

An optimal estimation of feet contact distributed normal reaction forces of walking bipeds

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Abstract—The motion of a walking biped has rich information about the contacts with the environment. This paper presents an optimal estimation method of the distributed normal reaction forces at the contact points on the feet soles of walking bipeds. The motion is acquired by employing the inertial measurement unit (IMU) and the joint-encoder readings into Newton-Euler dynamic equations without using any force sensor model. The quadratic programming optimization method is used. The validity of the proposed estimation method was confirmed by simulations on 3D dynamics model of the humanoid robot SURALP while walking.

Index Terms— Quadratic programming, reaction forces, inertial measurement unit (IMU), Newton-Euler equations.

I. INTRODUCTION

Humanoid robots [1] have attracted a great attention of many researchers due to the suitability of the biped structure for tasks in the human environment. In this field, the postural control has a crucial role for understanding the balance and locomotion of the biped. The knowledge of the zero moment point (ZMP) [2] and the ground reaction force [3] has significant importance in studying the stability of bipeds and developing several and powerful control strategies [3-5]. The ground reaction forces are measured using different sensors [6-8] embedded in the robot feet of the humanoid robot as in [1, 9-13].

An on-line assessment of the balance of the robot requires information of the robot dynamics and measurement data about the environmental interaction forces. Although, the use of expensive and numerous sensors on board could be regarded as a more reliable solution and achieves higher level of control, however, it would be a costly one. In the same context, it is not always applicable to use numerous sensors due to several limitations like the hardware and noise.

Obtaining estimated data based on models will indicate many parameters and variables. The estimated data can be used to judge whether the sensor readings are reliable or not. Also, they can replace the sensor if it has defective readings that would make the control laws diverge. For low cost humanoid robots and small applications, sensorless control based on estimated data is more likely and required.

These facts motivate the use of intelligent techniques and learning algorithms for the data estimation of the contact dynamics used in walking control algorithms [14, 15]. The joint torque information is used in [16] to estimate the ZMP for a standing robot without force measurement. In [17], the total reaction force is estimated based on the inertial measurement unit (IMU) and joint encoders, with no indication to the contact points and their corresponding forces. In [18], the pseudo inverse is used to estimate the contact force distribution based on known ZMP, however, the pseudo inverse may fail and doesn't fulfill the reaction force constraints.

In this paper, an optimal estimation method of the distributed reaction force for a walking biped is proposed. The number of the contact points and their locations are known in the foot frame. The constraint dynamic equations of the humanoid robot in translational motion are obtained. The quadratic programming optimization method is used to solve the constraint equations. The resulted solution is the total ground reaction force and its location (center of pressure CoP) within the foot geometry. This estimation is based on the readings of the IMU and the kinematics of the robot legs through the assembled joint-encoders. Then, while the foot is in contact, a constraint model is obtained to find the optimal force distribution at the contact points.

The rest of the paper is organized as follows: Section 2 describes the CoM dynamics and the estimation of the total reaction force at each foot with its location. Section 3 introduces the optimal estimation of the reaction forces at the contact points. Section 4 introduces the simulation platform and presents the simulation results of contact reaction forces estimation. The paper conclusion is in Section 5.

II. DYNAMICS OF CoM

The CoM dynamic equations of a humanoid robot in translational motion (1) are used to calculate the force and its location for each leg [19, 20]

$$\begin{aligned} F_R + F_L &= m(\ddot{c} - g) \\ f_R \times F_R + f_L \times F_L + M_R + M_L &= \dot{H} \end{aligned} \quad (1)$$

where, m is the robot mass, H the CoM angular momentum, c the CoM position, g the constant gravity acceleration vector pointing to the negative z - direction in the world frame. F_R , F_L , M_R and M_L are the reaction forces and moments at the right and left feet respectively. f_R and f_L the position of the right and left feet frame-origins respectively from the CoM. They are known by using the forward kinematics and the joints encoders for each leg. Equation (1) can be written in matrix form as

$$CN = d, \quad (2)$$

where

$$C = \begin{bmatrix} I_3 & I_3 & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ f_R \times & f_L \times & I_3 & I_3 \end{bmatrix}, N = \begin{bmatrix} F_R \\ F_L \\ M_R \\ M_L \end{bmatrix}, \quad (3)$$

and

$$d = \begin{bmatrix} m(\ddot{c} - g) \\ \dot{H} \end{bmatrix}, \quad (4)$$

where I_m and $\mathbf{0}_{m \times l}$ are the identity and the zero matrix of dimensions m and $m \times l$ respectively. The term $(\ddot{c} - g)$ is directly measured from the accelerometer and then transformed to the world frame. For rotationally stable robot, the angular momentum is considered constant i.e. $\dot{H} = 0$ [19]. The solution of (2) is subjected to constraints due to the leg in contact, friction and support polygon [12].

The leg in contact constraints are just activating or deactivating the leg parameters in binary way and can be written mathematically as

$$AN = b, \quad (5)$$

where

$$A = \begin{bmatrix} A_{right} & \mathbf{0}_{D \times D} & \mathbf{0}_{D \times 2D} \\ \mathbf{0}_{D \times D} & A_{left} & \mathbf{0}_{D \times 2D} \end{bmatrix}, b = \mathbf{0}_{4D \times 1}, \quad (6)$$

$$A_{right} = \begin{cases} I_D & \text{left contact} \\ \mathbf{0}_{D \times D} & \text{right contact} \end{cases}, \quad (7)$$

and

$$A_{left} = \begin{cases} I_D & \text{right contact} \\ \mathbf{0}_{D \times D} & \text{left contact} \end{cases}, \quad (8)$$

where D is the space dimension. For 3-dimensional space $D = 3$.

The support polygon constraints are

$$-\frac{\delta_x}{2} \leq \frac{-M_{y,i}}{F_{z,i}} \leq \frac{\delta_x}{2}, \quad -\frac{\delta_y}{2} \leq \frac{M_{x,i}}{F_{z,i}} \leq \frac{\delta_y}{2}, \quad (9)$$

where the subscripts x , y and z refer to the x -, y - and z - components respectively. The subscript i refers to the right or left leg. δ the foot dimension in the specified direction.

To guarantee that the feet will not slip, other restrictions do exist as friction constraints and written as

$$\begin{cases} |F_{x,i}|, |F_{y,i}| \leq \mu F_{z,i} \\ F_{z,i} \geq 0 \end{cases}, \quad (10)$$

where μ is the coefficient of friction.

The support polygon and friction constraints are written in matrix inequality form for both of the legs as

$$A_{eq} N \leq b_{eq}, \quad (11)$$

where

$$A_{eq} = \begin{pmatrix} A_1 & \mathbf{0}_{4 \times 3} & \mathbf{0}_{4 \times 1} \\ \mathbf{0}_{4 \times 3} & A_1 & \mathbf{0}_{4 \times 1} \\ A_3 & \mathbf{0}_{4 \times 3} & \mathbf{0}_{4 \times 6} \\ \mathbf{0}_{4 \times 3} & A_3 & \mathbf{0}_{4 \times 6} \end{pmatrix}, b_{eq} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}_{1 \times 16}, \quad (12)$$

with

$$A_1 = \begin{bmatrix} 0 & 0 & -\frac{\delta_y}{2} & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & \frac{\delta_y}{2} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{\delta_x}{2} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{\delta_x}{2} & 0 & 0 & 0 & 0 & -1 \end{bmatrix},$$

and

$$A_3 = \begin{bmatrix} 1 & 0 & \mu \\ -1 & 0 & \mu \\ 0 & 1 & \mu \\ 0 & -1 & \mu \end{bmatrix}.$$

The matrix forms of the constraints are used directly to estimate the vector \hat{N} by solving the constraint quadratic programming problem

$$\begin{aligned} \hat{N} &= \min_N N^T (C^T C + W) N - d^T C N \\ \text{s.t.} \quad & A_{eq} N \leq b_{eq} \\ & AN = b \end{aligned} \quad (13)$$

where W is a diagonal matrix. By the knowledge of the forces and moments from (13), the CoP is calculated for each foot using

$$CoP = \begin{bmatrix} M_y / F_z & -M_x / F_z & 0 \end{bmatrix}^T. \quad (14)$$

III. DISTRIBUTED CONTACT-POINTS FORCE ESTIMATION

The computed forces F_R and F_L are distributed among the contact points of the right and left feet respectively. The force distribution depends on the number of contact points n_R and n_L and their location for the right and left feet respectively. For simplicity, the number of contact points on the right foot is considered the same as for the left, i.e. $n_R = n_L = n$. The CoP coordinates (x_{CoP} and y_{CoP}) with F_z can be written as

$$\begin{bmatrix} F_{z,R} x_{CoP,R} \\ F_{z,R} y_{CoP,R} \\ F_{z,L} x_{CoP,L} \\ F_{z,L} y_{CoP,L} \end{bmatrix} = \begin{bmatrix} x_{R,1} & \cdots & x_{R,n} & 0_{n \times 1} \\ y_{R,1} & \cdots & y_{R,n} & 0_{n \times 1} \\ 0_{n \times 1} & x_{L,1} & \cdots & x_{L,n} \\ 0_{n \times 1} & y_{L,1} & \cdots & y_{L,n} \end{bmatrix} \begin{bmatrix} F_{z,1} \\ \vdots \\ F_{z,2n} \end{bmatrix}, \quad (15)$$

with

$$\begin{aligned} F_{z,R} &= \sum_{k=1}^{n_R} F_{z,k} \\ F_{z,L} &= \sum_{k=2n-n_R}^{2n} F_{z,k} \end{aligned}, \quad (16)$$

and

$$F_{z,k} \geq 0, \quad (17)$$

where x_k and y_k are the x - and y -positions of the contact point k that has the z - reaction force $F_{z,k}$.

Equation (15) can be written as in (2) with known C matrix and d vector and unknown N vector where

$$C = \begin{bmatrix} x_{R,1} & \cdots & x_{R,n} & 0_{n \times 1} \\ y_{R,1} & \cdots & y_{R,n} & 0_{n \times 1} \\ 0_{n \times 1} & x_{L,1} & \cdots & x_{L,n} \\ 0_{n \times 1} & y_{L,1} & \cdots & y_{L,n} \end{bmatrix}, N = \begin{bmatrix} F_{z,1} \\ \vdots \\ F_{z,2n} \end{bmatrix}, \quad (18)$$

and

$$d = \begin{bmatrix} F_{z,R} x_{CoP,R} \\ F_{z,R} y_{CoP,R} \\ F_{z,L} x_{CoP,L} \\ F_{z,L} y_{CoP,L} \end{bmatrix}. \quad (19)$$

Since C is a nonsquare matrix, the pseudo inverse of it can be calculated and then find N . However, the inverse suffers from singularities and doesn't fulfill the constraints in (16) and (17). While the optimal solution is still required, the same procedure as in (13) is used with the new inequality and equality constraints

$$\begin{aligned} A_{eq} &= -I_{2n}, b_{eq} = 0_{2n \times 1} \\ A &= \begin{bmatrix} 1_n & 0_n \\ 0_n & 1_n \end{bmatrix}, \end{aligned} \quad (20)$$

and

$$b = \begin{bmatrix} F_{z,R} & F_{z,L} \end{bmatrix}^T, \quad (21)$$

where 1_m is row vector of one's of dimension m .

IV. SIMULATION RESULTS

The simulations are carried on 12 degrees of freedom (DOF) biped model consists of two 6-DOF legs and a trunk connecting them. (Fig. 1). The details of contact modeling and simulation algorithm are in [21]. The coordinate frames are shown in Fig. 2. All the measurements and calculation are in the world frame. The transformation is done using the rotational matrix obtained by the author in [22]. The sampling time is 1ms, $W = 0.01I_8$ for the distributed normal reaction force estimation and $W = 0.01I_{12}$ for the normal reaction force estimation. To match our humanoid SURALP (Sabanci University Robotics Research Laboratory Platform) [1], four contact points are considered for each foot. They are placed at the corners with the corresponding coordinates shown in Table I as expressed in the foot frame.

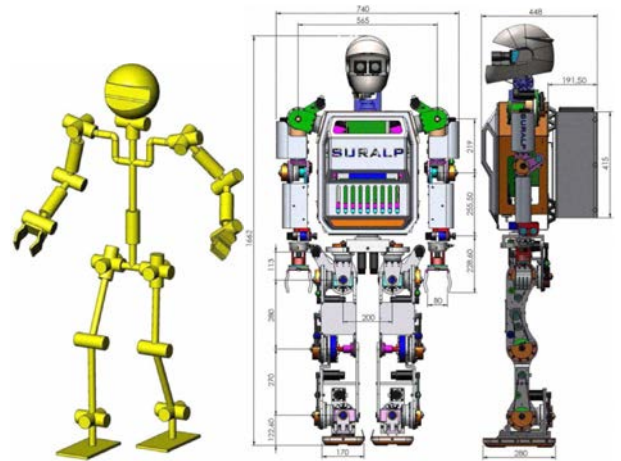


Fig. 1 The kinematic arrangement of SURALP

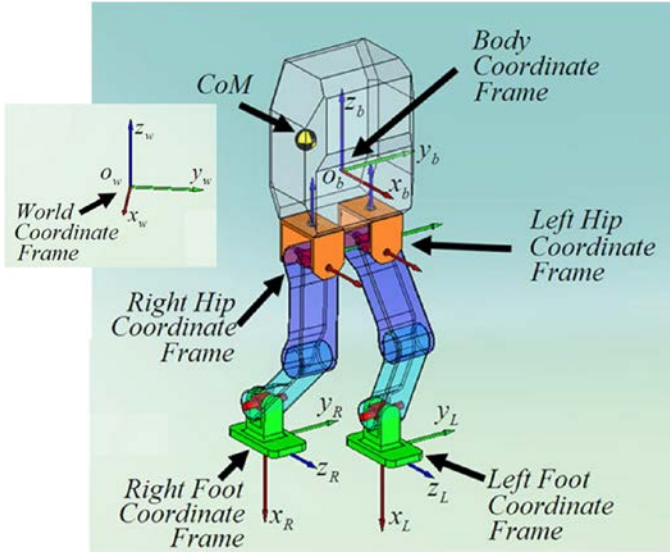


Fig. 2: Coordinate systems . o_w and o_b stand for the origins of the world and body coordinate frames, respectively. The foot coordinate frames are fixed to the foot soles.

The two feet are symmetrical. The walking foot trajectories are shown in Fig. 3. The biped starts walking after 0.5 sec, left single support (LS) then double support (DS) then right single support (RS) and so forth. The robot stops at the time instant 10.1 sec. The estimated contact point normal reaction forces for the right and left feet are shown in Fig. 4 and Fig. 5 respectively. As mentioned before, the foot contact is assumed to have an on/off binary strategy without transition, this explains the sudden zero and non zero reaction forces in both Fig. 4 and Fig. 5. This binary on/off strategy can be smoothed by using the CoM position to indicate the transition between the double and single support phases of walking. During the single support, with time 0.6 sec, one of the feet will have its maximum values while the other is zero. For the double support case, with time 0.9 sec, the reaction is distributed between the two feet. The simulations start with double support, left single support and so on. Since the estimated normal reaction forces have the sudden zero and nonzero changes and the true normal forces are noisy, they are smoothed using a Kalman filter [23] with zero input, identity matrices for the process and measurement matrices, and zero initial values. The comparison results show the accuracy of the estimation as in Fig. 6 and Fig. 7. The estimated forces are smooth and stable. The zero values at time zero is due to the zero initial value in Kalman filter.

Notes on the estimation method:

Note 1: The dynamic CoM model (1) considers neither the centrifugal nor the frictional forces of the legs. It assumes that the legs are massless or moving in a very low speed. Thus this method is applicable when the biped has a light legs weight and moving in a slow motion. For general bipeds, this method can be developed by including the inertial and centrifugal forces due to the legs. The estimation will be more accurate. However it needs more data about the CoM velocity. It also needs a method to calculate the angular acceleration for each joint in the legs. The simulations are carried on a heavy biped.

Its parameters are given in Table II. As a result of the aforementioned assumptions violations, an impulse forces and some estimation error occur as in Fig. 6 and Fig. 7. However, this error is bounded.

Note 2: Equation (13) has a considerable computational cost.

Note 3: In real applications the value of μ is important and might be inaccurate. It determines the friction forces between the feet and the walking surface. Thus it determines the maximum acceleration the robot can have.

Table I: Contact coordinates

Contact point	Position coordinates in the foot frame [x y z]
1	[0 0.0675 -0.11]
2	[0 0.0675 0.122]
3	[0 -0.0675 -0.11]
4	[0 -0.0675 0.122]

Table II: Robot Parameters

Parameter	Value
Upper leg length	280mm
Lower leg length	270mm
Sole-ankle distance	124mm
Foot dimensions	240mm×150mm
Upper arm length	219mm
Lower arm length	255mm
Robot weight	114 kg

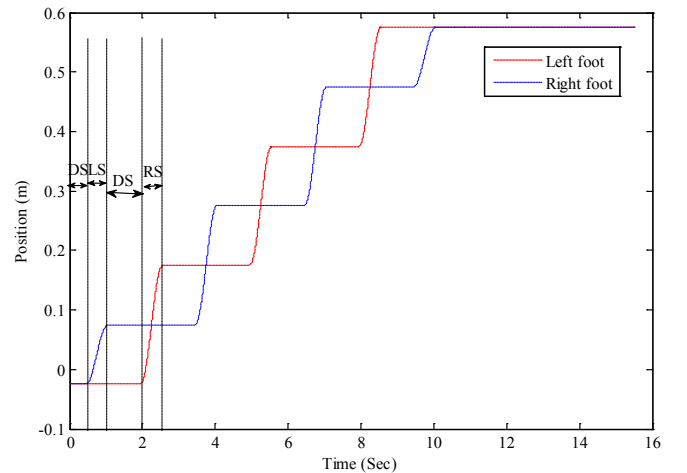


Fig. 3: Feet walking trajectories, DS stands for the double support phase, LS stands for the left leg single support phase, and RS stands for the right leg single support phase

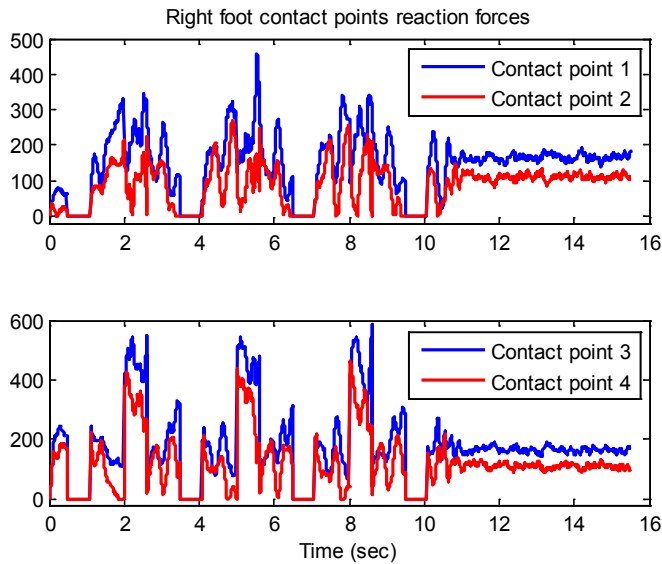


Fig. 4. The right foot contact points normal reaction forces based on the estimation algorithm

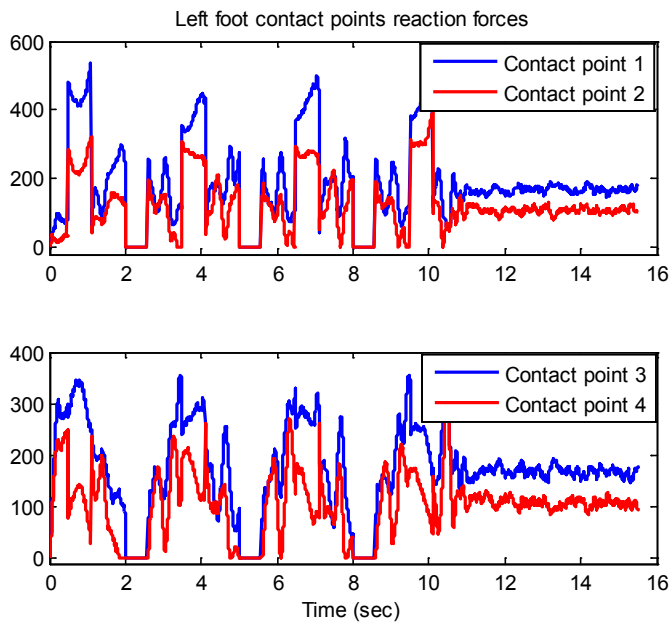


Fig. 5. The left foot contact points normal reaction forces based on the estimation algorithm

V. CONCLUSION

The optimal estimation of the distributed normal reaction forces using IMU and legs kinematics is shown. The method assumes a known but not limited number of contact points. The contact points have known positions in the feet reference frames. The quadratic programming optimization method is employed. The estimated results demonstrate their reliability and accuracy such that they can be used to detect the force sensor defect. Although the work is shown for two legs, it is still applicable and can be extended for more than two legs robots.

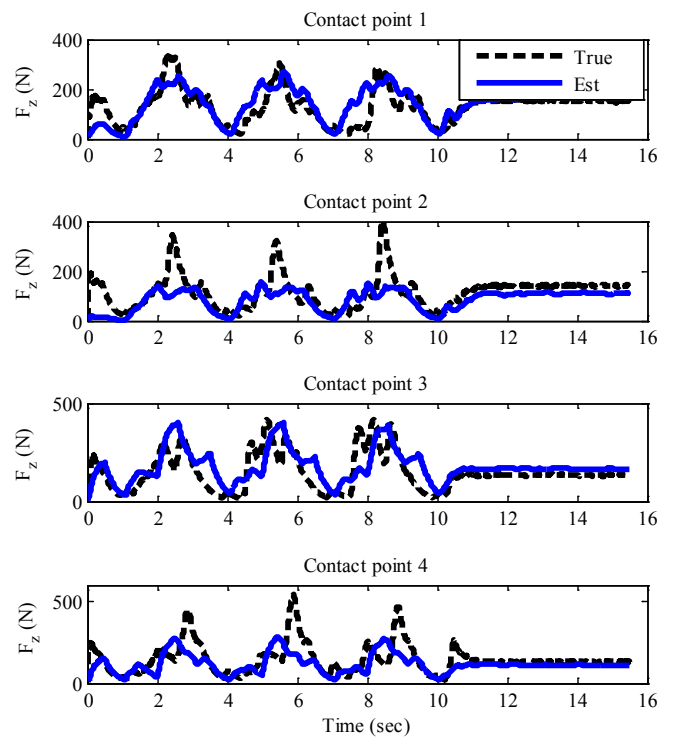


Fig. 6. Right foot contact points normal reaction forces

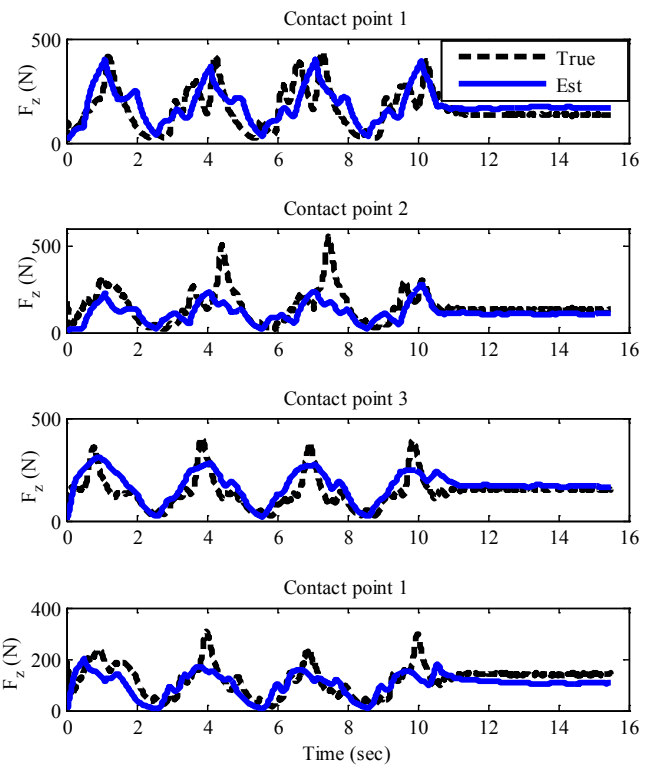


Fig. 7. Left foot contact points normal reaction forces

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