APPENDIX
DERIVATIONS of (11), (13), (17), (20), (21)

A. Probability Mass Function of Aggregated Active Period Duration

In this section we aim to derive the closed form formula of pmf of random variable (r.v.) Y which denotes the aggregated active period duration. Using the markov chain of our system model and applying the first step analysis, we will have the following equations:

$$T_1(1) = t_{10}, \tag{1}$$
$$T_1(k) = t_{11}T_1(k-1), \tag{2}$$
$$T_2(1) = t_{20}, \tag{3}$$
$$T_2(k) = t_{22}T_2(k-1) + t_{21}T_1(k-1), \tag{4}$$

where $T_i(.)$ denotes the number of steps that it takes to get to state zero, given that we are initially at state i(i= $1,2$), and t_{ij} denotes the transition probability from state i to state j. To obtain $T_1(k)$ in a closed formula, we rewrite (2) as follows:

$$\begin{aligned} T_1(k) &= t_{11}T_1(k-1) = t_{11}^2 T_1(k-2) = ... \\ &= t_{11}^{k-1}T_1(1) = t_{11}^{k-1}t_{10}, \quad k = 1,2,3,.... \end{aligned} \tag{5}$$

To obtain $T_2(k)$ in a closed formula, we rewrite (4) as follows:

$$\begin{aligned} T_2(k) &= t_{22}T_2(k-1) + t_{21}T_1(k-1) = t_{22}[t_{22}T_2(k-2) + t_{21}T_1(k-2)] + t_{21}T_1(k-1) \\ &= t_{22}^2 T_2(k-2) + t_{22}t_{21}T_1(k-2) + t_{21}T_1(k-1) = ... \\ &= t_{22}^{k-1}T_2(1) + t_{22}^{k-2}t_{21}T_1(1) + t_{22}^{k-3}t_{21}T_1(2) + ... + t_{21}T_1(k-1). \end{aligned} \tag{6}$$

Inserting (3) and (5) in (6) results in:

$$\begin{aligned} T_2(k) &= t_{22}^{k-1}t_{20} + t_{22}^{k-2}t_{21}t_{10} + t_{22}^{k-3}t_{21}t_{11}t_{10} + ... + t_{21}t_{11}^{k-2}t_{10} \\ &= t_{22}^{k-1}t_{20} + t_{21}t_{10}[t_{22}^{k-2} + t_{22}^{k-3}t_{11} + t_{22}^{k-4}t_{11}^2 + ... + t_{11}^{k-2}] \\ &= t_{22}^{k-1}t_{20} + t_{21}t_{10}t_{22}^{k-2}\sum_{r=2}^{k}\left(\frac{t_{11}}{t_{22}}\right)^{r-2} = t_{22}^{k-1}t_{20} + t_{21}t_{10}\left(\frac{t_{22}^{k-1} - t_{11}^{k-1}}{t_{22} - t_{11}}\right), \quad k = 1,2,3,.... \end{aligned} \tag{7}$$

Using (5) and (7), we obtain the aggregated active period length distribution as follows:

$$F_Y(y) = t_{11}^{y-1}t_{10}\psi_f + \left(t_{22}^{y-1}t_{20} + \frac{t_{21}t_{10}(t_{22}^{y-1} - t_{11}^{y-1})}{t_{22} - t_{11}}\right)\psi_m, \quad y = 1,2,3,... \tag{8}$$

where ψ_f, ψ_m denote the conditional probabilities that FBS or MBS is active, given that the system is in ON mode.

B. Probability of Packet Arrival During System's Active Period

We denote by p the probability of packet arrival during the aggregated active period, and by R the random variable of time duration from the beginning of an active period until the next packet arrival. The probability p denotes the event in which the time duration from the beginning of an active period until the next packet arrival is less than or equal to the duration of that active period. Considering that the packet arrival to the MDs buffer is modeled as a bernoulli process and with a success probability ς, we first obtain the pmf of r.v. R as follows:

$$F_R(r) = \varsigma(1-\varsigma)^{r-1}, \quad r = 1,2,3,.... \tag{9}$$

Using (8) and (9), we obtain the probability p as follows:

$$\begin{aligned}
p = \mathsf{Pr}(R \le Y) &= \sum_{y=1}^{\infty} F_Y(y) \sum_{r=1}^{y} F_R(r) \\
&= \sum_{y=1}^{\infty} F_Y(y) \sum_{r=1}^{y} \varsigma(1-\varsigma)^{r-1} = \sum_{y=1}^{\infty} F_Y(y)\varsigma[1+(1-\varsigma)+(1-\varsigma)^2+...+(1-\varsigma)^{y-1}] \\
&= \sum_{y=1}^{\infty} F_Y(y)\varsigma\frac{1-(1-\varsigma)^y}{\varsigma} = \sum_{y=1}^{\infty} F_Y(y) - \sum_{y=1}^{\infty} F_Y(y)(1-\varsigma)^y \\
&= \sum_{y=1}^{\infty}(t_{10}\psi_f - \frac{t_{21}t_{10}}{t_{22}-t_{11}}\psi_m)t_{11}^{y-1} + (t_{20} + \frac{t_{21}t_{10}}{t_{22}-t_{11}})\psi_m t_{22}^{y-1} \\
&\quad - \sum_{y=1}^{\infty}(t_{10}\psi_f - \frac{t_{21}t_{10}}{t_{22}-t_{11}}\psi_m)t_{11}^{y-1}(1-\varsigma)^y + (t_{20} + \frac{t_{21}t_{10}}{t_{22}-t_{11}})\psi_m t_{22}^{y-1}(1-\varsigma)^y \\
&= (t_{10}\psi_f - \frac{t_{21}t_{10}}{t_{22}-t_{11}}\psi_m)\frac{1}{1-t_{11}} + (t_{20} + \frac{t_{21}t_{10}}{t_{22}-t_{11}})\psi_m\frac{1}{1-t_{22}} \\
&\quad - (t_{10}\psi_f - \frac{t_{21}t_{10}}{t_{22}-t_{11}}\psi_m)\frac{1-\varsigma}{1-(1-\varsigma)t_{11}} - (t_{20} + \frac{t_{21}t_{10}}{t_{22}-t_{11}})\psi_m\frac{1-\varsigma}{1-(1-\varsigma)t_{22}} \\
&= (t_{10}\psi_f - \frac{t_{21}t_{10}}{t_{22}-t_{11}}\psi_m)(\frac{1}{1-t_{11}} - \frac{1-\varsigma}{1-(1-\varsigma)t_{11}}) + (t_{20} + \frac{t_{21}t_{10}}{t_{22}-t_{11}})\psi_m(\frac{1}{1-t_{22}} - \frac{1-\varsigma}{1-(1-\varsigma)t_{22}}).
\end{aligned} \tag{10}$$

C. Probability Generating Function of Random Variable C_k

We denote by C_k the time duration of active period number k given that this period ends before the arrival of the next packet, and let $U(z)$ be the probability generating function of r.v. C_k. To obtain the probability generating function $U(z)$, we first need to obtain the pmf of r.v. C_k. To this end, we first derive the probability $\mathsf{Pr}(C_k > m)$, and then we obtain the cdf of random variable C_k as $Z_{C_k}(m) = 1 - \mathsf{Pr}(C_k > m)$. Finally, from the obtained cdf, we will end up to the pmf of r.v. C_k as $F_{C_k}(m) = Z_{C_k}(m) - Z_{C_k}(m-1)$.

$$
\begin{aligned}
\mathsf{Pr}(C_k > m) = \mathsf{Pr}(Y_k > m \mid N_k) &= \frac{\mathsf{Pr}(Y_k > m,\, N_k)}{\mathsf{Pr}(N_k)} = \frac{\mathsf{Pr}(Y_k > m,\, N_{k-1},\, Y_k < R_k)}{\mathsf{Pr}(N_{k-1},\, Y_k < R_k)} \\
&= \frac{\mathsf{Pr}(m < Y_k < R_k)\mathsf{Pr}(N_{k-1})}{\mathsf{Pr}(Y_k < R_k)\mathsf{Pr}(N_{k-1})} = \frac{\sum_{r=m+2}^{\infty} F_{R_k}(r) \sum_{y=m+1}^{r\text{-}1} F_{Y_k}(y)}{\sum_{r=2}^{\infty} F_{R_k}(r) \sum_{y=1}^{r\text{-}1} F_{Y_k}(y)}.
\end{aligned}
\tag{11}
$$

In (11), if we set $m = 0$, the numerator and denominator will be the same. Hence, we first obtain the numerator and then set $m = 0$ in the obtained result to get the answer for the denominator.

$$
\begin{aligned}
\sum_{r=m+2}^{\infty} F_{R_k}(r) \sum_{y=m+1}^{r\text{-}1} F_{Y_k}(y) &= \sum_{r=m+2}^{\infty} F_{R_k}(r) \sum_{y=m+1}^{r\text{-}1} ((t_{10}\psi_f - \frac{t_{21}t_{10}}{t_{22}-t_{11}}\psi_m)t_{11}^{y-1} + (t_{20} + \frac{t_{21}t_{10}}{t_{22}-t_{11}})\psi_m t_{22}^{y-1}) \\
&= \frac{1}{1-t_{11}}(t_{10}\psi_f - \frac{t_{21}t_{10}}{t_{22}-t_{11}}\psi_m)(t_{11}^{m} - t_{11}^{r-1}) + \frac{1}{1-t_{22}}(t_{20} + \frac{t_{21}t_{10}}{t_{22}-t_{11}})\psi_m(t_{22}^{m} - t_{22}^{r-1}) \\
&= \frac{\varsigma}{1-t_{11}}(t_{10}\psi_f - \frac{t_{21}t_{10}}{t_{22}-t_{11}}\psi_m)\sum_{r=m+2}^{\infty}(t_{11}^{m} - t_{11}^{r-1}) \\
&\quad + \frac{\varsigma}{1-t_{22}}(t_{20} + \frac{t_{21}t_{10}}{t_{22}-t_{11}})\psi_m \sum_{r=m+2}^{\infty}(t_{22}^{m} - t_{22}^{r-1}) \\
&= \frac{\varsigma(1-\varsigma)}{1-t_{11}}(t_{10}\psi_f - \frac{t_{21}t_{10}}{t_{22}-t_{11}}\psi_m)(\frac{1}{1-(1-\varsigma)} - \frac{t_{11}}{1-t_{11}(1-\varsigma)})t_{11}^{m}(1-\varsigma)^m \\
&\quad + \frac{\varsigma(1-\varsigma)}{1-t_{22}}(t_{20} + \frac{t_{21}t_{10}}{t_{22}-t_{11}})\psi_m(\frac{1}{1-(1-\varsigma)} - \frac{t_{22}}{1-t_{22}(1-\varsigma)})t_{22}^{m}(1-\varsigma)^m \\
&= c_1 t_{11}^{m}(1-\varsigma)^m + c_2 t_{22}^{m}(1-\varsigma)^m,
\end{aligned}
\tag{12}
$$

where c_1, c_2 are:

$$
c_1 = \frac{\varsigma(1-\varsigma)}{1-t_{11}}(t_{10}\psi_f - \frac{t_{21}t_{10}}{t_{22}-t_{11}}\psi_m)(\frac{1}{1-(1-\varsigma)} - \frac{t_{11}}{1-t_{11}(1-\varsigma)}), \tag{13}
$$

$$
c_2 = \frac{\varsigma(1-\varsigma)}{1-t_{22}}(t_{20} + \frac{t_{21}t_{10}}{t_{22}-t_{11}})\psi_m(\frac{1}{1-(1-\varsigma)} - \frac{t_{22}}{1-t_{22}(1-\varsigma)}). \tag{14}
$$

By setting $m = 0$ in (12) we obtain the denominator of (11). Hence, we rewrite the (11) as follows:

$$
\begin{aligned}
\mathsf{Pr}(C_k > m) &= \frac{\sum_{r=m+2}^{\infty} F_{R_k}(r) \sum_{y=m+1}^{r\text{-}1} F_{Y_k}(y)}{\sum_{r=2}^{\infty} F_{R_k}(r) \sum_{y=1}^{r\text{-}1} F_{Y_k}(y)} \\
&= \frac{c_1 t_{11}^{m}(1-\varsigma)^m + c_2 t_{22}^{m}(1-\varsigma)^m}{c_1 + c_2}
\end{aligned}
\tag{15}
$$

From (15), we obtain the pmf of random variable C_k as follows:

$$
\begin{aligned}
F_{C_k}(m) &= Z_{C_k}(m) - Z_{C_k}(m-1) = (1 - \mathsf{Pr}(C_k > m)) - (1 - \mathsf{Pr}(C_k > m\text{-}1)) \\
&= \frac{c_1}{c_1+c_2}(1 - t_{11}(1-\varsigma))t_{11}^{m-1}(1-\varsigma)^{m-1} + \frac{c_2}{c_1+c_2}(1 - t_{22}(1-\varsigma))t_{22}^{m-1}(1-\varsigma)^{m-1}
\end{aligned}
\tag{16}
$$

Using (16) we obtain the probability generating function $U(z)$ as follows:

$$\begin{aligned}U(z) &= \mathsf{E}[z^C] = \sum_{r=1}^{\infty} z^r F_{C_k}(r)\\ &= \frac{c_1}{c_1+c_2}(1-t_{11}(1-\varsigma))\sum_{r=1}^{\infty} z^r t_{11}^{r-1}(1-\varsigma)^{r-1} + \frac{c_2}{c_1+c_2}(1-t_{22}(1-\varsigma))\sum_{r=1}^{\infty} z^r t_{22}^{r-1}(1-\varsigma)^{r-1}\\ &= \frac{c_1(1-t_{11}(1-\varsigma))z}{(c_1+c_2)(1-t_{11}(1-\varsigma)z)} + \frac{c_2(1-t_{22}(1-\varsigma))z}{(c_1+c_2)(1-t_{22}(1-\varsigma)z)}.\end{aligned} \tag{17}$$

D. Probability Generating Function of Random Variable D_k

We denote by D_k the time duration from the beginning of aggregated active period number k until the arrival of the next packet given that this packet has arrived in this active period, and let $W(z)$ be the probability generating function of r.v. D_k. To obtain the probability generating function $W(z)$, we first need to obtain the pmf of r.v. D_k. To this end, we first derive the probability $\mathsf{Pr}(D_k > m)$, and then we obtain the cdf of random variable D_k as $Z_{D_k}(m) = 1 - \mathsf{Pr}(D_k > m)$. Finally, from the obtained cdf, we will end up to the pmf of r.v. D_k as $F_{D_k}(m) = Z_{D_k}(m) - Z_{D_k}(m-1)$.

$$\begin{aligned}\mathsf{Pr}(D_k > m) &= \mathsf{Pr}(R_k > m \mid \phi_k) = \mathsf{Pr}(R_k > m \mid R_k \le Y_k, N_{k-1})\\ &= \frac{\mathsf{Pr}(R_k > m,\ R_k \le Y_k,\ N_{k-1})}{\mathsf{Pr}(R_k \le Y_k,\ N_{k-1})} = \frac{\mathsf{Pr}(m < R_k \le Y_k)\mathsf{Pr}(N_{k-1})}{\mathsf{Pr}(R_k \le Y_k)\mathsf{Pr}(N_{k-1})}\\ &= \frac{\mathsf{Pr}(m < R_k \le Y_k)}{\mathsf{Pr}(R_k \le Y_k)} = \frac{\sum_{y=m+1}^{\infty} F_{Y_k}(y)\sum_{r=m+1}^{y} F_{R_k}(r)}{\sum_{y=1}^{\infty} F_{Y_k}(y)\sum_{r=1}^{y} F_{R_k}(r)},\end{aligned} \tag{18}$$

where the term N_k denotes the event in which the next packet arrival does not happen in the first k active period. Note that the denominator in (18) is equal to p given in (10). Substituting (8) and (9) in (18) results in:

$$\begin{aligned}\mathsf{Pr}(D_k > m) &= \frac{\sum_{y=m+1}^{\infty} F_{Y_k}(y)\sum_{r=m+1}^{y} \varsigma(1-\varsigma)^{r-1}}{p} = \frac{\sum_{y=m+1}^{\infty} F_{Y_k}(y)(1-\varsigma)^m - \sum_{y=m+1}^{\infty} F_{Y_k}(y)(1-\varsigma)^y}{p}\\ &= \Big(\sum_{y=m+1}^{\infty}(t_{10}\psi_f - \frac{t_{21}t_{10}}{t_{22}-t_{11}}\psi_m)(1-\varsigma)^m t_{11}^{y-1} + (t_{20} + \frac{t_{21}t_{10}}{t_{22}-t_{11}})\psi_m(1-\varsigma)^m t_{22}^{y-1}\\ &\quad - \sum_{y=m+1}^{\infty}(t_{10}\psi_f - \frac{t_{21}t_{10}}{t_{22}-t_{11}}(1-\varsigma)^y\psi_m)t_{11}^{y-1}(1-\varsigma)^y + (t_{20} + \frac{t_{21}t_{10}}{t_{22}-t_{11}})\psi_m(1-\varsigma)^y t_{22}^{y-1}(1-\varsigma)^y\ \Big)/p\\ &= \Big((t_{10}\psi_f - \frac{t_{21}t_{10}}{t_{22}-t_{11}}\psi_m)\frac{t_{11}^m(1-\varsigma)^m}{1-t_{11}} + (t_{20} + \frac{t_{21}t_{10}}{t_{22}-t_{11}})\psi_m\frac{t_{22}^m(1-\varsigma)^m}{1-t_{22}}\\ &\quad - (t_{10}\psi_f - \frac{t_{21}t_{10}}{t_{22}-t_{11}}\psi_m)\frac{t_{11}^m(1-\varsigma)^{m+1}}{1-(1-\varsigma)t_{11}} - (t_{20} + \frac{t_{21}t_{10}}{t_{22}-t_{11}})\psi_m\frac{t_{22}^m(1-\varsigma)^{m+1}}{1-(1-\varsigma)t_{22}}\ \Big)/p\\ &= \frac{d_1 t_{11}^m(1-\varsigma)^m + d_2 t_{22}^m(1-\varsigma)^m}{d_1+d_2},\end{aligned} \tag{19}$$

where d_1 and d_2 are:

$$d_1 = (t_{10}\psi_f - \frac{t_{21}t_{10}}{t_{22}-t_{11}}\psi_m)(\frac{1}{1-t_{11}} - \frac{1-\varsigma}{1-(1-\varsigma)t_{11}}), \tag{20}$$

$$d_2 = (t_{20} + \frac{t_{21}t_{10}}{t_{22}-t_{11}})\psi_m(\frac{1}{1-t_{22}} - \frac{1-\varsigma}{1-(1-\varsigma)t_{22}}). \tag{21}$$

From (19), we obtain the pmf of random variable D_k as follows:

$$\begin{aligned}
F_{D_k}(m) &= Z_{D_k}(m) - Z_{D_k}(m-1) = (1-\mathsf{Pr}(D_k > m)) - (1-\mathsf{Pr}(D_k > m\text{-}1)) = \mathsf{Pr}(D_k > m\text{-}1) - \mathsf{Pr}(D_k > m) \\
&= \frac{d_1 t_{11}^{m-1}(1-\varsigma)^{m-1} + d_2 t_{22}^{m-1}(1-\varsigma)^{m-1}}{d_1+d_2} - \frac{d_1 t_{11}^{m}(1-\varsigma)^{m} + d_2 t_{22}^{m}(1-\varsigma)^{m}}{d_1+d_2} \\
&= \frac{d_1(1-t_{11}(1-\varsigma))}{d_1+d_2} t_{11}^{m-1}(1-\varsigma)^{m-1} + \frac{d_2(1-t_{22}(1-\varsigma))}{d_1+d_2} t_{22}^{m-1}(1-\varsigma)^{m-1}.
\end{aligned} \tag{22}$$

Using (22) we obtain the probability generating function $W(z)$ as follows:

$$\begin{aligned}
W(z) &= \mathsf{E}[z^D] = \sum_{r=1}^{\infty} z^r F_{D_k}(r) \\
&= \frac{d_1(1-t_{11}(1-\varsigma))}{d_1+d_2} \sum_{r=1}^{\infty} z^r t_{11}^{r-1}(1-\varsigma)^{r-1} + \frac{d_2(1-t_{22}(1-\varsigma))}{d_1+d_2} \sum_{r=1}^{\infty} z^r t_{22}^{r-1}(1-\varsigma)^{r-1} \\
&= \frac{d_1(1-t_{11}(1-\varsigma))z}{(d_1+d_2)(1-t_{11}(1-\varsigma)z)} + \frac{d_2(1-t_{22}(1-\varsigma))z}{(d_1+d_2)(1-t_{22}(1-\varsigma)z)}.
\end{aligned} \tag{23}$$

E. Probability Generating Function of Random Variable S_k

We denote by S_k the time duration of sleep period number k, i.e. the time duration during which both MBS and FBS are in sleep mode. Using the Markov chain of our system model, we obtain the pmf of sleep period as follows:

$$F_{S_k}(m) = t_{00}^{m-1}(t_{01}+t_{02}). \tag{24}$$

From this pmf we obtain the probability generating function of r.v. S_k as follows:

$$\begin{aligned}
W(z) &= \mathsf{E}[z^S] = \sum_{r=1}^{\infty} z^r F_{S_k}(r) \\
&= (t_{01}+t_{02}) \sum_{r=1}^{\infty} z^r t_{00}^{r-1} = \frac{(t_{01}+t_{02})z}{1-t_{00}z}
\end{aligned} \tag{25}$$