Economics of Collective Refusals to Supply *

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Abstract

This paper examines situations where vertically integrated firms refuse to supply an input to an independent competitor in the downstream market. The treatment of such cases by competition or regulatory authorities is often based on the assumption that such outcomes can only arise if there is collusion in the upstream markets. We argue that this is not always the case. In particular, we argue that proper antitrust or regulatory assessment of such cases must take into account the nature of competition, whether sales contracts are observable, the degree of contractual flexibility that is permitted, the substitutability of downstream products, and even the number of potential competitors in the downstream market.

Keywords: collective dominance, collective refusal to supply, tacit collusion, upstream competition.

JEL Classification: L13, L40, L41, L51.

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Introduction

This paper examines situations where vertically integrated firms refuse to supply an input to an independent firm that competes with them in the downstream market. Instances of collective refusals to supply have raised competition concerns in a variety of contexts. There have been cases where collective refusals to supply were believed to arise through tacit collusion when firms are collectively dominant. As discussed in the next section, in European mobile telecommunications markets, identification of such collective dominance in access and call origination markets have led some regulatory authorities to impose remedies, namely access and roaming regulations, on the collectively dominant firms. In the area of spectrum allocation, concerns have been raised that such behavior among collectively dominant firms may lead to spectrum hoarding. In other jurisdictions there have been cases where collective refusals to supply have been interpreted as arising from agreements or concerted practices[1] among oligopolists and have been condemned as infringements of bans on horizontal restrictions on competition.

We argue that these approaches of competition or regulatory authorities are implicitly based on models where a collective refusal to supply can only arise when the vertically integrated firms restrict competition in the upstream market through an explicit agreement or tacit collusion. In other words, in these models that implicitly guide competition law or regulatory enforcement, the assumption is that, in the context of a one-shot game, the best response of any firm to competitors who refuse to supply the input to a downstream competitor would be to supply the input. Hence, a collective refusal to supply would only arise (presumably) in a repeated game context as a result of collusive strategies.

We show that this basic assumption does not always hold and find that a collective refusal to supply by vertically integrated firms to independent competitors in a downstream market may constitute the Nash equilibrium of a one-shot game. We presume that such equilibria are taken to represent competitive cases not calling for ex-ante or ex-post intervention through competition law or sector specific regulation. An important implication is that the current treatment of collective refusals to deal in competition law and sector specific regulation are not consistent with the basic tenets that have guided either body of law. In particular, we will argue that proper antitrust or regulatory assessment of situations where vertically integrated firms refuse to supply downstream competitors requires a more detailed analysis of a number of issues including the nature of competition, whether sales contracts are observable, the characteristics of the downstream demand, the degree of contractual flexibility that is permitted, and even the number of potential competitors in the downstream market.

More specifically, we examine conditions under which collective refusals to supply may arise as the equilibrium of standard models of oligopoly where two vertically integrated firms (VIFs)

[1] European Union competition law differentiates between agreements and concerted practices but the distinction is not very relevant for the purposes of this paper. Henceforth we will use the term agreement to cover both agreements and concerted practices.

interact with an independent competitor in a downstream market. The independent firm has to procure an input from either of the VIFs in order to produce its product. Similar setups have been examined, among others, in Ordover and Schaffer (2007), Höffler and Schmidt (2008) and Bourreau et. al. (2011) which we discuss further below. An important contribution of our paper relative to this literature is that we explicitly consider the impact of *contract observability* and *complexity* on market outcomes.

We first examine the case where contracts are sufficiently sophisticated, that is, they are complicated enough to allow the maximization of the joint profits of a VIF and the downstream competitor to which the VIF supplies the input, in case a contractual relationship arises. Fairly general results can be obtained for the case of downstream quantity competition. When contracts are observable, the unique equilibrium entails both VIFs supplying the input. Hence, under these conditions an observation of joint refusal to supply would likely reflect coordination in a repeated game, for example, through trigger strategies. By contrast, when contracts are unobservable neither VIF supplies the input in the equilibrium of the stage game. In principle and according to the prevalent interpretation of collusion, such a collective refusal to supplying the input should not raise any competitive concerns.

The analysis is more complicated under price competition. We study the case where the input produced in the upstream market is homogeneous and the downstream products are differentiated. When contracts are sophisticated and observable, we find a general result that the equilibrium of the one-shot game entails at least one of the firms supplying the input. When contracts are sophisticated and unobservable, or observable but unsophisticated, collective refusal to supply may occur as the equilibrium of the one-shot game when downstream products are sufficiently substitutable. It turns out that the number of independent downstream firms that the VIFs can potentially contract with also affects when collective refusal to supply occurs as an equilibrium of the one shot game. We discuss the implications of these examples for competition and regulatory authorities.

A number of recent papers deal with related issues. Dewenter and Haucap (2006) study a similar situation but without any concern for the role of contracts or for upstream competition. Their work suggests that in a homogeneous good Cournot model collective refusal to deal can only arise as a collusive outcome. We show that this is due to their limited focus on contractual forms, and in particular, due to the observability of the contracts they consider. In other words, a collective refusal to deal can arise as the equilibrium of a simple one-shot game when upstream contracts are not publicly observable.

A growing strand of literature focuses on upstream competition. Ordover and Schaffer (2007) address basically the same questions we ask in this paper. They investigate how two VIFs would respond to demand for access to their upstream products by a non-integrated downstream rival. They, as we do, show that there may be equilibria of a one-shot interaction where neither VIF offers the input for sale. They, however, focus on observable linear wholesale contracts and use a stylized model of demand. In this paper, we extend their model in various directions. Most importantly, we consider both observable and unobservable contracts as well as more general contractual forms. We show that all of these features turn out to be crucial in determining when access to potential downstream competitors may be refused. In most of the cases, we can also derive our results using general demand specifications while we adopt a similar formulation to the one presented in Ordover and Schaffer (2007) only when we consider price competition with unobservable contracts.

Bourreau et. al. (2011) also examine the nature of competition between VIFs and non-integrated downstream competitors. The primary concern of their paper is to examine whether competition in the upstream market may be insufficient, in the sense that input is priced above marginal cost, a situation they refer to as partial foreclosure. In particular, they show that there may be equilibria where only one VIF supplies the input as well as those where both VIFs supply the input possibly at prices above marginal cost. They also show that an equilibrium where both VIFs refuse to supply the input ("complete foreclosure") is a possibility and its existence depends on the degree of substitutability between downstream products. Bourreau et. al. (2011) do not examine the effect of contract observability on the likelihood of complete foreclosure, which is one of our main concerns. In addition, while they consider only linear or two part tariff wholesale contracts, we allow the firms to employ more general contractual forms.

Höffler and Schmidt (2008) analyze a similar setup but pose a different question. They show that entry by a new downstream competitor need not lower downstream prices. The intuition is that the VIF that supplies the input has an incentive to keep its downstream price high in order to earn more in the wholesale market. However, their result only holds true when one of the VIFs is exogenously assigned to supply the downstream firm and/or downstream competition is spatial. With linear demands when the VIFs compete using linear wholesale prices, the unique equilibrium they derive entails marginal cost wholesale prices. Furthermore, they consider neither unobservable contracts nor the possibility of a collective refusal to supply.

Siciliani (2009) also discusses the possibility that a collective refusal to supply may arise as an equilibrium of a static game, although without a formal model. He states that there is an identification problem with tacit collusion because "observation of a common conduct among the alleged colluding firms could also be consistent with a situation where firms independently and rationally pursue their self-interest, without expecting to influence their competitors" (p. 689). He concludes that when investigating cases of collective refusal to supply, "the competition or regulatory authorities should instead thoroughly assess whether the observed refusal is a rational and independent decision consistent with a firm's unilateral incentives" (p. 718). In this paper we provide conditions under which observed refusal to supply is indeed a rational and independent decision consistent with firms' unilateral incentives.

The paper is organized as follows. In Section 1, we briefly discuss how collective refusals to supply have been treated under competition and regulatory policies and summarize several cases. In Section 2, we present a reduced form model to guide our subsequent analysis. In Section 3, we formally define the critical concepts of sufficiently sophisticated contracts and contract observability. Section 4, focusing on homogeneous product upstream and downstream markets and assuming Cournot competition in the downstream market, demonstrates how the assumptions on the observability of contracts can yield drastically different conclusions regarding whether non-collusive collective refusals to supply may arise as equilibrium phenomena. In Section 5, this time focusing on price competition with differentiated products, we show that the substitutability of products, the observability and the complexity of feasible contracts as well as the number of potential downstream competitors may alter the assessment of an observation of a collective refusal to supply. Section 6 concludes.

1 Policy towards Collective Refusals to Supply

In the European Union (EU), collective refusals to supply are most relevant for cases involving collective dominance.[2] The doctrine of collective dominance is closely inspired by the economic theory of tacit collusion. In particular, since the *Airtours* decision of the Court of First Instance (CFI), the finding of collective dominance requires, among others, that the dominant firms be able to mutually observe each others' behavior and that there are threats of credible and effective punishment that prevent firms from deviating from the common conduct.[3]

While the doctrine of collective dominance has rarely led to cases in the EU where firms have been condemned of abuse,[4] it has played a significant role in merger review and in the mobile telecommunications industry where a number of regulators have attempted to impose access or roaming obligations on the basis of a finding of collective dominance in the relevant markets.

The ComReg decision in Ireland to designate O_2 and Vodafone as jointly dominant in the wholesale mobile access and call origination market is a good example of the degree to which the doctrine of collective dominance relies on the theory of tacit collusion, and also how retail prices and offering access to upstream elements are treated as if they have similar strategic properties. In that decision, the conditions for establishing the existence of joint dominance included "the ability to coordinate", and "enforceability of compliance" (Doyle, 2006, p. 152). The ability to coordinate was assessed along two dimensions. The first was whether the competitors could coordinate on prices. The second was whether the competitors could coordinate on denying independent entities access to upstream products, in particular, wholesale airtime. Hence, prices

[2]Collective dominance, loosely speaking, refers to situations when oligopolists can coordinate their actions without resorting to agreements.

[3]In addition, there must not be any competitive constraints exercised by entities outside the parties engaged in collective dominance. See paragraph 62 of Judgment of the Court of First Instance of 6 June 2002 - Airtours plc v Commission of the European Communities, European Court Reports 2002 Page II-02585.

[4] A well known case is the European Commission's CEWAL decision where members of a shipping association were found to have abused their collective dominance by engaging in collective predatory practices to drive a main competitor out of the market. See Stroux (2004, pp. 101-105) for discussion.

and access to upstream products were treated as having similar strategic properties. The assumption, as summarized by Doyle (2006, p. 160) was that "if, for example, O_2 and Vodafone were not tacitly colluding, there would be a strong incentive to offer access to an entity capable of delivering higher profits to the upstream access provider." This is another way of saying that offering access when the competitor is not offering access is a best response in the stage game. A similar reasoning appears in the discussion of "enforceability of compliance". Building on the theory of tacit collusion with respect to prices, ComReg argued "the only deterrent or disciplinary mechanism required by Vodafone or O_2 to compel parallel behavior is the implicit threat of their reversion to the normal conditions of competition." (ComReg, 2004, p. 52). Regarding the specific dimension of providing access to wholesale products, ComReg stated: "If either did so, the other could retaliate in two ways – either by supplying similar services itself, or by cutting its retail prices. One or other, or both, responses could be rational, if the other firm undertook a course of action which reduced the retail price of services provided over its network. Overall it would appear that the potential reversion to the normal conditions of competition at the retail level constitutes a sufficiently serious disciplinary mechanism to perpetuate the consciously parallel market behavior being witnessed at present." (ibid). Implicit in this argument, again, is the belief that offering access when the competitor is not offering access is a best response in the stage game and hence the threat of retaliation is credible.[5]

Spain presents another example where a case of collective dominance by three operators was identified in the market for access and call origination. The analysis carried out by the Commission of the Telecommunications Market, CMT, carries a logic similar to the one described above. CMT argues that the three operators "Telefónica, Vodafone and Amena have adopted a common policy, which consists of a tacit agreement to persistently refuse to grant network access (rather than each operator acting unilaterally in its own self interest)."[6] Further, CMT argued that this tacit agreement was supported by retaliatory mechanisms against possible deviants. If a mobile network operator were to grant access, the other firms would retaliate by granting upstream access to other access seekers. An initial deviation and subsequent retaliation would result in an equilibrium with lower profits. So each operator had an incentive not to deviate in the first place. Again, the logic in this argument is that operators do not have a unilateral incentive not to provide upstream access.[7]

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[5]Eventually ComReg withdrew its decision on procedural grounds. See Doyle (2006, p. 143).

[6]European Commission, Case ES/2005/0330: Access and Call Origination on Public Mobile Telephone Networks in Spain: Comments pursuant to Article 7(3) of Directive 2002/21/EC.

[7]Another case has been observed in France. Initially the French telecommunications regulator (ART back then, now ARCEP) proposed to designate three operators (Orange, SFR and Bouygues Telecom) as jointly having significant market power in the market for access and call origination. The focal point of the alleged tacit collusion among the three mobile network operators consisted not simply of collective refusal to supply (as was the case in Ireland) but in the mobile operators deliberately not granting access to mobile virtual network operators (MVNOs) under terms and conditions that would enable them to exert significant competitive pressure at the retail level. The proposal was made even though there were a number of MVNO agreements in the market, hence

The issue of collective refusal to supply is also relevant for the ongoing discussion on how to allocate spectrum among operators. Spectrum access is essential for many communications services. Recently there is a move away from administrative methods of allocating spectrum towards more flexible and market based approaches. While this move is expected to lead to a more efficient allocation of the spectrum, it has also raised concerns that it may generate more scope for anti-competitive conduct. Anti-competitive conduct may arise if a company or a group of companies wish to hoard spectrum, or purchase spectrum in excess of technical needs so as to reduce access to spectrum by potential downstream competitors. In markets where multiple firms engage in such hoarding the situation becomes similar to collective refusal to supply in markets of mobile access and call origination.[8]

In the US, situations of joint refusal to supply are discussed under the topic of "group boycotts". The Federal Trade Commission discusses group boycotts in the following manner: "Any company may, on its own, refuse to do business with another firm, but an agreement among competitors not to do business with targeted individuals or businesses may be an illegal boycott, especially if the group of competitors working together has market power."[9] The term group boycott seems to cover many different market structures and forms of behavior, including not only joint refusals to supply by upstream firms to downstream competitors, but also situations where a group of (presumably not integrated) firms applying pressure on upstream suppliers to refuse to supply to their competitors in the downstream market, or where firms are refused acceptance to membership in trade organizations. As discussed by many legal scholars, this has generated quite a bit of confusion about the proper antitrust treatment of collective refusals to deal (Glazer 2002, Robinson 2002). One of the important points of controversy is whether concerted or group refusals to deal should be treated as a *per-se* violation of antitrust law. In practice, it is safe to say that concerted refusal to deal by upstream firms to downstream purchasers would be considered to be *per-se* illegal. In any case, most cases of group boycotts reflect strategic situations that are different from the one explored in this paper.[10]

ART's qualification. According to account provided by Cullen International (Cullen 2005) the Commission had "concerns as to whether the three network operators could all share a common understanding of such a complex focal point..." In any case, ARCEP later withdrew its notification because additional MVNO agreements were being implemented and the market could evolve to generate more competition.

[8] Cave (2010). See also ERG (2009).

[9] See http://www.ftc.gov/bc/antitrust/group_boycotts.shtm last accessed March 11, 2011.

[10] One case which does resemble the strategic situation examined in this paper is the 1945 case of Associated Press v US (326 US 1 (1945)). Associated Press (AP) had 1200 newspapers as its members. AP had its own reporters, in addition AP members furnished any news they had from their area and the AP then distributed the news to its members. AP bylaws prevented members from providing the news to non-members. The AP Board could admit new members as longs as the new member did not compete with an existing member. But if the applicant was a competitor of a member, then under the bylaws it had to either obtain a member's permission to join or win the support of the majority of AP members. The Supreme Court decision on the case stated that "inability to buy news from the largest news agency, or any one of its multitude of members, can have most serious effects on the publication of competitive newspapers"(see Elhauge and Geradin 2011 for a discussion).

A case of collective refusal to supply in the cement industry investigated by the Turkish Competition Authority (TCA) is closely related to the issues examined in this paper.[11] Cement industry typically contains two types of firms. Vertically integrated producers produce both clinker, which is used as an input in the production of cement, and cement itself. Producers that are not vertically integrated may purchase clinker from the upstream market, possibly from vertically integrated firms and grind them in their grinding plants and use it to produce cement. The two types of firms typically compete in the downstream cement market. In 2003, a cement producer in Turkey complained that two vertically integrated cement producers refused to supply it with clinker. The complainant and the vertically integrated firms were competitors in the downstream market. The TCA decided that the vertically integrated firms' behavior constituted a concerted action and fined them. In the TCA's approach, refusal to supply clinker was a result of concerted action. The argument was that absent concerted action between the clinker producers, each would have found it optimal to supply clinker to the downstream independent cement producer.

Another interesting decision by the TCA that deals with collective refusal to deal is related to roaming services in the mobile telecommunications industry in Turkey.[12] In that decision, the TCA concluded that mobile operators Turkcell and Telsim were jointly dominant in the GSM infrastructure services market, that the infrastructure owned by these operators constituted essential facilities during the entry of new operators into the GSM services market, and that the two incumbent operators abused their dominant position by refusing to provide national roaming services to the new entrant Is-Tim. The reasoning that led to the decision includes the argument that the interactions between the operators in the national roaming market resemble repeated games rather than one-shot games, and that the possibility of retaliation exists. In addition, the decision argues that while the incumbents were expected to compete with each other to provide roaming services, they chose not to and this choice acted as a barrier to entry. The decision further stated: "The reason that Turkcell and Telsim collectively choose to refuse the request [for roaming services] rather than obtain revenues by providing national roaming services lies in the fact that the revenue each would obtain by providing national roaming service is much lower than the revenue they expect to obtain in the long run if they act in a collective manner."

In all of the cases described above, the authorities' approach to collective refusal to supply is implicitly based on a model of oligopoly where each of the competing firms' best response is to supply when the rivals are refusing to supply. In this approach the market outcome of collective refusal to supply can only arise (presumably in a repeated interaction framework) if the competitors in the upstream market coordinate their behavior. In the next sections, we

The similarity with our model is that there is a collective agreement to withhold the provision of an input to competing downstream producers.

[11] Decision No. 05-05/42-17 dated 13.01.2005.

[12] Decision No. 03-40/432-186 dated 09.06.2003

show that this is not necessarily the case.

2 A Reduced Form Model

Consider an industry with two vertically integrated firms (VIFs), $i = 1, 2$. The VIFs produce a homogenous input good that is needed for the production of a downstream product. There is a third downstream competitor, $i = 3$, that can compete with the VIFs provided that it can obtain the input from one or both of the upstream firms. It takes one unit of the upstream input to produce one unit of the downstream product.

A one time (one-shot) interaction between these parties is composed of three stages. The timing of these stages is as follows:

- *Stage 1:* The VIF's offer upstream sales contracts to firm 3.
- *Stage 2:* Firm 3 decides which, if any, offer to accept.
- *Stage 3:* All firms that have/obtain the input compete in the downstream product market.

As stated in the introduction, we aim to investigate when and whether both VIFs refusing to supply the input to firm 3 can be an equilibrium of this game.[13] Let C denote a contract between one of the VIFs and the downstream competitor. In order to determine the equilibrium strategies of the VIFs, one needs to know:

1. The profits earned by the VIFs when they both refuse to sell to firm 3: $\hat{\Pi}_1$ and $\hat{\Pi}_2$
2. The profit of the VIF, say firm 2, that contracts with and sells to firm 3 with the optimal contract $\tilde{C}$ when the other VIF refuses to sell: $\tilde{\Pi}_2(\tilde{C})$
3. The profits earned by the VIFs when they compete at the downstream as well as the upstream market offering equilibrium contracts C_1^* and C_2^*: $\Pi_1^*(C_1^*)$ and $\Pi_2^*(C_2^*)$.

We will consider cases where contracts are sufficiently sophisticated (to be made precise in the next section) such that if a vertical sale takes place, the parties will agree on a contract that maximizes their joint profits conditional on the equilibrium behavior in the downstream market and the alternative offers.

Next we will start by describing the possible equilibria of the game in a general formulation. This will allow us to identify the conditions under which a refusal to deal by both VIFs can be called collusive. Then, in subsequent sections, we will solve the game for different modes of competition and market features and show that the possibility of a collusive outcome is highly dependent on these.

[13]In this setup a refusal to supply would simply correspond to a contract offer that included an infinitely high payment for the sale of the input.

First of all, note that whenever it is in the interest of one of the VIFs, say firm 2, not to sell the input when the other VIF also refuses to sell, that is, whenever $\tilde{\Pi}_2(\tilde{C}) \leq \hat{\Pi}_2$, both firms not selling is a Nash equilibrium. Even though the outcome of this equilibrium involves both firms 1 and 2 refusing to sell to firm 3, there is nothing anti-competitive about it. Both firms compete in good faith and simply find it unilaterally optimal to not sell the input to their downstream competitor. Obviously, there may be another equilibrium where both VIFs want to sell the input even when the above condition is satisfied. If one VIF decides to sell to firm 3, then it could very well be optimal for the other VIF to supply the input, too.

If, on the other hand, $\tilde{\Pi}_2(\tilde{C}) > \hat{\Pi}_2$, that is if firm 2 prefers to sell to firm 3, then both VIFs will compete in the upstream market. Notice that in an equilibrium where both VIFs are willing to sell the input to the third firm, they should obtain the same profit. This is true because otherwise one of the VIFs would have an incentive to adjust its offer and earn more. In other words, we must have $\Pi_1^*(C_1^*) = \Pi_2^*(C_2^*)$.

So far, we have considered the situation in the framework of a one-shot game. Now suppose that the one-shot game we described above is played infinitely many times. In each period the VIFs make offers to firm 3, and then all active firms compete at the downstream level. In this setup, there will be a scope for collusion if the VIFs can earn more by coordinating their strategies and by collectively refusing to supply the input. That is, whenever $\Pi_i^*(C_i) < \hat{\Pi}_i$ for $i = 1, 2$, firms may collude by, for example, adopting trigger strategies. In such an equilibrium of this repeated game, fear of future punishments in case of deviation allows the VIFs to sustain strategies that do not constitute a Nash equilibrium of the one-shot game.

When one considers collusive behavior, colluding on not supplying a downstream rival is only one possible dimension where the VIFs can coordinate their actions. They could also collude on downstream or upstream prices and output levels. Since collusion on prices and output levels are well studied in the literature, we abstract from these dimensions of collusion and focus only on the (binary) decision of whether to supply the downstream rival or not.

In summary, our definition of a collusive refusal to supply will refer to equilibria which can not be supported as the equilibrium outcome of a static, one-shot game, but which can result when firms adopt history dependent strategies that involve punishments in case of deviation from the collusive outcome. There will also be examples where upstream firms refuse to supply the downstream competitor in the equilibrium of a one-shot game. We will refer to such equilibria as competitive. These definitions are clearly debatable, however they are in line with the treatment of collusion in the literature on competition law.[14] We touch upon this issue further in Section 5.4 but leave a more thorough analysis for further research.

[14] See, for example, Motta (2004) and Werden (2005).

3 Sophisticated Contracts and Contract Observability

Sophisticated contracts allow the parties engaged in negotiations to achieve the maximum joint profit conditional on their rival's behavior. Furthermore, such contracts have an available instrument that allows the contracting parties to share this profit between themselves. The use of sophisticated contracts simplifies the analysis a great deal while it does not influence the economic mechanisms we intend to study. In addition, by focusing on such contracts, we avoid situations where the reasons behind a refusal to supply are inefficiencies in contracting.[15] In our analysis, we not only employ sophisticated contracts when their terms are observable by all parties prior to downstream competition, but also when only contracting parties observe the terms of the contract.[16]

As discussed above, whether an observation of a collective refusal to supply can be considered collusive depends on whether it is a best response for a VIF not to supply the downstream competitor when the rival VIF chooses not to supply. Based on this, we can, at first, restrict our focus on cases where given that VIF 1 refuses to sell to firm 3, VIF 2 contemplates whether it should sell to firm 3. The strategic effects of a contract between VIF 2 and firm 3 depend on whether its terms are observed by VIF 1. When these terms are observed by all parties—a case we term *observable contracts*, a contract signed between VIF 2 and firm 3 may provide a first mover advantage to the contracting parties as they can implement outcomes which otherwise are not possible, by committing to the terms of the contract. On the other hand, when the terms of the contract are observed only by the contracting parties, we refer to these contracts as *unobservable*. In such a situation, the equilibrium action chosen by VIF 1 is a best response against the actions of VIF 2 and firm 3 that are induced by the equilibrium contract.

We can now define a sophisticated contract under these two informational scenarios. Let $\pi_i(a_1, a_2, a_3)$ denote the profit and a_i the downstream action of firm i, $i = 1, 2, 3$. Suppose that VIF 2 enters in a contractual relationship with firm 3, while VIF 1 is known to not deal with firm 3. It is common knowledge whether firms 2 and 3 successfully sign a contract. However, the details of this contract may or may not be observable.

When the terms of the contract, C, are observable, firm 1 will formulate its strategy, a_1, conditional on these. Thus, in the second stage all three firms will choose their actions as a function of the agreed upon contract. Let us denote by $a_i(C)$ the equilibrium action chosen by firm i.

[15] Obviously, such inefficiencies may arise when the contracts that can be employed are restricted by competition authorities. Consequently, restrictions on feasible contracts may play a crucial role in assessing whether a case of collective refusal is collusive or not. We consider such restrictions on contracts further below.

[16] The degree of sophistication of contracts and their observability seem to have significant empirical and practical relevance. In the mobile electronic communications industry, for example, in many countries MVNO contracts are not public. Hausman and Sidak (2007, p. 397), in their analysis of the ComReg decision discussed above, mention that the terms of such contracts "are totally lacking in transparency, for the simple reason that the MVNO and the MNO negotiate them confidentially". Regarding the degree of sophistication of contracts, they state "[T]he agreements with which we are familiar contain complex terms with highly nonlinear prices".

Correspondingly, the contracting parties, firms 2 and 3, earn jointly $\pi_2(a_1(C), a_2(C), a_3(C)) + \pi_3(a_1(C), a_2(C), a_3(C))$. An important feature of a sophisticated contract is that it includes an instrument which can be used as a transfer between the contracting parties—for example, a fixed payment, which in turn implies that when firm 2 makes its contract offer, it does so with the objective of maximizing the joint profit of the contracting parties, i.e.

$$C_O^* = \underset{C}{\operatorname{argmax}}\, \pi_2(a_1(C), a_2(C), a_3(C)) + \pi_3(a_1(C), a_2(C), a_3(C))$$

with the corresponding profits of $\pi_O^* = \pi_2(a_1(C_O^*), a_2(C_O^*), a_3(C_O^*)) + \pi_3(a_1(C_O^*), a_2(C_O^*), a_3(C_O^*))$.

Let $a_1(a_2, a_3)$ be the best response action chosen by firm 1 against a_2 and a_3. Then, the maximum joint profits that the coalition of firms 2 and 3 can achieve is given by

$$\max_{a_2, a_3} \pi_2(a_1(a_2, a_3), a_2, a_3) + \pi_3(a_1(a_1, a_2), a_2, a_3).$$

Definition 1 ***(Sufficiently Sophisticated Observable Contracts)*** *A contract, C^*, when its details are observable by all firms before they choose their actions a_i, $i = 1, 2, 3$, is considered to be sufficiently sophisticated if*

$$\pi_O^* = \max_{a_2, a_3} \pi_2(a_1(a_2, a_3), a_2, a_3) + \pi_3(a_1(a_1, a_2), a_2, a_3)$$

By means of a sufficiently sophisticated *observable* contract, the contracting parties can implement their joint payoff as if they select the actions in order to maximize their joint profits before their rival gets to act. For example, in a homogeneous Cournot context, the contracting parties will be able to act as a Stackelberg leader if they have access to sufficiently sophisticated contracts and the terms of the agreed upon contract are observable by their downstream rival, firm 1. Hence, sufficiently sophisticated observable contracts provide contracting parties with a commitment device.

On the other hand, when contracts are unobservable, firms 2 and 3 agree upon a contract without being able to influence the action of firm 1, a_1. Then, for a given contract, C, and a belief about the equilibrium action of firm 1, a_1^*, they non-cooperatively choose their equilibrium actions as functions of these, $a_2(a_1^*, C)$ and $a_3(a_1^*, C)$. Note that here firms 2 and 3 observe C, but they have to act conditional on a belief about the equilibrium action of firm 1. Once again, we assume a sophisticated contract contains an available instrument which can be used for sharing the joint profits between the contracting parties. In this case, the contract offered by firm 2 will maximize the joint profits of firms 2 and 3, i.e.

$$C_U^* = \underset{C}{\operatorname{argmax}}\, \pi_2(a_1^*, a_2(a_1^*, C), a_3(a_1^*, C)) + \pi_3(a_1^*, a_2(a_1^*, C), a_3(a_1^*, C))$$

with the corresponding profits of

$$\pi_U^* = \pi_2(a_1^*, a_2(a_1^*, C_U^*), a_3(a_1^*, C_U^*)) + \pi_3(a_1^*, a_2(a_1^*, C_U^*), a_3(a_1^*, C_U^*)).$$

Firm 1 will choose its equilibrium action a_1^* as a best response against its belief about the equilibrium contract firms 2 and 3 agree upon and its beliefs about the corresponding downstream actions of the two firms, i.e.

$$a_1^* = \underset{a_1}{\operatorname{argmax}} \, \pi_1(a_1, a_2(a_1^*, C_U^*), a_3(a_1^*, C_U^*)).$$

The maximum profit that a decision maker can achieve by choosing a_2 and a_3, when these choices cannot be observed by firm 1, is simply the maximum joint profit against a belief about the action of firm 1 and is given by

$$\max_{a_2, a_3} \pi_2(a_1^*, a_2, a_3) + \pi_3(a_1^*, a_2, a_3).$$

Definition 2 ***(Sufficiently Sophisticated Unobservable Contracts)*** *A contract, C^*, when its details are unobservable to firm 1 but its existence is known before all firms choose their actions a_i, $i = 1, 2, 3$, is considered to be sufficiently sophisticated if*

$$\pi_U^* = \max_{a_2, a_3} \pi_2(a_1^*, a_2, a_3) + \pi_3(a_1^*, a_2, a_3)$$

By means of a sufficiently sophisticated *unobservable* contract, the contracting parties can implement their joint payoffs as if they select their actions in order to maximize their joint profits as a best response to the equilibrium action of firm 1. In other words, a sophisticated unobservable contract is capable of implementing the profits the contracting parties can achieve if they were to merge their downstream operations.

It is important to note that at this stage we do not consider possible restrictions competition policy may impose on the contracts that the firms may choose. Without such restrictions, it is straightforward to see that both sufficiently sophisticated observable and unobservable contracts exist. For example, a contract that prescribes prices or outputs (forcing contracts) can satisfy the conditions we have put forth in the above definitions. However, there are many other contractual forms that may achieve the same objectives. Our approach here abstracts from the limitations that may arise due to feasibility of contractual forms and investigates whether a collective refusal to supply can arise even in such an ideal environment. We will later return to the implications of restrictions on contractual forms in Section 5.2.2. Equipped with the two definitions above, we can proceed with our analysis of outcomes under different modes of competition.

4 Homogeneous Goods Cournot Competition

In order to focus on the strategic nature of contracts and abstract from contractual relationships that may arise due to cost saving motives, we will assume a constant returns to scale production technology both in the upstream and the downstream segments of the market.[17] Furthermore,

[17] When there are decreasing returns to scale, a firm may benefit from distributing its production (marketing) between two plants (outlets).

we assume that both firms produce a homogeneous input good at a marginal cost of c_I. One unit of the input good is then used in producing one unit of the final good at a marginal cost of c_O. The inverse demand in the downstream market is given by $P(Q)$ where Q is the total number of units of the final good available. We assume that the inverse demand function possesses the necessary properties so that a downstream Cournot equilibrium exists whenever two or three firms produce the final good. In this context, suppose that firm 1 produces the input good which is then transformed to the final good and sold by its downstream arm. Firm 2, on the other hand, in addition to producing the input good to supply its own downstream arm, considers contracting with firm 3 as well.

In this environment, when firm 2 can employ sufficiently sophisticated observable contracts, the coalition of firms 2 and 3 can replicate an outcome where they would earn as much as a firm acting as a Stackelberg leader.[18] If firm 2 did not contract with firm 3, both VIFs would produce the Cournot equilibrium quantities in a duopoly, and obtain the corresponding Cournot duopoly profits. Then, whether both firms refuse to supply the downstream rival in a static equilibrium depends on the comparison of Cournot duopoly and Stackelberg leader profits which in turn suggests that we should never observe collective refusals to supply in a homogeneous goods industry with constant returns to scale technologies. We formalize these arguments in the next proposition.

Proposition 1 *In a homogeneous goods industry where both upstream and downstream productions are characterized by constant returns to scale technologies, if firms have access to sufficiently sophisticated* observable *contracts and downstream competition is in quantities, all observed collective refusals to deal must be collusive.*

Proof. See the Appendix.

When contracts are observable, the contracting VIF and firm 3 collectively gain a first mover advantage. By committing on a contract they can also commit to a larger output, namely the Stackelberg leader quantity in a two firm industry. This, in turn, reduces the output of the non-contracting VIF, thereby increasing the combined profits of the two contracting firms. Since both VIFs desire to obtain this Stackelberg advantage, they ultimately compete away any gains that may arise from their ability to commit to a higher output. The only Nash equilibrium of the one-shot game involves both VIFs offering to supply the input to firm 3 yielding Stackelberg follower profits for both VIFs. This profit is less than the Cournot duopoly profit, which the VIFs would earn when they simultaneously announce their outputs.[19] Obviously, both VIFs would prefer an outcome where the third firm cannot obtain the input and remains out of the

[18]It should be noted that in the case of homogeneous goods and quantity competition, a simple two-part tariff wholesale contract will be sufficiently sophisticated. The contracting parties ultimately have to determine their total output which can be achieved by setting an appropriate per-unit wholesale price, and then the fixed fee can be used to share the resulting profit among the two contracting parties.

[19] See Appendix for a proof of this claim under very mild conditions.

market. In a setup where the one-shot game is repeated infinitely many times, the VIFs can earn more by coordinating their strategies and by collectively refusing to supply the input by adopting trigger strategies, provided that their discount rates are high enough. In other words, in a dynamic setup the VIFs do have incentives to collude on not supplying the third firm. In a world described by observable contracts, the striking implication for antitrust policy is then that every observed outcome where two rival VIFs do not supply the input to a downstream competitor is collusive and therefore anti-competitive.

The analysis above considered the case where any contract that is agreed upon becomes publicly known before firms compete in the downstream market. The nature of the strategic interaction would be considerably different if the non-contracting firm did not observe the terms of the contracts as they competed in the downstream market. In our base setting, where we analyze only one of the VIFs making an upstream offer to firm 3, the quantity choice of the non-contracting VIF would not depend on the unobservable contract agreed upon by its rivals. In turn, the contracting firms cannot affect the output choice of their non-contracting rival by their choice of the wholesale price.

Consider once again that VIF 1 refuses to contract with firm 3 and firms 2 and 3 are considering whether to agree on a contract. The sequence of events is as follows: VIF 2 makes a contract offer to firm 3. All firms observe whether there is an agreement or not, however the details of the contract are known only to VIF 2 and firm 3. Subsequently, all firms compete in the downstream market by selecting their production levels.

When contracts are unobservable, firms 2 and 3 will select the contract they sign as well as their output levels, as a best response to the expected output of firm 1, q_1^*. Given the fact that they can use a sophisticated contract, firms 2 and 3 will select their output levels in order to maximize their joint profit. That is, the contracting parties solve

$$\max_{q_2,q_3} \pi_2(q_1^*, q_2, q_3) + \pi_3(q_1^*, q_2, q_3) = (P(q_1^* + q_2 + q_3) - c_I - c_O)(q_2 + q_3)$$

which implies that the important magnitude to select is the total output by the firms, $q_T = q_2 + q_3$. Notice that given the constant returns to scale assumption, how this total output is distributed between the two downstream firms is irrelevant. In particular, firm 2 will be indifferent between an outcome where the whole output is produced in its own facilities—which can be achieved without a contract, and an outcome where some of it is produced by firm 3. Thus, what VIF 2 can achieve with or without a contract is the same. Indeed, if there were even minute costs of contracting, VIF 2 would be better off not contracting with firm 3.

From firm 1's perspective, regardless of whether firms 2 and 3 agree on a contract, the rivals will be expected to produce $q_2 + q_3 = q_2^d$, and hence as a best response, firm 1 will choose to produce $q_1^* = q_1^d$. The equilibrium outcome is one where the market price is equal to the duopoly price $P(q_1^d + q_2^d)$ as the total output will be $q_1^d + q_2^d$ regardless of whether all three firms or only the two VIFs are active.

Given that in equilibrium firm 2 may prefer not to supply firm 3 when it expects firm 1 not to supply (and since the same arguments apply when firms 1 and 2 switch roles), both firms not supplying firm 3 constitutes a Nash equilibrium. There are, of course, other equilibria where one or both firms engage in contractual relationships with firm 3. All these equilibria imply exactly the same payoffs, namely, the duopoly profits when the two VIFs make simultaneous output choices–the profits they would obtain by collectively refusing to supply. There is therefore no need to resort to employing tacitly collusive strategies and any observation of a collective refusal to supply in this case is the result of a static Nash equilibrium that represents competition in good faith. We summarize our findings under unobservable contracts in the next proposition.

Proposition 2 *In a homogeneous goods industry with constant returns to scale technologies both in the upstream and the downstream markets, if firms have access to sufficiently sophisticated* unobservable *contracts and downstream competition is in quantities, all observed collective refusals to deal are the result of competition in good faith.*

Proof. See the preceding discussion. ■

When contracts are unobservable, the contracting VIF cannot alter the output decision of its rival VIF by signing a wholesale contract. In this case, if the contracting VIF were to have two downstream plants, its best response would be to allocate a duopoly equilibrium quantity in anyway between these two plants. As a result, the total output in the market is simply the equilibrium output in a two firm Cournot duopoly where firms announce their output choices simultaneously. Therefore, being able contract with a third firm has no profit advantage—the resulting profits of the VIF would be the same with or without a contract. If contracting has costs, even if very small, both VIFs would be better off by unilaterally refusing to supply firm 3. One would then observe an outcome where both firms collectively refuse to supply their downstream rival. However, it is clear that in this case firms behave in good faith and there is no collusive behavior underlying this outcome.

The results laid out in Propositions 1 and 2 are in stark contrast with one another.[20] Under Cournot competition with observable contracts all observed collective refusals to supply must be collusive, however with unobservable contracts no collusive behavior is necessary to support the same outcome. When contracts are observable, but restricted in form—for example to be linear, it is possible to construct examples where refusing to supply firm 3 becomes a best response for firm 2 provided firm 1 refuses to supply as well. Thus, in judging whether an observed collective refusal to deal is collusive, one also needs to take into account the permitted contractual forms.

[20] Dewenter and Haucap (2006) study a linear demand model with zero marginal costs where contracts are restricted to be fixed payments only, and conclude that all observed collective refusals to supply should be considered collusive. Their result relies on the ability of the contracting parties to commit to producing after an agreement, hence their model is a specific version of our observable contracts case. We not only generalize their results, but also show that when contracts are not observable, significantly different conclusions emerge regarding the collusiveness of outcomes that exhibit collective refusals to supply.

Note that in the construction of our arguments the observability of contracts is used synonymously with commitment. Even when the terms of a contract are observable, if it is known that they are easily renegotiable, the implication would be that the strategic interaction would unravel in a manner that collective refusals would arise as an equilibrium outcome even without resorting to tacitly collusive behavior. Thus, from a practical point of view, if renegotiation of observable contracts is likely and feasible contracts are restricted by competition law, treating collective refusals to supply as a natural outcome of competition in good faith seems to be appropriate.

5 Differentiated Goods Price Competition

The rather clear cut results we find for the homogeneous goods case unfortunately do not carry over to situations with differentiated products and price competition. With differentiated products, even though introducing a new brand in the downstream market increases competition, when downstream brands are sufficiently differentiated—when consumers value variety—the potential consumer surplus also increases. This introduces a new motivation for a VIF to enter into a contractual relationship with a potential downstream competitor as the VIF may be able to appropriate some of the value generated by a new downstream variant. This new effect is going to strengthen the incentives of a VIF to supply a downstream rival when contracts are observable. These incentives may arise even when contracts are unobservable. It turns out that the characterization of these outcomes is not easily achieved in a general setting even when we focus on sufficiently sophisticated contracts. We will, therefore, first state a rather general and straightforward finding in the case of observable contracts and In subsections 5.2 and 5.3, by using a common linear demand formulation due to Shubik and Levitan (1971), we discuss various scenarios for the case of unobservable contracts.

Consider an industry characterized by a demand system for three downstream products, $q_i(p_1, p_2, p_3)$, $i = 1, 2, 3$. Once again, firms 1 and 2 are vertically integrated and have facilities to produce the input necessary for their final products. Given access to this input, which is assumed to be homogeneous, firm 3 can also compete in the downstream market. Firms compete in the downstream market by announcing their prices simultaneously. If firm 3 cannot participate in the downstream market, we assume that its price is set infinitely high, and the two remaining firms face the demand functions $q_i^d(p_1, p_2) = q_i(p_1, p_2, \infty)$, $i = 1, 2$.[21] As in the homogeneous goods setting, we assume that the input can be produced with a constant marginal cost of c_I and given access to the input, all three firms can produce the final good at a constant marginal cost of c_O.[22]

[21] Thus, we assume that $q_3(p_1, p_2, \infty) = 0$ for all p_1 and p_2.

[22] Once again, by focusing on constant returns to scale technologies we avoid contracts that may arise due to cost efficiency reasons. In our set-up, any contract between a VIF and the downstream competitor arises due to strategic reasons.

5.1 The Case of Sufficiently Sophisticated Observable Contracts

When firms have access to sufficiently sophisticated observable contracts, the contracting parties will be able to implement outcomes where they set the prices of their two downstream brands prior to competing with the rival VIF. On the other hand, in the absence of a contract the two VIFs will compete as Bertrand duopolists. In the next proposition, we establish that when one of the VIFs does not supply the input to firm 3, the best response of the other VIF is to offer a contract to firm 3. As a result, in equilibrium we should observe at least one VIF going into a contractual relationship with firm 3.

Proposition 3 *With differentiated goods and constant upstream and downstream marginal costs, when one vertically integrated firm refuses to supply the upstream input, the other vertically integrated firm always finds it profitable to sell to the downstream rival if they can sign a sufficiently sophisticated observable contract.*

Proof. See the Appendix.

The intuition is that if a VIF, even in a duopoly setting, can commit to a price by means of a contract before the rival VIF sets its price, it would do so and earn higher profits than it does under simultaneous moves Bertrand competition. If in addition a third downstream firm increases consumer value and the contract allows the VIF to appropriate some of this added value, there will be an even stronger incentive to engage in a contractual relationship. Clearly, if the non-contracting firm earns less than its vertically integrated rival, it would also have an incentive to make a contract offer and the two VIFs may compete upstream. In this case there is an equilibrium of the one-shot game where both VIFs make an offer to their downstream competitor and the downstream firm accepts one of these offers. However, depending on the underlying demand system and the costs of production, it is possible that the non-contracting VIF welcomes the contractual agreement between its rivals. An example of such a possibility can be found in Bourreau et. al. (2010). Therefore, there may be equilibria where firms 2 and 3 agree upon a contract, while firm 1 does not make a countering contract offer. Moreover, there may exist a case when firm 2 enters in a contractual relationship and the contract postulates a very high price for the product of firm 3, nevertheless allows firm 2 to commit to a moderately high price. Such a strategy may prove profitable when facing two downstream rivals makes firm 1 more aggressive than competing with a single rival. These outcomes depend on the specification of the underlying demand system, and hence escape a more general formulation.[23] Regardless, Proposition 3 implies that collective refusals to supply do not arise in a static setting when contracts are observable. When a collective refusal to supply is observed, concerns about the competitiveness of the market are warranted.

[23] In the Appendix, using a stylized demand system, we show that an equilibrium where firms 2 and 3 sign a contract prior to downstream competition and firm 3 does not produce in the downstream market may exist.

5.2 The Case of Unobservable Contracts with Linear Demands

When contracts are unobservable and there is one in place, the contracting parties (the supplying VIF and firm 3) prefer to sell positive quantities of the two downstream brands they control. Unlike in the homogeneous goods case, the fact that variety may be valuable implies that at least one of the VIFs may prefer to supply firm 3 even when contracts are unobservable. This fact may result in equilibria where the two VIFs compete in supplying firm 3, in which case we can once again claim that an observed collective refusal to deal must be collusive, contrary to our findings under the assumption of homogeneous goods quantity competition. On the other hand, there may also be an equilibrium where a VIF would prefer not to offer a contract to firm 3 when its rival does not offer a contract either. In this case, the resulting collective refusal to supply need not be collusive. Which one of these equilibria will arise hinges on the details of the underlying demand system.

Next, we proceed by assuming that the demand system is linear in prices, and characterize the nature of collective refusals to supply. Suppose that the downstream products of the VIFs and firm 3 are perceived to be imperfect substitutes by the consumers. The upstream product is still homogenous. Competition between active firms in the downstream market takes place in prices p_j, $j \in \{1, 2, 3\}$. For simplicity, and with little loss of generality given the linear demand system, we normalize marginal costs of production to zero for all firms both at the upstream and the downstream levels.

Acknowledging the fact that the assumption on the underlying demand system is likely to affect our findings in an important way, we proceed with a specific but commonly used demand model to highlight the potential role the degree of substitution between different downstream products, the complexity of feasible contracts, and the number of potential downstream competitors play in determining whether an observation of collective refusal to supply reflects collusion. Specifically, we adopt a version of the Shubik and Levitan (1971) model of linear demand.

Let a representative consumer of the downstream products have the following quasi-linear utility function:

$$U(q_1, q_2, q_3, y; \gamma) = \sum_{j=1}^{3} q_j - \frac{1}{2}\left(\sum_{j=1}^{3} q_j\right)^2 - \frac{3}{2(1+\beta)}\left[\sum_{j=1}^{3} q_j^2 - \frac{\left(\sum_{j=1}^{3} q_j\right)^2}{4}\right] + y \qquad (1)$$

where q_i, $i = 1, 2, 3$, are the quantities of the three downstream products consumed, and y is a composite good whose price is normalized to unity. It is straightforward to derive the corresponding demand functions which can be found in the Appendix. The only parameter of the demand functions, β, represents the substitutability of the downstream products. The degree of product differentiation decreases in β, and as $\beta \to \infty$ the products become perfect substitutes.

The first step in our analysis is to consider the case where both VIFs refuse to supply the input to firm 3. In this case we denote the equilibrium prices by $\{\hat{p}_1, \hat{p}_2\}$ and the profits by

$\{\hat{\Pi}_1(\hat{p}_1, \hat{p}_2), \hat{\Pi}_2(\hat{p}_1, \hat{p}_2)\}$. The derivations and the expressions are provided in the Appendix.

We proceed by analyzing the question of a collective refusal to deal under two subsections: when sufficiently sophisticated contracts are available and when they are not available. In practice, competition policy often restricts the nature of the feasible contracts, for example by disallowing minimum price restrictions. It turns out that contractual flexibility plays a major role in determining when a collective refusal to deal can be deemed collusive.

5.2.1 Sophisticated Unobservable Contracts

Once again we start the analysis by examining the best response of firm 2 when firm 1 refuses to supply the input. It is useful to recall the underlying informational structure when contracts are unobservable. In this case, firm 1 knows whether there is a contractual agreement between firms 2 and 3, but does not know the details of the contract. While firm 1 sets p_1, the coalition of firms 2 and 3 sets p_2 and p_3 to maximize their joint profits for a given p_1. The profit of firm 1 when it refuses to supply while firm 2 does supply the input is denoted by

$$\tilde{\Pi}_1(p_1, p_2, p_3) = p_1 q_1(p_1, p_2, p_3).$$

Similarly, the profit of firm 2, which appropriates the joint profit of the coalition of firms 2 and 3, is given by

$$\tilde{\Pi}_2(p_1, p_2, p_3) = p_2 q_2(p_1, p_2, p_3) + p_3 q_3(p_1, p_2, p_3).$$

The downstream prices and the profits corresponding to the equilibrium of this subgame, where firm 2 supplies and firm 1 refuses to supply, are denoted by $\{\tilde{p}_1, \tilde{p}_2, \tilde{p}_3\}$ and $\{\tilde{\Pi}_1(\tilde{p}_1, \tilde{p}_2, \tilde{p}_3), \tilde{\Pi}_2(\tilde{p}_1, \tilde{p}_2, \tilde{p}_3), \tilde{\Pi}_3(\tilde{p}_1, \tilde{p}_2, \tilde{p}_3)\}$. Note that these are functions of the only parameter of the model, β. The expressions are provided in the Appendix.

A simple comparison of the profit functions reveals that $\hat{\Pi}_2(\hat{p}_1, \hat{p}_2) > \tilde{\Pi}_2(\tilde{p}_1, \tilde{p}_2, \tilde{p}_3)$ for $\beta > 32.89$. That is, for a sufficiently high substitutability of the downstream products, firm 2 is better off refraining from contracting with firm 3, when it expects firm 1 to refuse to supply firm 3. In other words, both firms find it in their interest not to sell to firm 3, and a collective refusal to supply arises. Notice that this outcome prevails without any collusion since it is an equilibrium of the one-shot game. However, there is also another equilibrium where both VIFs would like to sell the input. Starting from a position where one VIF sells the input and the other does not, it is easy to show that the VIF that sells to firm 3 makes a higher profit.[24] As a result, the other VIF would like to make an offer to firm 3 as well, and in equilibrium both VIFs make identical competitive offers.

On the other hand, whenever $\beta < 32.89$, $\hat{\Pi}_2(\hat{p}_1, \hat{p}_2) \leq \tilde{\Pi}_2(\tilde{p}_1, \tilde{p}_2, \tilde{p}_3)$ and the best response of firm 2, when firm 1 refuses to supply, is to offer a contract to firm 3. In addition, it turns

[24]Note that in this case the contracting parties choose downstream prices as if they have merged. Using a linear demand as well as a more general demand specification, Davidson and Deneckere (1985) show that a merger always leads to higher total profits for the merging parties.

out that $\hat{\Pi}_1(\hat{p}_1, \hat{p}_2) > \tilde{\Pi}_1(\tilde{p}_1, \tilde{p}_2, \tilde{p}_3)$, for all $\beta > 0$. Note that at the duopoly equilibrium both VIFs earn equal profits and therefore $\tilde{\Pi}_1(\tilde{p}_1, \tilde{p}_2, \tilde{p}_3) < \tilde{\Pi}_2(\tilde{p}_1, \tilde{p}_2, \tilde{p}_3)$. Thus, expecting its rival to make an offer, firm 1 will also make an offer to firm 3. Firms 1 and 3 will compete away all the additional profit they make by obtaining a contract with firm 3, and in equilibrium they should each earn a profit of $\tilde{\Pi}_1(\tilde{p}_1, \tilde{p}_2, \tilde{p}_3)$ which was defined above. Given that this profit is less than what they could earn in a duopoly, the two VIFs have an interest in coordinating their decisions regarding whether or not to supply firm 3. In a scenario where both firms interact in a repeated fashion, they could sustain an outcome where they both refuse to sell by employing, for example, trigger strategies. Hence, when the downstream products are sufficiently differentiated, an observation of a collective refusal to deal must be collusive. We summarize these findings in the next proposition and the following corollary.[25]

Proposition 4 *In a market characterized by a linear differentiated products demand system and unobservable sophisticated contracts, for sufficiently differentiated downstream products ($\beta <$ 32.89), the only equilibrium of the one-shot game involves both VIFs making contract offers which yield identical profits. When downstream products are sufficiently close substitutes ($\beta >$ 32.89), in addition to the equilibrium mentioned above, the static game also has an equilibrium where neither VIF prefers to engage in a contractual relationship with the downstream firm.*

Corollary 1 *In a market characterized by a linear differentiated products demand system and unobservable sophisticated contracts, for sufficiently differentiated downstream products ($\beta <$ 32.89), an observation of a collective refusal to supply must be collusive when firms have access to sufficiently sophisticated unobservable contracts. In contrast, when downstream products are sufficiently close substitutes ($\beta >$ 32.89), a collective refusal to supply may arise as a result of competition in good faith.*

The above results suggest that in judging whether an observed collective refusal to supply is anti-competitive, the deciding authority must take the nature of product differentiation, in addition to whether contracts are observable or unobservable, into account. The implication of Corollary 1 is that when contracts are unobservable and downstream products are close substitutes, vertically integrated firms may prefer not to engage in contractual relationships with a downstream rival. However, there is another equilibrium where both VIFs supply the input. From the point of view of the VIFs, the equilibrium entailing refusal to supply dominates the equilibrium where both VIFs supply the input. In contrast, a utilitarian social planner would prefer the latter equilibrium. Given that the VIFs would naturally be expected to coordinate on the equilibrium with collective refusal to supply, what the stance of competition policy should be towards such coordination when there are multiple equilibria is an interesting question. We take up this issue in section 5.4.

[25]The proof of the proposition follows from the arguments above and straightforward algebraic calculations which are provided in the Appendix.

5.2.2 Unsophisticated Contracts

In deriving the result in Proposition 4, we relied on the fact that firms have access to sufficiently sophisticated contracts. Next, we would like to demonstrate that restrictions on contracts will also have an impact on the incentives of the VIFs to supply the input to a downstream rival. Suppose that firms are able to employ two-part tariff contracts only. Such a contract specifies a per unit wholesale price, w, and a fixed fee, F. Note that in the current environment two-part tariff contracts are not sufficiently sophisticated. The reason is simple. The downstream prices are functions of the per unit wholesale price w. Although firm 2 can commit to this wholesale price prior to downstream competition, it cannot commit to its own downstream price which would be necessary to implement the profit maximizing outcome for the contracting firms.[26] We maintain the assumption that details of a contract are observable only to the contracting parties while the existence of a contract is known by all market participants. To simplify the analysis, we now also assume that firm 3 lacks any bargaining power and thus the VIFs may offer their contracts on a take it or leave it basis.

Suppose that VIF 1 refuses to sell the input to firm 3, and VIF 2 makes a two-part tariff offer. The profit function of VIF 1 is simply given by

$$\Pi_1(p_1, p_2, p_3) = p_1 q_1(p_1, p_2, p_3).$$

On the other hand, the profit function of VIF 2 will consist of its downstream as well as upstream profits:

$$\Pi_2(p_1, p_2, p_3) = \underbrace{p_2 q_2(p_1, p_2, p_3)}_{\text{Downstream Profit}} + \underbrace{w q_3(p_1, p_2, p_3) + F}_{\text{Upstream Profit}}.$$

Finally, the profit function of firm 3 is given by

$$\Pi_3(p_1, p_2, p_3) = (p_3 - w) q_3(p_1, p_2, p_3) - F.$$

Each one of the three firms maximizes its profit function with respect to its price. However, the value of w is not known by VIF 1. Therefore, VIF 1 will formulate an equilibrium price as a best response to its equilibrium expectation of $\tilde{p}_2(\tilde{w})$ and $\tilde{p}_3(\tilde{w})$, and therefore, on the equilibrium expectation of $\tilde{w}$. At the same time, firms 2 and 3 know the value of w when they decide on their prices, while their prices also have to be formulated as a best response to their equilibrium expectation of firm 1's price, $\tilde{p}_1$.

Suppose firm 1 is expected to charge $\tilde{p}_1$ in equilibrium and firm 2 sets a wholesale price w. Denote by $p_2(\tilde{p}_1, w)$ and $p_3(\tilde{p}_1, w)$ the equilibrium prices that firms 2 and 3 offer in this scenario. Since by using F, firm 2 can appropriate the profits of firm 3, it will set w so as to maximize

[26] Note that, a per unit wholesale price contract —which has been used in the models of, among others, Ordover and Schaffer (2007), Höffler and Schmidt (2008) and Bourreau et. al. (2011), is a further restriction on this contractual form.

the joint profits of firms 2 and 3, that is

$$\tilde{w} \quad = \quad \underset{w}{\text{argmax}} \left[p_2(\tilde{p}_1, w) q_2(\tilde{p}_1, p_2(\tilde{p}_1, w), p_3(\tilde{p}_1, w)) + p_3(\tilde{p}_1, w) q_3(\tilde{p}_1, p_2(\tilde{p}_1, w), p_3(\tilde{p}_1, w)) \right].$$

Define equilibrium downstream prices chosen by firms 2 and 3 as $\tilde{p}_2 = p_2(\tilde{p}_1, \tilde{w})$ and $\tilde{p}_3 = p_3(\tilde{p}_1, \tilde{w})$. The equilibrium is established when firm 1 selects p_1 in order to maximize its profit given expectations about rivals' chosen strategies, i.e.

$$\tilde{p}_1 \quad = \quad \underset{p_1}{\text{argmax}} \quad p_1 q_1(p_1, \tilde{p}_2, \tilde{p}_3).$$

Cumbersome but straightforward calculations, presented in the Appendix, yield the equilibrium wholesale price, the corresponding equilibrium prices and profits. Based on these expressions, one can see that an observation of a collective refusal to deal is less likely to be collusive compared to the case with sufficiently sophisticated contracts. We summarize these findings in the next proposition.

Proposition 5 *In a market characterized by a linear differentiated demand system and unobservable, two part tariff contracts, for sufficiently differentiated downstream products ($\beta <$ 22.13), the only equilibrium of the one-shot game involves both VIFs offering identical competitive contracts to the downstream firm. When downstream products are sufficiently close substitutes ($\beta >$ 22.13), in addition to the equilibrium mentioned above, the one-shot game also has an equilibrium where neither VIF prefers to engage in a contractual relationship with the downstream rival.*

Corollary 2 *In a market characterized by a linear differentiated products demand system, when firms employ unobservable two part tariff contracts and for sufficiently differentiated downstream products ($\beta <$ 22.13), an observed collective refusal to supply must be collusive. In contrast, when downstream products are sufficiently close substitutes ($\beta >$ 22.13), a collective refusal to supply may arise as a result of competition in good faith.*

The suggestions of Proposition 5 and Corollary 2 are similar to those of Proposition 4 and Corollary 1. However, there is an interesting quantitative implication. Whenever $22.13 < \beta <$ 32.89, a collective refusal to supply is collusive if firms have access to sufficiently sophisticated contracts, while it may be a result of competition in good faith when contracts are restricted to two part tariffs. Note that, when the observed outcome is a collective refusal to supply, there are no contractual relationships and hence no information is available on the form of feasible contracts. Therefore, in judging the anti-competitiveness of an observation of a collective refusal to supply, a competition authority must engage in a counterfactual analysis of which types of contracts would have emerged in case the firms engaged in contractual relationships. Moreover, any restrictions which might arise due to competition laws should also be factored into this analysis. When downstream products are close enough substitutes, the issue of multiple equilibria arises also here. Once again, we postpone the discussion of equilibrium selection or coordination among VIFs until Section 5.4.

Interestingly, a similar result arises for observable contracts as well. In Proposition 3, we argued that when firms have access to sufficiently sophisticated observable contracts, we should not observe a collective refusal to deal. However, once contractual flexibility is restricted, the profitability of making offers might be reduced even though the contract provides the contracting firm with a commitment tool. All this in turn might imply that under some conditions both vertically integrated firms might prefer not making a contract offer as part of a static equilibrium and a collective refusal to supply might be the result of competition in good faith. In order to demonstrate this possibility, one can directly use the results of Höffler and Schmidt (2008) who solve for the equilibrium of the downstream price competition game in a setup that is exactly as ours except that the VIFs can only offer linear tariffs. Although they do not consider the issue of a refusal to supply in their paper, it is straightforward to show that for $\beta > 26.77$ both firms refusing to supply is a Nash equilibrium. On the other hand, they also show that there always exists an equilibrium where both VIFs offer a contract that entails marginal cost pricing, i.e. $w^* = 0$. It is also straightforward to show that both firms are collectively better off if they refuse to supply the input to the downstream competitor. Therefore, when only observable linear contracts are available, an observation of a collective refusal to deal is collusive for $\beta < 26.77$, while it may arise as a result of competition otherwise. Our next proposition contrasts our model under sufficiently sophisticated observable contracts with that of Höffler and Schmidt (2008) and establishes that the type of contracts that are employed by the VIFs plays a major role in deciding whether observed collective refusals to sell are due to collusive behavior.

Proposition 6 *Whenever* $26.77 < \beta$*, an observation of collective to refusal to supply*

1. *is collusive if firms have access to sufficiently sophisticated observable contracts,*

2. *can be the result of competition in good faith if firms can only employ observable linear wholesale prices.*

Proof. See preceding discussion.

According to Proposition 6, we can conclude that the downstream competitor will be foreclosed less often when sophisticated contracts are available, assuming that the VIFs compete in good faith. Alternatively, when we observe collective refusals to deal, it is more likely to be a result of collusion if firms could employ more sophisticated contracts.

In summary, the antitrust implications of our results are that when products are highly differentiated, a collective refusal to deal is collusive and remedies may be warranted. However, as mentioned earlier, the degree of product differentiation (or its inverse, the degree of product substitutability) that would deem such an action anti-competitive strongly depends on the type of contracts available to the VIFs. Interestingly, restrictions on contracts imposed by competition authorities may have perverse effects as for some parameters they make it more likely for VIFs to engage in (non-collusive) refusals to supply and may reduce consumer welfare.[27]

5.3 More Than One Downstream Competitor

Like in the case of the mobile phone services industry, there could potentially be more than one downstream firm that requires the upstream input in order to compete in the downstream market. In this subsection, we are going to extend the model of the preceding section to introduce a second potential downstream competitor, firm 4. The addition of a second downstream competitor complicates things considerably for a number of reasons. First of all, when contracts are unobservable, it is reasonable to expect that the contracting downstream firms will not observe each others' contracts either. As a consequence, one needs to specify the beliefs these downstream firms would hold regarding the other contracts if they were to observe an out of equilibrium offer from either VIF. Furthermore, if both integrated firms were to make offers, the two downstream competitors would find themselves in a non trivial strategic situation where they would need to decide which integrated firm to contract with based on their beliefs about what other contractual relationships will emerge. This second problem is present even when contracts are observable.

Since our goal here is just to point out that the number of potential competitors plays an important role on whether an observed collective refusal to supply can be considered collusive or not, we make a number of simplifying assumptions. We only consider the case of observable contracts in order to avoid dealing with off equilibrium beliefs. But in this case a simple variation on Proposition 3 would imply that when contracts are sufficiently sophisticated there should be no reason for a VIF to refrain from contracting with at least one of the downstream competitors. However, if contracts were observable but restricted, there may be situations where non-collusive collective refusals may arise as we show below by using two part tariff contracts. In addition, we will not consider upstream competition in supplying to downstream competitors in order to avoid studying additional strategic interactions that downstream firms have to engage in choosing their suppliers. Instead, we focus on what one of the VIFs would do given that the other VIF does not offer contracts to any downstream rival.

In order to derive the best response of a VIF when its rival is not supplying the input to the downstream competitors, one has to start by introducing a new demand system—one where there are potentially four products that consumers can choose from. Once again we employ a version of the Shubik and Levitan (1971) model of linear demand.

Let a representative consumer of the *four* downstream products have the following quasi-

[27] This is valid for $22.13 < \beta < 32.89$. Note that this situation resembles a result derived by Rey and Tirole (2007) who examine the interaction between a bottleneck owner and downstream firms and find that non-discrimination laws may allow the upstream monopolist to restore monopoly power.

linear utility function:

$$U(q_1, q_2, q_3, q_4, y; \gamma) = \sum_{j=1}^{4} q_j - \frac{1}{2}\left(\sum_{j=1}^{4} q_j\right)^2 - \frac{4}{2(1+\beta)}\left[\sum_{j=1}^{4} q_j^2 - \frac{\left(\sum_{j=1}^{4} q_j\right)^2}{4}\right] + y \qquad (2)$$

where q_i, $i = 1, 2, 3, 4$, are the quantities of the four downstream products consumed, and y is a composite good whose price is normalized to unity. It is straightforward to derive the corresponding demand functions, which can be found in the Appendix.

We are now in a position to derive the best-response of a VIF, say firm 2, when its rival VIF, firm 1, does not supply the input to their downstream competitors. In our analysis here, we restrict attention to two part tariff contracts of the form $F_i + w_i q_i$, $i = 3, 4$, where each downstream competitor pays a fixed fee F_i and a per unit wholesale price w_i. When contractual forms are restricted to two part tariffs, the coalition of firms 2, 3 and 4 cannot obtain the maximum possible profits since two part tariffs are not sufficiently sophisticated. The lack of commitment power to all downstream prices introduces the possibility that firm 2 may find it optimal not to make any offers at all.

Since the resulting expressions are complicated we relegate all the derivations to the Appendix and briefly summarize our steps here. We derive equilibrium outcomes under three scenarios: i) when firm 2 makes offers to both firms 3 and 4, ii) when firm 2 makes an offer to only firm 3 and not to firm 4, iii) when firm 2 makes no offers to the downstream competitors. Comparing equilibrium profits in cases i and ii, we establish that if firm 2 makes an offer, it will choose to make offers to both firms 3 and 4. By comparing the equilibrium profits under scenarios (i) and (iii), we establish that firm 2 is better off making no offers whenever the substitutability between downstream products is sufficiently high, namely whenever $\beta > 345.95$. Note that this critical value of β is not directly comparable to that of Proposition 4, because the two demand systems are not the same. However, as a by product of our analysis, we find the equilibrium outcome when firm 2 makes an offer only to firm 3 (scenario (ii)) even though the underlying demand system is based on four possible downstream products. This would be the relevant situation if there were only one potential downstream competitor . Comparing the equilibrium profits in scenario (ii) with that of scenario (iii), we find that firm 2 will be better off not making an offer to a downstream competitor, firm 3, whenever $\beta > 189.38$. We summarize these findings in the next proposition.

Proposition 7 *In a market characterized by a linear differentiated products demand system based on four products and observable two part tariffs contracts, suppose that VIF 1 does not make any contract offers to potential downstream competitors. When there are two potential downstream competitors, if firm 2 makes contract offers, it makes offers to both and will make these offers as long as $\beta < 345.95$. On the other hand, if there were only one downstream competitor, firm 2 makes an offer only when $\beta < 189.38$.*

Proof. See the Appendix.

Proposition 7 suggests that, whenever products are sufficiently substitutable, both VIFs may find it optimal not to deal with potential downstream competitors. Hence, an observation of a collective refusal to deal may arise as the outcome of a static interaction and is not necessarily collusive. The fact that the critical value of substitutability, β, depends on the number of potential downstream competitors is quite troublesome from a practical point of view. This result makes an observation of a refusal to supply by both VIFs much more likely to be the result of tacit collusion when there are two, rather than one, non-integrated rivals in the downstream market. This finding poses an interesting complication in terms of antitrust policy. Suppose that the antitrust authority observes that the VIFs do not supply the input to any downstream rivals, or equivalently, no other firms compete in the downstream market. Conducting an economic analysis of the relevant market, the authority obtains a reliable estimate of the corresponding substitution parameter, β. The question is: what critical value of β should this estimate be compared against? In order to be able to answer that question, one needs to know how many downstream firms would be active in the market, were the VIFs willing to supply the input. Unfortunately, without a clear answer to this counterfactual question, one cannot assert a certain type of conduct on the part of the VIFs.[28]

A second implication of our analysis can be derived by comparing Proposition 7 where contracts are restricted to be two part tariffs with Proposition 3 where contracts can be sufficiently sophisticated. In agreement with our findings for the case of unobservable contracts, this comparison suggests that when firms are not permitted to use sufficiently sophisticated contracts, an observation of a collective refusal to deal cannot be automatically considered to be collusive even when contracts are observable. With contractual restrictions the VIFs may prefer not to deal with downstream competitors even without resorting to tacitly collusive behavior.

5.4 A Discussion on Multiplicity of Static Equilibria

In the preceding analysis there were several instances where the one-shot game had multiple equilibria. Competition authorities are often not indifferent among these, and their preferences over such equilibria may be different from those of the firms involved. How should the authority proceed in such a case? Specifically, should authorities condemn coordination over equilibria that they deem undesirable? Should they apply remedies to direct firms to more desirable equilibria?

To the best of our best knowledge, this problem of "equilibrium selection" in competition and

[28] In our stylized environment, whenever substitutability is measured to fall between the two critical values we find, namely when $189.38 < \beta < 345.95$, to conclude whether an observation of collective refusal to deal is collusive or not, a competition authority needs to know the number of potential competitors. Although we have not studied the game when $\beta < 345.95$, it is clear that in this case the two VIFs will compete in supplying the two downstream competitors and a collective refusal to deal can arise only as a result of collusive behavior.

regulatory policy has not been subject to theoretical research, even though it arises in a number of circumstances. For example, models of tacit collusion in repeated oligopoly entail multiplicity of equilibria ranging from the cartel solution to the repeated play of the static equilibria. Firms may coordinate on the cartel solution while any of the equilibria with lower prices are preferable from a social point of view. The approach of most competition authorities and of the European Commission to pure "tacit collusion", i.e. coordination that does not entail any agreement, is that this does not constitute a horizontal infringement of competition law (or, in the context of the EU, an infringement of the Article 101 of the Treaty of the Functioning of the EU, TFEU). This is the case even though theoretically speaking successful tacit collusion at cartel prices does not generate any smaller consumer harm than a cartel. The main argument here is that such coordination simply consists of the implementation of rational independent profit maximizing strategies by individual firms and condemning them would be tantamount to condemning profit maximizing behavior.

The treatment of tacitly collusive behavior as an abuse of collective dominance (or as an infringement of Article 102 of TFEU, which bans abuse of dominant position) seems to depend on the nature of collusion. As mentioned earlier (see Footnote 4), there have been a few cases where collusion to exclude competitors has been condemned. By contrast, tacit collusion that simply results in high prices (or so-called exploitative abuse) has never been condemned in the European Union. Hence, when the result of tacit collusion is simply to raise prices, it seems that the European Commission does not condemn coordination over equilibria that are seen as undesirable.[29] We are not aware of any rigorous justification for this asymmetric treatment.

By contrast, as mentioned earlier, the European Commission as well as national competition or regulatory authorities have been more willing to deal with collective dominance through ex-ante instruments such as merger and access regulations. Again, in our context, recourse to such ex-ante remedies can in principle be construed as attempts by competition or regulatory authorities to push firms to select particular equilibria over others.

This brief review suggests that so far competition policy has not developed a rigorous approach to the problem of multiplicity of equilibria. We conclude that the way in which oligopoly theory informs competition and regulatory policy leaves some unanswered questions.

6 Conclusion

Competition policy towards horizontal agreements and coordination is guided by advances in economic theory of the last few decades, especially by those in game theory and the theory of tacit collusion. The basic tenet of the current approach has been that behavior that can be construed as the equilibrium strategy of a stage game is considered as competitive and is

[29]This tendency of not using Articles 101 and 102 of TFEU to challenge tacit collusion that raises prices does have its critics. See, for instance, Petit and Henry (2010).

not treated with any remedial action. By contrast, behavior that cannot be interpreted as an equilibrium of the stage game but which can only be supported by history-dependent strategies raises competitive or regulatory concerns.

Within the boundaries of this approach, the general conclusion of our analysis is that the antitrust or regulatory assessment of an observation of collective refusal to supply is not a straightforward task. In order to ascertain whether an observation of collective refusal to supply reflects the (competitive) equilibrium of a static game or the (collusive) equilibrium of a repeated game hinges on a number of factors. Such factors include whether contracts are observable and whether there are legal or other restrictions that limit the kind of contracts firms can offer. In addition, one has to consider counterfactuals such as, were the VIFs to sell, what type of contracts the VIFs would use with their downstream competitors, and the likely number of downstream firms they could contract with. Furthermore, the degree of substitution between differentiated products also matters. Even more fundamentally, the mode of competition could alter the conclusions. The analysis clearly suggests that a mere observation of collective refusal to supply should not automatically trigger an antitrust or regulatory remedy.

The analysis raises some additional questions. We have shown above that there are examples where the stage game has multiple equilibria. In one equilibrium, there is upstream competition, and in the other a collective refusal to supply obtains (albeit in good faith). How should competition policy react to such a situation? The current analytical tools at the disposal of competition policy do not seem to provide a satisfactory answer to this question. Any policy that aims to induce firms to play a given equilibrium must consider the effects of such a policy on prior investment incentives as well. We believe further research that addresses these issues is warranted.

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Appendix

Proof of Proposition 1

Given our definition of a sufficiently sophisticated observable contract, the contracting parties will be able to maximize their joint profits while taking into account the effect of their choices on the optimal response of firm 1. Let $q_1(q_2+q_3)$ denote the best response of firm 1, when firms 2 and 3 produce q_2+q_3. Then, firms 2 and 3 will select q_2 and q_3 in order to maximize their joint profits, that is

$$\max_{q_2,q_3}(P(q_1(q_2+q_3)+q_2+q_3)-c_I-c_O)(q_2+q_3)$$

or equivalently select total output as

$$q_S^* = \underset{q_S}{\operatorname{argmax}}(P(q_1(q_S)+q_S)-c_I-c_O)(q_S),$$

where $q_S = q_2+q_3$. This choice clearly delivers the contracting parties a total profit that is equal to the Stackelberg leader profits in a duopoly.

Let q_1^d and q_2^d denote the Cournot equilibrium choices of firms 1 and 2, respectively. Notice that when the contracting parties search for their optimal output, the Cournot equilibrium output is one of the feasible options. Thus, they can obtain the Cournot duopoly profits by simply choosing $q_S = q_2^d$, which in turn will induce firm 1 to produce its Cournot equilibrium quantity, $q_1(q_2^d) = q_1^d$, generating profits given by $(P(q_1(q_2^d)+q_2^d)-c_I-c_O)q_2^d$. It is easy to see that

$$[P(q_1(q_S^*)+q_S^*)-c_I-c_O]q_S^* \geq [P(q_1(q_2^d)+q_2^d)-c_I-c_O]q_2^d.$$

Apart from knife-edge cases where the Stackelberg leader profits and Cournot profits coincide, firm 2 will have an incentive to supply a third downstream rival when firm 1 does not. Hence, there is no equilibrium where both VIFs refuse to sell to firm 3.

When the two VIFs compete for selling to firm 3, and assuming that the contracts can entail a transfer—a fixed fee—between the contracting parties, in equilibrium, the VIF which contracts with the downstream rival makes a contract offer where the total profit of the contracting parties equals Stackelberg leader profits. At the same time, in equilibrium the contracting VIF cannot earn more than the non-contracting VIF. Therefore, both VIFs must earn the Stackelberg follower profits in equilibrium, i.e. $\Pi_1^* = \Pi_2^* = [P(q_1(q_S^*)+q_S^*)-c_I-c_O]q_1(q_S^*)$. Consequently, the downstream rival earns the difference between the Stackelberg leader and follower profits.

Under fairly general conditions it is easy to verify that $\Pi_2^* \leq [P(q_1(q_2^d)+q_2^d)-c_I-c_O]q_1(q_2^d)$, i.e. Stackelberg follower profits are less than the symmetric Cournot duopoly profits. Hence, the VIFs will have an incentive to tacitly collude in not supplying the downstream rival and obtain Cournot duopoly profits, for example by employing trigger strategies.

Proof of the claim in footnote 19

To show that the Stackelberg follower earns less than the Cournot duopoly profits, we must establish a few things. First, $q_S^* \geq q_2^d$. Suppose $\pi_L(q_F(q_L), q_L)$ denote the Stackelberg leader profits when it produces q_L units and the follower responds with $q_F(q_L)$. Then evaluating the first order condition at q_2^d, the Cournot equilibrium output, we have

$$\left.\frac{d\pi_L(q_F(q_L), q_L)}{dq_L}\right|_{q_L=q_2^d} = \left.\underbrace{\frac{\partial\pi_L(q_F(q_L), q_L)}{\partial q_F}}_{-}\underbrace{\frac{\partial q_F(q_L)}{\partial q_L}}_{-}\right|_{q_L=q_2^d} + \underbrace{\left.\frac{\partial\pi_L(q_F(q_L), q_L)}{\partial q_L}\right|_{q_L=q_2^d}}_{=0} \geq 0$$

Thus, the profit of the leader is maximized at $q_L \geq q_2^d$. Similarly, let $\pi_F(q_F(q_L), q_L)$ denote the Stackelberg follower profits. Note that, $\pi_F(q_F(q_2^d), q_2^d)$ is simply the Cournot duopoly profits. The derivative of the follower profits is given by

$$\frac{d\pi_F(q_F(q_L), q_L)}{dq_L} = \underbrace{\frac{\partial\pi_F(q_F(q_L), q_L)}{\partial q_F}}_{=0}\underbrace{\frac{\partial q_F(q_L)}{\partial q_L}}_{-} + \underbrace{\frac{\partial\pi_F(q_F(q_L), q_L)}{\partial q_L}}_{-} \leq 0.$$

Thus, the follower profits decreases with the leader's output. Hence, $\pi_F(q_F(q_S^*), q_S^*) \leq \pi_F(q_F(q_2^d), q_2^d)$, and the Stackelberg follower earns less than what it would in a Cournot duopoly.

Proof of Proposition 3

We start by assuming, without loss of generality, that firm 1 refuses to sell to firm 3 and consider the contracting incentives for firm 2. Let the duopoly equilibrium prices be given by p_1^d and p_2^d, respectively. Clearly, the best response function of firm 1 when only the two VIFs compete is given by

$$p_1^d(p_2) = \underset{p_1}{\operatorname{argmax}} \;\; (p_1 - c_I - c_O)q_1^d(p_1, p_2) = \underset{p_1}{\operatorname{argmax}} \;\; (p_1 - c_I - c_O)q_1(p_1, p_2, \infty)$$

On the other hand, if firms 2 and 3 can agree on a sufficiently sophisticated observable contract, firm 1's best response is obtained by

$$p_1(p_2, p_3) = \underset{p_1}{\operatorname{argmax}} \;\; (p_1 - c_I - c_O)q_1(p_1, p_2, p_3).$$

Then, firms 2 and 3 agree on a contract so that they can earn the maximum profit by setting p_2 and p_3 in a coordinated fashion taking into account the response of firm 1 in their pricing decisions. In this case, the optimal contract that firms 2 and 3 agree on implements prices such that

$$(p_2, p_3) = \underset{p_2, p_3}{\operatorname{argmax}} \;\; \Big((p_2 - c_I - c_O)q_2(p_1(p_2, p_3), p_2, p_3) + (p_3 - c_I - c_O)q_3(p_1(p_2, p_3), p_2, p_3)\Big).$$

Furthermore, note that the best response function of firm 1 in a duopoly is equivalent to its best response in the downstream market with three firms and where the price of firm 3 is set infinitely high, i.e. $p_1^d(p_2) = p_1(p_2, \infty)$. The equilibrium duopoly profit of firm 2 can then be written as

$$\pi_2^d = (p_2^d - c_I - c_O) q_2(p_1^d(p_2^d, \infty), p_2^d, \infty).$$

When all three firms are active in the downstream market, clearly $p_2 = p_2^d$ and $p_3 = \infty$ are in the set of feasible choices for the contracting parties. Thus, it must be true that

$$\begin{aligned}
\pi_2^d &= (p_2^d - c_I - c_O) q_2(p_1^d(p_2^d, \infty), p_2^d, \infty) \\
&\leq \max_{p_2} \Big((p_2 - c_I - c_O) q_2(p_1(p_2, \infty), p_2, \infty) \Big) \\
&\leq \max_{p_2, p_3} \Big((p_2 - c_I - c_O) q_2(p_1(p_2, p_3), p_2, p_3) + (p_3 - c_I - c_O) q_3(p_1(p_2, p_3), p_2, p_3) \Big),
\end{aligned}$$

and hence firm 2 will earn at least as much by contracting with firm 3.

Derivation of the equilibrium prices and profits in Sections 5.2 and 5.2.1, and the Proof of Proposition 4

By maximizing the utility given in equation (1) subject to the budget constraing $\left[\sum_{i=1}^{3} p_i q_i\right] + y = M$, with M denoting the income, the demand function for product j, when all three firms are active, is found to be

$$q_j(p_1, p_2, p_3) = \frac{1}{3}\left[1 - p_j - \beta\left(p_j - \frac{1}{3}(p_1 + p_2 + p_3)\right)\right]. \tag{3}$$

As noted by Höffler and Schmidt (2008), when only the first two products are available, it is necessary to derive the corresponding demand functions once again from the utility maximization problem of the representative consumer subject to the constraint that $q_3 = 0$. In this case, the demand functions for the products of the VIFs are

$$q_i(p_1, p_2) = \frac{1 + \beta}{3 + 2\beta}\left[1 - p_i - \frac{\beta}{3}(p_i - p_j)\right] \tag{4}$$

for $i \neq j \in \{1, 2\}$.

When neither VIF supplies the input, they both earn

$$\Pi_i(p_1, p_2) = p_i q_i(p_1, p_2)$$

with $i \in \{1, 2\}$. The equilibrium of the price competition game is symmetric and yields the following equilibrium prices

$$\hat{p}_1 = \hat{p}_2 = \frac{3}{\beta + 6}$$

which in turn yield equilibrium profits of

$$\hat{\Pi}_1(\hat{p}_1, \hat{p}_2) = \hat{\Pi}_2(\hat{p}_1, \hat{p}_2) = \frac{3(1 + \beta)(3 + \beta)}{(\beta + 6)^2(2 + 3\beta)}.$$

When firm 2 supplies the input to firm 3 while firm 1 does not, it is easy to verify that the resulting prices are given by

$$\begin{aligned}\tilde{p}_1 &= \frac{3+2\beta}{(3+\beta)^2-3}\\ \tilde{p}_2=\tilde{p}_3 &= \frac{6+5\beta}{2((3+\beta)^2-3)}\end{aligned}$$

with the corresponding profits of

$$\begin{aligned}\tilde{\Pi}_1(\tilde{p}_1,\tilde{p}_2,\tilde{p}_3) &= \frac{(3+2\beta)^3}{9((3+\beta)^2-3)^2}\\ \tilde{\Pi}_2(\tilde{p}_1,\tilde{p}_2,\tilde{p}_3) &= \frac{(6+5\beta)^2(3+\beta)}{18((3+\beta)^2-3)^2}.\end{aligned}$$

Given these expressions, a simple comparison implies that $\hat{\Pi}_2(\hat{p}_1,\hat{p}_2) > \tilde{\Pi}_2(\tilde{p}_1,\tilde{p}_2,\tilde{p}_3)$, whenever $\beta > 32.89$.

Proof of Proposition 5

The equilibrium wholesale price in this case is given by

$$\tilde{w} = \frac{\beta\ (5\,\beta+6)^2}{17\,\beta^4+145\ \ \beta^3+396\,\beta^2+468\,\beta+216}$$

and the corresponding equilibrium retail prices are given by

$$\begin{aligned}\tilde{p}_1 &= \frac{3\,(36+11\,\beta^3+66\,\beta+45\,\beta^2)}{17\,\beta^4+145\,\beta^3+396\,\beta^2+468\,\beta+216}\\ \tilde{p}_2 &= \frac{(3+2\,\beta)\left(19\,\beta^2+42\,\beta+36\right)}{17\,\beta^4+145\,\beta^3+396\,\beta^2+468\,\beta+216}\\ \tilde{p}_3 &= \frac{43\,\beta^3+162\,\beta^2+216\,\beta+108}{17\,\beta^4+145\,\beta^3+396\,\beta^2+468\,\beta+216}.\end{aligned}$$

With these prices and given that firm 2 can obtain the joint profits earned by the downstream arms of itself and firm 3 by using the fixed fee, the equilibrium profit of firm 2, when firm 1 does not make a contract offer to firm 3, is given by

$$\tilde{\Pi}_2 = \frac{1}{3}\,\frac{(3+2\,\beta)\,(3+\beta)\left(7\,\beta^2+16\,\beta+12\right)\left(83\,\beta^3+315\,\beta^2+432\,\beta+216\right)}{\left(17\,\beta^4+145\,\beta^3+396\,\beta^2+468\,\beta+216\right)^2}$$

while firm 1 earns in equilibrium

$$\tilde{\Pi}_1 = \frac{\left(36+66\,\beta+11\,\beta^3+45\,\beta^2\right)^2(3+2\,\beta)}{\left(17\,\beta^4+145\,\beta^3+396\,\beta^2+468\,\beta+216\right)^2}.$$

Comparing $\tilde{\Pi}_2$ to what firm 2 can earn in case it does not make a contract offer and compete with firm 1 in a duopoly, we find that firm 2 will be better off by not entering in a contractual relationship with firm 3 whenever $\beta > 22.13$.

On the other hand, for smaller values β, firm 2 is better off if it makes a contract offer to firm 3. Since the equilibrium profit of firm 1 is less than $\tilde{\Pi}_2$ in this case, it can induce firm 3 to accept a similar two part tariff contract offer with the same wholesale price, but a slightly lower fixed fee, leaving some profit to firm 3. The two VIFs will compete in this fashion until such undercutting is no longer profitable, namely, when both VIFs earn a profit equal to $\tilde{\Pi}_1$—the equilibrium profits earned by the noncontracting VIF. That is, $\Pi_1^* = \Pi_2^* = \tilde{\Pi}_1$. The difference, $\tilde{\Pi}_2 - \tilde{\Pi}_1$ is offered via the fixed fee to firm 3, hence $\Pi_3^* = \tilde{\Pi}_2 - \tilde{\Pi}_1$.

It is straightforward to verify that the equilibrium profits of VIFs 1 and 2, in case they compete in supplying firm 3 with two part tariff wholesale contracts are below what they would earn if they were to jointly refuse to sell to firm 3 and compete in a duopoly. Thus, when $\beta < 22.13$, the two firms will have incentives to collude in refusing to sell the input to firm 3.

Derivation of the demand functions in Section 5.3 and Proof of Proposition 7

By maximizing the utility given in equation (2) subject to the budget constraing $\left[\sum_{i=1}^4 p_i q_i\right] + y = M$, with M denoting the income, the demand function for product j, when all four firms are active, is found to be

$$q_j^4(p_1,p_2,p_3,p_4) = \frac{1}{4}\left[1 - p_j - \beta\left(p_j - \frac{1}{4}(p_1+p_2+p_3+p_4)\right)\right]. \tag{5}$$

The demand functions when only three firms are active can be found by solving the utility maximization problem subject to $q_4 = 0$ and are given by

$$q_j^3(p_1,p_2,p_3) = \frac{1+\beta}{4+3\beta} - \frac{(1+\beta)p_j}{4} - \frac{\beta(1+\beta)}{4(4+3\beta)}(p_1+p_2+p_3) \qquad j=1,2,3. \tag{6}$$

Similarly, when only two firms are active the resulting demand system can be found by solving the utility maximization problem subject to $q_3 = q_4 = 0$ and is given by

$$q_j^2(p_1,p_2) = \frac{1+\beta}{2(2+\beta)} - \frac{(1+\beta)p_j}{4} - \frac{\beta(1+\beta)}{8(2+\beta)}(p_1+p_2) \qquad j=1,2. \tag{7}$$

Given the demand functions in equations (5), (6) and (7), we first consider the case where there are two potential competitors and firm 1 does not make any offers to either of them. Moreover, firm 2 is restricted to use two part tariff contracts. Note also that we maintain the assumption that both upstream and downstream marginal costs are zero. With these assumptions, the profits of the four firms can be written as

$$\begin{aligned}
\Pi_1(p_1,p_2,p_3,p_4) &= p_1 q_1^4(p_1,p_2,p_3,p_4) \\
\Pi_2(p_1,p_2,p_3,p_4) &= p_2 q_2^4(p_1,p_2,p_3,p_4) + w_3 q_3^4(p_1,p_2,p_3,p_4) + F_3 + w_4 q_4^4(p_1,p_2,p_3,p_4) + F_4 \\
\Pi_3(p_1,p_2,p_3,p_4) &= (p_3 - w_3) q_3^4(p_1,p_2,p_3,p_4) - F_3 \\
\Pi_4(p_1,p_2,p_3,p_4) &= (p_4 - w_4) q_4^4(p_1,p_2,p_3,p_4) - F_4.
\end{aligned}$$

Downstream equilibrium prices can be found by simply solving the four first order conditions with respect to downstream prices. The resulting downstream equilibrium prices as functions of the wholesale prices w_3 and w_4 are given by

$$\begin{aligned}
p_1^*(w_3, w_4) &= \frac{4(\beta(1+\beta)(w_3+w_4)+7\beta+8)}{(7\beta+8)(3\beta+8)} \\
p_2^*(w_3, w_4) &= \frac{\beta(7\beta+12)(w_3+w_4)+28\beta+32)}{(7\beta+8)(3\beta+8)} \\
p_3^*(w_3, w_4) &= p_2^*(w_3, w_4) + \frac{2(2+\beta)w_3 - \beta w_4}{(7\beta+8)(3\beta+8)} \\
p_4^*(w_3, w_4) &= p_2^*(w_3, w_4) + \frac{2(2+\beta)w_4 - \beta w_3}{(7\beta+8)(3\beta+8)}
\end{aligned} \tag{8}$$

Taking these equilibrium prices into account in the downstream market, VIF 2 will set F_3 and F_4 to the downstream profits of the two competitors inducing them to just accept its offer. Thus, in choosing the per unit wholesale prices, firm 2 will maximize the joint profits of the three firms with respect to w_3 and w_4. Substituting the equilibrium prices from 8 above in the sum of the profits of firms 2, 3 and 4, and maximizing the resulting expression with respect to w_3 and w_4 yields the symmetric optimal choices given by

$$w_3^* = w_4^* = \frac{2(7\beta+8)(15\beta+16)}{51\beta^4+664\beta^3+1968\beta^2+2304\beta+1024}.$$

Using this one can calculate the equilibrium profits of VIF 2 as

$$\Pi_2(p_1(w_3^*, w_4^*), p_2(w_3^*, w_4^*), p_3(w_3^*, w_4^*), p_4(w_3^*, w_4^*)) = \frac{101\beta^3+350\beta^2+432\beta+192}{51\beta^4+664\beta^3+1968\beta^2+2304\beta+1024}$$

Next, suppose there is only a single potential downstream competitor. In this case as well, we will assume that VIF 1 makes no offers. The relevant demand system is given by (6). In this case, the profits of the three firms can be written as

$$\begin{aligned}
\Pi_1(p_1, p_2, p_3) &= p_1 q_1^3(p_1, p_2, p_3, p_4) \\
\Pi_2(p_1, p_2, p_3) &= p_2 q_2^3(p_1, p_2, p_3, p_4) + w q_3^3(p_1, p_2, p_3, p_4) + F \\
\Pi_3(p_1, p_2, p_3) &= (p_3 - w) q_3^3(p_1, p_2, p_3, p_4) - F
\end{aligned}$$

where w denotes the per unit wholesale price firm 2 charges to firm 3, while F denotes the fixed fee firm 3 has to pay as a part of the contract. Downstream equilibrium prices can be found by simply solving the three first order conditions with respect to downstream prices. The resulting downstream equilibrium prices as functions of the wholesale prices w_3 and w_4 are given by

$$\begin{aligned}
p_1^*(w) &= \frac{\beta(4+3\beta)w+20\beta+32}{2(5\beta+8)(\beta+4)} \\
p_2^*(w) &= \frac{\beta(12+5\beta)w+20\beta+32}{2(5\beta^2+28\beta+32)} \\
p_3^*(w) &= p_2^*(w) + \frac{(4+\beta)w}{5\beta+8}
\end{aligned} \tag{9}$$

Clearly, VIF 2 will set the fixed fee F to extract the profit of firm 3 and then choose the wholesale price w to maximize the aggregate profits of firms 2 and 3. Solving this problem yields the optimal wholesale price which is given by

$$w^* = \frac{2\beta\,(3\beta+4)\,(8+5\beta)}{7\beta^4+108\beta^3+448\beta^2+768\beta+512}.$$

Using this the profits firm 2 makes amounts to

$$\Pi_2(p_1(w^*), p_2(w^*), p_3(w^*)) = \frac{(1+\beta)\left(83\beta^3+420\,\beta^2+768\beta+512\right)}{2\,(3\beta+4)\,(7\beta^4+108\beta^3+448\beta^2+768\beta+512)}$$

Finally consider the situation where firm 2 makes no offers to either of the downstream competitors and the two VIFs are the only ones present in the downstream market. In this case, the relevant demand system is given by equation (7) and the firm profits are given by:

$$\begin{aligned}\Pi_1(p_1,p_2) &= p_1 q_1^2(p_1,p_2)\\ \Pi_2(p_1,p_2) &= p_2 q_2^2(p_1,p_2).\end{aligned}$$

It is straightforward to derive the equilibrium prices as

$$p_1^* = p_2^* = \frac{4}{8+\beta}$$

and the corresponding equilibrium profits are symmetric as well and given by

$$\Pi_1(p_1^*, p_2^*) = \Pi_2(p_1^*, p_2^*) = \frac{2(4+\beta)(1+\beta)}{(8+\beta)^2(2+\beta)}.$$

Proving the first statement in the proposition that if firm 2 makes any offer at all it will make it to both downstream competitors, requires us to compare profits under scenario i and scenario ii. Although this comparison results in a complicated expression, given that $\beta > 0$, it is unambiguously positive. Formally,

$$\Pi_2(p_1(w_3^*,w_4^*), p_2(w_3^*,w_4^*), p_3(w_3^*,w_4^*), p_4(w_3^*,w_4^*)) - \Pi_2(p_1(w^*), p_2(w^*), p_3(w^*)) =$$
$$\frac{262144+1048576\beta+1814528\beta^2+1748992\,\beta^3+1009728\beta^4+347536\beta^5+65372\beta^6+5039\beta^7+9\beta^8}{2\,(51\beta^4+664\beta^3+1968\beta^2+2304\beta+1024)\,(3\beta+4)\,(7\beta^4+108\beta^3+448\beta^2+768\beta+512)} > 0,$$

Therefore, if firm 2 prefers to contract with downstream competitors, it will do so with both of them.

Given that firm 2 contracts with both firms, we next explore if and when contracting is in the interest of firm 2. Clearly, contracting would be preferable when VIF 2 earns higher profits under scenario i when compared with the profits under scenario iii where the two VIFs compete in the downstream market. This comparison results in

$$\begin{aligned}&\Pi_2(p_1(w_3^*,w_4^*), p_2(w_3^*,w_4^*), p_3(w_3^*,w_4^*), p_4(w_3^*,w_4^*)) - \Pi_2(p_1^*, p_2^*) =\\ &\qquad -\frac{\beta^6-330\beta^5-5444\beta^4-24896\beta^3-48896\beta^2-45056\beta-16384}{(51\beta^4+664\beta^3+1968\beta^2+2304\beta+1024)\,(\beta+8)^2\,(2+\beta)}\end{aligned}$$

which is positive for small values of β and negative for larger values of β. Hence, for sufficiently large values of β, namely for $\beta > 345.95$, VIF 2 is better off not offering any contracts to the downstream competitors when expecting VIF 1 to make no contract offers as well. A collective refusal to deal arises as an equilibrium outcome of the static strategic interaction between VIFs 1 and 2.

Were the second potential competitor, firm 4, absent from the market, VIFs 2's only alternative would be to contract with firm 3 or not. Comparing profits under scenario ii and scenario iii yields

$$\Pi_2(p_1(w^*), p_2(w^*), p_3(w^*)) - \Pi_2(p_1^*, p_2^*) = \\ -\frac{(1+\beta)\left(\beta^6 - 170\beta^5 - 3560\beta^4 - 20480\beta^3 - 52736\beta^2 - 65536\beta - 32768\right)}{2\,(3\beta+4)\,(7\beta^4 + 108\beta^3 + 448\beta^2 + 768\beta + 512)\,(\beta+8)^2\,(2+\beta)}$$

which once again is positive for small values of β and negative for larger values of β. Hence, for sufficiently large values of β, namely for $\beta > 189.38$, VIF 2 is better off not offering any contracts to firm 3 when expecting VIF 1 to make no contract offers as well. A collective refusal to deal arises as an equilibrium outcome of the static strategic interaction between VIFs 1 and 2 as well. Clearly, the likelihood of such a collective refusal to supply arising as the equilibrium outcome of a static strategic interaction is higher when there is one potential competitor than when there are two.

Proof of the claim in footnote 23

Notice that a sufficiently sophisticated observable contract gives firm 2 (together with the contracting party, firm 3) the opportunity to commit to the downstream prices p_2 and p_3. Using the demand functions in equation (3), we can solve for the subgame perfect equilibrium of the game where firms 2 and 3 commit to their retail prices, p_2 and p_3, in stage 1, and firm 1 sets p_1 after observing these prices. The profit of firm 1 is simply

$$\Pi_1(p_1, p_2, p_3) = p_1 q_1(p_1, p_2, p_3)$$

which in turn implies that the best response function is given by

$$p_1(p_2, p_3) = \frac{3 + \beta(p_2 + p_3)}{2(3 + 2\beta)}.$$

Firms 2 and 3, therefore, choose p_2 and p_3 to maximize

$$\begin{aligned} \Pi_{23}(p_2, p_3) &= \Pi_2(p_1(p_2, p_3), p_2, p_3) + \Pi_3(p_1(p_2, p_3), p_2, p_3) \\ &= p_2 q_2(p_1(p_2, p_3), p_2, p_3) + p_2 q_2(p_1(p_2, p_3), p_2, p_3) \end{aligned}$$

resulting in optimal prices of

$$\breve{p}_2 = \breve{p}_3 = \frac{3(6 + 5\beta)}{4(\beta^2 + 9\beta + 9)}.$$

Computing the total profits of the contracting parties we obtain

$$\Pi_{23}(\breve{p}_2, \breve{p}_3) = \frac{(6 + 5\beta)^2}{8\,(\beta^2 + 9\beta + 9)\,(3 + 2\beta)}$$

Now consider the following contract where firms 2 and 3 agree on and commit to a p_2, while agreeing to set $p_3 = \infty$. Or practically, in return for some payment firm 3 agrees not to compete in the downstream market. Under this scenario, the contract between firms 2 and 3 only allows firm 2 to commit to a price in the downstream market against firm 1. In this case, the relevant demand functions in the downstream market are given in equation (4). The profit function of firm 1 is given by

$$\Pi_1(p_1, p_2, \infty) = \Pi_1(p_1, p_2) = p_1\left(\frac{1+\beta}{3+2\beta}\left[1 - p_1 - \frac{\beta}{3}(p_1 - p_2)\right]\right)$$

which in turn implies a best response function of

$$p_1(p_2) = \frac{3+\beta p_2}{2(3+\beta)}.$$

Then, given their commitment to $p_3 = \infty$, the coalition of the contracting firms will select p_2 in order to maximize

$$\Pi_2(p_1(p_2), p2, \infty) = \Pi_2(p_1(p_2), p_2) = p_2\left(\frac{1+\beta}{3+2\beta}\left[1 - p_2 - \frac{\beta}{3}(p_2 - p_1(p_2))\right]\right)$$

implying an optimal price of

$$\breve{\breve{p}}_2 = \frac{9(2+\beta)}{2(18+12\,\beta+\beta^2)}$$

with corresponding profits of

$$\Pi_2(p_1(\breve{\breve{p}}_2), \breve{\breve{p}}_2) = \frac{27}{8}\frac{(2+\beta)^2\,(1+\beta)}{(\beta+3)\,(3+2\beta)\,(18+12\beta+\beta^2)}.$$

The difference between $\Pi_2(p_1(\breve{\breve{p}}_2), \breve{\breve{p}}_2)$ and $\Pi_{23}(\breve{p}_2, \breve{p}_3)$ is given by

$$\Pi_2(p_1(\breve{\breve{p}}_2), \breve{\breve{p}}_2) - \Pi_{23}(\breve{p}_2, \breve{p}_3) = \frac{\beta^4 - 30\beta^3 - 261\beta^2 - 540\beta - 324}{8\,(\beta^2+9\,\beta+9)\,\left(18+12\beta+\ \beta^2\right)(\beta+3)}$$

which is negative for small values of β and positive for large values of β. Specifically, for $\beta > 37.38$, the two contracting parties are better off by using the contract to commit to a price for product 2 as well as not to sell the product 3 in the downstream market.

Interestingly under this scenario, firm 1 earns

$$\Pi_1(p_1(\breve{\breve{p}}_2), \breve{\breve{p}}_2) = \frac{3\left(36+30\,\beta+5\,\beta^2\right)^2(1+\beta)}{16\,(18+12\,\beta+\beta^2)^2\,(3+2\,\beta)\,(\beta+3)}$$

which can be shown to be larger than $\Pi_2(p_1(\breve{\breve{p}}_2), \breve{\breve{p}}_2)$. Thus, in this particular case, both VIFs 1 and 2 will earn more when compared with a situation where neither of them enters into a contractual relationship with firm 3. VIF 2 also earns more when compared with the situation where the two VIFs compete in the downstream market. Firm 3 will earn part of the profits of the coalition as well. Thus, all firms are better off here, although consumers will face higher downstream prices and have access to only two downstream products.