International Journal of Information Technology & Decision Making
Vol. 3, No. 2 (2004) 321–335

MULTICRITERIAL RANKING APPROACH FOR EVALUATING BANK BRANCH PERFORMANCE

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14 ranking methods based on multiple criteria are suggested for evaluating the performance of the bank branches. The methods are explained via an illustrative example, and some of them are applied to a real-life data for 23 retail bank branches in a large-scale private Turkish commercial bank.

Keywords: Bank branches; efficiency measurement; ranking methods; multicriterial evaluation.

1. Introduction

Banks are multi-product firms and therefore operate simultaneously in different financial markets. In a country, the spatial distribution of economic and financial activity is not homogeneous. Instead, the level of development of financial markets as well as the intensity of financial transactions vary considerably among regions.

On the other hand, such differences are also not uniform among the types of financial services offered by banks. In one region, the economic environment may be conducive for the development of corporate financing activities, while in the other, for individual banking.

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Banks, in order to enhance their efficiency, periodically evaluate the performances of their branches, i.e. the bodies which are responsible for carrying out banking activities at the local level. It is generally accepted that such a comparative evaluation should not be carried out on a single scale, namely, profit. The final aim of a private bank is to increase its net worth. This is not always equivalent to profit maximization, and bank management define a set of targets without confining themselves to only profit. Such targets give signals about the long-term profitability of the bank as well as the sources of the current profit. Therefore, banks evaluate their overall as well as branch performances in a multi-scale framework which refers to performance at different markets. Such an approach requires comparative evaluations of performances of the branches over a set of activities. Since branches are located at different environments, comparative evaluations are meaningful if the local environmental characteristics are taken into account.

In this paper, a set of ranking methods to evaluate branch performances given in a multicriterial form are suggested.[a] These methods are grouped with respect to their information content. Although it is practically impossible for the management to define an objective function for the bank over all conceivable outcomes of its branches, it is assumed that the management is (or should be) capable of choosing among a small set of alternative rankings of branches, once the necessary information concerning the logic of the construction of each ranking is known.

The paper is structured as follows. Section 2 gives 5 groups (14 methods) for evaluating branch performance. Section 3 provides a real-life example of evaluation of 23 branches with respect to 4 indices. Making use of 5 methods out of the suggested 14 (one from each group), we construct 5 rankings, and then compare them with each other. Section 4 concludes the paper.

2. Multicriterial Methods

Hereafter, branches will be denoted as $x, y, z, \ldots$, and the set of branches will be denoted as A.

Suppose we obtain an evaluation of a branch with respect to several indices, i.e.

$$P_1(x), P_2(x), \ldots, P_k(x).$$

Below, we illustrate all methods via an example of evaluation of 5 branches with respect to 4 indices which show the share of operations of the branch in total

[a]Bank profitability and evaluation of the performance at branch level has drawn much attention of the researchers. However, all these methods as far as we know suggest rather cardinal approach to the problem. A comprehensive review of these studies had been done by Berger *et al.*[6] Other studies in the field can be found in Oral and Yolalan,[14] Giokas,[11] Sherman and Ladino,[15] Camanho and Dyson,[7] Golany and Storbeck,[12] Zenios *et al.*[16]

Table 1. Evaluation of branches.

Branch #	D	C	S	FX
2	1.71	0.68	0.82	0.51
10	1.62	1.50	1.09	1.21
16	1.07	1.06	1.58	0.57
17	1.31	1.16	0.26	0.21
21	1.66	1.73	0.62	0.24

operations of the bank. We choose the following indices:

(1) branch i's in total deposits of the bank, D;
(2) branch i's in total credits of the bank, C;
(3) branch i's in total securities trading volume of the bank, S;
(4) branch i's in total FX trading volume of the bank, FX.

The corresponding evaluations are given in Table 1 in which numbers have been multiplied to some positive constant not to work with very small numbers.

Before we give the methods of evaluation let us introduce some notions which are used below.

Let us construct the relation R such that

$$xRy \quad \text{iff} \quad \{\forall l\, P_l(x) \geq P_l(y) + \varepsilon_l \text{ and } \exists l_0 \text{ s.t. } P_{l_0}(x) > P_{l_0}(y) + \varepsilon_{l_0}\}$$

where x and y are bank branches, P_l is the lth index according to which the branch is evaluated, $l = 1, \ldots, k; \varepsilon_l$ is a parameter of "sensitivity" — a threshold which corresponds to each index[b] l. The relation R can be interpreted as "to be better than", i.e. xRy means "x is better than y", or "branch x performed better than y". It is constructed as follows: xRy if for any criterion the branch x has higher or equal values than y taking into account sensitivity ε, and at least for one criterion x has strictly higher values than y, again taking into account ε. The relation R is called generalized Pareto relation. It is constructed over indices $\{P_l(x)\}$, $l = 1, \ldots, k$, and it is strict partial order, i.e. irreflexive and transitive binary relation.

For the example above, the relation R constructed using $\varepsilon = 0.05$ is given in Table 2.

It can be easily seen that branch #10 has greater values of all indices than branch #17, hence in the intersection of the raw for branch #10 with the column for branch #17 there is 1; the same is true for branch #21. The branches #2, 10, 16, and 21 are at the Pareto frontier, but the branch #17 is Pareto dominated by the branches #10 and 21.

[b]An introduction to the scheme of evaluation of branches performance, the parameter ε is crucial and can be explained via an example. Suppose we have three branches x, y, and z, which we evaluate with respect to the time deposits per worker, D. Suppose we obtain the following numbers: $D(x) = \$100,000; D(y) = \$100,200; D(z) = \$70,000$. It is natural to consider then that the branches x and y show the same performance, while the branch z shows lower performance. Thus, ε in this case is greater than \$200 and less than \$30,000.

Table 2. The relation R.

*	2	10	16	17	21
2	0	0	0	0	0
10	0	0	0	1	0
16	0	0	0	0	0
17	0	0	0	0	0
21	0	0	0	1	0

However, we cannot compare branches with each other using the relation R since one can meet the situation when xRy but neither xRz nor zRy for some branch z (in this case they say that it does not satisfy negative transitivity). Indeed, in the example above the branch #10 is "better" than branch #17, but branch #2 cannot be compared with those two branches (see values of indices in Table 1).

The usual approach here is to approximate the relation R via some weak order W — irreflexive, transitive and negatively transitive binary relation. In the latter case, for any two branches, either one is better than the other, or both of them are equal in terms of final evaluation of performances. In this case, the best group of objects will be denoted as $C_1(A)$. Then, after one excludes the best branches from comparison, applying the same procedure, the second best group of objects can be found. This group of objects are denoted as $C_2(A)$. Continuing the process, one can obtain the sequence of sets $C_3(A), C_4(A)$, etc.

Let us now define the upper contour set $D(x)$ for the branch x as the set of branches which are better than x with respect to R, i.e.

$$D(x) = \{y \mid yRx\}.$$

Analogously, let us define the lower contour set $L(x)$ for the branch x as the set of branches which are worse than x with respect to R, i.e.

$$L(x) = \{y \mid xRy\}.$$

Now everything is ready to give methods of constructing the resulting weak order W by either the relation R or directly by the set of indices $\{P_l(x)\}, l = 1, \dots, k$. Different versions of these methods are given in Copeland,[9] Larichev,[13] Fishburn,[10] and Aleskerov.[1,2]

2.1. *Methods based on upper and lower contour sets*

(1) *Ranking with respect to upper contour sets*
Construct binary relations R_1(which is equal to R), $R_2, \dots, R_s$ such that

$$xR_ly \Leftrightarrow D_{l-1}(x) \subset D_{l-1}(y), \quad l = 2, \dots, s$$

where $D_{l-1}(x) = \{z \mid zR_{l-1}x\}$.

Then it can be shown that at some step s a binary relation R_s is a weak order.

Table 3. Upper contour sets for R_1 and R_2.

Branch #	D_1	D_2
2	∅	∅
10	∅	∅
16	∅	∅
17	{10,21}	{2,10,16,21}
21	∅	∅

Table 4. Lower contour sets for R_1 and R_2.

Branch #	D_1	D_2
2	∅	∅
10	{17}	{2,16,17}
16	∅	∅
17	∅	∅
21	{17}	{2,16,17}

The rule is illustrated via the example. For the relation R given in Table 2, let us construct the relations $R_1 = R$ and R_2. The upper contour sets are given in Table 3.

It is readily seen that the relation R_2 is a weak order, and the first best branches are $C_1(A) = \{2, 10, 16, 21\}$, and the second best is only one branch $C_2(A) = \{17\}$.

(2) *Ranking with respect to lower contour sets*
This rule is completely analogous to the previous one, the only difference is that we construct R_l with respect to $L_{l-1}(i)$, i.e.

$$xR_ly \Leftrightarrow L_{l-1}(x) \supset L_{l-1}(y), \quad l = 2, \ldots, s$$

where $L_{l-1}(x) = \{z \mid xR_{l-1}z\}$.

For the relation R given in Table 2, let us construct the relations $R_1 = R$ and R_2. The lower contour sets are given in Table 4.

It can be seen that the relation R_2 is a weak order, and the first best branches are $C_1(A) = \{10, 21\}$, and the second best branches constitute the set $C_2(A) = \{2, 16, 17\}$.

(3) *Ranking with respect to upper and lower contour sets*
In this case the relation R_l is constructed as follows:

$$xR_ly \Leftrightarrow D_{l-1}(x) \subseteq D_{l-1}(y) \quad \text{and} \quad L_{l-1}(x) \supseteq L_{l-1}(y)$$
$$\text{and either} \quad D_{l-1}(x) \subset D_{l-1}(y) \quad \text{or} \quad L_{l-1}(x) \supset L_{l-1}(y).$$

For the relation R given in Table 2, the upper and lower contour sets for $R_1 = R$ and R_2 are given below.

Again, the relation R_2 is a weak order, and the first best branches are $C_1(A) = \{10, 21\}$, the second best is the set $C_2(A) = \{2, 16\}$, and the third best is the branch $C_3(A) = \{17\}$.

Table 5. Upper and lower contour sets for R_1 and R_2.

Branch #	D_1	L_1	D_2	L_2
2	∅	∅	{10,21}	{17}
10	∅	{17}	∅	{2,16,17}
16	∅	∅	{10,21}	{17}
17	{10,21}	∅	{2,16,17,21}	∅
21	∅	{17}	∅	{2,16,17}

Table 6. Function u.

Branch #	$u(\cdot)$
2	0
10	1
16	0
17	−2
21	1

(4) *Ranking with respect to cardinalities of upper and lower contour sets*
Define $u(x)$ as the difference of cardinalities of lower and upper contour sets of x in R, i.e. $u(x) = |L(x)| - |D(x)|$. Then the function $u(x)$ define natural order on the set of objects (branches).

For our example using Table 5 one can construct the function u.

Then we obtain the same ranking as in the previous case.

(5) *Ranking with respect to cardinalities of lower contour sets*
Define $u(x)$ as the cardinality of lower contour set of x in R, i.e. $u(x) = |L(x)|$. Then again the function $u(x)$ define natural order on the set of branches.

2.2. *Methods based on tournament matrices*

(6) *Maximin procedure*
Construct a matrix S^+ such that $\forall x, y \in A$, $S^+ = \{n(x,y)\}$, with $n(x,x) = +\infty$, and

$$n(x,y) = \{l \mid P_l(x) > P_l(y) + \varepsilon_l\}.$$

The rows and columns of the matrix S^+ correspond to the set of branches in A. Such matrix can be called a generalized tournament matrix. In words, at the intersection of xth row and yth column the number is put $n(x,y)$ equal to the number of criteria in which branch x has higher values than the branch y, taking into account the measurement error.

Choose row minima from every row (for every branch). For any $z \in A$, this number (row minimum) shows the performance of z against its "toughest" contestant. Then choose the branch to which the maximum of row minima correspond, that is, choose the branch which performed best against its contestants, i.e.

$$x \in C_1(A) \quad \text{iff} \quad n(x,y) = \max_{a \in A} \left\{ \min_{b \in A} \{n(a,b)\} \right\} \quad \text{for some } y \in A.$$

Then exclude x from the set A and repeat procedure.

Let us illustrate the rule for the data given in Table 1. The tournament matrix for this case is as follows ($\varepsilon_l = 0.06$ for all $l = 1, \ldots, 4$).

Then $C_1(A) = \{10\}$. Excluding branch #10 gives us the next best $C_2(A) = \{16\}$, then $C_3(A) = \{2\}$, $C_4(A) = \{21\}$, and $C_5(A) = \{17\}$.

(7) *Minimax procedure*
Construct a matrix S^- such that $\forall x, y \in A, S^- = \{n(x,y)\}$, with $n(x,x) = -\infty$, where rows and columns correspond to the set of branches in A. Then choose column maxima from every column (for every branch). For any $z \in A$, this number (column maximum) shows the worst performance of z against its "toughest" contestant. Then choose the branch to which the minimum of column maxima correspond, i.e. choose the branch which performed best against its contestants,

$$x \in C_1(A) \quad \text{iff} \quad n(x,y) = \min_{b \in A} \left\{ \max_{a \in A} \{n(a,b)\} \right\} \quad \text{for some } y \in A.$$

Then exclude x from the set A and repeat procedure.

(8) *Maximization of wins*
Since $n(x,y)$ shows the number of "wins" of branch x over branch y, the index

$$w(x) = \sum_{y, y \neq x} n(x,y)$$

will express the total number of wins of the branch x over other branches. Then the function $w(x)$ defines natural order on the set A. Consider the example in Table 7. Then $w(x)$ is as follows.

Finally, $C_1(A) = \{10\}$, $C_2(A) = \{2, 16, 21\}$, and $C_3(A) = \{17\}$.

(9) *Minimization of losses*
We can calculate the index

$$l(y) = \sum_{x, x \neq y} n(x,y)$$

Table 7. Tournament matrix.

*	2	10	16	17	21	min
2	∞	1	1	3	2	1
10	3	∞	3	4	2	2
16	2	1	∞	2	2	1
17	1	0	2	∞	0	1
21	1	1	2	3	∞	1

Table 8. Function of wins.

*	2	10	16	17	21	$w(x)$
2	—	1	1	3	2	7
10	3	—	3	4	2	12
16	2	1	—	2	2	7
17	1	0	2	—	0	3
21	1	1	2	3	—	7

Table 9. Function of losses.

*	2	10	16	17	21
2	—	1	1	3	2
10	3	—	3	4	2
16	2	1	—	2	2
17	1	0	2	—	0
21	1	1	2	3	—
l	7	3	8	12	6

which will express the total number of losses of the branch y to all branches. Then the function $l(x)$ defines natural order on the set A. Again, consider the example in Table 7. Then $l(x)$ is as follows.

Finally, $C_1(A) = \{10\}$, $C_2(A) = \{21\}$, $C_3(A) = \{2\}$, $C_4(A) = \{16\}$, and $C_5(A) = \{17\}$.

2.3. *Methods based on value function*

(10) *Borda counts*

Consider the branch $x \in A$ and assign to x a score $r_i(x)$, which is the cardinality of lower contour set of x in $P_i(x)$, i.e.

$$r_i(x) = |L_i(x)| = |\{b \in A : P_i(x) > P_i(b) + \varepsilon_i\}|.$$

The sum of these scores through every $i \in N$ is called the Borda count of a branch, i.e.

$$r(x) = \sum_{i=1}^{n} r_i(x).$$

Let us construct scores for the branches in our example. The corresponding scores are given in Table 10 which is constructed with the assumption that $\varepsilon = 0.05$ for all indices.

Then

$$x \in C_1(A) \Leftrightarrow [\forall y \in A,\ r(x) \geq r(y)]$$
$$x \in C_2(A) \Leftrightarrow [\forall y \in A \backslash C_1(A),\ r(x) \geq r(y)]$$

etc.

For our example, then $C_1(A) = \{10\}$, $C_2(A) = \{2, 16, 21\}$, and $C_3(A) = \{17\}$.

Table 10. Borda counts.

Branch #	r_1	r_2	r_3	r_4	r
2	3	0	2	2	7
10	2	3	3	4	12
16	0	1	4	2	7
17	1	2	0	0	3
21	2	4	1	0	7

Table 11. Averages of Borda counts.

Branch #	r_1	r_2	r_3	r_4	r
2	2	0	2	2	6
16	0	1	3	3	7
17	1	2	0	0	3
21	2	3	1	0	6

(11) *Averages of Borda counts*
Compute the Borda counts for each branch in A. Then compute the average of these counts. Eliminate those branches, which have lower scores than the average value. Compute the new Borda counts from the contracted set of branches. Then eliminate another one similarly and go on like this until there is no branch to eliminate from the contracted set.

In other words, first one should compute

$$\bar{r} = \left(\sum_{a \in A} r(a)\right) / |A|.$$

Then eliminate $c \in A$ if $r(c) < \bar{r}$ and construct $X = \{a \in A : r(a) \geq \bar{r}\}$.

Then apply the same procedure to X.

Continue with the procedure by contracting the set in consideration to obtain $C_1(A)$. Then exclude $C_1(A)$ from A, and apply the procedure again to obtain $C_2(A)$, etc.

Let us apply this procedure to our example (see Table 10). In this case, $\bar{r} = 36/5 = 7.2$. Then immediately one can obtain that $C_1(A) = \{10\}$. After exclusion of the branch #10 from the set A, the following Table 11 will be obtained.

Again, calculate $\bar{r} = 5.5$. Hence, $X = \{2, 16, 21\}$, and we can obtain $C_2(A) = \{16\}$. Analogously, $C_3(A) = \{2, 21\}$, and $C_4(A) = \{17\}$.

2.4. *q-Paretian procedures*

Construct according to each $\{P_l(x)\}, l = 1, \ldots, k$, a binary relation W_l as follows[c]

$$xW_l y \quad \text{iff} \quad P_l(x) > P_l(y) + \varepsilon_l.$$

[c]Such relation W_l is in general an interval order.[1,10]

Table 12. Matrix for W_1.

*	2	10	16	17	21
2	0	1	1	1	0
10	0	0	1	1	0
16	0	0	0	0	0
17	0	0	0	0	0
21	0	0	1	1	0

Table 13. Upper contour sets for $W_1 - W_4$.

Branch #	$D(x, W_1)$	$D(x, W_2)$	$D(x, W_3)$	$D(x, W_4)$
2	$\emptyset$	{10,16,17,21}	{10,16}	{10,16}
10	{2}	{21}	{16}	$\emptyset$
16	{2,10,21}	{17,21}	$\emptyset$	{10}
17	{2,10,21}	{21}	{2,10,16,21}	{2,10,16}
21	$\emptyset$	$\emptyset$	{2,10,16}	{2,10,16}

Thus, we obtain a set of k relations $\{W_1, \ldots, W_k\}$. Using them we construct now a partition of a set of branches which represent first best, second best, $\ldots$, sth best branches. To do this we first use the upper contour set $D(x, W_l)$ of object x in the relation W_l:

$$D(x, W_l) = \{y \mid yW_lx\}.$$

Before we explain the procedure let us construct the relation W_1 and $D(x, W_1)$ for all branches in A with respect to W_1. The matrix for W_1 is given in Table 12.

The sets $D(x, W_i)$ for all indices i are given in Table 13.

(12) *q-Paretian rule*

First, we define the function $f(i, q)$ as follows

$$f(i, q) = \{y \mid \text{card}(D(y, W_i) \leq q\}.$$

Then

$$C_1(A) = \bigcap_{i=1}^{k} f(i, q)$$

where A is a set of all branches.

Then, we first check the case with $q = 0$. If $C_1(A)$ is empty, $C_1(A) = \emptyset$, then q is increasing to 1, etc.[d]

Otherwise speaking, first we check if we do have a branch which is the best with respect to all indices. If yes, then we go to the second step — exclude the first best and find new "first best" on the contracted set of branches. Otherwise, we increase q to 1, i.e. put $q = 1$, and begin the procedure again.

[d]In this form, it is so-called Condorcet practical rule (see, e.g. Ref. 5).

For our example, we obtain $C_1(A) = \{10\}$ with $q = 2$; $C_2(A) = \{16, 21\}$ with $q = 3$; $C_3(A) = \{2, 17\}$ with $q = 2$.

(13) (s, q)-*Paretian rule*
The rule is analogous to the previous one with the following addition. We check q-Pareto optimal elements not for the whole set of indices but to each subset of indices of cardinality s, e.g. for any 3 indices out of 4, etc. Formally, this rule can be written down as follows:

$$C_1(A) = \bigcup_{I} \bigcap_{l \in I, \mathrm{card}(I) = s} f(l, q).$$

Then we define $C_2(A)$ on the set $A \backslash C_1(A)$, etc.

It is worth mentioning here that other q-Paretian rules can be used.[1,2]

2.5. *Approximation of Pareto relation*

Since a (polynomial) construction of a weak order which approximates the given strict partial order in terms of some distance is still an unsolved problem, the solution can be obtained by "branches and bounds" method for small numbers of objects (bank branches). The other way is to use special approximation technique (see Ref. 3). Below, we give an algorithm to construct a ranking.

(14) *Partition via maximal elements*
This algorithm is based on partition of the set A with respect to maximal elements of Pareto relation R. It constructs the sets $C_1(A)$, $C_2(A), \ldots$ as follows:

$$\begin{aligned}
C_1(A) &= \{y \in A \mid \nexists x \in A \text{ such that } xRy\} \\
C_2(A) &= \{y \in A \backslash C_1(A) \mid \nexists x \in A \backslash C_1(A) \text{ such that } xRy\} \\
&\vdots \\
C_s(A) &= \left\{ y \in A \backslash \bigcup_{l=1}^{s-1} C_l(A) \mid \nexists x \in A \backslash \bigcup_{l=1}^{s-1} C_l(A) \text{ such that } xRy \right\}.
\end{aligned}$$

For our example above, the set $C_1(A) = \{2, 10, 16, 21\}$, and $C_2(A) = \{17\}$.

Remark. All methods above are of ordinal type, i.e. they take into account only ordinal information (relations "greater — smaller") about performance of the branch on separate indices. This is very important since in this case final result is stable to small changes of initial data — if relation "$\geq$" is not violated, any changes of values of indices are permitted.

3. Multicriterial Evaluation: An Example

The Yapı Kredi Bank for which this study was done[e] is a bank employing around 10,000 personnel in more than 400 branches of different sizes that are located in different cities of Turkey. From the viewpoint of the assets, the Bank is one of the

[e] All data used are of 1995.

largest private commercial banks in Turkey. Among the branches of the bank a majority are deposit oriented retail branches.

The values of the indices considered above for 23 bank branches from one "environmental" group[f] are given in Table 15.

We construct five rankings (taking a method from each group) according to

(a) cardinalities of lower and upper contour sets (Method 4);
(b) maximization of wins by tournament matrix (Method 8);
(c) Borda counts (Method 10);
(d) (3,q)-Pareto rule (Method 13);
(e) partition via maximal elements (Method 14).

They are given in Table 16.

Table 15. Values of indices for branches #1–23.

Branch #	D	C	S	FX	Branch #	D	C	S	FX
1	3.42	1.26	0.81	0.51	**13**	2.78	0.81	1.65	0.71
2	1.71	0.68	0.82	0.51	**14**	2.36	2.20	1.23	1.68
3	3.72	0.64	3.38	1.22	**15**	2.13	3.93	3.07	1.47
4	2.08	1.85	0.68	1.30	**16**	1.07	1.06	1.58	0.57
5	3.63	1.63	1.68	0.98	**17**	1.31	1.16	0.26	0.21
6	5.83	2.71	3.34	2.99	**18**	2.24	0.95	1.21	1.08
7	1.97	5.78	4.06	1.55	**19**	2.66	5.49	0.92	2.64
8	3.12	1.12	1.91	0.22	**20**	2.93	1.08	2.60	0.67
9	4.02	2.15	2.74	2.98	**21**	1.66	1.73	0.62	0.24
10	1.62	1.50	1.09	1.21	**22**	4.06	0.71	0.81	0.70
11	1.93	1.00	0.19	1.61	**23**	3.11	1.54	0.98	3.85
12	2.72	0.66	2.87	0.88					

Table 16. Rankings of branches.[g]

Branch #	Method (a)	(b)	(c)	(d)	(e)	Branch #	Method (a)	(b)	(c)	(d)	(e)
1	15	14	8	12	3	13	11	13	15	10	2
2	22	20	23	16	3	14	4	7	13	7	2
3	9	8	3	6	1	15	5	4	10	4	1
4	16	10	16	13	3	16	20	17	21	14	3
5	8	7	5	5	2	17	23	19	22	17	4
6	1	1	1	1	1	18	12	14	17	11	2
7	3	3	11	3	1	19	6	5	6	9	1
8	17	12	9	8	2	20	13	9	12	14	2
9	2	2	2	2	1	21	21	18	19	15	4
10	18	14	18	9	3	22	14	15	4	12	2
11	19	16	20	15	3	23	7	6	7	6	1
12	10	11	14	8	2						

[f] In another study (see Ref. 4), the branches of the Bank were grouped with respect to the similarities in the environmental conditions (regional and urban) the branches are operating in. Thus, the chosen branches are homogeneous in terms of the environmental conditions which make comparison of these branches more reasonable.
[g] All rankings were obtained by using the sensitivity threshold $\varepsilon = 0.05$.

3.1. *Comparison of the results*

Let W_1 and W_2 be two rankings, and $\|w^1_{ij}\|$ and $\|w^2_{ij}\|$ be their adjacent matrices.[h] To compare the rankings W_1 and W_2 we will use well-known measures: Hamming's distance and pseudo-metric. Hemming's distance $d(W_1, W_2)$ between W_1 and W_2 is defined as follows:

$$d(W_1, W_2) = \frac{1}{n \cdot (n-1)} \cdot \sum_{i,j} |w^1_{ij} - w^2_{ij}|.$$

Pseudo-metric which shows only the existence of "inverted" pairs of objects (i.e. aW_1b and bW_2a) can be defined as

$$\rho(W_1, W_2) = \frac{1}{n \cdot (n-1)} \cdot \sum_{i,j} w^1_{ij} \bullet w^2_{ji}$$

where $x \bullet y$ for boolean (0-1) variables is defined as follows

$$x \bullet y = \begin{cases} 1, & \text{iff } x = 1 \text{ and } y = 1 \\ 0, & \text{otherwise.} \end{cases}$$

In Table 17, we give the comparison of how far are those rankings from each other using the Hemming's metric. One can see that the two nearest rankings are those obtained via $(3, q)$-Pareto rule (method 13) and partition with respect to maximal elements (method 14).

The results of comparison of rankings via pseudo-metric are given in Table 18. Here the nearest rankings are those obtained via cardinalities of lower and upper contour sets (method 4) and Borda counts (method 10).

Table 17. Hemming distances between rankings.

Methods	(a)	(b)	(c)	(d)	(e)
(a)	—	0.13	0.16	0.20	0.20
(b)		—	0.24	0.19	0.19
(c)			—	0.19	0.16
(d)				—	0.08
(e)					—

Table 18. Values of pseudo-metric between rankings.

Method	(a)	(b)	(c)	(d)	(e)
(a)	—	0.02	0.01	0.13	0.11
(b)		—	0.11	0.10	0.09
(c)			—	0.11	0.14
(d)				—	0.06
(e)					—

[h]The adjacent matrix $\|w^2_{ij}\|$ of a ranking W is constructed as follows: $w_{ij} = 1$ iff the alternative i is above the alternative j in W.

Since both measures are symmetric only half of the tables is given. It should be kept in mind that the two methods within the same group can in general give rankings which are far from each other. However, we can say that all obtained rankings are pretty close to each other both in terms of Hemming's distance and pseudo-metric.

4. Conclusion

The problem of evaluating the branch performance with respect to bank objectives in a multi-dimensional framework is a necessary, albeit, difficult task that the management has to deal with. One solution of this problem is to invite the management to define an objective function over a large set of variables to distinguish various banking services and also different environmental conditions. Such a procedure requires the management to be equipped with an unrealistically large information processing capability.

In this paper, an alternative approach, which considerably decreases the informational requirements to management, is proposed.

In order to apply this alternative approach, the bank management is expected to form a choice function over the norms which characterizes the ranking methods discussed. The management may lean to place higher emphasis in punishing mistakes (for example, using Borda counts) or may be eager to praise success (for example, using Ranking with respect to upper contour set). Once this decision is made, the ranking of the branches according to the banks objectives can be determined by applying the appropriate method.

We can report that YKB is efficiently using the Borda count method for six years to evaluate the monthly basis of its branches' performances.

It is worth noting that the method suggested can be used in different aggregation problems, i.e. not only in the evaluation of bank branches performance.

Acknowledgments

The authors gratefully acknowledge the favorable research environment in which this project has been fulfilled, as well as partial financial support given by Yapı Kredi Bank. We appreciate a kind invitation by Professesor Yong Shi to contribute this article to the journal. F. Aleskerov thanks National Training Foundation and Social Science Institutions Capacity Building Program for the partial financial support (grant #A1/011/5/1.13).

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