

# W-band Silicon Dielectric Measurement

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**Abstract**—In this paper, a free space based method for measurement of silicon dielectric constant is presented at W band frequencies. The dielectric constant of silicon is calculated using measured phase information of the transmitted signal (S21) through Silicon sheet of 500um thickness. Measured dielectric constant for silicon wafer has numerous fluctuations which come from multiple reflection and diffraction effect due to sharp edges.

## I. INTRODUCTION

There is uncertainty about the dielectric constant of substrates such as silicon at W-band frequencies. Silicon dielectric uncertainty effect becomes very important for the design and simulations of RF circuits and antennas on silicon substrates. Designing an antenna on a substrate with inaccurate value of dielectric constant will change antenna performance and shift the resonance frequency in practice which causes large discrepancies between simulated and measured results. Simple and applicable methods for measuring the permittivity of the substrate are always in great interest of the microwave circuit and antenna designers.

There are various methods for measuring dielectric constant including dielectric waveguide, cavity resonator, open resonator and free space method which is more suitable for wideband measurement [1]. In free space method, the dielectric constant is measured based on either the transmission method or metal backed method [2-3]. Metal backed method is better for very thin dielectric samples. However, some disadvantages such as edge diffraction and multiple reflections are associated with the free space method. Edge diffraction can be minimized by using spot focus antenna which produce Gaussian beam by mean of lens and also taking the sample size large enough, while time domain filtering and LRL (line, reflect, line) calibration technique can be used for multiple reflection problem.

Using amplitude and phase information it is possible to derive both real and imaginary parts of dielectric constant. However, it is possible to derive only the real part just using the phase information of either transmitted or reflected signal. In this work, two approaches are applied to bring out the real part of silicon dielectric constant based on transmission and reflection type measurement method. Note that no calibration and time domain filtering is utilized in the measurements. The transmitter and receiver antennas which are used in the measurement set-up have wide beamwidth far away from Gaussian beam. Only the phase information of the transmitted signal is used to determine the real part of the permittivity.

## II. THEORY

Travel time of EM wave is different for different medias and this time difference can be interpreted as phase difference in frequency domain. A slab of silicon wafer with dielectric constant of  $\epsilon_1$  is placed in the far fields of the two horn antennas as shown in Fig. 1. Thickness of the DUT is supposed to be  $d_1$ , while  $d_0+d_1$  is the total distance between the two antennas. Impedance variation in the direction of wave propagation will cause multi-reflections inside the silicon sheet and infinite series of rays with different phase difference will be picked up by the receiver antenna (Ant. 2 in Fig. 1). S21 between two antennas can be calculated for the configuration of the Fig. 1 which is confined to the air ( $\epsilon_0$ ) at both sides of the DUT. Analytical relation for S21 is obtained as follow,

$$\begin{aligned} S_{21} &= e^{-jk_0d_0} T_{21} e^{-jk_1d_1} T_{12} + e^{-jk_0d_0} T_{21} e^{-j3k_1d_1} \Gamma_{12} \Gamma_{12} T_{12} + \dots \\ &= e^{-j(k_0d_0+k_1d_1)} T_{21} T_{12} (1 + \Gamma^2 e^{-j2k_1d_1} + \Gamma^4 e^{-j4k_1d_1} + \dots) \quad (1) \\ &= e^{-j(k_0d_0+k_1d_1)} T_{21} T_{12} / (1 - \Gamma^2 e^{-j2k_1d_1}), \quad |\Gamma| < 1 \end{aligned}$$

where  $T_{21}$  and  $T_{12}$  are transmission coefficients from air to dielectric and vice versa, respectively.

$\Gamma_{12}$  is reflection coefficient from dielectric to air and is defined as:

$$\Gamma = \Gamma_{12} = \frac{\sqrt{\epsilon_r} - 1}{\sqrt{\epsilon_r} + 1} \quad (2)$$

and

$$k_i = \omega \sqrt{\mu_0 \epsilon_i}, \quad i = 0, 1 \quad (3)$$

which is wave-number in each of the media.

Accurate measured phase between transceiver antennas will be

$$\begin{aligned} \varphi_{S_{21}} &= -(k_0d_0 + k_1d_1) \\ &\quad - \tan^{-1}(\sin(2k_1d_1) / (\Gamma^{-2} - \cos(2k_1d_1))) \quad (4) \end{aligned}$$

which is deduced from relation (1). We can use phase of the measured S21 and (4) with determined  $d_0+d_1$  and  $d_1$  to calculate the permittivity of silicon at the measured frequency. In this case, accurate values of measured phase, transceiver

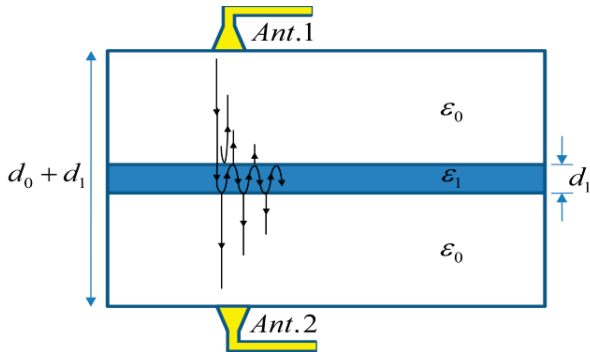


Fig. 1. Schematic of the measurement setup.

antennas spacing and dielectric thickness is needed to achieve accuracy in the calculated  $\epsilon_r$ .

In the proposed method, phase of S21 is measured in two cases; 1) Silicon sheet is placed between the antennas. 2) Silicon is absent between the antennas.

Phase difference between two cases can be calculated by subtracting relation in (4) when the silicon is between the antennas and silicon is replaced by air (by setting  $\epsilon_r=1$  or  $\Gamma=0$  in (4)).

$$\Delta\varphi = \varphi_{S_{21}}|_{air} - \varphi_{S_{21}}|_{sub.} = (k_1 - k_0)d_1 + \tan^{-1}(\sin(2k_1d_1)/(\Gamma^{-2} - \cos(2k_1d_1))) \quad (5)$$

The calibrated formula is independent of the spacing between the antennas. If multiple reflections are neglected, one can obtain a closed form expression for the solution which is given by

$$\Delta\varphi = (k_1 - k_0)d_1 = 2\pi fd_1(\sqrt{\epsilon_r} - 1)/c \quad (6)$$

by replacing  $\Gamma=0$  in (5). Although evaluated  $\epsilon_r$  in (6) cannot achieve a good approximation due to neglected multiple reflections, but, (6) can easily be solved for relative dielectric constant as given in (7).

$$\epsilon_r = (1 + c\Delta\varphi/(2\pi fd_1))^2 \quad (7)$$

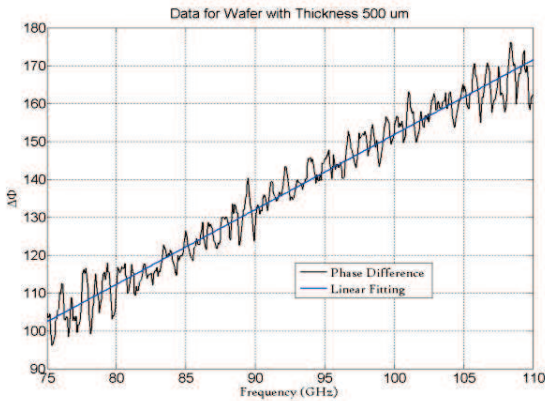


Fig. 2. Measured phase difference versus frequency.

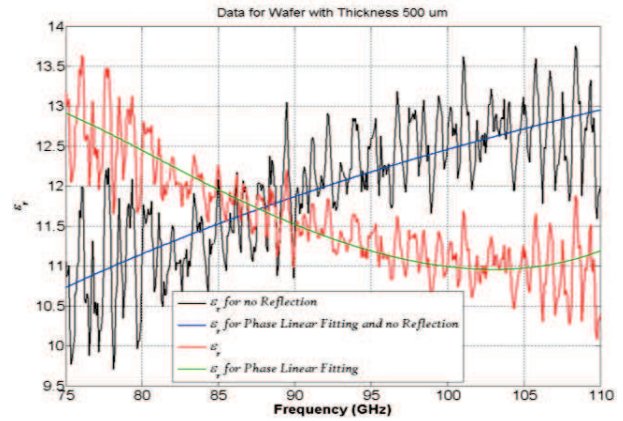


Fig. 3. Calculated dielectric constant.

### III. MEASURED RESULTS

Measured phase difference  $\Delta\varphi$  for 500um thick silicon wafer and corresponding linear line which is fitted for minimum error are shown in Fig. 2. Calculated permittivity using Eqn. 5 (red and green line) and Eqn. 7 (black and blue line) are presented in Fig. 3. The effect of reflection is presented in Fig. 3, where the dielectric constant for accurate calculation is descending versus frequency, while ascending for when the reflection is neglected. Fluctuations in the measured  $\Delta\varphi$  which are aroused from multiple reflections of the surrounding environment cause to instability in the calculated dielectric constant accordingly. The measured dielectric constant is around 11.9 which is used by designers in simulations.

### IV. CONCLUSION

A dielectric constant of silicon slab is measured at W-band using free space method. Real goal of this experiment is to obtain the correct value of silicon permittivity which is used as the substrate in designing of on-chip antenna, and consequently reduce frequency shift coming out of non-precise dielectric constant. However, there is some deviation in measured result due to multiple reflections and diffraction. To improve the result absorber materials can be used to minimize unwanted reflections. Moreover, time gating technique can be applied to further improve of the results.

### ACKNOWLEDGEMENT

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