Medium) | Model_1 | 91462 | 142573 | 234035 | 11% | 78012.9 | 142573 | 220585.9 |
| | Model_2 | 117779 | 142539 | 260318 | | 88289.4 | 142539 | 230828.4 |

Table 5.1: Accessibility statistics for Model_1 and Model_2 under different parameter settings

Due to the structure of the objective function of Model_2, enforcing equitable accessibility in the second echelon can also impact the optimal delivery decisions along the first echelon (i.e., the optimal delivery amounts sent from the LDCs to PODs). For instance, a tighter upper bound on the accessibility threshold (such as reducing the threshold to 22 from 44 in the second echelon) when $c = 2.5$, $B = M$, and $\tau^{(1)} = 2.18$ results in higher alterations in IOFV-I (the relative change smaller than $5.58\%$) value than IOFV-II (the relative change smaller than $0.0008\%$) for Model_2. On the other hand, both the relative changes in IOFV-I and IOFV-II are less than $0.004\%$ for Model_1. Furthermore, we observe that Model_1 performs in a similar manner in terms of IOFV, even though it is confined with a more restrictive accessibility threshold in the second echelon. Similarly, for $c = 2.5$, $B = M$, and $\tau^{(1)} = 2.20$, when $\tau^{(2)}$ is decreased from 60 to 44, the relative change in IOFV-II of Model_2 is quite bigger than that of Model_1 (i.e., the relative changes in IOFV-II for Model_1 and Model_2 are $2.53\%$ and $0.001\%$, respectively).

**Observation 3** *As the capacity limitations of the PODs decrease, the relative gap between the proposed models also decreases. Furthermore, we observe that IOFV-II of Model_2 is further sensitive to the changes in the capacity limitations of the PODs compared to Model_1.*

We can see from Table 5.1 that the relative difference between two proposed models is easily noticed when more restrictive limitations are applied to the POD capacities. In other words, Model_1 performs much better than Model_2 in settings where the upper bounds on the POD capacities are much tighter. For instance, the relative gap between two models is $6.06\%$, for $c = 3$, $\tau^{(1)} = 2.18$, $\tau^{(2)} = 44$, and $B = M$. For $c = 2.5$, the relative gap becomes $8.85\%$, and when $c = 2$, it further increases to $9.19\%$. Additionally, we observe that ideal accessibility metrics of both models (e.g., IOFV-I, IOFV-II, IOFV) increase as the capacity limitations on the PODs are more restrictive. Similarly, the individual accessibility metrics (e.g., I, II, (I+II)) of the proposed models increase as the POD capacities get smaller.

**Observation 4** *When the available budget gets tighter, the relative gap between the proposed models becomes even larger.*

As observed from Table 5.1, the budget significantly impacts IOFV for both models as it limits the logistics costs associated with supply delivery between depots (from the global depot and/or reallocation between LDCs), and opening new LDCs. For instance, for $c = 2$, $\tau^{(1)} = 2.18$, $\tau^{(2)} = 44$, and $B = L$, the relative difference between Model_1 and Model_2 becomes $13.99\%$. By the same parameter settings except the medium budget category, we observe that the relative gap between two models decreases to $8.42\%$. Furthermore, IOFV-II of Model_2 is the most affected accessibility metric by a limited budget. We also note that enforcing limited capacities on the PODs as well as limited budget for the relief operations cause a more discernible relative gap between two models.

**Observation 5** *While the focus on ensuring more equitable accessibility in the first echelon increases, the relative gap between the proposed models generally increases. However, we also detect few problem instances which indicate the otherwise.*

The impacts of imposing more demanding equity threshold for accessibility in the first echelon are ambiguous in terms of the relative gap between the models, as we observe changes in different directions in IOFVs of both models (increase/decrease in IOFV). For instance, for $c = 2.5$, $\tau^{(1)} = 2.40$, $\tau^{(2)} = 44$, and $B = M$, the relative gap between Model_1 and Model_2 is $8.23\%$. When only the value of the parameter $\tau^{(1)}$ is set to 2.20, the relative gap becomes $10\%$. On the contrary, from the latter to the setting where $\tau^{(1)} = 2.18$, the relative gap is decreased to $9\%$. Thus, we cannot conclude a pattern in the relative gap in terms of IOFV for comparison purposes while conducting sensitivity analysis for the parameter $\tau^{(1)}$.

The results in Table 5.1 implies that Model_1 always performs better than Model_1 in terms of IOFV, particularly when $\alpha$ takes value of 1 for all PODs. As suggested in Remark 1, there are few remedies to resolve this problem. However, both suggestions have their own shortcomings. The first suggestion causes non-convex optimization problem, whereas the latter does not guarantee a better solution in terms of IOFV. Therefore, we discuss the performance of Model_1 based on the model-specific accessibility and equity metrics for the rest of this section. Furthermore, we present the computational performance of Model_1 in more detail, whereas we illustrate few samples of the computational performance for Model_2.

Here we analyze Model_1 with respect to its model-specific accessibility metrics (e.g., (I), (II), (I+II) in Table 5.1), and list the observations related to this particular model.

**Observation 6** *Model_1 is generally robust to alterations in the parameter $\tau^{(1)}$ in terms of the metrics I, II, and (I+II). Particularly, the metric II increases as the focus on ensuring equitable accessibility in the first echelon increases.*

The impacts of smaller values for $\tau^{(1)}$ with respect to the accessibility metrics I, II, and (I+II) of Model_1 are not evident in Table 5.1. In fact, we do not observe any increase/decrease in the metric I for the given problem instances. However, the impacts of the parameter $\tau^{(1)}$ on the metric II reflect on the total aggregated accessibility metric (I+II), although the fluctuations in the metric II as well as the metric (I+II) are minor. For instance, given $c = 2.5$, $\tau^{(2)} = 44$, and $B = M$, if the value of $\tau^{(2)}$ decreases to $2.10$ from $2.40$, the metric I stays the same, while the metric II increases. Therefore, we conclude that imposing more equitable accessibility in the second echelon directly affects the accessibility in that echelon for Model_1.

**Observation 7** *For Model_1, the parameter $\tau^{(2)}$ affects the metric II in different directions; however, it does not incline any alterations in the metric I.*

We observe that focusing more on equitable accessibility in the second echelon impacts the expected total weighted accessibility of the demand points from PODs, yet a decrease in the parameter $\tau^{(2)}$ does not necessarily trigger an increase in the metric II. Furthermore, the problem instances in Table 5.1 indicate no effect on the metric I. For instance, for $\tau^{(1)} = 2.18$, $c = 2.5$, and $B = M$, when $\tau^{(2)}$ is decreased to 22 from $44$, only the metric II contributes to the difference in the expected total weighted accessibility (the metric (I+II)) between two problem settings. Similarly, for $\tau^{(1)} = 2.20$, $c = 3$, and $B = M$, when $\tau^{(2)}$ is decreased to $44$ from $60$, the metric I does not change, whereas the metric II decreases.

**Observation 8** *A more restrictive setting for the capacity limitations of PODs and the available budget usually give rise to an increase in the metric (I+II).*

As expected, we observe from Table 5.1 that the expected total weighted accessibility metric defined particularly for Model_1 (i.e., the metric (I+II)) increases as tight capacities for the PODs and limited budget are enforced. For instance, the largest value for the metric (I+II) appears for the problem instance where $c = 2$ and $B = M$. We also observe that as the capacity limitations get more restrictive and/or the budget becomes insufficient, the metric (I+II) increases.

As discussed in the introduction, Noyan et al. (2013) study three alternate supply allocations, which focus on *i)* proportional distribution at the POD level (Model_PD), *ii)* maximum proportion of unsatisfied demand at the model (Model_TD), and *iii)* the hybrid approach that we utilize in both models (Model_Hybrid). In the study of Noyan et al.

(2013), it is analytically and empirically shown that Model_Hybrid achieves higher levels of accessibility and equity simultaneously (see Noyan et al. (2013) for more details). Motivated by this result, we opt for Model_Hybrid for computational analysis and performance. Nevertheless, we carry out the other supply allocation approaches for Model_1 for a particular problem instance in order to demonstrate the effectiveness of the selected policy. Note that we can easily modify SLMNRDR in order to model the other supply allocation policies discussed in *i* and *ii*. Model_PD is obtained by enforcing the decision variables $\beta_j^s \leq 0$, whereas we simply set the parameter $\epsilon = 0$ to attain the formulation of Model_TD.

Here we describe the performance metrics in terms of unsatisfied demand in order to compare the performances of three alternate supply allocation policies. Following the study of Noyan et al. (2013), we compare the proposed models with respect to the proportion of unsatisfied demand (PUD). We evaluate equity in supply chain based on the maximum proportion of unsatisfied demand over PODs (MPUD). Furthermore, we measure the average proportion of unsatisfied demand across PODs (APUD) while evaluating the PUD in the network. In order to compare the proposed models, we illustrate the empirical cumulative distribution functions (CDFs) of the random APUD and MPUD for a particular problem instance for each model in Figure 5.1 and 5.2, respectively. Furthermore, for this problem instance, Figure 5.3 and 5.4 show the expected and maximum PUD at each selected POD, respectively.

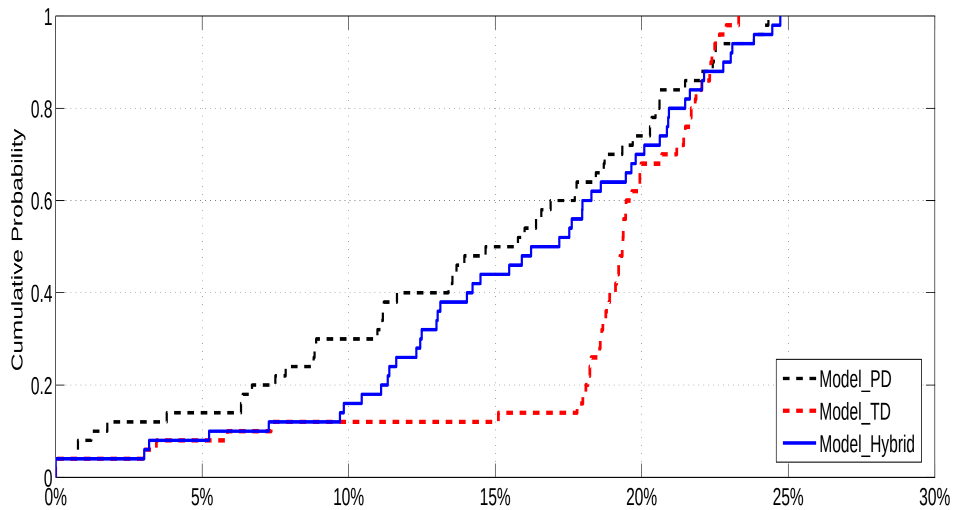


Figure 5.1: Empirical cumulative distribution function of APUD
$(\tau^{(1)} = 2.10,\ \tau^{(1)} = 22,\ c = 2.5,\ B = M)$

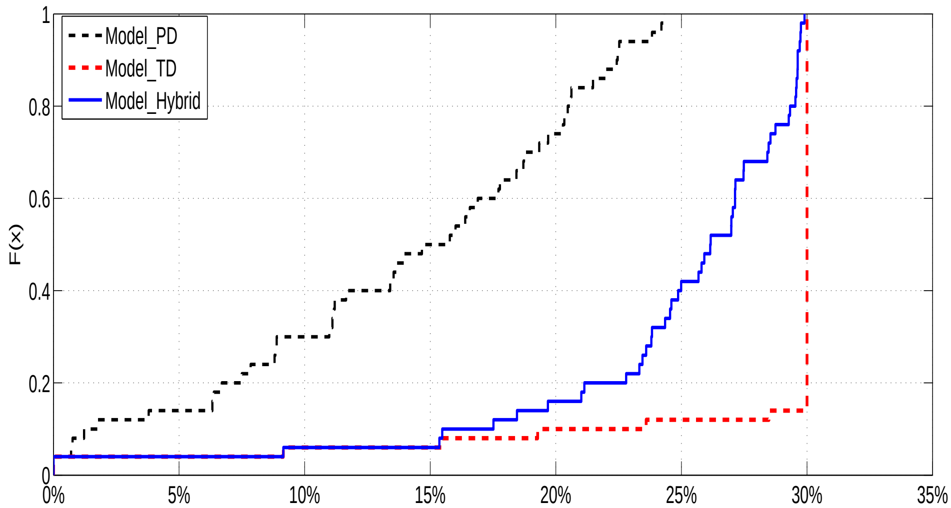


Figure 5.2: Empirical cumulative distribution function of MPUD
$(\tau^{(1)} = 2.10,\ \tau^{(1)} = 22,\ c = 2.5,\ B = M)$

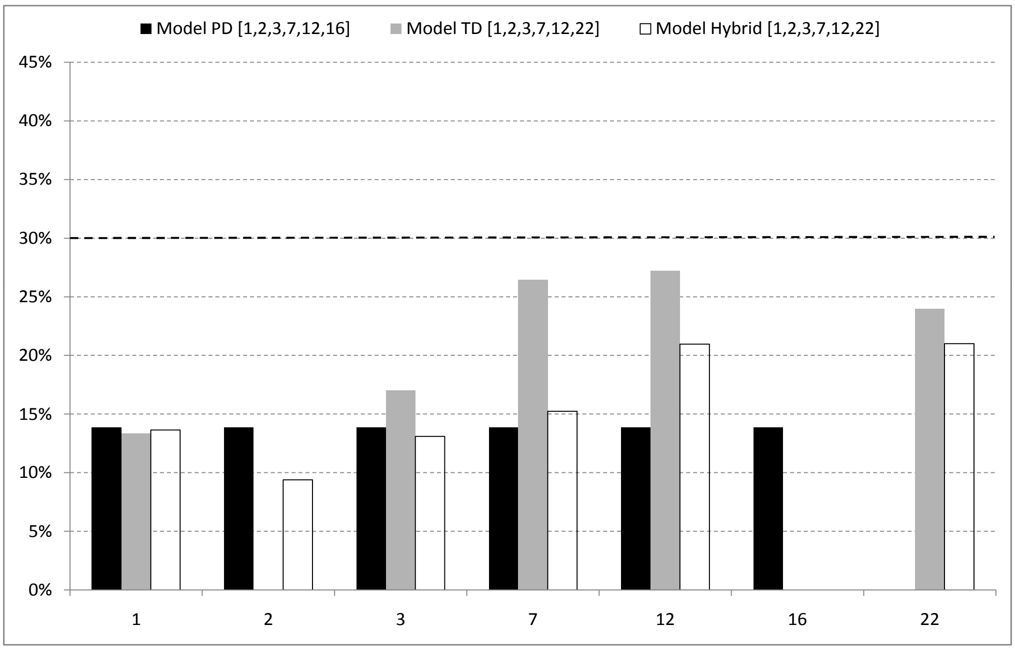


Figure 5.3: Expected PUD for each selected POD; selected PODs are listed in the legend and x-axis
$(\tau^{(1)} = 2.10,\ \tau^{(1)} = 22,\ c = 2.5,\ B = M)$.

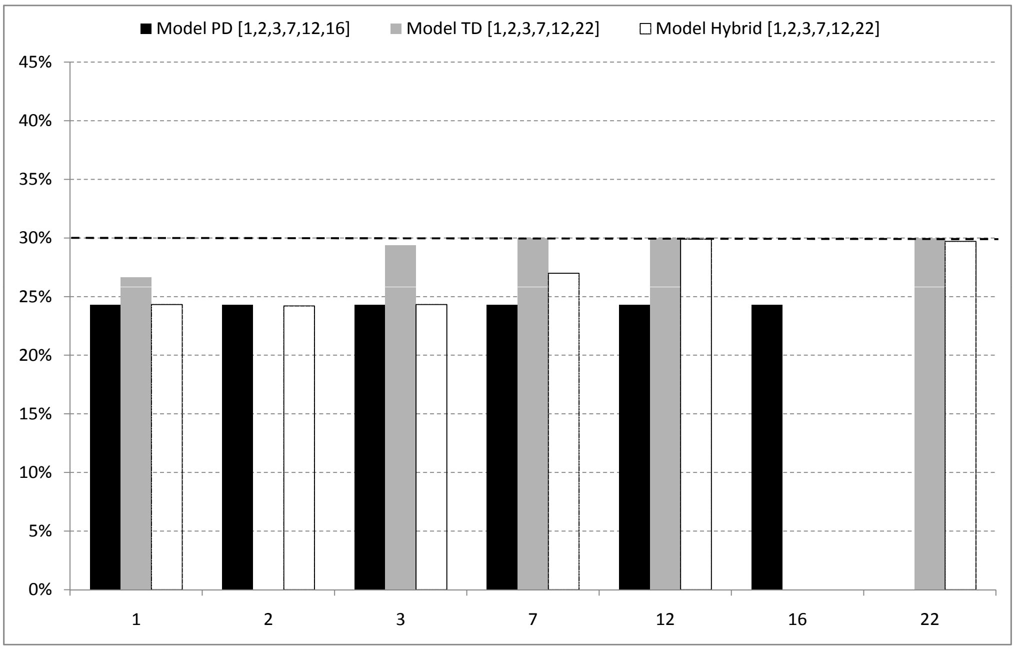


Figure 5.4: Maximum PUD for each selected POD; selected PODs are listed in the legend and x-axis ($\tau^{(1)} = 2.10,\ \tau^{(1)} = 22,\ c = 2.5,\ B = M$).

Figure 5.2 shows that the random MPUD for Model_PD is the dominating one among the other models. In addition, Model_PD also generally performs better than Model_TD with respect to APUD, yet in Figure 5.1 the stochastic dominance of Model_PD over Model_TD is not observed. Figure 5.3 and 5.4 demonstrate that Model_PD is also the superior model in contrast to other models in terms of the PUD. We generally observe smaller values for the expected and maximum PUD at selected PODs as opposed to other models. However, there are also some PODs which indicate the otherwise.

## 5.3 Computational Performance of the Proposed Algorithm

We perform a computational study on larger instances, particularly $|I| = 30$ and $|S| = 50, 100, 200, 500$, in order to illustrate the effectiveness of the proposed integer L-shaped algorithm. In this computational study, we only focus on Model_Hybrid, which performs better in terms of both proposed metrics. Following the schemes (5.1) and (5.2) in Section 5.1; we select low, medium and high values for the $\tau^{(1)}$ and $\tau^{(2)}$ parameters, respectively. Likewise, we choose a set of $c$ parameter values which associate with low, medium and high capacity limitations for PODs. Additionally, low, medium and high values of the budget parameter $B$ are considered in this computational study. We set $\kappa^{(1)} = 6$, $\kappa^{(2)} = 8$, whereas $\rho = 0.30$ for all problem instances presented in Table 5.3 and 5.4.

We benchmark our solution approach against the mixed-integer solver `CPLEX`. All the optimization problems are modeled with the AMPL mathematical programming language. The DEFs of Model_1 and Model_2 were solved by `IBM ILOG CPLEX 12.6.0` within the absolute optimality gap of 0.00001, and the relative optimality gap of 0.004 and 0.005 as the stopping criteria, respectively. The algorithm IntLS was implemented in C++ through using the Concert Technology component library of `IBM ILOG CPLEX 12.6.0`, and the parallelization were carried out by the Boost C++ Libraries (Version 1.55.0). All runs were executed on 4 threads of a Lenovo(R) workstation with two Intel® Xeon® 2.30 GHz CE5-2630 CPUs and 64 GB memory running on Microsoft Windows Server 8.1 Pro x64 Edition. All reported times are elapsed times, and the time limit is set to 7200 seconds.

As the problem instances get larger, solving the large-scaled DEFs by a standard mixed integer programming solver becomes more difficult. Even though we experiment on a smaller network ($|I| = 30$), we observe that `CPLEX` could not provide optimal solutions within the prescribed time for larger scenarios. In particular, the weakness of `CPLEX` is more pronounced for larger instances of Model_2 as seen from Table 5.4. In such cases, we calculate the relative optimality gap by using the best known upper bound on the objective value found by `CPLEX`. Let $\bar{\text{Obf}}$ denote the best lower bound on the first-stage objective function value provided by `CPLEX`, when the prescribed time limit is reached. $\text{Obf}^*$ denotes the best available objective function value within the time limit, which defines a lower bound on the objective value. Then, we define an upper bound on the relative optimality gap as follows:

$$\text{UBROG} = \frac{\text{Obf}^* - \bar{\text{Obf}}}{\bar{\text{Obf}}}.$$

We refer to the enhanced integer L-shaped algorithm with additional features (e.g., starting solutions, alternating cuts, scenario prioritization) as IntLS*, whereas we denote the simple version of the integer L-shaped algorithm (in particular, without the listed features) as IntLS. Table 5.3 and 5.4 exhibit the results related to the computational performances of IntLS* and `CPLEX`, whereas the computational results related to IntLS are illustrated only in Table 5.3. More specifically, these tables show the elapsed solution times, the UBROG values that are greater than the prescribed relative optimality gaps particular to the proposed models, the relative reductions in the elapsed solution times by IntLS* with respect to `CPLEX` and by IntLS* with respect to IntLS, if available. From Table 5.3 and 5.4, it is evident that `CPLEX` finds high-quality solutions when it terminates within the prescribed time limit. In such cases, UBROG values associated with those solutions are smaller than the relative optimality gap specific to the model in consideration. However, `CPLEX` terminates with no feasible solution within the specified time limit for larger instances. Specifically, the performance of `CPLEX` for Model_2 is poor for the majority of the problem instances compared to Model_1, since the mathematical programming formulation of this problem is more complex. In these cases, we find the best available upper bound on the optimal objective value to calculate a valid upper bound on the optimality gap; however, these upper bounds are quite high. Unlike `CPLEX`, our solution algorithm (both versions; enhanced and simple) usually obtained optimal solutions in shorter times. When our algorithm terminates due to the specified time limit, in general, `CPLEX` was not able to find a feasible solution. We also demonstrate the superiority of IntLS* in Table 5.3. In most of the problem instances, IntLS* showed better performance in terms of the elapsed solution times. Therefore, we only provide IntLS*-related computational performance metrics for Model_2 (excluding the computational performance of IntLS) in Table 5.4. Furthermore, as discussed in Section 5.2, we only present a few samples of the computational analysis for Model_2. We also present the individual contributions of the additional features, particularly for Model_1 in Table 5.2. This table illustrates the relative decrease in the elapsed solution times achieved by each feature through adding the features one by one to the simple version of the integer L-shaped algorithm, which is IntLS. For instance, in Table 5.2 Alternating Cuts column indicates the relative improvements in the elapsed solution times, when this particular feature is added to the integer L-shaped algorithm where only starting solutions feature is enabled. We remark that the listed features are added in the particular order of the columns as in Table 5.2. The results for Model_1 in Table 5.3 indicate that the parameters $c$ and $B$ have also significant impacts on the computational performance. The smaller values of $\tau^{(1)}$ and $\tau^{(2)}$ usually lead to a decrease in the elapsed solution times, since such alterations reduce the size of the feasible region. Moreover, the impacts of these parameters on the computational performance increase with large number of scenarios.

In summary, the computational study demonstrates that in general our enhanced inte-

| $(\tau^{(1)}, \tau^{(2)}, c, B)$ | $\|S\|$ | Starting Solutions | Alternating Cuts | Scenario Prioritization |
|---|---|---|---|---|
| (2.18,44,2,Medium) | 50 | 2.3% | 48.2% | - |
| | 100 | - | - | 19.4% |
| | 200 | 14.0% | - | 27.2% |
| | 500 | - | 11.7% | - |
| (2.18,22,2.5,Medium) | 50 | 20.0% | 42.8% | - |
| | 100 | - | - | 97.2% |
| | 200 | - | 16.5% | 78.5% |
| | 500 | - | - | 94.6% |
| (2.10,44,2.5,Medium) | 50 | 39.4% | 1.4% | - |
| | 100 | 56.1% | - | - |
| | 200 | 30.1% | - | 35.6% |
| | 500 | 12.9% | 36.7% | - |
| (2.18,44,2.5,Low) | 50 | 17.1% | 16.0% | 38.5% |
| | 100 | 28.0% | - | 95.2% |
| | 200 | - | 56.5% | 21.3% |
| | 500 | 1.1% | 9.9% | - |
| (2.20,60,3,High) | 50 | 67.5% | - | - |
| | 100 | 64.1% | 9.3% | 15.0% |
| | 200 | - | 57.6% | 6.4% |
| | 500 | 35.2% | 8.5% | 46.8% |
| (2.18,44,2.5,Medium) | 50 | 34.6% | 0.8% | 42.1% |
| | 100 | 95.2% | - | 95.2% |
| | 200 | - | 57.6% | 12.7% |
| | 500 | 39.1% | 12.4% | - |

Table 5.2: Improvement percentages in the elapsed solution times by the additional features for Model_1

-: No relative improvement with respect to the elapsed solution time.

ger L-shaped algorithm produces better computational performance compared to `CPLEX`, especially for larger scenarios. Additionally, the enhanced version of the integer L-shaped method performs better than its simpler version for the majority of the problem instances. However, we note that the computational performance presented in this study is not conclusive. That is, we cannot assert that our solution algorithm is superior than the benchmark solver `CPLEX`, since we are not able to provide results on the computational performance for larger number of nodes (i.e., $|I| = 60,\ 94$). That is because, we encountered infeasibility for a few problem instances where $|I| = 60$. For the other problem instances, our solution algorithm reached to its prescribed time limit. Furthermore, we note that the additional features which are proposed to improve the performance of the enhanced integer L-shaped algorithm as discussed in Section 4.2 do not perform better than the simple version of the solution algorithm for all test instances. Along these lines, we intend to test our solution algorithm on a more reasonable parameter setting (also including the larger networks for the computational performance), and improve the computational performance of the proposed solution algorithm as a future work. Accordingly, we can evaluate the computational performance of the proposed integer L-shaped algorithm more comprehensively.

| $(\tau^{(1)}, \tau^{(2)}, c, B)$ | $\lvert S\rvert$ | IntLS | IntLS* | CPLEX | Rel. Reduc. |
|---|---|---|---|---|---|
| (2.18,44,2,Medium) | 50 | 31.41 | 17.71 | 43.73 | 59.5 |
| | 100 | 1033.67 | 901.62 | 102.54 | - |
| | 200 | 104.29 | 79.19 | 1070.6 | 92.6 |
| | 500 | 1482 | 7201.57 △ | 7218.45 *[▲] | 0.2 |
| (2.18,22,2.5,Medium) | 50 | 30.09 | 18.12 | 35.89 | 49.5 |
| | 100 | 29.06 | 32.36 | 77.51 | 58.3 |
| | 200 | 154.5 | 33.89 | 204.96 | 83.5 |
| | 500 | 4427.18 | 385.8 | 2623.06 | 85.3 |
| (2.10,44,2.5,Medium) | 50 | 28.45 | 18.3 | 46.05 | 60.3 |
| | 100 | 88.38 | 42.54 | 100.82 | 57.8 |
| | 200 | 63.68 | 42.01 | 584.77 | 92.8 |
| | 500 | 311.09 | 247.01 | 4388.28 | 94.4 |
| (2.18,44,2.5,Low) | 50 | 28.95 | 12.4 | 45.05 | 72.5 |
| | 100 | 61.51 | 39.63 | 186.07 | 78.7 |
| | 200 | 103.5 | 38.38 | 291.54 | 86.8 |
| | 500 | 200.67 | 184.78 | 4067.92 | 95.5 |
| (2.20,60,3,High) | 50 | 41.95 | 18.03 | 34.61 | 47.9 |
| | 100 | 88.44 | 24.51 | 82.53 | 70.3 |
| | 200 | 42.73 | 24.52 | 307.28 | 92.0 |
| | 500 | 226.61 | 71.4 | 2061.95 | 96.5 |
| (2.18,44,2.5,Medium) | 50 | 33.91 | 12.74 | 46.58 | 72.6 |
| | 100 | 911.82 | 39.58 | 235.44 | 83.2 |
| | 200 | 95.87 | 43.59 | 298.81 | 85.4 |
| | 500 | 342.18 | 320.31 | 4027.07 | 92.0 |

Table 5.3: Model_1- Elapsed solution times, the UBROG values (%), and the relative reduction in IntLS* solution times with respect to CPLEX (%), $\lvert I\rvert = 30,\ \rho = 0.3$.

UBROG values are reported in [ ]; the values above 5000% are indicated with ▲ and the values below 1% are not reported. The elapsed solution times of IntLS* which are higher than those of IntLS are indicated with △.

†: Time limit with integer feasible solution.

*: Time limit with no integer feasible solution.

-: No Relative Reduction in the elapsed solution times.

| $(\tau^{(1)}, \tau^{(2)}, c, B)$ | $\lvert S \rvert$ | IntLS* | CPLEX | Rel. Reduc. |
|---|---|---|---|---|
| (2.18,44,2.5,Medium) | 50 | 1204.19 | 1789.99 | 32.7 |
| | 100 | 7200.37 † | 7200.26 † | - |
| | 200 | 3307.86 | 7226.63 † [▲] | 54.2 |
| | 500 | 7225.24 † | 7200 * [▲] | - |
| (2.18,22,2.5,Medium) | 50 | 7135.63 | 1784 | - |
| | 100 | 7205.06 † | 4806.99 | - |
| | 200 | 7205.28 † | 7221.85 * [▲] | - |
| | 500 | 7232.83 † | 7228.52 * [▲] | - |
| (2.10,44,2.5,Medium) | 50 | 603.51 | 1590.23 | 62.0 |
| | 100 | 7200.08 † | 7087.95 † | - |
| | 200 | 4023.06 | 7222.95 * [▲] | 44.3 |
| | 500 | 7228.37 † | 7237.08 * [▲] | - |

Table 5.4: Model_2- Elapsed solution times, the UBROG values (%), and the relative reduction in IntLS* solution times with respect to CPLEX (%), $|I| = 30,\ \rho = 0.3$.

UBROG values are reported in [ ]; the values above 5000% are indicated with ▲ and the values below 1% are not reported. The elapsed solution times of IntLS* which are higher than those of IntLS are indicated with △.

†: Time limit with integer feasible solution.

*: Time limit with no integer feasible solution.

-: No Relative Reduction in the elapsed solution times.

# Chapter 6

# Conclusion and Future Work

We have introduced a new last mile relief network design problem with resource reallocation, which addresses the critical concerns of relief organizations. In particular, the proposed corresponding optimization models capture the uncertain aspects of the post-disaster environment, and address accessibility, equity, and budget issues. In terms of incorporating equity, we use the concept of coverage sets based on the accessibility scores, and enforce a previously proposed hybrid supply allocation policy that can achieve high levels of equity and accessibility simultaneously. In addition, we introduce new accessibility metrics considering the multi-echelon network structure. More specifically, we develop two types of aggregated accessibility metrics. The first one is based on a direct weighted summation of the accessibility scores associated with the used links. Alternatively, the second one involves a more elaborate expression for the accessibility of the PODs from the LDCs. Unfortunately, according to our computational study, the second and more elaborate modeling approach turns out to be less effective in terms of our way of quantifying accessibility compared to the simpler version.

As part of our ongoing research, we investigate how to improve the proposed modeling approaches for incorporating accessibility into optimization models. In particular, we intend to enhance the second model to mitigate its drawbacks. In addition, developing risk-averse versions of the proposed models is a topic of future research. Such models would incorporate the decision makers' risk preferences, and provide them with risk-averse decisions that would perform better in the presence of high variability compared to their risk-neutral versions.

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